Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/7.2.5-Inverse-hyperbolic-cosine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [296]. This is test number [191].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.66 (295)	% 0.34 (1)
Mathematica	% 97.3 (288)	% 2.7 (8)
Maple	% 66.55 (197)	% 33.45 (99)
Maxima	% 17.23 (51)	% 82.77 (245)
Fricas	% 41.22 (122)	% 58.78 (174)
Sympy	% 24.32 (72)	% 75.68 (224)
Giac	% 26.69 (79)	% 73.31 (217)

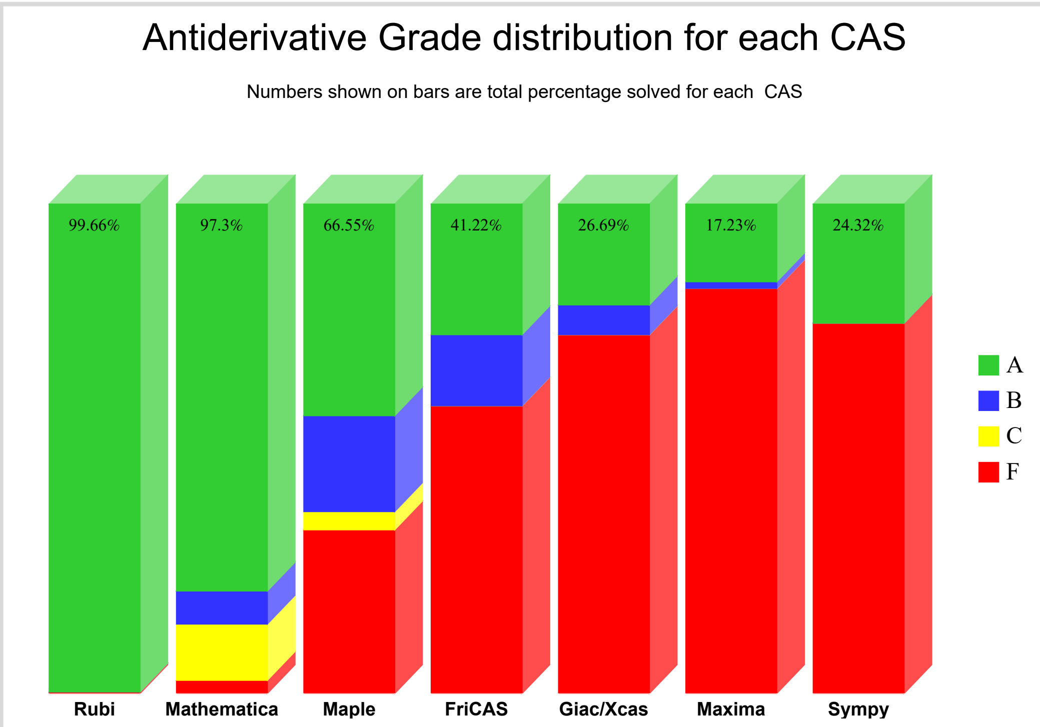
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

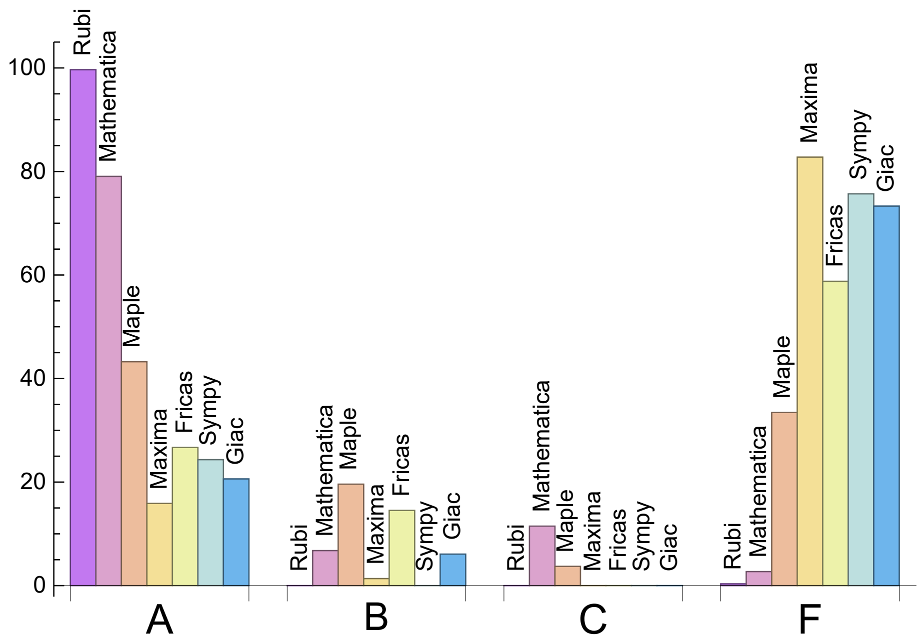
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.66	0.	0.	0.34
Mathematica	79.05	6.76	11.49	2.7
Maple	43.24	19.59	3.72	33.45
Maxima	15.88	1.35	0.	82.77
Fricas	26.69	14.53	0.	58.78
Sympy	24.32	0.	0.	75.68
Giac	20.61	6.08	0.	73.31

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.45	199.76	0.85	161.	1.
Mathematica	3.97	279.92	1.13	159.	0.97
Maple	0.21	380.64	1.61	201.	1.53
Maxima	0.65	85.2	0.79	26.	0.65
Fricas	1.76	532.1	3.63	270.	2.6
Sympy	2.65	311.53	1.48	85.	1.05
Giac	0.77	155.33	1.41	82.	1.21

1.4 list of integrals that has no closed form antiderivative

{30, 31, 34, 35, 36, 37, 39, 40, 47, 48, 78, 82, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 268, 272, 273, 288, 289}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {76, 77, 98, 109, 118, 120, 126, 128, 269, 270, 271}

Mathematica {3, 12, 13, 25, 26, 32, 33, 38, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 91, 92, 94, 96, 98, 109, 110, 112, 118, 119, 120, 121, 126, 127, 128, 129, 136, 137, 138, 139, 140, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 231, 232, 233, 234, 235, 239, 244, 245, 251, 252, 253, 256, 257, 258, 260, 261, 262, 265, 266, 267, 275, 276, 277, 278, 279, 280, 281, 282, 283, 290}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

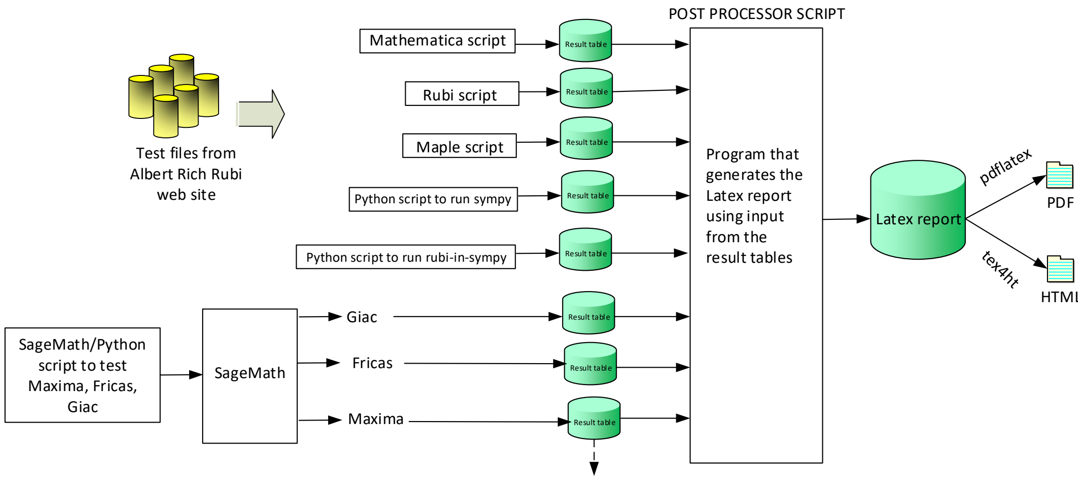
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296 }

B grade: { }

C grade: { }

F grade: { 61}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 53, 54, 55, 58, 59, 60, 62, 63, 64, 66, 67, 68, 71, 72, 73, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 163, 164, 169, 171, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 275, 276, 277, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296 }

B grade: { 125, 127, 128, 129, 157, 162, 165, 166, 167, 168, 170, 172, 177, 183, 189, 195, 239, 278, 291, 294 }

C grade: { 7, 12, 13, 20, 25, 26, 46, 49, 50, 51, 52, 56, 57, 61, 65, 69, 70, 74, 88, 89, 90, 198, 199, 200, 201, 202, 203, 204, 205, 279, 280, 281, 282, 283 }

F grade: { 79, 80, 215, 221, 269, 270, 271, 274}

2.1.3 Maple

A grade: { 3, 4, 5, 8, 9, 10, 12, 16, 17, 18, 23, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 53, 56, 58, 59, 60, 61, 62, 63, 64, 68, 69, 78, 82, 85, 86, 88, 93, 95, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 110, 112, 113, 115, 117, 130, 131, 132, 133, 134, 135, 139, 140, 141, 145, 146, 147, 151, 152, 153, 159, 164, 169, 173, 179, 185, 191, 197, 199, 201, 203, 205, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 232, 233, 234, 235, 236, 237, 238, 239, 243, 250, 268, 270, 271, 272, 273, 274, 288, 289, 290, 292, 294 }

B grade: { 1, 2, 6, 7, 13, 14, 15, 19, 20, 21, 22, 26, 54, 55, 57, 65, 66, 67, 70, 71, 72, 73, 74, 83, 84, 87, 89, 90, 94, 96, 105, 107, 111, 114, 116, 118, 120, 122, 123, 124, 125, 126, 128, 136, 137, 138, 142, 143, 144, 148, 149, 150, 269, 278, 281, 282, 283, 291 }

C grade: { 45, 46, 198, 200, 202, 204, 275, 276, 277, 279, 280 }

F grade: { 11, 24, 38, 49, 50, 51, 52, 75, 76, 77, 79, 80, 81, 91, 92, 119, 121, 127, 129, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 284, 285, 286, 287, 293, 295, 296 }

2.1.4 Maxima

A grade: { 1, 2, 3, 14, 15, 16, 30, 31, 34, 35, 39, 40, 41, 42, 43, 44, 78, 82, 86, 97, 135, 141, 159, 164, 169, 173, 179, 185, 191, 197, 230, 232, 233, 234, 236, 237, 242, 243, 249, 250, 268, 272, 273, 288, 289, 292, 293

B grade: { 100, 102, 111, 238 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 235, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 294, 295, 296 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 8, 9, 10, 14, 15, 16, 21, 22, 23, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 49, 78, 82, 83, 84, 85, 86, 96, 97, 135, 141, 147, 153, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 232, 233, 234, 236, 237, 241, 242, 243, 249, 268, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 288, 289, 292, 296 }

B grade: { 5, 6, 7, 18, 19, 20, 50, 51, 52, 88, 89, 90, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 111, 113, 114, 115, 116, 117, 122, 123, 124, 125, 238, 240, 247, 248, 250, 293, 294, 295 }

C grade: { }

F grade: { 4, 11, 12, 13, 17, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 87, 91, 92, 98, 109, 110, 112, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 235, 239, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 284, 285, 286, 287, 290, 291 }

2.1.6 Sympy

A grade: { 1, 2, 3, 8, 9, 10, 14, 15, 16, 21, 22, 23, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 82, 83, 84, 85, 86, 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 113, 114, 115, 116, 117, 122, 123, 124, 125, 135, 141, 147, 159, 164, 179, 185, 191, 215, 216, 217, 218, 221, 222, 223, 224, 227, 230, 234, 288, 289, 292 }

B grade: { }

C grade: { }

F grade: { 4, 5, 6, 7, 11, 12, 13, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 225, 226, 228, 229, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 293, 294, 295, 296 }

2.1.7 Giac

A grade: { 1, 2, 3, 14, 15, 16, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 49, 78, 82, 83, 84, 85, 88, 89, 135, 141, 147, 153, 159, 179, 185, 191, 197, 214, 215, 217, 219, 220, 221, 223, 225, 226, 227, 230, 232, 233, 234, 236, 237, 243, 275, 276, 277, 278, 279, 288, 289 }

B grade: { 5, 18, 86, 90, 93, 94, 95, 96, 97, 238, 250, 280, 281, 282, 283, 292, 293, 294 }

C grade: { }

F grade: { 4, 6, 7, 8, 9, 10, 11, 12, 13, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 87, 91, 92, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 218, 222, 224, 228, 229, 231, 235, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 284, 285, 286, 287, 290, 291, 295, 296 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	153	351	333	329	258	225
normalized size	1	1.	0.84	1.92	1.82	1.8	1.41	1.23
time (sec)	N/A	0.153	0.249	0.032	1.199	1.921	1.791	1.2

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	233	201	230	155	174
normalized size	1	1.	0.92	1.89	1.63	1.87	1.26	1.41
time (sec)	N/A	0.103	0.168	0.013	1.087	2.002	0.832	1.167

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	73	107	123	143	80	117
normalized size	1	1.	0.75	1.1	1.27	1.47	0.82	1.21
time (sec)	N/A	0.042	0.073	0.012	1.168	2.023	0.368	1.177

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	176	295	0	0	0	0
normalized size	1	1.	0.99	1.66	0.	0.	0.	0.
time (sec)	N/A	0.271	0.012	0.075	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	92	126	0	903	0	286
normalized size	1	1.	1.11	1.52	0.	10.88	0.	3.45
time (sec)	N/A	0.088	0.095	0.041	0.	2.312	0.	1.389

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	190	338	0	2040	0	0
normalized size	1	1.	1.44	2.56	0.	15.45	0.	0.
time (sec)	N/A	0.135	0.185	0.025	0.	2.959	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	244	1108	0	3479	0	0
normalized size	1	1.	1.25	5.68	0.	17.84	0.	0.
time (sec)	N/A	0.237	0.551	0.023	0.	6.393	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	191	329	0	513	371	0
normalized size	1	1.	0.57	0.99	0.	1.54	1.11	0.
time (sec)	N/A	1.464	0.256	0.072	0.	1.985	3.555	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	131	207	0	358	223	0
normalized size	1	1.	0.61	0.96	0.	1.67	1.04	0.
time (sec)	N/A	0.996	0.187	0.06	0.	1.985	1.5	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	105	100	0	220	110	0
normalized size	1	1.	0.86	0.82	0.	1.8	0.9	0.
time (sec)	N/A	0.651	0.077	0.042	0.	1.941	0.717	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	252	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.426	0.157	0.135	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	848	374	0	0	0	0
normalized size	1	1.	3.27	1.44	0.	0.	0.	0.
time (sec)	N/A	0.582	3.3	0.1	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	936	766	0	0	0	0
normalized size	1	1.	2.66	2.18	0.	0.	0.	0.
time (sec)	N/A	0.694	4.457	0.185	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	193	408	382	468	323	401
normalized size	1	1.	1.01	2.14	2.	2.45	1.69	2.1
time (sec)	N/A	0.145	0.291	0.006	1.19	2.368	2.06	1.465

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	142	274	238	324	197	266
normalized size	1	1.	1.08	2.08	1.8	2.45	1.49	2.02
time (sec)	N/A	0.105	0.199	0.006	1.11	2.315	1.028	1.359

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	117	123	146	197	105	170
normalized size	1	1.	1.1	1.16	1.38	1.86	0.99	1.6
time (sec)	N/A	0.044	0.093	0.006	1.154	2.273	0.417	1.263

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	183	314	0	0	0	0
normalized size	1	1.	0.94	1.61	0.	0.	0.	0.
time (sec)	N/A	0.264	0.133	0.038	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	121	145	0	1008	0	308
normalized size	1	1.	1.38	1.65	0.	11.45	0.	3.5
time (sec)	N/A	0.056	0.192	0.004	0.	2.5	0.	1.354

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	184	361	0	2261	0	0
normalized size	1	1.	1.33	2.62	0.	16.38	0.	0.
time (sec)	N/A	0.102	0.372	0.004	0.	3.42	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	259	1137	0	3889	0	0
normalized size	1	1.	1.28	5.63	0.	19.25	0.	0.
time (sec)	N/A	0.164	0.939	0.004	0.	7.67	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	386	791	0	1010	750	0
normalized size	1	1.	0.97	1.99	0.	2.54	1.88	0.
time (sec)	N/A	1.689	0.824	0.065	0.	2.525	5.14	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	360	517	0	687	461	0
normalized size	1	1.	1.39	2.	0.	2.65	1.78	0.
time (sec)	N/A	1.148	0.612	0.052	0.	2.331	2.453	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	174	245	0	406	240	0
normalized size	1	1.	1.16	1.63	0.	2.71	1.6	0.
time (sec)	N/A	0.756	0.39	0.043	0.	2.301	1.028	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	285	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	0.242	0.03	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	959	536	0	0	0	0
normalized size	1	1.	3.44	1.92	0.	0.	0.	0.
time (sec)	N/A	0.61	4.463	0.063	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	1089	1170	0	0	0	0
normalized size	1	1.	2.87	3.08	0.	0.	0.	0.
time (sec)	N/A	0.747	8.402	0.112	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	287	394	0	0	0	0
normalized size	1	1.	0.73	1.	0.	0.	0.	0.
time (sec)	N/A	1.17	0.568	0.141	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	187	254	0	0	0	0
normalized size	1	1.	0.76	1.04	0.	0.	0.	0.
time (sec)	N/A	0.704	0.317	0.1	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	98	120	0	0	0	0
normalized size	1	1.	0.84	1.03	0.	0.	0.	0.
time (sec)	N/A	0.338	0.137	0.066	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.206	0.198	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.376	0.221	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	366	304	649	0	0	0	0
normalized size	1	0.98	0.81	1.74	0.	0.	0.	0.
time (sec)	N/A	0.751	2.306	0.174	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	186	157	285	0	0	0	0
normalized size	1	0.98	0.83	1.5	0.	0.	0.	0.
time (sec)	N/A	0.475	2.43	0.106	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	9.89	0.208	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	98.74	0.227	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	6.279	3.002	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.322	0.184	3.13	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	177	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.224	3.666	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.359	1.266	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.775	1.105	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	216	255	520	590	503	478
normalized size	1	1.	0.58	0.69	1.41	1.59	1.36	1.29
time (sec)	N/A	0.459	0.282	0.03	1.137	2.335	25.8	1.159

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	154	176	347	406	328	325
normalized size	1	1.	0.58	0.66	1.3	1.52	1.23	1.22
time (sec)	N/A	0.35	0.196	0.012	1.117	2.221	9.391	1.134

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	103	113	208	269	199	203
normalized size	1	1.	0.57	0.62	1.15	1.49	1.1	1.12
time (sec)	N/A	0.189	0.146	0.011	1.121	2.176	2.935	1.117

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	60	62	100	154	90	107
normalized size	1	1.	0.71	0.74	1.19	1.83	1.07	1.27
time (sec)	N/A	0.071	0.06	0.009	1.15	2.357	0.743	1.132

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	481	375	214	0	0	0	0
normalized size	1	1.	0.78	0.44	0.	0.	0.	0.
time (sec)	N/A	0.758	0.373	0.623	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	774	774	687	1632	0	0	0	0
normalized size	1	1.	0.89	2.11	0.	0.	0.	0.
time (sec)	N/A	1.101	1.223	1.255	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	3.042	0.256	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	1.896	0.224	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	551	0	0	649	0	111
normalized size	1	1.	5.74	0.	0.	6.76	0.	1.16
time (sec)	N/A	0.193	2.966	0.146	0.	2.48	0.	1.197

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	609	0	0	1270	0	0
normalized size	1	1.	3.38	0.	0.	7.06	0.	0.
time (sec)	N/A	0.176	1.977	0.151	0.	2.73	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	655	0	0	2221	0	0
normalized size	1	1.	2.43	0.	0.	8.26	0.	0.
time (sec)	N/A	0.805	3.214	0.153	0.	3.537	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	723	0	0	3546	0	0
normalized size	1	1.	1.96	0.	0.	9.61	0.	0.
time (sec)	N/A	1.014	6.17	0.145	0.	5.566	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	713	713	491	1213	0	0	0	0
normalized size	1	1.	0.69	1.7	0.	0.	0.	0.
time (sec)	N/A	1.484	1.997	0.544	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	479	356	855	0	0	0	0
normalized size	1	1.	0.74	1.78	0.	0.	0.	0.
time (sec)	N/A	1.176	1.208	0.425	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	251	461	0	0	0	0
normalized size	1	1.	0.98	1.81	0.	0.	0.	0.
time (sec)	N/A	0.546	1.055	0.389	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	785	785	1121	1072	0	0	0	0
normalized size	1	1.	1.43	1.37	0.	0.	0.	0.
time (sec)	N/A	3.379	4.05	0.347	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	918	918	1139	1956	0	0	0	0
normalized size	1	1.	1.24	2.13	0.	0.	0.	0.
time (sec)	N/A	3.561	7.052	0.309	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1029	1029	901	1638	0	0	0	0
normalized size	1	1.	0.88	1.59	0.	0.	0.	0.
time (sec)	N/A	2.057	4.536	0.52	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	725	725	623	1177	0	0	0	0
normalized size	1	1.	0.86	1.62	0.	0.	0.	0.
time (sec)	N/A	1.693	2.865	0.44	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	398	398	432	656	0	0	0	0
normalized size	1	1.	1.09	1.65	0.	0.	0.	0.
time (sec)	N/A	0.757	1.701	0.373	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	F(-2)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	1270	0	3068	1965	0	0	0	0
normalized size	1	0.	2.42	1.55	0.	0.	0.	0.
time (sec)	N/A	3.853	11.717	0.27	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1385	1385	1802	2116	0	0	0	0
normalized size	1	1.	1.3	1.53	0.	0.	0.	0.
time (sec)	N/A	2.518	7.974	0.622	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1015	1015	1282	1540	0	0	0	0
normalized size	1	1.	1.26	1.52	0.	0.	0.	0.
time (sec)	N/A	2.106	7.348	0.526	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	644	877	0	0	0	0
normalized size	1	1.	1.13	1.54	0.	0.	0.	0.
time (sec)	N/A	0.905	6.303	0.465	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1744	1744	6244	4234	0	0	0	0
normalized size	1	1.	3.58	2.43	0.	0.	0.	0.
time (sec)	N/A	4.779	20.71	0.368	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	478	478	405	859	0	0	0	0
normalized size	1	1.	0.85	1.8	0.	0.	0.	0.
time (sec)	N/A	1.288	2.054	0.323	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	267	559	0	0	0	0
normalized size	1	1.	0.93	1.94	0.	0.	0.	0.
time (sec)	N/A	0.914	1.582	0.258	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	172	239	0	0	0	0
normalized size	1	1.	1.26	1.76	0.	0.	0.	0.
time (sec)	N/A	0.472	0.667	0.225	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	932	754	0	0	0	0
normalized size	1	1.	2.55	2.07	0.	0.	0.	0.
time (sec)	N/A	0.703	1.866	0.153	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	523	523	1115	1978	0	0	0	0
normalized size	1	1.	2.13	3.78	0.	0.	0.	0.
time (sec)	N/A	0.836	5.8	0.223	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	549	549	353	1238	0	0	0	0
normalized size	1	1.	0.64	2.26	0.	0.	0.	0.
time (sec)	N/A	1.599	1.853	0.319	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	459	459	281	879	0	0	0	0
normalized size	1	1.	0.61	1.92	0.	0.	0.	0.
time (sec)	N/A	1.267	1.078	0.276	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	178	123	498	0	0	0	0
normalized size	1	1.25	0.87	3.51	0.	0.	0.	0.
time (sec)	N/A	0.307	0.376	0.227	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	773	773	1203	2484	0	0	0	0
normalized size	1	1.	1.56	3.21	0.	0.	0.	0.
time (sec)	N/A	1.857	9.715	0.255	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	204	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.471	0.632	0.322	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	200	242	204	0	0	0	0	0
normalized size	1	1.21	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.625	0.132	0.376	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	260	248	219	0	0	0	0	0
normalized size	1	0.95	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.695	2.06	0.439	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.955	0.17	1.755	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	774	774	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.599	4.248	1.726	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	600	600	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.129	1.842	1.412	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	246	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.346	0.02	0.192	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.063	0.279	0.925	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	121	308	0	262	255	220
normalized size	1	1.	0.8	2.03	0.	1.72	1.68	1.45
time (sec)	N/A	0.187	0.176	0.033	0.	2.273	1.932	1.203

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	101	207	0	216	170	178
normalized size	1	1.	0.97	1.99	0.	2.08	1.63	1.71
time (sec)	N/A	0.119	0.114	0.01	0.	2.343	0.887	1.207

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	87	120	0	178	104	151
normalized size	1	1.	0.97	1.33	0.	1.98	1.16	1.68
time (sec)	N/A	0.064	0.073	0.01	0.	2.26	0.384	1.208

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	56	36	41	135	46	126
normalized size	1	1.	1.37	0.88	1.	3.29	1.12	3.07
time (sec)	N/A	0.016	0.036	0.001	1.174	2.315	0.226	1.176

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	153	436	0	0	0	0
normalized size	1	1.	1.17	3.33	0.	0.	0.	0.
time (sec)	N/A	0.249	0.013	0.081	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	83	97	0	771	0	99
normalized size	1	1.	1.3	1.52	0.	12.05	0.	1.55
time (sec)	N/A	0.083	0.107	0.014	0.	2.612	0.	1.201

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	136	181	0	1091	0	230
normalized size	1	1.	1.28	1.71	0.	10.29	0.	2.17
time (sec)	N/A	0.102	0.264	0.02	0.	2.673	0.	1.191

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	162	397	0	1305	0	459
normalized size	1	1.	1.05	2.58	0.	8.47	0.	2.98
time (sec)	N/A	0.172	0.296	0.02	0.	2.716	0.	1.197

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	110	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.161	0.151	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	111	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.1	0.134	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	103	78	0	608	527	1110
normalized size	1	1.	0.76	0.58	0.	4.5	3.9	8.22
time (sec)	N/A	0.086	0.118	0.013	0.	2.466	4.696	2.843

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	115	359	0	495	394	807
normalized size	1	1.	0.97	3.02	0.	4.16	3.31	6.78
time (sec)	N/A	0.071	0.125	0.004	0.	2.425	2.357	2.458

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	67	0	365	258	548
normalized size	1	1.	0.73	0.69	0.	3.76	2.66	5.65
time (sec)	N/A	0.063	0.072	0.007	0.	2.42	1.101	2.139

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	81	162	0	257	148	331
normalized size	1	1.	1.08	2.16	0.	3.43	1.97	4.41
time (sec)	N/A	0.039	0.155	0.004	0.	2.418	0.454	1.804

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	61	41	47	154	51	135
normalized size	1	1.	1.33	0.89	1.02	3.35	1.11	2.93
time (sec)	N/A	0.024	0.049	0.003	1.223	2.16	0.197	1.142

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	69	111	0	0	0	0
normalized size	1	1.	0.85	1.37	0.	0.	0.	0.
time (sec)	N/A	0.107	0.062	0.031	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	76	88	0	312	0	0
normalized size	1	1.	1.36	1.57	0.	5.57	0.	0.
time (sec)	N/A	0.052	0.108	0.006	0.	2.53	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	65	159	257	0	0
normalized size	1	1.	0.83	0.98	2.41	3.89	0.	0.
time (sec)	N/A	0.054	0.056	0.011	1.761	2.413	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	101	120	0	610	0	0
normalized size	1	1.	1.02	1.21	0.	6.16	0.	0.
time (sec)	N/A	0.065	0.207	0.004	0.	2.781	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	86	76	351	448	0	0
normalized size	1	1.	0.83	0.73	3.38	4.31	0.	0.
time (sec)	N/A	0.069	0.08	0.004	1.314	2.595	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	136	152	0	909	0	0
normalized size	1	1.	0.99	1.11	0.	6.64	0.	0.
time (sec)	N/A	0.087	0.265	0.005	0.	3.226	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	220	294	0	1331	1268	0
normalized size	1	1.	1.01	1.35	0.	6.11	5.82	0.
time (sec)	N/A	0.485	0.323	0.044	0.	2.64	10.204	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	212	822	0	1034	916	0
normalized size	1	1.	1.14	4.42	0.	5.56	4.92	0.
time (sec)	N/A	0.436	0.266	0.042	0.	2.506	6.3	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	168	202	0	767	610	0
normalized size	1	1.	1.12	1.35	0.	5.11	4.07	0.
time (sec)	N/A	0.323	0.227	0.038	0.	2.342	2.668	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	167	334	0	533	335	0
normalized size	1	1.	1.52	3.04	0.	4.85	3.05	0.
time (sec)	N/A	0.251	0.217	0.036	0.	2.384	1.189	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	105	100	0	333	143	0
normalized size	1	1.	1.64	1.56	0.	5.2	2.23	0.
time (sec)	N/A	0.125	0.084	0.003	0.	2.374	0.479	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	118	117	140	263	0	0	0	0
normalized size	1	0.99	1.19	2.23	0.	0.	0.	0.
time (sec)	N/A	0.19	0.393	0.031	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	161	290	0	0	0	0
normalized size	1	1.	1.46	2.64	0.	0.	0.	0.
time (sec)	N/A	0.243	0.721	0.049	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	81	194	309	713	0	0
normalized size	1	1.	0.88	2.11	3.36	7.75	0.	0.
time (sec)	N/A	0.214	0.205	0.063	1.774	2.761	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	251	381	0	0	0	0
normalized size	1	1.	1.35	2.05	0.	0.	0.	0.
time (sec)	N/A	0.387	0.956	0.086	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	404	602	0	2325	2518	0
normalized size	1	1.	1.06	1.58	0.	6.09	6.59	0.
time (sec)	N/A	0.723	0.586	0.05	0.	2.776	23.048	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	359	1554	0	1773	1828	0
normalized size	1	1.	1.17	5.06	0.	5.78	5.95	0.
time (sec)	N/A	0.625	0.526	0.044	0.	2.66	13.486	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	296	396	0	1285	1173	0
normalized size	1	1.	1.13	1.51	0.	4.9	4.48	0.
time (sec)	N/A	0.472	0.393	0.043	0.	2.548	6.599	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	244	605	0	891	685	0
normalized size	1	1.	1.39	3.46	0.	5.09	3.91	0.
time (sec)	N/A	0.359	0.302	0.036	0.	2.514	2.931	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	168	180	0	548	282	0
normalized size	1	1.	1.47	1.58	0.	4.81	2.47	0.
time (sec)	N/A	0.184	0.173	0.056	0.	2.43	1.162	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	217	471	0	0	0	0
normalized size	1	1.	1.36	2.96	0.	0.	0.	0.
time (sec)	N/A	0.216	0.531	0.035	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	327	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.362	1.096	0.092	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	266	375	0	0	0	0
normalized size	1	1.	1.62	2.29	0.	0.	0.	0.
time (sec)	N/A	0.369	1.043	0.067	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	504	0	0	0	0	0
normalized size	1	1.	1.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.594	2.167	0.184	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	562	2465	0	2631	2876	0
normalized size	1	1.	1.49	6.54	0.	6.98	7.63	0.
time (sec)	N/A	1.174	0.806	0.05	0.	2.879	27.364	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	475	632	0	1894	1889	0
normalized size	1	1.	1.54	2.05	0.	6.13	6.11	0.
time (sec)	N/A	0.835	0.588	0.045	0.	2.639	12.776	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	360	933	0	1287	1027	0
normalized size	1	1.	1.72	4.46	0.	6.16	4.91	0.
time (sec)	N/A	0.562	0.469	0.04	0.	2.61	6.278	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	261	275	0	784	444	0
normalized size	1	1.	2.02	2.13	0.	6.08	3.44	0.
time (sec)	N/A	0.284	0.252	0.033	0.	2.384	2.426	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	308	727	0	0	0	0
normalized size	1	1.	1.6	3.79	0.	0.	0.	0.
time (sec)	N/A	0.266	0.762	0.033	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	872	0	0	0	0	0
normalized size	1	1.	3.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.413	2.345	0.086	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	398	605	0	0	0	0
normalized size	1	1.	2.04	3.1	0.	0.	0.	0.
time (sec)	N/A	0.426	2.218	0.066	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	432	432	1213	0	0	0	0	0
normalized size	1	1.	2.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.842	8.805	0.187	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	209	151	194	0	0	0	0
normalized size	1	0.98	0.71	0.91	0.	0.	0.	0.
time (sec)	N/A	0.429	0.25	0.155	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	109	134	0	0	0	0
normalized size	1	1.	0.75	0.92	0.	0.	0.	0.
time (sec)	N/A	0.334	0.179	0.079	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	137	102	130	0	0	0	0
normalized size	1	0.97	0.72	0.92	0.	0.	0.	0.
time (sec)	N/A	0.298	0.155	0.066	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	61	66	0	0	0	0
normalized size	1	1.	0.88	0.96	0.	0.	0.	0.
time (sec)	N/A	0.16	0.079	0.032	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	49	60	0	0	0	0
normalized size	1	1.	0.84	1.03	0.	0.	0.	0.
time (sec)	N/A	0.09	0.113	0.027	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.861	0.18	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	259	293	665	0	0	0	0
normalized size	1	0.98	1.11	2.53	0.	0.	0.	0.
time (sec)	N/A	0.394	2.054	0.204	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	230	418	0	0	0	0
normalized size	1	1.	1.18	2.14	0.	0.	0.	0.
time (sec)	N/A	0.298	2.166	0.115	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	187	150	374	0	0	0	0
normalized size	1	0.98	0.79	1.96	0.	0.	0.	0.
time (sec)	N/A	0.276	1.702	0.099	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	110	170	0	0	0	0
normalized size	1	1.	1.	1.55	0.	0.	0.	0.
time (sec)	N/A	0.148	0.799	0.048	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	94	143	139	0	0	0	0
normalized size	1	0.96	1.46	1.42	0.	0.	0.	0.
time (sec)	N/A	0.252	0.991	0.041	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	7.333	0.167	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	323	323	993	0	0	0	0
normalized size	1	0.99	0.99	3.04	0.	0.	0.	0.
time (sec)	N/A	1.111	1.271	0.237	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	186	624	0	0	0	0
normalized size	1	1.	0.73	2.46	0.	0.	0.	0.
time (sec)	N/A	0.899	0.563	0.138	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	311	223	557	0	0	0	0
normalized size	1	1.23	0.88	2.21	0.	0.	0.	0.
time (sec)	N/A	0.799	0.664	0.116	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	127	254	0	0	0	0
normalized size	1	1.	0.78	1.56	0.	0.	0.	0.
time (sec)	N/A	0.509	0.325	0.056	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	109	207	0	0	0	0
normalized size	1	1.	0.83	1.57	0.	0.	0.	0.
time (sec)	N/A	0.26	0.409	0.059	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.68	0.172	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	427	424	1375	0	0	0	0
normalized size	1	0.99	0.98	3.19	0.	0.	0.	0.
time (sec)	N/A	1.104	1.996	0.272	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	330	860	0	0	0	0
normalized size	1	1.	0.92	2.39	0.	0.	0.	0.
time (sec)	N/A	0.887	1.313	0.164	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	348	272	777	0	0	0	0
normalized size	1	0.99	0.77	2.21	0.	0.	0.	0.
time (sec)	N/A	0.965	1.291	0.141	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	195	353	0	0	0	0
normalized size	1	1.	0.89	1.62	0.	0.	0.	0.
time (sec)	N/A	0.511	0.994	0.066	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	170	144	295	0	0	0	0
normalized size	1	0.98	0.83	1.7	0.	0.	0.	0.
time (sec)	N/A	0.445	0.635	0.073	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	14.	0.184	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	342	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.951	0.707	0.346	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	223	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.793	0.506	0.254	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	245	245	237	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.759	0.482	0.244	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	437	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.575	2.369	0.128	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	110	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.35	0.182	0.118	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	2.042	0.189	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	558	0	0	0	0	0
normalized size	1	1.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.412	3.727	0.249	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	592	0	0	0	0	0
normalized size	1	1.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	1.11	2.418	0.241	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	1144	0	0	0	0	0
normalized size	1	1.	5.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.687	7.854	0.127	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	290	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.363	0.707	0.12	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	1.408	0.201	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	968	0	0	0	0	0
normalized size	1	1.	2.06	0.	0.	0.	0.	0.
time (sec)	N/A	2.239	10.863	0.252	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	1008	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	1.751	9.192	0.242	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	1846	0	0	0	0	0
normalized size	1	1.	6.86	0.	0.	0.	0.	0.
time (sec)	N/A	1.108	9.005	0.141	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	494	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.609	3.309	0.122	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	1.061	0.237	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	1523	0	0	0	0	0
normalized size	1	1.	2.99	0.	0.	0.	0.	0.
time (sec)	N/A	2.142	13.572	0.244	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	288	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	1.276	5.44	0.132	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	765	0	0	0	0	0
normalized size	1	1.	3.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.642	7.416	0.127	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	1.084	0.209	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	319	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.622	0.548	0.402	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	205	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	0.39	0.267	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	216	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.459	0.343	0.246	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	306	0	0	0	0	0
normalized size	1	1.	2.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	1.329	0.13	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	110	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.103	0.	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.075	0.188	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	396	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.644	1.452	0.332	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	265	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.449	0.979	0.251	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	265	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.45	1.544	0.237	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	314	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	6.382	0.129	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	145	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.354	0.469	0.118	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.083	0.224	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	615	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.765	3.233	0.331	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	391	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	1.441	2.345	0.251	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	391	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	1.237	2.936	0.234	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	687	0	0	0	0	0
normalized size	1	1.	3.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.78	5.478	0.128	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	219	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.409	0.169	0.119	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.088	0.227	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	552	552	654	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	1.776	4.659	0.329	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	441	441	445	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	1.406	3.014	0.25	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	452	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	1.475	2.786	0.237	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	916	0	0	0	0	0
normalized size	1	1.	3.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.784	4.526	0.128	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	243	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.627	0.747	0.121	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.092	0.231	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	150	277	0	0	0	0
normalized size	1	1.	0.79	1.47	0.	0.	0.	0.
time (sec)	N/A	0.144	0.327	0.064	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	180	218	0	0	0	0
normalized size	1	1.	1.07	1.29	0.	0.	0.	0.
time (sec)	N/A	0.133	0.264	0.023	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	109	254	0	0	0	0
normalized size	1	1.	0.75	1.75	0.	0.	0.	0.
time (sec)	N/A	0.111	0.451	0.02	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	131	194	0	0	0	0
normalized size	1	1.	1.03	1.53	0.	0.	0.	0.
time (sec)	N/A	0.097	0.435	0.016	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	138	0	0	0	0
normalized size	1	1.	0.9	1.33	0.	0.	0.	0.
time (sec)	N/A	0.088	0.177	0.019	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	92	119	0	0	0	0
normalized size	1	1.	1.1	1.42	0.	0.	0.	0.
time (sec)	N/A	0.084	0.155	0.016	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	94	269	0	0	0	0
normalized size	1	1.	0.63	1.79	0.	0.	0.	0.
time (sec)	N/A	0.121	0.139	0.026	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	94	201	0	0	0	0
normalized size	1	1.	0.72	1.55	0.	0.	0.	0.
time (sec)	N/A	0.111	0.151	0.027	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0
normalized size	1	1.08	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.321	0.505	0.345	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0
normalized size	1	1.08	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.328	0.464	0.343	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0
normalized size	1	1.08	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.326	0.476	0.348	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0
normalized size	1	1.08	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.316	0.436	0.37	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	163	140	0	0	0	0	0
normalized size	1	1.08	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.289	0.328	0.412	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	161	140	0	0	0	0	0
normalized size	1	1.08	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	0.29	0.361	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0
normalized size	1	1.08	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.312	0.358	0.354	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0
normalized size	1	1.08	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.359	0.355	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.32	100.013	0.355	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	180.002	0.384	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	22.32	0.24	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.303	30.667	0.353	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.322	35.8	0.377	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	115.254	0.399	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.315	70.616	0.37	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.309	180.001	0.424	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.28	14.431	0.256	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	84	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	37.614	0.352	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.315	36.873	0.353	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	163.198	0.358	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	3.736	1.961	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	1.782	1.86	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	218	178	0	0	0	0	0
normalized size	1	1.06	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.319	0.429	2.108	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	124	106	0	0	0	0	0
normalized size	1	1.05	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.192	1.967	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	1.139	0.881	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	50	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.042	0.056	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	79	75	76	128	0	82
normalized size	1	1.	0.68	0.64	0.65	1.09	0.	0.7
time (sec)	N/A	0.07	0.054	0.035	1.021	2.179	0.	1.162

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	74	65	62	112	0	76
normalized size	1	1.	0.86	0.76	0.72	1.3	0.	0.88
time (sec)	N/A	0.05	0.038	0.005	1.021	1.957	0.	1.167

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	64	49	45	92	29	65
normalized size	1	1.	1.28	0.98	0.9	1.84	0.58	1.3
time (sec)	N/A	0.031	0.023	0.003	1.018	2.033	0.344	1.168

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	46	65	0	0	0	0
normalized size	1	1.	1.	1.41	0.	0.	0.	0.
time (sec)	N/A	0.064	0.035	0.031	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	29	26	73	0	61
normalized size	1	1.	1.	0.72	0.65	1.82	0.	1.52
time (sec)	N/A	0.026	0.014	0.003	1.542	2.174	0.	1.121

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	49	35	41	97	0	84
normalized size	1	1.	0.64	0.46	0.54	1.28	0.	1.11
time (sec)	N/A	0.041	0.022	0.004	1.535	1.995	0.	1.137

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	38	23	173	0	30
normalized size	1	1.	1.92	1.58	0.96	7.21	0.	1.25
time (sec)	N/A	0.009	0.046	0.025	1.538	2.153	0.	1.104

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	179	91	0	0	0	0
normalized size	1	1.	2.98	1.52	0.	0.	0.	0.
time (sec)	N/A	0.065	0.469	0.049	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	264	0	0	626	0	0
normalized size	1	1.	1.82	0.	0.	4.32	0.	0.
time (sec)	N/A	0.036	0.22	0.132	0.	2.083	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	171	0	0	441	0	0
normalized size	1	1.	1.37	0.	0.	3.53	0.	0.
time (sec)	N/A	0.063	0.118	0.112	0.	2.085	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	104	0	173	277	0	0
normalized size	1	1.	1.44	0.	2.4	3.85	0.	0.
time (sec)	N/A	0.014	0.061	0.125	1.296	2.029	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	37	37	59	132	0	88
normalized size	1	1.	0.76	0.76	1.2	2.69	0.	1.8
time (sec)	N/A	0.039	0.059	0.004	1.033	1.982	0.	1.124

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	118	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.127	0.083	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	130	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.881	0.066	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	152	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.458	0.064	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	264	0	0	626	0	0
normalized size	1	1.	1.8	0.	0.	4.26	0.	0.
time (sec)	N/A	0.034	0.225	0.113	0.	2.182	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	171	0	0	441	0	0
normalized size	1	1.	1.55	0.	0.	4.01	0.	0.
time (sec)	N/A	0.046	0.12	0.117	0.	2.109	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	104	0	173	277	0	0
normalized size	1	1.	1.42	0.	2.37	3.79	0.	0.
time (sec)	N/A	0.014	0.061	0.121	1.326	2.235	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	37	59	132	0	95
normalized size	1	1.	1.	1.12	1.79	4.	0.	2.88
time (sec)	N/A	0.017	0.023	0.006	0.996	2.146	0.	1.114

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	86	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.13	0.066	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	141	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.699	0.063	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	168	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.644	0.072	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	311	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	3.306	0.075	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	254	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.657	0.066	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	210	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.292	0.068	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	166	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.305	0.067	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	242	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	1.046	0.066	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	273	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.971	0.069	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	301	301	291	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	1.221	0.063	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	277	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.569	0.065	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	221	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.534	0.064	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	178	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.25	0.063	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.254	0.066	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	209	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.973	0.067	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	238	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.847	0.065	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	260	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	1.018	0.063	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.133	0.588	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F(-1)	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	265	265	0	870	0	0	0	0
normalized size	1	1.	0.	3.28	0.	0.	0.	0.
time (sec)	N/A	0.221	0.367	0.942	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-1)	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	196	197	0	491	0	0	0	0
normalized size	1	1.01	0.	2.51	0.	0.	0.	0.
time (sec)	N/A	0.195	0.977	0.007	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F(-1)	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	0	207	0	0	0	0
normalized size	1	1.	0.	1.56	0.	0.	0.	0.
time (sec)	N/A	0.119	2.978	0.008	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.142	0.324	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	5.091	0.319	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	115	0	0	0	0
normalized size	1	1.	0.	1.51	0.	0.	0.	0.
time (sec)	N/A	0.073	0.674	0.02	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	138	376	0	327	0	348
normalized size	1	1.	0.84	2.28	0.	1.98	0.	2.11
time (sec)	N/A	0.161	0.089	0.047	0.	1.891	0.	1.276

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	119	288	0	271	0	271
normalized size	1	1.	1.03	2.5	0.	2.36	0.	2.36
time (sec)	N/A	0.121	0.124	0.01	0.	1.977	0.	1.264

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	93	194	0	217	0	176
normalized size	1	1.	1.39	2.9	0.	3.24	0.	2.63
time (sec)	N/A	0.07	0.153	0.012	0.	1.99	0.	1.271

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	31	31	69	147	0	174	0	82
normalized size	1	1.	2.23	4.74	0.	5.61	0.	2.65
time (sec)	N/A	0.017	0.027	0.007	0.	2.003	0.	1.322

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	141	156	0	647	0	159
normalized size	1	1.	1.41	1.56	0.	6.47	0.	1.59
time (sec)	N/A	0.092	0.092	0.013	0.	2.074	0.	1.24

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	140	237	0	852	0	317
normalized size	1	1.	1.28	2.17	0.	7.82	0.	2.91
time (sec)	N/A	0.074	0.152	0.014	0.	2.224	0.	1.288

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	142	236	0	818	0	471
normalized size	1	1.	1.03	1.71	0.	5.93	0.	3.41
time (sec)	N/A	0.109	0.24	0.016	0.	2.017	0.	1.399

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	179	374	0	1035	0	695
normalized size	1	1.	0.95	1.98	0.	5.48	0.	3.68
time (sec)	N/A	0.156	0.156	0.014	0.	2.044	0.	1.643

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	198	603	0	1337	0	1148
normalized size	1	1.	0.83	2.53	0.	5.62	0.	4.82
time (sec)	N/A	0.201	0.254	0.017	0.	2.08	0.	2.062

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	198	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.738	0.369	0.009	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	136	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	0.215	0.008	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.128	0.007	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.035	0.005	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.134	0.007	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.436	0.006	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	53	85	0	0	0	0
normalized size	1	1.	0.88	1.42	0.	0.	0.	0.
time (sec)	N/A	0.09	0.043	0.039	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	19	78	0	0	0	0
normalized size	1	1.	6.33	26.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.054	0.14	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	50	151	61	143
normalized size	1	1.	0.93	0.83	0.93	2.8	1.13	2.65
time (sec)	N/A	0.052	0.033	0.001	0.973	2.083	1.215	1.175

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	0	53	443	0	154
normalized size	1	1.	0.91	0.	0.96	8.05	0.	2.8
time (sec)	N/A	0.054	0.046	0.065	0.972	2.191	0.	1.184

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	175	91	0	583	0	161
normalized size	1	1.	3.02	1.57	0.	10.05	0.	2.78
time (sec)	N/A	0.093	0.428	0.043	0.	2.461	0.	1.765

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	282	0	0
normalized size	1	1.	1.	0.	0.	4.55	0.	0.
time (sec)	N/A	0.119	0.154	0.236	0.	2.227	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	80	0	0
normalized size	1	1.	1.	0.	0.	1.48	0.	0.
time (sec)	N/A	0.109	0.089	0.217	0.	2.12	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [65] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.	12	0.417
2	A	4	4	1.	12	0.333
3	A	4	4	1.	10	0.4
4	A	8	5	1.	12	0.417
5	A	3	3	1.	12	0.25
6	A	4	4	1.	12	0.333
7	A	6	6	1.	12	0.5
8	A	18	7	1.	14	0.5
9	A	13	7	1.	14	0.5
10	A	9	7	1.	12	0.583
11	A	10	6	1.	14	0.429
12	A	10	7	1.	14	0.5
13	A	13	10	1.	14	0.714
14	A	5	5	1.	16	0.312
15	A	4	4	1.	16	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	4	4	1.	14	0.286
17	A	8	5	1.	16	0.312
18	A	3	3	1.	16	0.188
19	A	4	4	1.	16	0.25
20	A	6	6	1.	16	0.375
21	A	18	7	1.	18	0.389
22	A	13	7	1.	18	0.389
23	A	9	7	1.	16	0.438
24	A	10	6	1.	18	0.333
25	A	10	7	1.	18	0.389
26	A	13	10	1.	18	0.556
27	A	27	7	1.	18	0.389
28	A	17	6	1.	18	0.333
29	A	11	7	1.	16	0.438
30	A	0	0	0.	0	0.
31	A	0	0	0.	0	0.
32	A	19	7	0.98	18	0.389
33	A	11	7	0.98	16	0.438
34	A	0	0	0.	0	0.
35	A	0	0	0.	0	0.
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	3	3	1.	16	0.188
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	6	6	1.	14	0.429
42	A	6	6	1.	14	0.429
43	A	6	6	1.	14	0.429
44	A	3	3	1.	12	0.25
45	A	18	6	1.	14	0.429
46	A	26	9	1.	14	0.643
47	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	0	0	0.	0	0.
49	A	7	8	1.	16	0.5
50	A	8	10	1.	16	0.625
51	A	9	11	1.	16	0.688
52	A	10	12	1.	16	0.75
53	A	16	12	1.	31	0.387
54	A	13	8	1.	31	0.258
55	A	8	6	1.	29	0.207
56	A	23	22	1.	31	0.71
57	A	38	22	1.	31	0.71
58	A	24	17	1.	31	0.548
59	A	20	12	1.	31	0.387
60	A	12	9	1.	29	0.31
61	F	0	0	N/A	0	N/A
62	A	30	20	1.	31	0.645
63	A	26	15	1.	31	0.484
64	A	14	10	1.	29	0.345
65	A	38	31	1.	31	1.
66	A	13	7	1.	31	0.226
67	A	9	7	1.	31	0.226
68	A	6	5	1.	29	0.172
69	A	10	7	1.	31	0.226
70	A	13	10	1.	31	0.323
71	A	19	14	1.	31	0.452
72	A	17	12	1.	31	0.387
73	A	5	6	1.25	29	0.207
74	A	25	17	1.	31	0.548
75	A	7	5	1.	30	0.167
76	A	7	5	1.21	35	0.143
77	A	7	5	0.95	37	0.135
78	A	0	0	0.	0	0.
79	A	14	11	1.	35	0.314

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	12	10	1.	33	0.303
81	A	9	7	1.	25	0.28
82	A	0	0	0.	0	0.
83	A	6	6	1.	10	0.6
84	A	5	5	1.	10	0.5
85	A	5	5	1.	8	0.625
86	A	3	3	1.	6	0.5
87	A	9	6	1.	10	0.6
88	A	4	4	1.	10	0.4
89	A	5	5	1.	10	0.5
90	A	7	7	1.	10	0.7
91	A	7	6	1.	14	0.429
92	A	7	6	1.	15	0.4
93	A	8	5	1.	21	0.238
94	A	7	6	1.	21	0.286
95	A	6	5	1.	21	0.238
96	A	5	5	1.	19	0.263
97	A	4	3	1.	10	0.3
98	A	7	7	1.	21	0.333
99	A	5	5	1.	21	0.238
100	A	4	4	1.	21	0.19
101	A	6	6	1.	21	0.286
102	A	6	5	1.	21	0.238
103	A	8	6	1.	21	0.286
104	A	9	7	1.	23	0.304
105	A	8	6	1.	23	0.261
106	A	7	7	1.	23	0.304
107	A	6	6	1.	21	0.286
108	A	4	4	1.	12	0.333
109	A	8	8	0.99	23	0.348
110	A	9	7	1.	23	0.304
111	A	5	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	11	9	1.	23	0.391
113	A	19	8	1.	23	0.348
114	A	14	8	1.	23	0.348
115	A	12	8	1.	23	0.348
116	A	8	7	1.	21	0.333
117	A	6	4	1.	12	0.333
118	A	9	9	1.	23	0.391
119	A	11	8	1.	23	0.348
120	A	9	9	1.	23	0.391
121	A	15	11	1.	23	0.478
122	A	16	6	1.	23	0.261
123	A	13	8	1.	23	0.348
124	A	9	6	1.	21	0.286
125	A	6	4	1.	12	0.333
126	A	10	9	1.	23	0.391
127	A	13	9	1.	23	0.391
128	A	10	10	1.	23	0.435
129	A	21	12	1.	23	0.522
130	A	14	7	0.98	23	0.304
131	A	11	7	1.	23	0.304
132	A	11	7	0.97	23	0.304
133	A	8	7	1.	21	0.333
134	A	5	5	1.	12	0.417
135	A	0	0	0.	0	0.
136	A	13	6	0.98	23	0.261
137	A	10	6	1.	23	0.261
138	A	10	6	0.98	23	0.261
139	A	6	6	1.	21	0.286
140	A	6	6	0.96	12	0.5
141	A	0	0	0.	0	0.
142	A	26	9	0.99	23	0.391
143	A	20	9	1.	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	18	10	1.23	23	0.435
145	A	11	10	1.	21	0.476
146	A	7	7	1.	12	0.583
147	A	0	0	0.	0	0.
148	A	24	8	0.99	23	0.348
149	A	17	8	1.	23	0.348
150	A	18	10	0.99	23	0.435
151	A	9	9	1.	21	0.429
152	A	8	7	0.98	12	0.583
153	A	0	0	0.	0	0.
154	A	21	9	1.	25	0.36
155	A	16	9	1.	25	0.36
156	A	16	9	1.	25	0.36
157	A	11	9	1.	23	0.391
158	A	8	7	1.	14	0.5
159	A	0	0	0.	0	0.
160	A	27	11	1.	25	0.44
161	A	24	12	1.	25	0.48
162	A	13	11	1.	23	0.478
163	A	9	8	1.	14	0.571
164	A	0	0	0.	0	0.
165	A	29	11	1.	25	0.44
166	A	26	12	1.	25	0.48
167	A	14	11	1.	23	0.478
168	A	10	8	1.	14	0.571
169	A	0	0	0.	0	0.
170	A	35	13	1.	25	0.52
171	A	16	11	1.	23	0.478
172	A	11	8	1.	14	0.571
173	A	0	0	0.	0	0.
174	A	20	8	1.	25	0.32
175	A	15	8	1.	25	0.32

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	15	8	1.	25	0.32
177	A	10	8	1.	23	0.348
178	A	7	6	1.	14	0.429
179	A	0	0	0.	0	0.
180	A	19	7	1.	25	0.28
181	A	14	7	1.	25	0.28
182	A	14	7	1.	25	0.28
183	A	8	7	1.	23	0.304
184	A	8	7	1.	14	0.5
185	A	0	0	0.	0	0.
186	A	36	10	1.	25	0.4
187	A	26	10	1.	25	0.4
188	A	24	11	1.	25	0.44
189	A	13	11	1.	23	0.478
190	A	9	8	1.	14	0.571
191	A	0	0	0.	0	0.
192	A	34	9	1.	25	0.36
193	A	23	9	1.	25	0.36
194	A	24	11	1.	25	0.44
195	A	11	10	1.	23	0.435
196	A	10	8	1.	14	0.571
197	A	0	0	0.	0	0.
198	A	8	6	1.	23	0.261
199	A	8	6	1.	23	0.261
200	A	6	6	1.	23	0.261
201	A	6	6	1.	23	0.261
202	A	4	4	1.	23	0.174
203	A	4	4	1.	23	0.174
204	A	7	7	1.	23	0.304
205	A	7	7	1.	23	0.304
206	A	3	3	1.08	25	0.12
207	A	3	3	1.08	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	3	3	1.08	25	0.12
209	A	3	3	1.08	25	0.12
210	A	3	3	1.08	25	0.12
211	A	3	3	1.08	25	0.12
212	A	3	3	1.08	25	0.12
213	A	3	3	1.08	25	0.12
214	A	0	0	0.	0	0.
215	A	0	0	0.	0	0.
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	0	0	0.	0	0.
219	A	0	0	0.	0	0.
220	A	0	0	0.	0	0.
221	A	0	0	0.	0	0.
222	A	0	0	0.	0	0.
223	A	0	0	0.	0	0.
224	A	0	0	0.	0	0.
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.
228	A	3	3	1.06	23	0.13
229	A	5	5	1.05	21	0.238
230	A	0	0	0.	0	0.
231	A	5	5	1.	10	0.5
232	A	7	5	1.	10	0.5
233	A	6	5	1.	8	0.625
234	A	5	5	1.	6	0.833
235	A	5	5	1.	10	0.5
236	A	3	3	1.	10	0.3
237	A	4	4	1.	10	0.4
238	A	3	3	1.	4	0.75
239	A	5	5	1.	10	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	3	2	1.	14	0.143
241	A	6	5	1.	14	0.357
242	A	2	2	1.	14	0.143
243	A	5	4	1.	12	0.333
244	A	1	1	1.	14	0.071
245	A	1	1	1.	14	0.071
246	A	2	2	1.	14	0.143
247	A	3	2	1.	14	0.143
248	A	5	4	1.	14	0.286
249	A	2	2	1.	14	0.143
250	A	4	3	1.	12	0.25
251	A	1	1	1.	14	0.071
252	A	1	1	1.	14	0.071
253	A	2	2	1.	14	0.143
254	A	2	2	1.	16	0.125
255	A	2	2	1.	16	0.125
256	A	1	1	1.	16	0.062
257	A	1	1	1.	16	0.062
258	A	1	1	1.	16	0.062
259	A	2	2	1.	16	0.125
260	A	2	2	1.	16	0.125
261	A	2	2	1.	16	0.125
262	A	2	2	1.	16	0.125
263	A	1	1	1.	16	0.062
264	A	1	1	1.	16	0.062
265	A	1	1	1.	16	0.062
266	A	2	2	1.	16	0.125
267	A	2	2	1.	16	0.125
268	A	0	0	0.	0	0.
269	A	8	8	1.	40	0.2
270	A	7	7	1.01	40	0.175
271	A	6	7	1.	38	0.184

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	0	0	0.	0	0.
273	A	0	0	0.	0	0.
274	A	6	6	1.	10	0.6
275	A	5	4	1.	12	0.333
276	A	5	4	1.	12	0.333
277	A	5	4	1.	10	0.4
278	A	5	4	1.	8	0.5
279	A	9	8	1.	12	0.667
280	A	9	8	1.	12	0.667
281	A	7	5	1.	12	0.417
282	A	8	6	1.	12	0.5
283	A	10	7	1.	12	0.583
284	A	37	8	1.	14	0.571
285	A	27	8	1.	14	0.571
286	A	17	8	1.	12	0.667
287	A	7	4	1.	10	0.4
288	A	0	0	0.	0	0.
289	A	0	0	0.	0	0.
290	A	7	7	1.	19	0.368
291	A	2	2	1.	20	0.1
292	A	4	4	1.	12	0.333
293	A	4	4	1.	14	0.286
294	A	5	5	1.	10	0.5
295	A	2	2	1.	26	0.077
296	A	2	2	1.	26	0.077

Chapter 3

Listing of integrals

3.1 $\int (d+ex)^3 \cosh^{-1}(cx)\,dx$

Optimal. Leaf size=183

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{\left(24c^2d^2e^2+8c^4d^4+3e^4\right)\cosh^{-1}(cx)}{32c^4e}-\frac{\sqrt{cx-1}\sqrt{cx+1}(d}{16c}$$

```
[Out] (-7*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(d + e*x)^3)/(16*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*(19*c^
2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - ((8*c^4*d^4 + 24*c^
2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*ArcCosh[c*x])/(4
*e)
```

Rubi [A] time = 0.153374, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5802, 100, 153, 147, 52}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{\left(24c^2d^2e^2+8c^4d^4+3e^4\right)\cosh^{-1}(cx)}{32c^4e}-\frac{\sqrt{cx-1}\sqrt{cx+1}(d}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*ArcCosh[c*x],x]
```

```
[Out] (-7*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(d + e*x)^3)/(16*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*(19*c^
2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - ((8*c^4*d^4 + 24*c^
```

```
2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*ArcCosh[c*x])/(4
*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (d+ex)^3\cosh^{-1}(cx)\,dx = \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \frac{c\int \frac{(d+ex)^4}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4e}$$

$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} + \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \frac{\int \frac{(d+ex)^2\left(4c^2d^2+3e^2+7c^2dex\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{16ce}$$

$$= -\frac{7d\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} + \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \int$$

$$= -\frac{7d\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(4d\left(19c^2\right.\right.}{}$$

$$= -\frac{7d\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(4d\left(19c^2\right.\right.}{}$$

Mathematica [A] time = 0.248598, size = 153, normalized size = 0.84

$$-\frac{c\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(72d^2ex+96d^3+32de^2x^2+6e^3x^3\right)+e^2(64d+9ex)\right)-24c^4x\cosh^{-1}(cx)\left(6d^2ex+4d^3+4de^2x^2\right.}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*ArcCosh[c*x],x]
```

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2
*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)) - 24*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x
^2 + e^3*x^3)*ArcCosh[c*x] + 9*(8*c^2*d^2*e + e^3)*Log[c*x + Sqrt[-1 + c*x]
*Sqrt[1 + c*x]])/(96*c^4)
```

Maple [B] time = 0.032, size = 351, normalized size = 1.9

$$\frac{e^3\mathrm{arccosh}(cx)\,x^4}{4} + e^2\mathrm{arccosh}(cx)\,x^3d + \frac{3\,e\mathrm{arccosh}(cx)\,x^2d^2}{2} + \mathrm{arccosh}(cx)\,xd^3 + \frac{\mathrm{arccosh}(cx)\,d^4}{4\,e} - \frac{e^3x^3}{16\,c}\sqrt{cx-1}\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*arccosh(c*x),x)
```

```
[Out] 1/4*e^3*arccosh(c*x)*x^4+e^2*arccosh(c*x)*x^3*d+3/2*e*arccosh(c*x)*x^2*d^2+
arccosh(c*x)*x*d^3+1/4/e*arccosh(c*x)*d^4-1/16/c*e^3*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*x^3-1/3/c*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d-1/4/e*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-3/4/c*e*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*d^2*x-1/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3-3/4/c^2*e*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-
3/32/c^3*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-2/3/c^3*e^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*d-3/32/c^4*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x
+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.19868, size = 333, normalized size = 1.82

$$-\frac{1}{96}\left(\frac{6\sqrt{c^2x^2-1}e^3x^3}{c^2}+\frac{32\sqrt{c^2x^2-1}de^2x^2}{c^2}+\frac{72\sqrt{c^2x^2-1}d^2ex}{c^2}+\frac{96\sqrt{c^2x^2-1}d^3}{c^2}+\frac{72\,d^2e\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/96*(6*sqrt(c^2*x^2 - 1)*e^3*x^3/c^2 + 32*sqrt(c^2*x^2 - 1)*d*e^2*x^2/c^2
 + 72*sqrt(c^2*x^2 - 1)*d^2*e*x/c^2 + 96*sqrt(c^2*x^2 - 1)*d^3/c^2 + 72*d^2
*e*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2) + 9*sqrt(c^
2*x^2 - 1)*e^3*x/c^4 + 64*sqrt(c^2*x^2 - 1)*d*e^2/c^4 + 9*e^3*log(2*c^2*x +
 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c + 1/4*(e^3*x^4 + 4*d*e^2
*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*arccosh(c*x)
```

Fricas [A] time = 1.92111, size = 329, normalized size = 1.8

$$\frac{3\left(8\,c^4e^3x^4+32\,c^4de^2x^3+48\,c^4d^2ex^2+32\,c^4d^3x-24\,c^2d^2e-3\,e^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(6\,c^3e^3x^3+32\,c^3de^2x^2+96\,c^3\right.}{96\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*c^4*e^3*x^4 + 32*c^4*d*e^2*x^3 + 48*c^4*d^2*e*x^2 + 32*c^4*d^3*x
 - 24*c^2*d^2*e - 3*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*c^3*e^3*x^3 + 32
*c^3*d*e^2*x^2 + 96*c^3*d^3 + 64*c*d*e^2 + 9*(8*c^3*d^2*e + c*e^3)*x)*sqrt(
c^2*x^2 - 1))/c^4
```

Sympy [A] time = 1.79107, size = 258, normalized size = 1.41

$$\begin{cases} d^3x\operatorname{acosh}(cx) + \frac{3d^2ex^2\operatorname{acosh}(cx)}{2} + de^2x^3\operatorname{acosh}(cx) + \frac{e^3x^4\operatorname{acosh}(cx)}{4} - \frac{d^3\sqrt{c^2x^2-1}}{c} - \frac{3d^2ex\sqrt{c^2x^2-1}}{4c} - \frac{de^2x^2\sqrt{c^2x^2-1}}{3c} - \frac{e^3x^3\sqrt{c^2x^2-1}}{16c} \\ \frac{i\pi\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*acosh(c*x),x)
```

```
[Out] Piecewise((d**3*x*acosh(c*x) + 3*d**2*e*x**2*acosh(c*x)/2 + d*e**2*x**3*aco
sh(c*x) + e**3*x**4*acosh(c*x)/4 - d**3*sqrt(c**2*x**2 - 1)/c - 3*d**2*e*x*
sqrt(c**2*x**2 - 1)/(4*c) - d*e**2*x**2*sqrt(c**2*x**2 - 1)/(3*c) - e**3*x*
*3*sqrt(c**2*x**2 - 1)/(16*c) - 3*d**2*e*acosh(c*x)/(4*c**2) - 2*d*e**2*sqr
t(c**2*x**2 - 1)/(3*c**3) - 3*e**3*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*e**3
*acosh(c*x)/(32*c**4), Ne(c, 0)), (I*pi*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*
x**3 + e**3*x**4/4)/2, True))
```

Giac [A] time = 1.20013, size = 225, normalized size = 1.23

$$\frac{1}{4}(xe+d)^4e^{(-1)}\log\left(cx+\sqrt{c^2x^2-1}\right) - \frac{1}{96}\left(\sqrt{c^2x^2-1}\left(\left(2x\left(\frac{3xe^4}{c}+\frac{16de^3}{c}\right)+\frac{9\left(8c^5d^2e^2+c^3e^4\right)}{c^6}\right)x+\frac{32\left(3c^5d^3e+2c^3\right.}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/4*(x*e + d)^4*e^(-1)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/96*(sqrt(c^2*x^2 -
1)*((2*x*(3*x*e^4/c + 16*d*e^3/c) + 9*(8*c^5*d^2*e^2 + c^3*e^4)/c^6)*x + 32
*(3*c^5*d^3*e + 2*c^3*d*e^3)/c^6) - 3*(8*c^4*d^4 + 24*c^2*d^2*e^2 + 3*e^4)*
log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^3*abs(c)))*e^(-1)
```

3.2 $\int (d+ex)^2 \cosh^{-1}(cx)\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{1}{6}d\left(\frac{3e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}(d+ex)^2}{9c}+\frac{\cosh^{-1}(cx)(d+e}{3e}$$

```
[Out] -(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(9*c) - (Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (d*((2*d^2)/e + (3*e
)/c^2)*ArcCosh[c*x])/6 + ((d + e*x)^3*ArcCosh[c*x])/(3*e)
```

Rubi [A] time = 0.103075, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5802, 100, 147, 52}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{1}{6}d\left(\frac{3e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}(d+ex)^2}{9c}+\frac{\cosh^{-1}(cx)(d+e}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*ArcCosh[c*x],x]
```

```
[Out] -(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(9*c) - (Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (d*((2*d^2)/e + (3*e
)/c^2)*ArcCosh[c*x])/6 + ((d + e*x)^3*ArcCosh[c*x])/(3*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (d+ex)^2\cosh^{-1}(cx)\,dx = \frac{(d+ex)^3\cosh^{-1}(cx)}{3e} - \frac{c\int \frac{(d+ex)^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} + \frac{(d+ex)^3\cosh^{-1}(cx)}{3e} - \frac{\int \frac{(d+ex)\left(3c^2d^2+2e^2+5c^2dex\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{9ce}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3} + \frac{(d+ex)^3\cos}{3e}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3} - \frac{1}{6}d\left(\frac{2d^2}{e} + \frac{3}{c}\right.$$

Mathematica [A] time = 0.168389, size = 113, normalized size = 0.92

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)+4e^2\right)-6c^3x\cosh^{-1}(cx)\left(3d^2+3dex+e^2x^2\right)+9cde\log\left(cx+\sqrt{cx-1}\sqrt{cx\right.}}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*ArcCosh[c*x],x]
```

```
[Out] -(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))
 - 6*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCosh[c*x] + 9*c*d*e*Log[c*x + Sqr
t[-1 + c*x]*Sqrt[1 + c*x]])/(18*c^3)
```

Maple [B] time = 0.013, size = 233, normalized size = 1.9

$$\frac{e^2 \operatorname{arccosh}(cx)\, x^3}{3} + e \operatorname{arccosh}(cx)\, x^2 d + \operatorname{arccosh}(cx)\, x d^2 + \frac{\operatorname{arccosh}(cx)\, d^3}{3e} - \frac{e^2 x^2}{9c}\sqrt{cx-1}\sqrt{cx+1} - \frac{d^3}{3e}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*arccosh(c*x),x)
```

```
[Out] 1/3*e^2*arccosh(c*x)*x^3+e*arccosh(c*x)*x^2*d+arccosh(c*x)*x*d^2+1/3/e*arcc
osh(c*x)*d^3-1/9/c*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-1/3/e*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-1/2/c*e*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*d*x-1/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-1/2/c^2*e*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-2/9
/c^3*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Maxima [A] time = 1.08668, size = 201, normalized size = 1.63

$$-\frac{1}{18}\left(\frac{2\sqrt{c^2x^2-1}e^2x^2}{c^2} + \frac{9\sqrt{c^2x^2-1}dex}{c^2} + \frac{18\sqrt{c^2x^2-1}d^2}{c^2} + \frac{9de\log\left(2c^2x + 2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2} + \frac{4\sqrt{c^2x^2-1}e^2}{c^4}\right)c + \frac{1}{3}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(c^2*x^2 - 1)*e^2*x^2/c^2 + 9*sqrt(c^2*x^2 - 1)*d*e*x/c^2 + 18
*sqrt(c^2*x^2 - 1)*d^2/c^2 + 9*d*e*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c
^2))/(sqrt(c^2)*c^2) + 4*sqrt(c^2*x^2 - 1)*e^2/c^4)*c + 1/3*(e^2*x^3 + 3*d*
e*x^2 + 3*d^2*x)*arccosh(c*x)
```

Fricas [A] time = 2.00196, size = 230, normalized size = 1.87

$$\frac{3\left(2c^3e^2x^3+6c^3dex^2+6c^3d^2x-3cde\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2c^2e^2x^2+9c^2dex+18c^2d^2+4e^2\right)\sqrt{c^2x^2-1}}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*c^3*e^2*x^3 + 6*c^3*d*e*x^2 + 6*c^3*d^2*x - 3*c*d*e)*log(c*x + s
qrt(c^2*x^2 - 1)) - (2*c^2*e^2*x^2 + 9*c^2*d*e*x + 18*c^2*d^2 + 4*e^2)*sqrt
(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 0.832006, size = 155, normalized size = 1.26

$$\begin{cases} d^2x\operatorname{acosh}(cx)+dex^2\operatorname{acosh}(cx)+\frac{e^2x^3\operatorname{acosh}(cx)}{3}-\frac{d^2\sqrt{c^2x^2-1}}{c}-\frac{dex\sqrt{c^2x^2-1}}{2c}-\frac{e^2x^2\sqrt{c^2x^2-1}}{9c}-\frac{de\operatorname{acosh}(cx)}{2c^2}-\frac{2e^2\sqrt{c^2x^2-1}}{9c^3} & \text{for } c \\ \frac{i\pi\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)}{2} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*acosh(c*x),x)
```

```
[Out] Piecewise((d**2*x*acosh(c*x) + d*e*x**2*acosh(c*x) + e**2*x**3*acosh(c*x)/3
 - d**2*sqrt(c**2*x**2 - 1)/c - d*e*x*sqrt(c**2*x**2 - 1)/(2*c) - e**2*x**2
*sqrt(c**2*x**2 - 1)/(9*c) - d*e*acosh(c*x)/(2*c**2) - 2*e**2*sqrt(c**2*x**
2 - 1)/(9*c**3), Ne(c, 0)), (I*pi*(d**2*x + d*e*x**2 + e**2*x**3/3)/2, True
))
```

Giac [A] time = 1.16729, size = 174, normalized size = 1.41

$$\frac{1}{3}(xe+d)^3e^{(-1)}\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{1}{18}\left(\sqrt{c^2x^2-1}\left(x\left(\frac{2xe^3}{c}+\frac{9de^2}{c}\right)+\frac{2\left(9c^3d^2e+2ce^3\right)}{c^4}\right)-\frac{3\left(2c^2d^3+3de^2\right)\log\left(\right.}{c|c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/3*(x*e + d)^3*e^(-1)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/18*(sqrt(c^2*x^2 -
1)*(x*(2*x*e^3/c + 9*d*e^2/c) + 2*(9*c^3*d^2*e + 2*c*e^3)/c^4) - 3*(2*c^2*d
^3 + 3*d*e^2)*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c*abs(c)))*e^(-1)
```

3.3 $\int (d+ex)\cosh^{-1}(cx)\,dx$

Optimal. Leaf size=97

$$-\frac{1}{4}\left(\frac{e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}(d+ex)}{4c}+\frac{\cosh^{-1}(cx)(d+ex)^2}{2e}-\frac{3d\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

```
[Out] (-3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
d + e*x))/(4*c) - (((2*d^2)/e + e/c^2)*ArcCosh[c*x])/4 + ((d + e*x)^2*ArcCo
sh[c*x])/(2*e)
```

Rubi [A] time = 0.0417405, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5802, 90, 80, 52}

$$-\frac{1}{4}\left(\frac{e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}(d+ex)}{4c}+\frac{\cosh^{-1}(cx)(d+ex)^2}{2e}-\frac{3d\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*ArcCosh[c*x],x]
```

```
[Out] (-3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
d + e*x))/(4*c) - (((2*d^2)/e + e/c^2)*ArcCosh[c*x])/4 + ((d + e*x)^2*ArcCo
sh[c*x])/(2*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int (d+ex)\cosh^{-1}(cx)\,dx &= \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{c\int \frac{(d+ex)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e} \\
&= -\frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} + \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{\int \frac{2c^2d^2+e^2+3c^2dex}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4ce} \\
&= -\frac{3d\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} + \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{1}{4}\left(\frac{2cd^2}{e}+\frac{e}{c}\right)\int \\
&= -\frac{3d\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} - \frac{1}{4}\left(\frac{2d^2}{e}+\frac{e}{c^2}\right)\cosh^{-1}(cx) + \frac{(d+ex)^2\,\text{cc}}{2e}
\end{aligned}
$$

Mathematica [A] time = 0.0731214, size = 73, normalized size = 0.75

$$
-\frac{-2c^2x\cosh^{-1}(cx)(2d+ex)+c\sqrt{cx-1}\sqrt{cx+1}(4d+ex)+2e\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)}{4c^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*ArcCosh[c*x],x]
```

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d + e*x) - 2*c^2*x*(2*d + e*x)*ArcCosh[
c*x] + 2*e*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(4*c^2)
```

Maple [A] time = 0.012, size = 107, normalized size = 1.1

$$\frac{\operatorname{arccosh}(cx)\,x^2e}{2}+\operatorname{arccosh}(cx)\,xd-\frac{ex}{4c}\sqrt{cx-1}\sqrt{cx+1}-\frac{d}{c}\sqrt{cx-1}\sqrt{cx+1}-\frac{e}{4c^2}\sqrt{cx-1}\sqrt{cx+1}\ln\left(cx+\sqrt{c^2x^2-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*arccosh(c*x),x)
```

```
[Out] 1/2*arccosh(c*x)*x^2*e+arccosh(c*x)*x*d-1/4/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e
*x-d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2
*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.16758, size = 123, normalized size = 1.27

$$-\frac{1}{4}c\left(\frac{\sqrt{c^2x^2-1}ex}{c^2}+\frac{4\sqrt{c^2x^2-1}d}{c^2}+\frac{e\log\left(2c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)+\frac{1}{2}\left(ex^2+2dx\right)\operatorname{arcosh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/4*c*(sqrt(c^2*x^2 - 1)*e*x/c^2 + 4*sqrt(c^2*x^2 - 1)*d/c^2 + e*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)) + 1/2*(e*x^2 + 2*d*x)*
arccosh(c*x)
```

Fricas [A] time = 2.02282, size = 143, normalized size = 1.47

$$\frac{\left(2c^2ex^2+4c^2dx-e\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}(cex+4cd)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/4*((2*c^2*e*x^2 + 4*c^2*d*x - e)*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*
x^2 - 1)*(c*e*x + 4*c*d))/c^2
```

Sympy [A] time = 0.367506, size = 80, normalized size = 0.82

$$\begin{cases} dx\operatorname{acosh}(cx) + \frac{ex^2\operatorname{acosh}(cx)}{2} - \frac{d\sqrt{c^2x^2-1}}{c} - \frac{ex\sqrt{c^2x^2-1}}{4c} - \frac{e\operatorname{acosh}(cx)}{4c^2} & \text{for } c \neq 0 \\ \frac{i\pi\left(dx+\frac{ex^2}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*acosh(c*x),x)
```

```
[Out] Piecewise((d*x*acosh(c*x) + e*x**2*acosh(c*x)/2 - d*sqrt(c**2*x**2 - 1)/c -
 e*x*sqrt(c**2*x**2 - 1)/(4*c) - e*acosh(c*x)/(4*c**2), Ne(c, 0)), (I*pi*(d
*x + e*x**2/2)/2, True))
```

Giac [A] time = 1.17712, size = 117, normalized size = 1.21

$$\frac{1}{2}\left(x^2e + 2\,dx\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{1}{4}\sqrt{c^2x^2-1}\left(\frac{xe}{c} + \frac{4\,d}{c}\right) + \frac{e\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{4\,c|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/4*sqrt(c^2*x^2 - 1)*(x
*e/c + 4*d/c) + 1/4*e*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c*abs(c))
```

3.4 $\int \frac{\cosh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=178

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(c}}{\sqrt{c^2d^2-e^2}}\right.}{e}$$

```
[Out] -ArcCosh[c*x]^2/(2*e) + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2
 - e^2]))]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

Rubi [A] time = 0.271444, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5800, 5562, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(c}}{\sqrt{c^2d^2-e^2}}\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x),x]
```

```
[Out] -ArcCosh[c*x]^2/(2*e) + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2
 - e^2]))]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{x\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \text{Subst}\left(\int \frac{e^x x}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x x}{cd+\sqrt{c^2d^2-e^2}+ \ldots}\right.$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} - \frac{\text{Subst}\left(\int \log \ldots\right.}{\ldots}$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{\log\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\text{Li}_2\left(-\frac{ee^{\cosh^{-1}\ldots}}{cd-\sqrt{c^2 \ldots}}\right.}{e}$$

Mathematica [A] time = 0.0124997, size = 176, normalized size = 0.99

$$\frac{\text{PolyLog}\left(2,\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+c\ldots}\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x),x]
```

```
[Out] -ArcCosh[c*x]^2/(2*e) + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 - e^2])]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

Maple [A] time = 0.075, size = 295, normalized size = 1.7

$$-\frac{(\text{arccosh}\,(cx))^2}{2e}+\frac{\text{arccosh}\,(cx)}{e}\ln\left(\left(-\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}\right)\left(-cd+\sqrt{c^2d^2-e^2}\right)^{-1}\right)+\frac{\text{arccosh}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d),x)
```

```
[Out] -1/2*arccosh(c*x)^2/e+1/e*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))+1/e*arccosh(c*x)*
ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*
d^2-e^2)^(1/2)))+1/e*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^
2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+1/e*dilog((-(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\,(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(c*x)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcosh}\,(cx)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{acosh}\,(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d),x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arcosh}\,(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arccosh(c*x)/(e*x + d), x)
```

3.5 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^2} dx$

Optimal. Leaf size=83

$$\frac{2c\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\sqrt{cd+e}}-\frac{\cosh^{-1}(cx)}{e(d+ex)}$$

```
[Out] -(ArcCosh[c*x]/(e*(d + e*x))) + (2*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/
(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rubi [A] time = 0.0877095, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5802, 93, 208}

$$\frac{2c\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\sqrt{cd+e}}-\frac{\cosh^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^2,x]
```



```
[Out] -(ArcCosh[c*x]/(e*(d + e*x))) + (2*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/
(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{c\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{e}\\ &= -\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e}\\ &= -\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right)}{\sqrt{cd-e}e\sqrt{cd+e}}\end{aligned}$$

Mathematica [A] time = 0.0953047, size = 92, normalized size = 1.11

$$\frac{\frac{c\left(\log(d+ex)-\log\left(-\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}+c^2dx+e\right)\right)}{\sqrt{c^2d^2-e^2}}-\frac{\cosh^{-1}(cx)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^2,x]
```

```
[Out] (-(ArcCosh[c*x]/(d + e*x)) + (c*(Log[d + e*x] - Log[e + c^2*d*x - Sqrt[c^2*
d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/Sqrt[c^2*d^2 - e^2])/e
```

Maple [A] time = 0.041, size = 126, normalized size = 1.5

$$-\frac{c\,\text{arccosh}\,(cx)}{(cxe+cd)\,e}-\frac{c}{e^2}\sqrt{cx-1}\sqrt{cx+1}\ln\left(-2\,\frac{1}{cxe+cd}\left(c^2dx-\sqrt{c^2x^2-1}\sqrt{\frac{c^2d^2-e^2}{e^2}}e+e\right)\right)\frac{1}{\sqrt{\frac{c^2d^2-e^2}{e^2}}}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^2,x)
```

```
[Out] -c/(c*e*x+c*d)/e*arccosh(c*x)-c/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-2*(c^2*
d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))/((c^2*d^2
-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31234, size = 903, normalized size = 10.88

$$\left[\frac{\left(c^2d^2e-e^3\right)x\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2d^2-e^2}\left(cdex+cd^2\right)\log\left(\frac{c^3d^2x+cde+\sqrt{c^2d^2-e^2}\left(c^2dx+e\right)+\left(c^2d^2+\sqrt{c^2d^2-e^2}cd-e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right)+}{c^2d^4e-d^2e^3+\left(c^2d^3e^2-de^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((c^2*d^2*e - e^3)*x*log(c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*d^2 - e^2)*(c
*d*e*x + c*d^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e)
+ (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) +
 (c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^
2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), ((c^2*d^2*e - e^3)*x*log(c*x
+ sqrt(c^2*x^2 - 1)) - 2*sqrt(-c^2*d^2 + e^2)*(c*d*e*x + c*d^2)*arctan(-(sq
rt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d)
)/(c^2*d^2 - e^2)) + (c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x)*log(-c*x + sqr
t(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**2,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**2, x)
```

Giac [B] time = 1.38854, size = 286, normalized size = 3.45

$$\left(\frac{e^{(-1)}\log\left(\left|-c^2d+\sqrt{c^2d^2-e^2}|c|\right|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{\sqrt{c^2d^2-e^2}}-\frac{e^{(-1)}\log\left(\left|-c^2d+\sqrt{c^2d^2-e^2}\left(\sqrt{c^2-\frac{2c^2d}{xe+d}+\frac{c^2d^2}{(xe+d)^2}-\frac{e^2}{(xe+d)^2}}+\frac{\sqrt{c^2d^2e^2-e^4}e^{(-}}{xe+d}\right.\right.\right.}{\sqrt{c^2d^2-e^2}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (e^(-1)*log(abs(-c^2*d + sqrt(c^2*d^2 - e^2)*abs(c)))*sgn(1/(x*e + d))/sqrt
(c^2*d^2 - e^2) - e^(-1)*log(abs(-c^2*d + sqrt(c^2*d^2 - e^2)*(sqrt(c^2 - 2
*c^2*d/(x*e + d) + c^2*d^2/(x*e + d)^2 - e^2/(x*e + d)^2) + sqrt(c^2*d^2*e^
2 - e^4)*e^(-1)/(x*e + d))))/(sqrt(c^2*d^2 - e^2)*sgn(1/(x*e + d))))*c - e^
(-1)*log(c*x + sqrt(c^2*x^2 - 1))/(x*e + d)
```

3.6 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^3} \, dx$

Optimal. Leaf size=132

$$-\frac{c\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}$$

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c^2*d^2 - e^2)*(d + e*x)) - ArcCosh[c
*x]/(2*e*(d + e*x)^2) + (c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[
c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```

Rubi [A] time = 0.134899, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5802, 96, 93, 208}

$$-\frac{c\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^3,x]
```

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c^2*d^2 - e^2)*(d + e*x)) - ArcCosh[c
*x]/(2*e*(d + e*x)^2) + (c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[
c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(cx)}{(d+ex)^3}\,dx &= -\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{c\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{2e}\\
&= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(c^3d\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{2e\left(c^2d^2-e^2\right)}\\
&= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(c^3d\right)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e\left(c^2d^2-e^2\right)}\\
&= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.185377, size = 190, normalized size = 1.44

$$
\frac{c(d+ex)\left(-e\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}-c^2d(d+ex)\log\left(-\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}+c^2dx+e\right)+c^2d(d+ex)\log(d+ \cdots}{2e(cd-e)(cd+e)\sqrt{c^2d^2-e^2}(d+ex)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^3,x]
```

```
[Out] (-((c^2*d^2 - e^2)^(3/2)*ArcCosh[c*x]) + c*(d + e*x)*(-(e*Sqrt[c^2*d^2 - e^
2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + c^2*d*(d + e*x)*Log[d + e*x] - c^2*d*(d
+ e*x)*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
)/(2*(c*d - e)*e*(c*d + e)*Sqrt[c^2*d^2 - e^2]*(d + e*x)^2)
```

Maple [B] time = 0.025, size = 338, normalized size = 2.6

$$-\frac{c^2 \operatorname{arccosh}(cx)}{2(cxe+cd)^2 e} - \frac{c^4 x d}{2e(cd+e)(cd-e)(cxe+cd)}\sqrt{cx-1}\sqrt{cx+1}\ln\left(-2\frac{1}{cxe+cd}\left(c^2dx - \sqrt{c^2x^2-1}\sqrt{\frac{c^2d^2-e^2}{e^2}}e+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2/(c*e*x+c*d)^2/e*arccosh(c*x)-1/2*c^4/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)
/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*
x*d-1/2*c^4/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-
e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*(
(c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2-1/2*c^2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.959, size = 2040, normalized size = 15.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4)*x^2 + (c^3*d^3*e
^2*x^2 + 2*c^3*d^4*e*x + c^3*d^5)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*
e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - sqrt(c^2*d^2 - e^2)*c*d
- e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + 2*(c^4*d^5*e - c^2*d^3*e^3)*x - ((c^
4*d^4*e^2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5
)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (c^4*d^6 - 2*c^2*d^4*e^2 + d^2*e^4 + (c
^4*d^4*e^2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^
5)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (c^3*d^5*e - c*d^3*e^3 + (c^3*d^4*e^2
 - c*d^2*e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 +
(c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^
4 + d^3*e^6)*x), -1/2*(c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4)*
x^2 + 2*(c^3*d^3*e^2*x^2 + 2*c^3*d^4*e*x + c^3*d^5)*sqrt(-c^2*d^2 + e^2)*ar
ctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e
*x + c*d))/(c^2*d^2 - e^2)) + 2*(c^4*d^5*e - c^2*d^3*e^3)*x - ((c^4*d^4*e^2
 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5)*x)*log(
c*x + sqrt(c^2*x^2 - 1)) - (c^4*d^6 - 2*c^2*d^4*e^2 + d^2*e^4 + (c^4*d^4*e^
2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5)*x)*log
(-c*x + sqrt(c^2*x^2 - 1)) + (c^3*d^5*e - c*d^3*e^3 + (c^3*d^4*e^2 - c*d^2*
e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*
e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e
^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**3,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.7 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^4} dx$

Optimal. Leaf size=195

$$-\frac{c\sqrt{cx-1}\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{c^3d\sqrt{cx-1}\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}$$

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*(c^2*d^2 - e^2)*(d + e*x)^2) - (c^3*d*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - ArcCo
sh[c*x]/(3*e*(d + e*x)^3) + (c^3*(2*c^2*d^2 + e^2)*ArcTanh[(Sqrt[c*d + e]*S
qrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(5/2)*e*(c*d +
e)^(5/2))
```

Rubi [A] time = 0.237232, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5802, 103, 151, 12, 93, 208}

$$-\frac{c\sqrt{cx-1}\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{c^3d\sqrt{cx-1}\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^4,x]
```

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*(c^2*d^2 - e^2)*(d + e*x)^2) - (c^3*d*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - ArcCo
sh[c*x]/(3*e*(d + e*x)^3) + (c^3*(2*c^2*d^2 + e^2)*ArcTanh[(Sqrt[c*d + e]*S
qrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(5/2)*e*(c*d +
e)^(5/2))
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{c\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}\,dx}{3e}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} - \frac{c\int \frac{-2c^2d+c^2ex}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{6e\left(c^2d^2-e^2\right)}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{c\int \frac{c^2\left(2c^2d^2+e^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{6e\left(c^2d^2-e^2\right)^2}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{\left(c^3\left(2c^2d^2+e^2\right)\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+e}}{6e\left(c^2d^2-e^2\right)^2}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{\left(c^3\left(2c^2d^2+e^2\right)\right)\text{Subst}\left(\int \frac{1}{cd-e-(cd}\right.}{3e\left(c^2d^2-e^2\right)^2}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+c}}{\sqrt{cd-e}\sqrt{-1+c}}\right.}{3(cd-e)^{5/2}e(cd+e)^{5/2}}$$

Mathematica [C] time = 0.550665, size = 244, normalized size = 1.25

$$\frac{1}{6}\left(\frac{c\sqrt{cx-1}\sqrt{cx+1}\left(e^2-c^2d(4d+3ex)\right)}{\left(e^2-c^2d^2\right)^2(d+ex)^2} - \frac{ic^3\left(2c^2d^2+e^2\right)\log\left(\frac{12e^2(e-cd)^2(cd+e)^2\left(\sqrt{cx-1}\sqrt{cx+1}\sqrt{e^2-c^2d^2}-ic^2dx-ie\right)}{c^3\sqrt{e^2-c^2d^2}\left(2c^2d^2+e^2\right)(d+ex)}\right)}{e(e-cd)^2(cd+e)^2\sqrt{e^2-c^2d^2}} - \frac{2\cosh^{-1}(c}{e(d+ex)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^4,x]
```

```
[Out] ((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2 - c^2*d*(4*d + 3*e*x)))/((-(c^2*d^2)
+ e^2)^2*(d + e*x)^2) - (2*ArcCosh[c*x])/(e*(d + e*x)^3) - (I*c^3*(2*c^2*d^
2 + e^2)*Log[(12*e^2*(-(c*d) + e)^2*(c*d + e)^2*((-I)*e - I*c^2*d*x + Sqrt[
-(c^2*d^2) + e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*Sqrt[-(c^2*d^2) + e^2
]*(2*c^2*d^2 + e^2)*(d + e*x))])/(e*(-(c*d) + e)^2*(c*d + e)^2*Sqrt[-(c^2*d
^2) + e^2]))/6
```

Maple [B] time = 0.023, size = 1108, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3/(c*e*x+c*d)^3/e*arccosh(c*x)-1/3*c^7*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(
c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c
*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)
/(c*e*x+c*d))*x^2*d^2-2/3*c^7/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/
2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*
x*d^3-1/6*c^5*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*
d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(
c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x^2-1/3*c^7/e^
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e
^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)
*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^4-1/2*c^5*e*(c*x+1)^(1/2)*(c
*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*x*d-1/3*c^5*e*(c*x+
1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2
)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*
d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d-2/3*c^5*(c*x+1)^(1/2)*(c*x-1)^(1/
2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d^2-1/6*c^5*(c*x+1)^(1/2)*(c
*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^
2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^
2)^(1/2)*e+e)/(c*e*x+c*d))*d^2+1/6*c^3*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d
+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.39307, size = 3479, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(3*c^6*d^9 - 3*c^4*d^7*e^2 + 3*(c^6*d^6*e^3 - c^4*d^4*e^5)*x^3 + 9*(c
^6*d^7*e^2 - c^4*d^5*e^4)*x^2 - (2*c^5*d^8 + c^3*d^6*e^2 + (2*c^5*d^5*e^3 +
 c^3*d^3*e^5)*x^3 + 3*(2*c^5*d^6*e^2 + c^3*d^4*e^4)*x^2 + 3*(2*c^5*d^7*e +
c^3*d^5*e^3)*x)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 -
 e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^
2 - 1))/(e*x + d)) + 9*(c^6*d^8*e - c^4*d^6*e^3)*x - 2*((c^6*d^6*e^3 - 3*c^
4*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^3 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c
^2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^8*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^
2*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(c^6*d^9 - 3*c^4*d^7*e^2 + 3*c^2
*d^5*e^4 - d^3*e^6 + (c^6*d^6*e^3 - 3*c^4*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^
3 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c^2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^
8*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 -
 1)) + (4*c^5*d^8*e - 5*c^3*d^6*e^3 + c*d^4*e^5 + 3*(c^5*d^6*e^3 - c^3*d^4*
e^5)*x^2 + (7*c^5*d^7*e^2 - 8*c^3*d^5*e^4 + c*d^3*e^6)*x)*sqrt(c^2*x^2 - 1)
)/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4 - 3
*c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d^8*
e^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3*c^
2*d^7*e^6 - d^5*e^8)*x), -1/6*(3*c^6*d^9 - 3*c^4*d^7*e^2 + 3*(c^6*d^6*e^3 -
 c^4*d^4*e^5)*x^3 + 9*(c^6*d^7*e^2 - c^4*d^5*e^4)*x^2 + 2*(2*c^5*d^8 + c^3*
d^6*e^2 + (2*c^5*d^5*e^3 + c^3*d^3*e^5)*x^3 + 3*(2*c^5*d^6*e^2 + c^3*d^4*e^
4)*x^2 + 3*(2*c^5*d^7*e + c^3*d^5*e^3)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqr
t(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))
/(c^2*d^2 - e^2)) + 9*(c^6*d^8*e - c^4*d^6*e^3)*x - 2*((c^6*d^6*e^3 - 3*c^4
*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^3 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c^
2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^8*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^2
*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(c^6*d^9 - 3*c^4*d^7*e^2 + 3*c^2*
d^5*e^4 - d^3*e^6 + (c^6*d^6*e^3 - 3*c^4*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^3
 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c^2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^8
*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 -
1)) + (4*c^5*d^8*e - 5*c^3*d^6*e^3 + c*d^4*e^5 + 3*(c^5*d^6*e^3 - c^3*d^4*e
^5)*x^2 + (7*c^5*d^7*e^2 - 8*c^3*d^5*e^4 + c*d^3*e^6)*x)*sqrt(c^2*x^2 - 1))
/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4 - 3*
c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d^8*e
^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3*c^2
*d^7*e^6 - d^5*e^8)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**4,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.8 $\int (d+ex)^3 \cosh^{-1}(cx)^2 \, dx$

Optimal. Leaf size=334

$$-\frac{3d^2e\cosh^{-1}(cx)^2}{4c^2}+\frac{4de^2x}{3c^2}-\frac{4de^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{3c^3}+\frac{3e^3x^2}{32c^2}-\frac{3e^3\cosh^{-1}(cx)^2}{32c^4}-\frac{3e^3x\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}}{16c^3}$$

```
[Out] 2*d^3*x + (4*d*e^2*x)/(3*c^2) + (3*d^2*e*x^2)/4 + (3*e^3*x^2)/(32*c^2) + (2
*d*e^2*x^3)/9 + (e^3*x^4)/32 - (2*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[
c*x])/c - (4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3) - (3*
d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (3*e^3*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(16*c^3) - (2*d*e^2*x^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c) - (e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*A
rcCosh[c*x])/(8*c) - (d^4*ArcCosh[c*x]^2)/(4*e) - (3*d^2*e*ArcCosh[c*x]^2)/
(4*c^2) - (3*e^3*ArcCosh[c*x]^2)/(32*c^4) + ((d + e*x)^4*ArcCosh[c*x]^2)/(4
*e)
```

Rubi [A] time = 1.46354, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{3d^2e\cosh^{-1}(cx)^2}{4c^2}+\frac{4de^2x}{3c^2}-\frac{4de^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{3c^3}+\frac{3e^3x^2}{32c^2}-\frac{3e^3\cosh^{-1}(cx)^2}{32c^4}-\frac{3e^3x\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d^3*x + (4*d*e^2*x)/(3*c^2) + (3*d^2*e*x^2)/4 + (3*e^3*x^2)/(32*c^2) + (2
*d*e^2*x^3)/9 + (e^3*x^4)/32 - (2*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[
c*x])/c - (4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3) - (3*
d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (3*e^3*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(16*c^3) - (2*d*e^2*x^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c) - (e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*A
rcCosh[c*x])/(8*c) - (d^4*ArcCosh[c*x]^2)/(4*e) - (3*d^2*e*ArcCosh[c*x]^2)/
(4*c^2) - (3*e^3*ArcCosh[c*x]^2)/(32*c^4) + ((d + e*x)^4*ArcCosh[c*x]^2)/(4
*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
```

```
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \frac{c\int \frac{(d+ex)^4\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e}$$

$$= \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \frac{c\int \left(\frac{d^4\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4d^3ex\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{6d^2e^2x^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4de^3x^3\cosh^{-1}(c}{\sqrt{-1+cx}\sqrt{1+cx}}\right.}{2e}$$

$$= \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \left(2cd^3\right)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(cd^4\right)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e} - \left(3cd^2e\right)$$

$$= -\frac{2d^3\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{3d^2ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} - \frac{2de^2x^2\sqrt{-1+cx}}{}$$

$$= 2d^3x + \frac{3}{4}d^2ex^2 + \frac{2}{9}de^2x^3 + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{4de^2\sqrt{-1+cx}\sqrt{1+c}}{3c^3}$$

$$= 2d^3x + \frac{4de^2x}{3c^2} + \frac{3}{4}d^2ex^2 + \frac{3e^3x^2}{32c^2} + \frac{2}{9}de^2x^3 + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{4d}{}$$

Mathematica [A] time = 0.256192, size = 191, normalized size = 0.57

$$\frac{c^2x\left(c^2\left(216d^2ex+576d^3+64de^2x^2+9e^3x^3\right)+3e^2(128d+9ex)\right)-6c\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)\left(c^2\left(72d^2ex+96d^3+3\right.\right.}{288c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*ArcCosh[c*x]^2,x]
```

```
[Out] (c^2*x*(3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*e*x + 64*d*e^2*x^2 +
 9*e^3*x^3)) - 6*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(
96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))*ArcCosh[c*x] + 9*(-24*c^2*
d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*ArcCos
```

```
h[c*x]^2)/(288*c^4)
```

Maple [A] time = 0.072, size = 329, normalized size = 1.

$$\frac{1}{288\,c^4}\left(72\,(\mathrm{arccosh}\,(cx))^2\,c^4x^4e^3+288\,(\mathrm{arccosh}\,(cx))^2\,c^4x^3de^2+432\,(\mathrm{arccosh}\,(cx))^2\,c^4x^2d^2e+288\,(\mathrm{arccosh}\,(cx))^2\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*arccosh(c*x)^2,x)
```

```
[Out] 1/288/c^4*(72*arccosh(c*x)^2*c^4*x^4*e^3+288*arccosh(c*x)^2*c^4*x^3*d*e^2+4
32*arccosh(c*x)^2*c^4*x^2*d^2*e+288*arccosh(c*x)^2*c^4*x*d^3-36*arccosh(c*x
)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^3*e^3-192*arccosh(c*x)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*c^3*x^2*d*e^2-432*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*
x*d^2*e-576*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*d^3-216*arccosh(c*
x)^2*c^2*d^2*e-54*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x*e^3-384*arcc
osh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*d*e^2+9*c^4*x^4*e^3+64*c^4*d*e^2*x^3
+216*c^4*d^2*e*x^2+576*x*c^4*d^3-27*arccosh(c*x)^2*e^3+27*c^2*x^2*e^3+384*x
*c^2*d*e^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(e^3x^4+4\,de^2x^3+6\,d^2ex^2+4\,d^3x\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2-\int\frac{\left(c^3e^3x^6+4\,c^3de^2x^5-6\,cd^2ex^2-4\,cd^3x+\left(6\,c^3d^2e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2 - integrate(1/2*(c^3*e^3*x^6 + 4*c^3*d*e^2*x^5 - 6*c*d^2*
e*x^2 - 4*c*d^3*x + (6*c^3*d^2*e - c*e^3)*x^4 + 4*(c^3*d^3 - c*d*e^2)*x^3 +
 (c^2*e^3*x^5 + 4*c^2*d*e^2*x^4 + 6*c^2*d^2*e*x^3 + 4*c^2*d^3*x^2)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2
*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A] time = 1.98508, size = 513, normalized size = 1.54

$$\frac{9\,c^4e^3x^4 + 64\,c^4de^2x^3 + 27\left(8\,c^4d^2e + c^2e^3\right)x^2 + 9\left(8\,c^4e^3x^4 + 32\,c^4de^2x^3 + 48\,c^4d^2ex^2 + 32\,c^4d^3x - 24\,c^2d^2e - 3\,e^3\right)\log\left(cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*c^4*e^3*x^4 + 64*c^4*d*e^2*x^3 + 27*(8*c^4*d^2*e + c^2*e^3)*x^2 +
9*(8*c^4*e^3*x^4 + 32*c^4*d*e^2*x^3 + 48*c^4*d^2*e*x^2 + 32*c^4*d^3*x - 24*
c^2*d^2*e - 3*e^3)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 6*(6*c^3*e^3*x^3 + 32*c
^3*d*e^2*x^2 + 96*c^3*d^3 + 64*c*d*e^2 + 9*(8*c^3*d^2*e + c*e^3)*x)*sqrt(c^
2*x^2 - 1)*log(c*x + sqrt(c^2*x^2 - 1)) + 192*(3*c^4*d^3 + 2*c^2*d*e^2)*x)/
c^4
```

Sympy [A] time = 3.55514, size = 371, normalized size = 1.11

$$\begin{cases} d^3x\operatorname{acosh}^2(cx) + 2d^3x + \frac{3d^2ex^2\operatorname{acosh}^2(cx)}{2} + \frac{3d^2ex^2}{4} + de^2x^3\operatorname{acosh}^2(cx) + \frac{2de^2x^3}{9} + \frac{e^3x^4\operatorname{acosh}^2(cx)}{4} + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{c^2x^2-1}\operatorname{acosh}(cx}{c} \\ -\frac{\pi^2\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d**3*x*acosh(c*x)**2 + 2*d**3*x + 3*d**2*e*x**2*acosh(c*x)**2/2
+ 3*d**2*e*x**2/4 + d*e**2*x**3*acosh(c*x)**2 + 2*d*e**2*x**3/9 + e**3*x**4
*acosh(c*x)**2/4 + e**3*x**4/32 - 2*d**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/c -
 3*d**2*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(2*c) - 2*d*e**2*x**2*sqrt(c**2*
x**2 - 1)*acosh(c*x)/(3*c) - e**3*x**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(8*c)
 - 3*d**2*e*acosh(c*x)**2/(4*c**2) + 4*d*e**2*x/(3*c**2) + 3*e**3*x**2/(32*
c**2) - 4*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 3*e**3*x*sqrt(c*
*2*x**2 - 1)*acosh(c*x)/(16*c**3) - 3*e**3*acosh(c*x)**2/(32*c**4), Ne(c, 0
)), (-pi**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)/4, True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 \operatorname{arcosh}(cx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*arccosh(c*x)^2, x)
```

3.9 $\int (d+ex)^2 \cosh^{-1}(cx)^2 \, dx$

Optimal. Leaf size=215

$$-\frac{de\cosh^{-1}(cx)^2}{2c^2}+\frac{4e^2x}{9c^2}-\frac{4e^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{9c^3}-\frac{d^3\cosh^{-1}(cx)^2}{3e}-\frac{2d^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{dex\sqrt{cx}}{}$$

```
[Out] 2*d^2*x + (4*e^2*x)/(9*c^2) + (d*e*x^2)/2 + (2*e^2*x^3)/27 - (2*d^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (4*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*ArcCosh[c*x])/(9*c^3) - (d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/
c - (2*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(9*c) - (d^3*ArcC
osh[c*x]^2)/(3*e) - (d*e*ArcCosh[c*x]^2)/(2*c^2) + ((d + e*x)^3*ArcCosh[c*x
]^2)/(3*e)
```

Rubi [A] time = 0.996076, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{de\cosh^{-1}(cx)^2}{2c^2}+\frac{4e^2x}{9c^2}-\frac{4e^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{9c^3}-\frac{d^3\cosh^{-1}(cx)^2}{3e}-\frac{2d^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{dex\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*ArcCosh[c*x]^2,x]
```


```
[Out] 2*d^2*x + (4*e^2*x)/(9*c^2) + (d*e*x^2)/2 + (2*e^2*x^3)/27 - (2*d^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (4*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*ArcCosh[c*x])/(9*c^3) - (d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/
c - (2*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(9*c) - (d^3*ArcC
osh[c*x]^2)/(3*e) - (d*e*ArcCosh[c*x]^2)/(2*c^2) + ((d + e*x)^3*ArcCosh[c*x
]^2)/(3*e)
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \frac{(2c)\int \frac{(d+ex)^3\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$

$$= \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \frac{(2c)\int \left(\frac{d^3\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3d^2ex\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3de^2x^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{e^3x^3\cosh^{-1}(c}{\sqrt{-1+cx}\sqrt{1+c}}\right.}{3e}$$

$$= \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \left(2cd^2\right)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(2cd^3\right)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e} - (2cde)$$

$$= -\frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{dex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{2e^2x^2\sqrt{-1+cx}\sqrt{1}}{9c}$$

$$= 2d^2x + \frac{1}{2}dex^2 + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{4e^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(c}{9c^3}$$

$$= 2d^2x + \frac{4e^2x}{9c^2} + \frac{1}{2}dex^2 + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{4e^2\sqrt{-1+cx}\sqrt{1+cx}\,c}{9c^3}$$

Mathematica [A] time = 0.186733, size = 131, normalized size = 0.61

$$\frac{cx\left(c^2\left(108d^2+27dex+4e^2x^2\right)+24e^2\right)+9\cosh^{-1}(cx)^2\left(2c^3x\left(3d^2+3dex+e^2x^2\right)-3cde\right)-6\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*ArcCosh[c*x]^2,x]
```

```
[Out] (c*x*(24*e^2 + c^2*(108*d^2 + 27*d*e*x + 4*e^2*x^2)) - 6*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))*ArcCosh[c*x] + 9*(-
3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcCosh[c*x]^2)/(54*c^3)
```

Maple [A] time = 0.06, size = 207, normalized size = 1.

$$\frac{1}{54\,c^3}\left(18\,(\mathrm{arccosh}\,(cx))^2\,c^3x^3e^2 + 54\,(\mathrm{arccosh}\,(cx))^2\,c^3x^2de + 54\,(\mathrm{arccosh}\,(cx))^2\,c^3xd^2 - 12\,\mathrm{arccosh}\,(cx)\,\sqrt{cx-1}\sqrt{cx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*arccosh(c*x)^2,x)
```

```
[Out] 1/54/c^3*(18*arccosh(c*x)^2*c^3*x^3*e^2+54*arccosh(c*x)^2*c^3*x^2*d*e+54*ar
ccosh(c*x)^2*c^3*x*d^2-12*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2*
e^2-54*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x*d*e-108*arccosh(c*x)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*d^2-27*arccosh(c*x)^2*c*d*e-24*arccosh(c*x)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2+4*e^2*c^3*x^3+27*c^3*x^2*d*e+108*x*c^3*d^2
+24*c*x*e^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2-\int\frac{2\left(c^3e^2x^5+3\,c^3dex^4-3\,cdex^2-3\,cd^2x+\left(3\,c^3d^2-ce^2\right)x^3+\right.}{3\left(c^3x^3+\left(c^2x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - integrate(2/3*(c^3*e^2*x^5 + 3*c^3*d*e*x^4 - 3*c*d*e*x^2 - 3*c*d^2*x +
(3*c^3*d^2 - c*e^2)*x^3 + (c^2*e^2*x^4 + 3*c^2*d*e*x^3 + 3*c^2*d^2*x^2)*sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
 (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A] time = 1.98544, size = 358, normalized size = 1.67

$$\frac{4\,c^3e^2x^3+27\,c^3dex^2+9\left(2\,c^3e^2x^3+6\,c^3dex^2+6\,c^3d^2x-3\,cde\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(2\,c^2e^2x^2+9\,c^2dex+18\,c^2d^2\right.}{54\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*c^3*e^2*x^3 + 27*c^3*d*e*x^2 + 9*(2*c^3*e^2*x^3 + 6*c^3*d*e*x^2 + 6
*c^3*d^2*x - 3*c*d*e)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 6*(2*c^2*e^2*x^2 + 9
*c^2*d*e*x + 18*c^2*d^2 + 4*e^2)*sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x^2 -
```

```
1)) + 12*(9*c^3*d^2 + 2*c*e^2)*x)/c^3
```

Sympy [A] time = 1.49982, size = 223, normalized size = 1.04

$$\begin{cases} d^2x\operatorname{acosh}^2(cx) + 2d^2x + dex^2\operatorname{acosh}^2(cx) + \frac{dex^2}{2} + \frac{e^2x^3\operatorname{acosh}^2(cx)}{3} + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{dex\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{2e^2x \\ -\frac{\pi^2\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d**2*x*acosh(c*x)**2 + 2*d**2*x + d*e*x**2*acosh(c*x)**2 + d*e*x
**2/2 + e**2*x**3*acosh(c*x)**2/3 + 2*e**2*x**3/27 - 2*d**2*sqrt(c**2*x**2
- 1)*acosh(c*x)/c - d*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 2*e**2*x**2*sq
rt(c**2*x**2 - 1)*acosh(c*x)/(9*c) - d*e*acosh(c*x)**2/(2*c**2) + 4*e**2*x/
(9*c**2) - 4*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3), Ne(c, 0)), (-pi*
*2*(d**2*x + d*e*x**2 + e**2*x**3/3)/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \operatorname{arcosh}(cx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*arccosh(c*x)^2, x)
```

3.10 $\int (d+ex)\cosh^{-1}(cx)^2\,dx$

Optimal. Leaf size=122

$$-\frac{e\cosh^{-1}(cx)^2}{4c^2}-\frac{d^2\cosh^{-1}(cx)^2}{2e}+\frac{\cosh^{-1}(cx)^2(d+ex)^2}{2e}-\frac{2d\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}}{2c}$$

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (d^2*ArcCosh[c*x]^2)/(
2*e) - (e*ArcCosh[c*x]^2)/(4*c^2) + ((d + e*x)^2*ArcCosh[c*x]^2)/(2*e)
```

Rubi [A] time = 0.650873, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{e\cosh^{-1}(cx)^2}{4c^2}-\frac{d^2\cosh^{-1}(cx)^2}{2e}+\frac{\cosh^{-1}(cx)^2(d+ex)^2}{2e}-\frac{2d\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (d^2*ArcCosh[c*x]^2)/(
2*e) - (e*ArcCosh[c*x]^2)/(4*c^2) + ((d + e*x)^2*ArcCosh[c*x]^2)/(2*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
```

```
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)\cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - \frac{c\int \frac{(d+ex)^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e}$$
$$= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - \frac{c\int \left(\frac{d^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2dex\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{e^2x^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{e}$$
$$= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - (2cd)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(cd^2\right)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e} - (ce)\int -$$
$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} - \frac{d^2\cosh^{-1}(cx)^2}{2e} + \frac{(d}{}$$
$$= 2dx + \frac{ex^2}{4} - \frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} - \frac{d^2\cosh^{}}{2e}$$

Mathematica [A] time = 0.0767002, size = 105, normalized size = 0.86

$$\frac{e\left(2c^2x^2-1\right)\cosh^{-1}(cx)^2}{4c^2} + dx\cosh^{-1}(cx)^2 - \frac{2d\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{2c} + 2dx + \frac{e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) + d*x*ArcCosh[c*x]^2 + (
e*(-1 + 2*c^2*x^2)*ArcCosh[c*x]^2)/(4*c^2)
```

Maple [A] time = 0.042, size = 100, normalized size = 0.8

$$\frac{1}{c}\left(\frac{e}{4c}\left(2\,(\mathrm{arccosh}\,(cx))^2c^2x^2 - 2\,\mathrm{arccosh}\,(cx)\,cx\sqrt{cx-1}\sqrt{cx+1} - (\mathrm{arccosh}\,(cx))^2 + c^2x^2\right) + d\left((\mathrm{arccosh}\,(cx))^2cx - 2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*arccosh(c*x)^2,x)
```

```
[Out] 1/c*(1/4*e*(2*arccosh(c*x)^2*c^2*x^2-2*arccosh(c*x)*c*x*(c*x-1)^(1/2)*(c*x+
1)^(1/2)-arccosh(c*x)^2+c^2*x^2)/c+d*(arccosh(c*x)^2*c*x-2*arccosh(c*x)*(c*
```

```
x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(ex^2 + 2\,dx\right)\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)^2 - \int \frac{\left(c^3ex^4 + 2\,c^3dx^3 - cex^2 - 2\,cdx + \left(c^2ex^3 + 2\,c^2dx^2\right)\sqrt{cx+1}\sqrt{cx-1}\right)\log\left(\right.}{c^3x^3 + \left(c^2x^2 - 1\right)\sqrt{cx+1}\sqrt{cx-1} - cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - integrate((c
^3*e*x^4 + 2*c^3*d*x^3 - c*e*x^2 - 2*c*d*x + (c^2*e*x^3 + 2*c^2*d*x^2)*sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
(c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A] time = 1.94098, size = 220, normalized size = 1.8

$$\frac{c^2ex^2 + 8\,c^2dx + \left(2\,c^2ex^2 + 4\,c^2dx - e\right)\log\left(cx + \sqrt{c^2x^2-1}\right)^2 - 2\,\sqrt{c^2x^2-1}(cex + 4\,cd)\log\left(cx + \sqrt{c^2x^2-1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*e*x^2 + 8*c^2*d*x + (2*c^2*e*x^2 + 4*c^2*d*x - e)*log(c*x + sqrt(c
^2*x^2 - 1))^2 - 2*sqrt(c^2*x^2 - 1)*(c*e*x + 4*c*d)*log(c*x + sqrt(c^2*x^2
 - 1)))/c^2
```

Sympy [A] time = 0.717199, size = 110, normalized size = 0.9

$$\begin{cases} dx\operatorname{acosh}^2(cx) + 2dx + \frac{ex^2\operatorname{acosh}^2(cx)}{2} + \frac{ex^2}{4} - \frac{2d\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{ex\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{2c} - \frac{e\operatorname{acosh}^2(cx)}{4c^2} & \text{for}\ c \neq 0 \\ -\frac{\pi^2\left(dx + \frac{ex^2}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d*x*acosh(c*x)**2 + 2*d*x + e*x**2*acosh(c*x)**2/2 + e*x**2/4 -
2*d*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(
2*c) - e*acosh(c*x)**2/(4*c**2), Ne(c, 0)), (-pi**2*(d*x + e*x**2/2)/4, Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\,\mathrm{arcosh}\,(cx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*arccosh(c*x)^2, x)
```

3.11 $\int \frac{\cosh^{-1}(cx)^2}{d+ex}\,dx$

Optimal. Leaf size=272

$$\frac{2\cosh^{-1}(cx)\mathrm{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2\cosh^{-1}(cx)\mathrm{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}-\frac{2\mathrm{PolyLog}\left(3,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2\mathrm{Poly}}{}$$

```
[Out] -ArcCosh[c*x]^3/(3*e) + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - S
qrt[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d +
 Sqrt[c^2*d^2 - e^2])])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])
/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCo
sh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*x])/(
c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rubi [A] time = 0.42594, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5800, 5562, 2190, 2531, 2282, 6589}

$$\frac{2\cosh^{-1}(cx)\mathrm{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2\cosh^{-1}(cx)\mathrm{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}-\frac{2\mathrm{PolyLog}\left(3,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2\mathrm{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x),x]
```

```
[Out] -ArcCosh[c*x]^3/(3*e) + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - S
qrt[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d +
 Sqrt[c^2*d^2 - e^2])])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])
/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCo
sh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*x])/(
c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^2\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x x^2}{cd+\sqrt{c^2d^2-e^2}+\cdots}\right.$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} - \frac{2\,\text{Subst}\left(\cdots\right.}{}$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{2\cosh^{-1}(\cdots}{}$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{2\cosh^{-1}(\cdots}{}$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{2\cosh^{-1}(\cdots}{}$$

Mathematica [A] time = 0.157307, size = 252, normalized size = 0.93

$$-\frac{-6\cosh^{-1}(cx)\text{PolyLog}\left(2, \frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right) - 6\cosh^{-1}(cx)\text{PolyLog}\left(2, -\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right) + 6\text{PolyLog}\left(3, \frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right) + 6\text{PolyL}\cdots}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x),x]
```

```
[Out] -(ArcCosh[c*x]^3 - 3*ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[
c^2*d^2 - e^2])] - 3*ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[
c^2*d^2 - e^2])] - 6*ArcCosh[c*x]*PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + S
qrt[c^2*d^2 - e^2])] - 6*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d
+ Sqrt[c^2*d^2 - e^2]))] + 6*PolyLog[3, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c
^2*d^2 - e^2])] + 6*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e
^2]))])/(3*e)
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{(\text{arccosh}\,(cx))^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d),x)
```

```
[Out] int(arccosh(c*x)^2/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(c*x)^2/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d),x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(cx)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arccosh(c*x)^2/(e*x + d), x)
```

3.12 $\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=259

$$\frac{2c\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\cosh^{-1}(cx)\log}{e\sqrt{c^2d}}$$

```
[Out] -(ArcCosh[c*x]^2/(e*(d + e*x))) + (2*c*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*ArcCosh[c*
x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2
 - e^2]) + (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])
)])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + S
qrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2])
```

Rubi [A] time = 0.58165, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5802, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{2c\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\cosh^{-1}(cx)\log}{e\sqrt{c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x)^2,x]
```

```
[Out] -(ArcCosh[c*x]^2/(e*(d + e*x))) + (2*c*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*ArcCosh[c*
x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2
 - e^2]) + (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])
)])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + S
qrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^2}\,dx = -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(2c)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(2c)\,\mathrm{Subst}\left(\int \frac{x}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{e^x x}{e+2cde^x+ee^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{e^x x}{2cd-2\sqrt{c^2d^2-e^2}+2ee^x}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{c^2d^2-e^2}} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{e^x x}{2cd+2\sqrt{c^2d^2-e^2}+2}\right.}{\sqrt{c^2d^2-}}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{(2c)\,\mathrm{S}}{}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{(2c)\,\mathrm{S}}{}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} + \frac{2c\mathrm{Li}_2}{e}$$

Mathematica [C] time = 3.3005, size = 848, normalized size = 3.27

$$-\frac{c\left(\frac{\cosh^{-1}(cx)^2}{cd+cex} + \frac{2\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right)-2i\cos^{-1}\left(-\frac{cd}{e}\right)\tan^{-1}\left(\frac{(e-cd)\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right)+\left(\cos^{-1}\left(-\frac{cd}{e}\right)+2\left(\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\right.}{\sqrt{e^2-c}}\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x)^2,x]
```

```
[Out] -((c*(ArcCosh[c*x]^2/(c*d + c*e*x) + (2*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*C
oth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*Arc
Tan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-
```

```
((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e
^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))
*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d +
c*e*x])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])
/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[
-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[
2]*Sqrt[e]*Sqrt[c*d + c*e*x])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) +
e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e +
 I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqr
t[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTa
n[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d +
e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(
c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2,
((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[
ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/
2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^
2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]
*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[-(c^2*d^2) + e^2]))/e)
```

Maple [A] time = 0.1, size = 374, normalized size = 1.4

$$-\frac{c\left(\operatorname{arccosh}(cx)\right)^2}{e\,(cxe+cd)}+2\,\frac{c\operatorname{arccosh}(cx)}{e\sqrt{c^2d^2-e^2}}\ln\left(\frac{-\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)-2\,\frac{c\operatorname{arccosh}(cx)}{e\sqrt{c^2d^2-e^2}}\ln\left(\frac{\left(cx+\sqrt{cx\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d)^2,x)
```

```
[Out] -c*arccosh(c*x)^2/e/(c*e*x+c*d)+2*c/e*arccosh(c*x)/(c^2*d^2-e^2)^(1/2)*ln((
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^
2-e^2)^(1/2)))-2*c/e*arccosh(c*x)/(c^2*d^2-e^2)^(1/2)*ln(((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+2*c/
e/(c^2*d^2-e^2)^(1/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*
d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))-2*c/e/(c^2*d^2-e^2)^(1/2)*dilog
(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^
2-e^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}(cx)^2}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}^2(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d)**2,x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.13 $\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=352

$$\frac{c^3 d \,\mathrm{PolyLog}\left(2,-\frac{e e^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{c^3 d \,\mathrm{PolyLog}\left(2,-\frac{e e^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}+\frac{c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)}-\frac{c\sqrt{-\frac{1-cx}{cx+1}}(cx+1)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3 d \cos\cdots}{\cdots}$$

```
[Out] -((c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*ArcCosh[c*x])/((c^2*d^2 - e^2)*
(d + e*x))) - ArcCosh[c*x]^2/(2*e*(d + e*x)^2) + (c^3*d*ArcCosh[c*x]*Log[1
+ (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)
) - (c^3*d*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^
2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) +
(c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c
^2*d^2 - e^2)^(3/2)) - (c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c
^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2))
```

Rubi [A] time = 0.694363, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5802, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{c^3 d \,\mathrm{PolyLog}\left(2,-\frac{e e^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{c^3 d \,\mathrm{PolyLog}\left(2,-\frac{e e^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}+\frac{c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)}-\frac{c\sqrt{-\frac{1-cx}{cx+1}}(cx+1)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3 d \cos\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x)^3,x]
```

```
[Out] -((c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*ArcCosh[c*x])/((c^2*d^2 - e^2)*
(d + e*x))) - ArcCosh[c*x]^2/(2*e*(d + e*x)^2) + (c^3*d*ArcCosh[c*x]*Log[1
+ (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)
) - (c^3*d*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^
2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) +
(c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c
^2*d^2 - e^2)^(3/2)) - (c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c
^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2))
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^3}\,dx = -\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\,\text{Subst}\left(\int \frac{x}{(cd+e\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\,\text{Subst}\left(\int \frac{\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2d^2-e^2} + \frac{\left(c^3d\right)}{}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{cd+x}\,dx, x, cex\right)}{e\left(c^2d^2-e^2\right)} + \frac{\left(2c^3d\right)\text{Subst}\left(\int \frac{}{e+}\right.}{e\,(}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)} + \frac{\left(2c^3d\right)\text{Subst}\left(\int \frac{e^x x}{2cd-2\sqrt{c^2d^2-e^2}+2ee}\right.}{\left(c^2d^2-e^2\right)^{3/2}}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}} - \frac{c^3d\cosh^{-1}}{e}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}} - \frac{c^3d\cosh^{-1}}{e}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}} - \frac{c^3d\cosh^{-1}}{e}$$

Mathematica [C] time = 4.45733, size = 936, normalized size = 2.66

$$c^2\left(-\frac{\cosh^{-1}(cx)^2}{2e(cd+cex)^2} - \frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)}{(cd-e)(cd+e)(cd+cex)} + \frac{\log\left(\frac{ex}{d}+1\right)}{c^2d^2e-e^3} + \frac{cd\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right) - 2i\,\text{co}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x)^3,x]
```

```
[Out] c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/((c*d - e)*(c*d
+ e)*(c*d + c*e*x))) - ArcCosh[c*x]^2/(2*e*(c*d + c*e*x)^2) + Log[1 + (e*x)
/d]/(c^2*d^2*e - e^3) + (c*d*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d
) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)]
+ 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcT
an[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-
(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d + c*e*x])] +
(ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2
*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2)
+ e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*
Sqrt[c*d + c*e*x])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c
^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2
) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d)
+ e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d)
+ e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I
*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*S
qrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x
]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - Pol
yLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^
2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCo
sh[c*x]/2]))])))/(e*(-(c^2*d^2) + e^2)^(3/2)))
```

Maple [B] time = 0.185, size = 766, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d)^3,x)
```

```
[Out] -c^3*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x+c^4*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*x^2-1/2*c^4*arccosh(c*x)^
2/e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*d^2-c^3*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d^2
-e^2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*d+2*c^4*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d
^2-e^2)*x*d+c^4*arccosh(c*x)/e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*d^2+1/2*c^2*arcc
osh(c*x)^2*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)+c^3/e/(c^2*d^2-e^2)^(3/2)*arccosh(
c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*
d+(c^2*d^2-e^2)^(1/2)))*d-c^3/e/(c^2*d^2-e^2)^(3/2)*arccosh(c*x)*ln(((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(
1/2)))*d+c^3/e/(c^2*d^2-e^2)^(3/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

```
)*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))*d-c^3/e/(c^2*d^2-e
^2)^(3/2)*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2
))/(c*d+(c^2*d^2-e^2)^(1/2)))*d+c^2/e/(c^2*d^2-e^2)*ln(2*c*d*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+e)-2*c^2/e/(c^2*d
^2-e^2)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcosh}\left(cx\right)^{2}}{e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}^{2}\left(cx\right)}{\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d)**3,x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.14 $\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=191

$$\frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{b\left(24c^2d^2e^2+8c^4d^4+3e^4\right)}{32c^4e}$$

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(d + e*x)^3)/(16*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*
(19*c^2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - (b*(8*c^4*d^4
 + 24*c^2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*A
rcCosh[c*x]))/(4*e)
```

Rubi [A] time = 0.144522, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5802, 100, 153, 147, 52}

$$\frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{b\left(24c^2d^2e^2+8c^4d^4+3e^4\right)}{32c^4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(d + e*x)^3)/(16*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*
(19*c^2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - (b*(8*c^4*d^4
 + 24*c^2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*A
rcCosh[c*x]))/(4*e)
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{4e}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} + \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)}{4e} - \frac{b\int \frac{(d+ex)^2(4c^2d^2+3e^2-}{\sqrt{-1+cx}\sqrt{1+}}}{16ce}$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} + \frac{(d+ex)^4 \left(a+b\right.}{4e}$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} - \frac{b\sqrt{-1+cx}\sqrt{1+}}{}$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} - \frac{b\sqrt{-1+cx}\sqrt{1+}}{}$$

Mathematica [A] time = 0.290879, size = 193, normalized size = 1.01

$$\frac{24ac^4x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right) - bc\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(72d^2ex+96d^3+32de^2x^2+6e^3x^3\right)+e^2(64d+9ex)\right)+2}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x
^2 + 6*e^3*x^3)) + 24*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*A
rcCosh[c*x] - 9*b*e*(8*c^2*d^2 + e^2)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x
]])/(96*c^4)
```

Maple [B] time = 0.006, size = 408, normalized size = 2.1

$$\frac{ae^3x^4}{4} + ade^2x^3 + \frac{3\,ad^2ex^2}{2} + axd^3 + \frac{ad^4}{4\,e} + \frac{b\mathrm{arccosh}\,(cx)\,e^3x^4}{4} + b\mathrm{arccosh}\,(cx)\,de^2x^3 + \frac{3\,b\mathrm{arccosh}\,(cx)\,d^2ex^2}{2} + b\mathrm{arcc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*a*e^3*x^4+a*d*e^2*x^3+3/2*a*d^2*e*x^2+a*x*d^3+1/4*a/e*d^4+1/4*b*arccosh
(c*x)*e^3*x^4+b*arccosh(c*x)*d*e^2*x^3+3/2*b*arccosh(c*x)*d^2*e*x^2+b*arcco
sh(c*x)*x*d^3+1/4*b/e*arccosh(c*x)*d^4-1/4*b/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-1/16/c*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*e^3*x^3-1/3*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d*e^2-3/4*b*d^2*e
*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3-3/4/
c^2*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1
)^(1/2))-3/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x-2/3*b/c^3*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*d*e^2-3/32/c^4*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-
1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.19032, size = 382, normalized size = 2.

$$\frac{1}{4}\,ae^3x^4 + ade^2x^3 + \frac{3}{2}\,ad^2ex^2 + \frac{3}{4}\left(2\,x^2\,\mathrm{arcosh}\,(cx) - c\left(\frac{\sqrt{c^2x^2-1}x}{c^2} + \frac{\log\left(2\,c^2x + 2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)bd^2e + \frac{1}{3}\left(3\,x^3\,\mathrm{arco}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(s
qrt(c^2)*c^2)))*b*d^2*e + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^
2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d*e^2 + 1/32*(8*x^4*arccosh(c*x) - (2*s
qrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sq
rt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*e^3 + a*d^3*x + (c*x*arcco
sh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

Fricas [A] time = 2.36774, size = 468, normalized size = 2.45

$$\frac{24\,ac^4e^3x^4 + 96\,ac^4de^2x^3 + 144\,ac^4d^2ex^2 + 96\,ac^4d^3x + 3\left(8\,bc^4e^3x^4 + 32\,bc^4de^2x^3 + 48\,bc^4d^2ex^2 + 32\,bc^4d^3x - 24\,bc^2d^2e\right.}{96\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c^
4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 + 32
```

```
*b*c^4*d^3*x - 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*
b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 + 64*b*c*d*e^2 + 9*(8*b*c
^3*d^2*e + b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 2.06011, size = 323, normalized size = 1.69

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{acosh}(cx) + \frac{3bd^2ex^2\operatorname{acosh}(cx)}{2} + bde^2x^3\operatorname{acosh}(cx) + \frac{be^3x^4\operatorname{acosh}(cx)}{4} - \frac{bd^3\sqrt{c^2x^2-1}}{c} - \\ \left(a + \frac{i\pi b}{2}\right)\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*acosh(c*x) + 3*b*d**2*e*x**2*acosh(c*x)/2 + b*d*e**2*x**3*acosh(c*x
) + b*e**3*x**4*acosh(c*x)/4 - b*d**3*sqrt(c**2*x**2 - 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 - 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 - 1)/(16*c) - 3*b*d**2*e*acosh(c*x)/(4*c**2) - 2*b*d*
e**2*sqrt(c**2*x**2 - 1)/(3*c**3) - 3*b*e**3*x*sqrt(c**2*x**2 - 1)/(32*c**3
) - 3*b*e**3*acosh(c*x)/(32*c**4), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x + 3*d
**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))
```

Giac [A] time = 1.4652, size = 401, normalized size = 2.1

$$\left(x\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)bd^3 + ad^3x + \frac{1}{32}\left(8\,ax^4 + \left(8\,x^4\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(\sqrt{c^2x^2-1}x\left(\frac{2\,x^2}{c^2} + \frac{3}{c^4}\right) - \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^3 + a*d^3*x + 1/
32*(8*a*x^4 + (8*x^4*log(c*x + sqrt(c^2*x^2 - 1)) - (sqrt(c^2*x^2 - 1)*x*(2
*x^2/c^2 + 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^4*abs(c)))
*c)*b)*e^3 + 1/3*(3*a*d*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x
^2 - 1)^(3/2) + 3*sqrt(c^2*x^2 - 1))/c^3)*b*d)*e^2 + 3/4*(2*a*d^2*x^2 + (2*
x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*x/c^2 - log(abs(-x*
abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*d^2)*e
```

3.15 $\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=132

$$\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{bd\left(\frac{3e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{6e}-\frac{b\sqrt{cx-1}\sqrt{cx+}}{9c}$$

```
[Out] -(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(9*c) - (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (b*d*(2*d^2 + (3
*e^2)/c^2)*ArcCosh[c*x])/(6*e) + ((d + e*x)^3*(a + b*ArcCosh[c*x]))/(3*e)
```

Rubi [A] time = 0.104595, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5802, 100, 147, 52}

$$\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{bd\left(\frac{3e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{6e}-\frac{b\sqrt{cx-1}\sqrt{cx+}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(9*c) - (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (b*d*(2*d^2 + (3
*e^2)/c^2)*ArcCosh[c*x])/(6*e) + ((d + e*x)^3*(a + b*ArcCosh[c*x]))/(3*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\cosh^{-1}(cx)\right)dx = \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} + \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)\left(3c^2d^2+2e^2+\cdots\right.}{\sqrt{-1+cx}\sqrt{1+c\cdots}}}{9ce}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3} + \frac{(d\cdots}{\cdots}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3} - \frac{bd\cdots}{\cdots}$$

Mathematica [A] time = 0.199151, size = 142, normalized size = 1.08

$$ad^2x + adex^2 + \frac{1}{3}ae^2x^3 - \frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)+4e^2\right)}{18c^3} - \frac{bde\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)}{2c^2} + \frac{1}{3}bx\,\mathrm{co}\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*d^2*x + a*d*e*x^2 + (a*e^2*x^3)/3 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*e^
2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)))/(18*c^3) + (b*x*(3*d^2 + 3*d*e*x +
 e^2*x^2)*ArcCosh[c*x])/3 - (b*d*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
/(2*c^2)
```

Maple [B] time = 0.006, size = 274, normalized size = 2.1

$$\frac{ax^3e^2}{3} + ax^2de + axd^2 + \frac{ad^3}{3e} + \frac{b\operatorname{arccosh}(cx)\,x^3e^2}{3} + b\operatorname{arccosh}(cx)\,x^2de + b\operatorname{arccosh}(cx)\,xd^2 + \frac{bd^3\operatorname{arccosh}(cx)}{3e} - \frac{bd^3}{3e}\sqrt{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/3*a*x^3*e^2+a*x^2*d*e+a*x*d^2+1/3*a/e*d^3+1/3*b*arccosh(c*x)*x^3*e^2+b*ar
ccosh(c*x)*x^2*d*e+b*arccosh(c*x)*x*d^2+1/3*b/e*arccosh(c*x)*d^3-1/3*b/e*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-1
/9*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*e^2-1/2*b*d*e*x*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-1/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-1/2/c^2*b*e*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-2/9*b/c^3*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*e^2
```

Maxima [A] time = 1.11011, size = 238, normalized size = 1.8

$$\frac{1}{3}ae^2x^3 + adex^2 + \frac{1}{2}\left(2x^2\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x}{c^2} + \frac{\log\left(2c^2x + 2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)bde + \frac{1}{9}\left(3x^3\operatorname{arcosh}(cx) - c\left(\ \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d*
e + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2
 - 1)/c^4))*b*e^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^2/
```

c

Fricas [A] time = 2.31549, size = 324, normalized size = 2.45

$$\frac{6\,ac^3e^2x^3 + 18\,ac^3dex^2 + 18\,ac^3d^2x + 3\left(2\,bc^3e^2x^3 + 6\,bc^3dex^2 + 6\,bc^3d^2x - 3\,bcde\right)\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(2\,bc^2e^2x^2 + 9\right.}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2*
x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x - 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2 -
 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 + 4*b*e^2)*sqrt(c^2*
x^2 - 1))/c^3
```

Sympy [A] time = 1.02843, size = 197, normalized size = 1.49

$$\begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{acosh}(cx) + bdex^2\operatorname{acosh}(cx) + \frac{be^2x^3\operatorname{acosh}(cx)}{3} - \frac{bd^2\sqrt{c^2x^2-1}}{c} - \frac{bdex\sqrt{c^2x^2-1}}{2c} - \frac{be^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{bc}{} \\ \left(a + \frac{i\pi b}{2}\right)\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acosh(c*x) + b*
d*e*x**2*acosh(c*x) + b*e**2*x**3*acosh(c*x)/3 - b*d**2*sqrt(c**2*x**2 - 1)
/c - b*d*e*x*sqrt(c**2*x**2 - 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 - 1)/(9
*c) - b*d*e*acosh(c*x)/(2*c**2) - 2*b*e**2*sqrt(c**2*x**2 - 1)/(9*c**3), Ne
(c, 0)), ((a + I*pi*b/2)*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [A] time = 1.35861, size = 266, normalized size = 2.02

$$\left(x\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \frac{\sqrt{c^2x^2 - 1}}{c}\right)bd^2 + ad^2x + \frac{1}{9}\left(3\,ax^3 + \left(3\,x^3\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \frac{\left(c^2x^2 - 1\right)^{\frac{3}{2}} + 3\,\sqrt{c^2x^2 - 1}}{c^3}\right)b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d^2 + a*d^2*x + 1/
9*(3*a*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 - 1)) - ((c^2*x^2 - 1)^(3/2) + 3
*sqrt(c^2*x^2 - 1))/c^3)*b)*e^2 + 1/2*(2*a*d*x^2 + (2*x^2*log(c*x + sqrt(c^
2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*x/c^2 - log(abs(-x*abs(c) + sqrt(c^2*x^2
 - 1)))/(c^2*abs(c))))*b*d)*e
```

3.16 $\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{b\left(\frac{e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}(d+ex)}{4c}-\frac{3bd\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(d + e*x))/(4*c) - (b*(2*d^2 + e^2/c^2)*ArcCosh[c*x])/(4*e) + ((d + e*x)
^2*(a + b*ArcCosh[c*x]))/(2*e)
```

Rubi [A] time = 0.0444908, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5802, 90, 80, 52}

$$\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{b\left(\frac{e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}(d+ex)}{4c}-\frac{3bd\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(d + e*x))/(4*c) - (b*(2*d^2 + e^2/c^2)*ArcCosh[c*x])/(4*e) + ((d + e*x)
^2*(a + b*ArcCosh[c*x]))/(2*e)
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)dx=\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{(bc)\int\frac{(d+ex)^2}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{2e}$$

$$=-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{b\int\frac{2c^2d^2+e^2+3c^2dex}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{4ce}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}$$

$$=-\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c}-\frac{b\left(2d^2+\frac{e^2}{c^2}\right)\cosh^{-1}(cx)}{4e}+\frac{(d}{}$$

Mathematica [A] time = 0.0928296, size = 117, normalized size = 1.1

$$adx+\frac{1}{2}aex^2-\frac{be\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{2c^2}-\frac{bd\sqrt{cx-1}\sqrt{cx+1}}{c}+bdx\cosh^{-1}(cx)+\frac{1}{2}bex^2\cosh^{-1}(cx)-\frac{bex\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*e*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])/(4*c) + b*d*x*ArcCosh[c*x] + (b*e*x^2*ArcCosh[c*x])/
```

```
2 - (b*e*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(2*c^2)
```

Maple [A] time = 0.006, size = 123, normalized size = 1.2

$$\frac{ax^2e}{2}+adx+\frac{b\text{arccosh}(cx)\,x^2e}{2}+b\text{arccosh}(cx)\,xd-\frac{bex}{4c}\sqrt{cx-1}\sqrt{cx+1}-\frac{bd}{c}\sqrt{cx-1}\sqrt{cx+1}-\frac{be}{4c^2}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arccosh(c*x)*x^2*e+b*arccosh(c*x)*x*d-1/4*b*e*x*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c-b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*b*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [A] time = 1.15359, size = 146, normalized size = 1.38

$$\frac{1}{2}aex^2+\frac{1}{4}\left(2x^2\text{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)be+adx+\frac{\left(cx\,\text{arcosh}(cx)-\sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*
c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*b*e + a*d*x + (c*x
*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d/c
```

Fricas [A] time = 2.27267, size = 197, normalized size = 1.86

$$\frac{2ac^2ex^2+4ac^2dx+\left(2bc^2ex^2+4bc^2dx-be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-(bcex+4bcd)\sqrt{c^2x^2-1}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x - b*e)*log(
c*x + sqrt(c^2*x^2 - 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 - 1))/c^2
```

Sympy [A] time = 0.417238, size = 105, normalized size = 0.99

$$\begin{cases} adx + \frac{aex^2}{2} + bdx \operatorname{acosh}(cx) + \frac{bex^2 \operatorname{acosh}(cx)}{2} - \frac{bd\sqrt{c^2x^2-1}}{c} - \frac{bex\sqrt{c^2x^2-1}}{4c} - \frac{be \operatorname{acosh}(cx)}{4c^2} & \text{for } c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(dx + \frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*acosh(c*x) + b*e*x**2*acosh(c*x)/2 -
b*d*sqrt(c**2*x**2 - 1)/c - b*e*x*sqrt(c**2*x**2 - 1)/(4*c) - b*e*acosh(c*x
)/(4*c**2), Ne(c, 0)), ((a + I*pi*b/2)*(d*x + e*x**2/2), True))
```

Giac [A] time = 1.26328, size = 170, normalized size = 1.6

$$\left(x \log\left(cx + \sqrt{c^2x^2 - 1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)bd + adx + \frac{1}{4}\left(2\,ax^2 + \left(2\,x^2 \log\left(cx + \sqrt{c^2x^2-1}\right) - c\left(\frac{\sqrt{c^2x^2-1}x}{c^2} - \frac{\log\left(\left|-x|c| + \sqrt{\right.\right.}{c^2|c|}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d + a*d*x + 1/4*(2
*a*x^2 + (2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*x/c^2 -
 log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b)*e
```

3.17 $\int \frac{a+b\cosh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=195

$$\frac{b\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{b\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\left(a+b\cosh^{-1}(\right.}{}$$

```
[Out] -(a + b*ArcCosh[c*x])^2/(2*b*e) + ((a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCos
h[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])*Log[1 + (e*
E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (b*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (b*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rubi [A] time = 0.264315, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{b\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\left(a+b\cosh^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x),x]
```

```
[Out] -(a + b*ArcCosh[c*x])^2/(2*b*e) + ((a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCos
h[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])*Log[1 + (e*
E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (b*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (b*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \text{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd+\cdots}\right.$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\cdots\right.}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\cdots\right.}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\cdots\right.}{e}$$

Mathematica [A] time = 0.133431, size = 183, normalized size = 0.94

$$\frac{2b^2\text{PolyLog}\left(2,\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right)+2b^2\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)-\left(a+b\cosh^{-1}(cx)\right)\left(a-2b\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)-2b\log\right.}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 + (e*E^ArcCosh[c*x]
)/(c*d - Sqrt[c^2*d^2 - e^2])] - 2*b*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt
[c^2*d^2 - e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 - e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/(2*b*e)
```

Maple [A] time = 0.038, size = 314, normalized size = 1.6

$$\frac{a\ln(cxe+cd)}{e}-\frac{b\left(\operatorname{arccosh}(cx)\right)^2}{2e}+\frac{b\operatorname{arccosh}(cx)}{e}\ln\left(\left(-\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}\right)\left(-cd+\sqrt{c^2d^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e-1/2*b/e*arccosh(c*x)^2+b/e*arccosh(c*x)*ln((-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2))
)+b/e*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2
)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+b/e*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+b/e*dilog((-(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^
2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{ex+d}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e*
x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{acosh}\,(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d), x)
```

3.18 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^2} dx$

Optimal. Leaf size=88

$$\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\sqrt{cd+e}}-\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}$$

```
[Out] -((a + b*ArcCosh[c*x])/(e*(d + e*x))) + (2*b*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[
1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rubi [A] time = 0.0564455, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5802, 93, 208}

$$\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\sqrt{cd+e}}-\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(e*(d + e*x))) + (2*b*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[
1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{e}\\ &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e}\\ &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right)}{\sqrt{cd-e}e\sqrt{cd+e}}\end{aligned}$$

Mathematica [A] time = 0.191636, size = 121, normalized size = 1.38

$$-\frac{\frac{a}{d+ex}-\frac{bc\log(d+ex)}{\sqrt{c^2d^2-e^2}}+\frac{bc\log\left(-\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}+c^2dx+e\right)}{\sqrt{c^2d^2-e^2}}+\frac{b\cosh^{-1}(cx)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a/(d + e*x) + (b*ArcCosh[c*x])/(d + e*x) - (b*c*Log[d + e*x])/Sqrt[c^2*d
^2 - e^2] + (b*c*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]])/Sqrt[c^2*d^2 - e^2])/e)
```

Maple [A] time = 0.004, size = 145, normalized size = 1.7

$$-\frac{ca}{(cxe+cd)\,e}-\frac{b\text{arccosh}\,(cx)}{(cxe+cd)\,e}-\frac{bc}{e^2}\sqrt{cx-1}\sqrt{cx+1}\ln\left(-2\,\frac{1}{cxe+cd}\left(c^2dx-\sqrt{c^2x^2-1}\sqrt{\frac{c^2d^2-e^2}{e^2}}e+e\right)\right)\frac{1}{\sqrt{\frac{c^2d^2-e^2}{e^2}}}\frac{1}{\sqrt{c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arccosh(c*x)-c*b/e^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/
(c*e*x+c*d))/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50001, size = 1008, normalized size = 11.45

$$\left[-\frac{ac^2d^3 - ade^2 - \left(bc^2d^2e - be^3\right)x\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(bcdex + bcd^2\right)\sqrt{c^2d^2-e^2}\log\left(\frac{c^3d^2x + cde + \sqrt{c^2d^2-e^2}\left(c^2dx+e\right) + \left(c^2d^2 + \sqrt{\cdots}\right.}{ex+d}\right.}{c^2d^4e - d^2e^3 + \left(c^2d^3e^2 - de^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 - (b*c^2*d^2*e - b*e^3)*x*log(c*x + sqrt(c^2*x^2 - 1
)) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqr
t(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*
sqrt(c^2*x^2 - 1))/(e*x + d)) - (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3
)*x)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d
*e^4)*x), -(a*c^2*d^3 - a*d*e^2 - (b*c^2*d^2*e - b*e^3)*x*log(c*x + sqrt(c^
2*x^2 - 1)) + 2*(b*c*d*e*x + b*c*d^2)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c
^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^
2*d^2 - e^2)) - (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*log(-c*x +
sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{(d + ex)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**2, x)
```

Giac [B] time = 1.35395, size = 308, normalized size = 3.5

$$\Bigg(\Bigg(\Bigg(\frac{e^{(-1)} \log\Big(\Big|-c^2 d + \sqrt{c^2 d^2 - e^2}|c|\Big|\Big) \operatorname{sgn}\Big(\frac{1}{xe+d}\Big)}{\sqrt{c^2 d^2 - e^2}} - \frac{e^{(-1)} \log\Bigg(\Bigg|-c^2 d + \sqrt{c^2 d^2 - e^2}\Bigg(\sqrt{c^2 - \frac{2c^2 d}{xe+d} + \frac{c^2 d^2}{(xe+d)^2} - \frac{e^2}{(xe+d)^2}} + \frac{\sqrt{c^2 d^2 e^2 - e^4}e^{(}}{xe+d}}{\sqrt{c^2 d^2 - e^2}\operatorname{sgn}\Big(\frac{1}{xe+d}\Big)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] ((e^(-1)*log(abs(-c^2*d + sqrt(c^2*d^2 - e^2)*abs(c)))*sgn(1/(x*e + d))/sqr
t(c^2*d^2 - e^2) - e^(-1)*log(abs(-c^2*d + sqrt(c^2*d^2 - e^2)*(sqrt(c^2 -
2*c^2*d/(x*e + d) + c^2*d^2/(x*e + d)^2 - e^2/(x*e + d)^2) + sqrt(c^2*d^2*e
^2 - e^4)*e^(-1)/(x*e + d))))/(sqrt(c^2*d^2 - e^2)*sgn(1/(x*e + d))))*c - e
^(-1)*log(c*x + sqrt(c^2*x^2 - 1))/(x*e + d))*b - a*e^(-1)/(x*e + d)
```

3.19 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}$$

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*
ArcCosh[c*x])/(2*e*(d + e*x)^2) + (b*c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 +
c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```

Rubi [A] time = 0.10192, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5802, 96, 93, 208}

$$-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^3,x]
```


```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*
ArcCosh[c*x])/(2*e*(d + e*x)^2) + (b*c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 +
c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^3}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{2e}\\
&= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{2e\left(c^2d^2-e^2\right)}\\
&= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e\left(c^2d^2-e^2\right)}\\
&= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.372045, size = 184, normalized size = 1.33

$$
\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\log(d+ex)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc^3d\log\left(-\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}+c^2dx+e\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{b\cosh^{-1}}{e(d+e}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)
*(d + e*x)) - (b*ArcCosh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*
(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]])/(e*(c^2*d^2 - e^2)^(3/2)))/2
```

Maple [B] time = 0.004, size = 361, normalized size = 2.6

$$-\frac{c^2a}{2\,(cxe+cd)^2\,e}-\frac{c^2b\mathrm{arccosh}\,(cx)}{2\,(cxe+cd)^2\,e}-\frac{c^4bxd}{2\,e\,(cd+e)\,(cd-e)\,(cxe+cd)}\sqrt{cx-1}\sqrt{cx+1}\ln\left(-2\,\frac{1}{cxe+cd}\left(c^2dx-\sqrt{c^2x^2-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arccosh(c*x)-1/2*c^4*b
/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-
e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)
/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d-1/2*c^4*b/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)
/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*
d^2-1/2*c^2*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.42041, size = 2261, normalized size = 16.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*((a + b)*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
 - b*c^2*d^2*e^4)*x^2 + (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*s
qrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x +
e) + (c^2*d^2 - sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)
) + 2*(b*c^4*d^5*e - b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 +
 b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt
(c^2*x^2 - 1)) - (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2
- 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5
)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^3*d^5*e - b*c*d^3*e^3 + (b*c^3*d^
4*e^2 - b*c*d^2*e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4
*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2
*d^5*e^4 + d^3*e^6)*x), -1/2*((a + b)*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + a*d
^2*e^4 + (b*c^4*d^4*e^2 - b*c^2*d^2*e^4)*x^2 + 2*(b*c^3*d^3*e^2*x^2 + 2*b*c
^3*d^4*e*x + b*c^3*d^5)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*
sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2))
+ 2*(b*c^4*d^5*e - b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b
*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt(c
^2*x^2 - 1)) - (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 -
2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*
x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^3*d^5*e - b*c*d^3*e^3 + (b*c^3*d^4*
e^2 - b*c*d^2*e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e
^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d
^5*e^4 + d^3*e^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(cx)}{(d + ex)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(ex + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d)^3, x)
```

3.20 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{bc^3d\sqrt{cx-1}\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}$$

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*(c^2*d^2 - e^2)*(d + e*x)^2) - (b*c^
3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - (
a + b*ArcCosh[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(2*c^2*d^2 + e^2)*ArcTanh[(S
qrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(
5/2)*e*(c*d + e)^(5/2))
```

Rubi [A] time = 0.164191, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5802, 103, 151, 12, 93, 208}

$$-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{bc^3d\sqrt{cx-1}\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^4,x]
```


```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*(c^2*d^2 - e^2)*(d + e*x)^2) - (b*c^
3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - (
a + b*ArcCosh[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(2*c^2*d^2 + e^2)*ArcTanh[(S
qrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(
5/2)*e*(c*d + e)^(5/2))
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}\,dx}{3e}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{(bc)\int \frac{-2c^2d+c^2ex}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{6e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc)\int \frac{c^2\left(2c^2d^2+e^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}}{6e\left(c^2d^2-e^2\right)^2}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(bc^3\left(2c^2d^2+e^2\right)\right)\int \frac{}{\sqrt{-}}}{6e\left(c^2d^2-\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(bc^3\left(2c^2d^2+e^2\right)\right)\text{Subs}}{3e\left(\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}}{3(cd-e)^{5/2}e(cd}$$

Mathematica [C] time = 0.939228, size = 259, normalized size = 1.28

$$-\frac{\frac{2a+\frac{bce\sqrt{cx-1}\sqrt{cx+1}(d+ex)\left(c^2d(4d+3ex)-e^2\right)}{\left(e^2-c^2d^2\right)^2}}{(d+ex)^3}+\frac{ibc^3\left(2c^2d^2+e^2\right)\log\left(\frac{12e^2(e-cd)^2(cd+e)^2\left(\sqrt{cx-1}\sqrt{cx+1}\sqrt{e^2-c^2d^2}-ic^2dx-ie\right)}{bc^3\sqrt{e^2-c^2d^2}\left(2c^2d^2+e^2\right)(d+ex)}\right)}{(e-cd)^2(cd+e)^2\sqrt{e^2-c^2d^2}}+\frac{2b\cosh^{-1}(cx)}{(d+ex)^3}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^4,x]
```

```
[Out] -((2*a + (b*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)*(-e^2 + c^2*d*(4*d +
 3*e*x)))/(-(c^2*d^2) + e^2)^2)/(d + e*x)^3 + (2*b*ArcCosh[c*x])/(d + e*x)^
3 + (I*b*c^3*(2*c^2*d^2 + e^2)*Log[(12*e^2*(-(c*d) + e)^2*(c*d + e)^2*((-I)
*e - I*c^2*d*x + Sqrt[-(c^2*d^2) + e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(b*c
^3*Sqrt[-(c^2*d^2) + e^2]*(2*c^2*d^2 + e^2)*(d + e*x))])/((-(c*d) + e)^2*(c
*d + e)^2*Sqrt[-(c^2*d^2) + e^2]))/(6*e)
```

Maple [B] time = 0.004, size = 1137, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3*a/(c*e*x+c*d)^3/e-1/3*c^3*b/(c*e*x+c*d)^3/e*arccosh(c*x)-1/3*c^7*b
*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^
2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*
((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x^2*d^2-2/3*c^7*b/e*(c*x+1)^(1/
2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2
)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^
2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d^3-1/3*c^7*b/e^2*(c*x+1)^(1/2)*(c*x-1)^(
1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e
^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2
)*e+e)/(c*e*x+c*d))*d^4-1/2*c^5*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*
d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*x*d-2/3*c^5*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/
(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d^2-1/6*c^5*b*e^2*(c*x+1)^(1/2)
*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/
(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)
/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x^2-1/3*c^5*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(
c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c
*e*x+c*d)^2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)
/(c*e*x+c*d))*x*d-1/6*c^5*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(
c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*ln(-2*
(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2+
1/6*c^3*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*
e*x+c*d)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 - 3*c*d^2*e^2*x^2 -
c*d^3*e*x + (3*c^3*d^2*e^2 - c*e^4)*x^4 + (c^3*d^3*e - 3*c*d*e^3)*x^3 + (c^
2*e^4*x^5 + 3*c^2*d*e^3*x^4 - 3*d^2*e^2*x - d^3*e + (3*c^2*d^2*e^2 - e^4)*x
```

```
^3 + (c^2*d^3*e - 3*d*e^3)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x
) + 2*(c^6*d^3 + 3*c^4*d*e^2)*log(e*x + d)/(c^6*d^6*e - 3*c^4*d^4*e^3 + 3*c
^2*d^2*e^5 - e^7) - (3*c^6*d^6 - 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e
^2 - c^2*e^6)*x^2 + (5*c^6*d^5*e - 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x - 2*(c^6*
d^6 - 3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 - e^6)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (c^6*d^6 + 3*c^5*d^5*e + 3*c^4*d^4*e^2 + c^3*d^3*e^3 + (c^6*d^3*e^
3 + 3*c^5*d^2*e^4 + 3*c^4*d*e^5 + c^3*e^6)*x^3 + 3*(c^6*d^4*e^2 + 3*c^5*d^3
*e^3 + 3*c^4*d^2*e^4 + c^3*d*e^5)*x^2 + 3*(c^6*d^5*e + 3*c^5*d^4*e^2 + 3*c^
4*d^3*e^3 + c^3*d^2*e^4)*x)*log(c*x + 1) + (c^6*d^6 - 3*c^5*d^5*e + 3*c^4*d
^4*e^2 - c^3*d^3*e^3 + (c^6*d^3*e^3 - 3*c^5*d^2*e^4 + 3*c^4*d*e^5 - c^3*e^6
)*x^3 + 3*(c^6*d^4*e^2 - 3*c^5*d^3*e^3 + 3*c^4*d^2*e^4 - c^3*d*e^5)*x^2 + 3
*(c^6*d^5*e - 3*c^5*d^4*e^2 + 3*c^4*d^3*e^3 - c^3*d^2*e^4)*x)*log(c*x - 1))
/(c^6*d^9*e - 3*c^4*d^7*e^3 + 3*c^2*d^5*e^5 - d^3*e^7 + (c^6*d^6*e^4 - 3*c^
4*d^4*e^6 + 3*c^2*d^2*e^8 - e^10)*x^3 + 3*(c^6*d^7*e^3 - 3*c^4*d^5*e^5 + 3*
c^2*d^3*e^7 - d*e^9)*x^2 + 3*(c^6*d^8*e^2 - 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 -
 d^2*e^8)*x))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Fricas [B] time = 7.67019, size = 3889, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a + 3*b)*c^6*d^9 - 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 - 2*
a*d^3*e^6 + 3*(b*c^6*d^6*e^3 - b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 - b*c^
4*d^5*e^4)*x^2 - (2*b*c^5*d^8 + b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 + b*c^3*d^
3*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 + b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e + b
*c^3*d^5*e^3)*x)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2
- e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x
^2 - 1))/(e*x + d)) + 9*(b*c^6*d^8*e - b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^3
 - 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b*
c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^6
*e^3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*
c^6*d^9 - 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 - b*d^3*e^6 + (b*c^6*d^6*e^3 -
3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b*c^4
*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^6*e^
3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^
5*d^8*e - 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 - b*c^3*d^4*e^5)
*x^2 + (7*b*c^5*d^7*e^2 - 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 -
1))/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4 -
```

```
3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d^
8*e^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3*
c^2*d^7*e^6 - d^5*e^8)*x), -1/6*((2*a + 3*b)*c^6*d^9 - 3*(2*a + b)*c^4*d^7*
e^2 + 6*a*c^2*d^5*e^4 - 2*a*d^3*e^6 + 3*(b*c^6*d^6*e^3 - b*c^4*d^4*e^5)*x^3
 + 9*(b*c^6*d^7*e^2 - b*c^4*d^5*e^4)*x^2 + 2*(2*b*c^5*d^8 + b*c^3*d^6*e^2 +
 (2*b*c^5*d^5*e^3 + b*c^3*d^3*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 + b*c^3*d^4*e^4
)*x^2 + 3*(2*b*c^5*d^7*e + b*c^3*d^5*e^3)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(
sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*
d))/(c^2*d^2 - e^2)) + 9*(b*c^6*d^8*e - b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^
3 - 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b
*c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^
6*e^3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b
*c^6*d^9 - 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 - b*d^3*e^6 + (b*c^6*d^6*e^3 -
 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b*c^
4*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^6*e
^3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (4*b*c
^5*d^8*e - 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 - b*c^3*d^4*e^5
)*x^2 + (7*b*c^5*d^7*e^2 - 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 -
 1))/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4
- 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d
^8*e^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3
*c^2*d^7*e^6 - d^5*e^8)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\operatorname{acosh}{(cx)}}{(d + ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\operatorname{arcosh}(cx) + a}{(ex + d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)/(e*x + d)^4, x)
```

3.21 $\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=398

$$-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{4bde^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{3e^3\left(a+b\cosh^{-1}(cx)\right)^2}{32c^4}-\frac{3be^3x\sqrt{cx-1}\sqrt{cx+}}{16}$$

```
[Out] 2*b^2*d^3*x + (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 + (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(2*c) - (3*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x]))/(16*c^3) - (2*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(8*c) - (d^4*(a + b*ArcCosh[c*x])^2)/(4*e) - (3*d^2*e
*(a + b*ArcCosh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcCosh[c*x])^2)/(32*c^4)
+ ((d + e*x)^4*(a + b*ArcCosh[c*x])^2)/(4*e)
```

Rubi [A] time = 1.68906, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{4bde^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{3e^3\left(a+b\cosh^{-1}(cx)\right)^2}{32c^4}-\frac{3be^3x\sqrt{cx-1}\sqrt{cx+}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] 2*b^2*d^3*x + (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 + (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(2*c) - (3*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x]))/(16*c^3) - (2*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(8*c) - (d^4*(a + b*ArcCosh[c*x])^2)/(4*e) - (3*d^2*e
*(a + b*ArcCosh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcCosh[c*x])^2)/(32*c^4)
+ ((d + e*x)^4*(a + b*ArcCosh[c*x])^2)/(4*e)
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
```

```
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e}$$

$$= \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \left(\frac{d^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4d^3ex\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{6d^2 \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \left(2bcd^3\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(bcd^4\right)\int \frac{a+b \cdots}{\sqrt{-\cdots}}}{2 \cdots}$$

$$= -\frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cos \cdots\right.}{2c}$$

$$= 2b^2d^3x + \frac{3}{4}b^2d^2ex^2 + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{\cdots}\right.}{c}$$

$$= 2b^2d^3x + \frac{4b^2de^2x}{3c^2} + \frac{3}{4}b^2d^2ex^2 + \frac{3b^2e^3x^2}{32c^2} + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{-1+c \cdots}}{\cdots}$$

Mathematica [A] time = 0.823838, size = 386, normalized size = 0.97

$$\frac{c\left(72a^2c^3x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)-6ab\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(72d^2ex+96d^3+32de^2x^2+6e^3x^3\right)+e^2(64d+9ex)\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*
```

```
d*e^2*x^2 + 6*e^3*x^3)) + b^2*c*x*(3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 2
16*d^2*e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*c*(-24*a*c^3*x*(4*d^3 + 6*d^
2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d
+ 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)))*ArcCosh[c
*x] + 9*b^2*(-24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x
^2 + e^3*x^3))*ArcCosh[c*x]^2 - 54*a*b*e*(8*c^2*d^2 + e^2)*Log[c*x + Sqrt[-
1 + c*x]*Sqrt[1 + c*x]])/(288*c^4)
```

Maple [B] time = 0.065, size = 791, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -3/2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d^2*e-3/16/c^4*a*b*e^
3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))-2
/3/c*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d-3/2/c*a*b*e*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*d^2*x-2/3/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d*
e^2-1/2*a*b/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2
*x^2-1)^(1/2))+2*b^2*d^3*x+1/32*b^2*e^3*x^4-3/32/c^4*b^2*arccosh(c*x)^2*e^3
+b^2*arccosh(c*x)^2*x*d^3+1/4*b^2*arccosh(c*x)^2*x^4*e^3+a^2*e^2*x^3*d+3/2*
a^2*e*x^2*d^2-2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3-2/c*a*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3+2*a*b*e^2*arccosh(c*x)*x^3*d+3*a*b*e*arccos
h(c*x)*x^2*d^2+1/4*a^2/e*d^4-3/2/c^2*a*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2
*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+1/4*a^2*e^3*x^4+a^2*x*d^3-1/8/c
*a*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-3/16/c^3*a*b*e^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*x-1/8/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*e^3-3/1
6/c^3*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e^3-4/3/c^3*b^2*arccos
h(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e^2-4/3/c^3*a*b*e^2*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*d-3/4/c^2*b^2*arccosh(c*x)^2*d^2*e+1/2*a*b/e*arccosh(c*x)*d^4+2*a
*b*arccosh(c*x)*x*d^3+b^2*arccosh(c*x)^2*x^3*d*e^2+3/2*b^2*arccosh(c*x)^2*x
^2*d^2*e+1/2*a*b*e^3*arccosh(c*x)*x^4+3/4*b^2*d^2*e*x^2+3/32*b^2*e^3*x^2/c^
2+2/9*b^2*d*e^2*x^3+4/3*b^2*d*e^2*x/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a^2e^3x^4 + a^2de^2x^3 + b^2d^3x\,\operatorname{arcosh}(cx)^2 + \frac{3}{2}a^2d^2ex^2 + \frac{3}{2}\left(2x^2\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}x}{c^2} + \frac{\log\left(2c^2x + 2\sqrt{c^2x^2-1}\sqrt{\right.}}{\sqrt{c^2}c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + b^2*d^3*x*arccosh(c*x)^2 + 3/2*a^2*d^2*e*
x^2 + 3/2*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b*d^2*e + 2/3*(3*x^3*arc
cosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d*
e^2 + 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*
x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*
c^4))*c)*a*b*e^3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d
^3*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^3/c + 1/4*(b^2*e^3*x^
4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
))^2 - integrate(1/2*(b^2*c^3*e^3*x^6 + 4*b^2*c^3*d*e^2*x^5 - 4*b^2*c*d*e^2
*x^3 - 6*b^2*c*d^2*e*x^2 + (6*c^3*d^2*e - c*e^3)*b^2*x^4 + (b^2*c^2*e^3*x^5
 + 4*b^2*c^2*d*e^2*x^4 + 6*b^2*c^2*d^2*e*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x +
1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A] time = 2.52465, size = 1010, normalized size = 2.54

$$9\left(8a^2+b^2\right)c^4e^3x^4+32\left(9a^2+2b^2\right)c^4de^2x^3+27\left(8\left(2a^2+b^2\right)c^4d^2e+b^2c^2e^3\right)x^2+9\left(8b^2c^4e^3x^4+32b^2c^4de^2x^3+48b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27*
(8*(2*a^2 + b^2)*c^4*d^2*e + b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32*b
^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x - 24*b^2*c^2*d^2
*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 96*(3*(a^2 + 2*b^2)*c^4*d^
3 + 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3 + 144
*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x - 72*a*b*c^2*d^2*e - 9*a*b*e^3 - (6*b
^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 + 64*b^2*c*d*e^2 + 9
*(8*b^2*c^3*d^2*e + b^2*c*e^3)*x)*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2
 - 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*d^3 + 64*
a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e + a*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] time = 5.14033, size = 750, normalized size = 1.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*acosh(c*x) + 3*a*b*d**2*e*x**2*acosh(c*x) + 2*a*b*d
*e**2*x**3*acosh(c*x) + a*b*e**3*x**4*acosh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 - 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 - 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 - 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 - 1)/(8*c) - 3*a*b*
d**2*e*acosh(c*x)/(2*c**2) - 4*a*b*d*e**2*sqrt(c**2*x**2 - 1)/(3*c**3) - 3*
a*b*e**3*x*sqrt(c**2*x**2 - 1)/(16*c**3) - 3*a*b*e**3*acosh(c*x)/(16*c**4)
+ b**2*d**3*x*acosh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*acosh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*acosh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*acosh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 - 1)*ac
osh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(8*c) - 3*b**2*d**2*e*acosh(c
*x)**2/(4*c**2) + 4*b**2*d*e**2*x/(3*c**2) + 3*b**2*e**3*x**2/(32*c**2) - 4
*b**2*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 3*b**2*e**3*x*sqrt(c
**2*x**2 - 1)*acosh(c*x)/(16*c**3) - 3*b**2*e**3*acosh(c*x)**2/(32*c**4), N
e(c, 0)), ((a + I*pi*b/2)**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3
*x**4/4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (b \operatorname{arcosh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arccosh(c*x) + a)^2, x)
```

3.22 $\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=259

$$-\frac{de\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2}-\frac{4be^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e}-\frac{2bd^2\sqrt{cx-1}\sqrt{cx+1}\,\big(a+}{c}$$

```
[Out] 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (2*b*e^2*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (d^3*(a + b*ArcCosh[c*x])^2)
/(3*e) - (d*e*(a + b*ArcCosh[c*x])^2)/(2*c^2) + ((d + e*x)^3*(a + b*ArcCosh
[c*x])^2)/(3*e)
```

Rubi [A] time = 1.14838, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{de\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2}-\frac{4be^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e}-\frac{2bd^2\sqrt{cx-1}\sqrt{cx+1}\,\big(a+}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (2*b*e^2*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (d^3*(a + b*ArcCosh[c*x])^2)
/(3*e) - (d*e*(a + b*ArcCosh[c*x])^2)/(2*c^2) + ((d + e*x)^3*(a + b*ArcCosh
[c*x])^2)/(3*e)
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
```

```
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int\left(\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3d^2ex\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3a}{} \right.}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \left(2bcd^2\right)\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(2bcd^3\right)\int\frac{a+}{\sqrt{}}}{3}$$

$$= -\frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bdex\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}\right.}{c}$$

$$= 2b^2d^2x + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{4be^2}{}$$

$$= 2b^2d^2x + \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(c\right.}{c}$$

Mathematica [A] time = 0.611559, size = 360, normalized size = 1.39

$$a^2d^2x + a^2dex^2 + \frac{1}{3}a^2e^2x^3 - \frac{b\cosh^{-1}(cx)\left(b\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)+4e^2\right)-6ac^3x\left(3d^2+3dex+e^2x\right.\right.}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] a^2*d^2*x + 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + a^2*d*e*x^2 + (b^2*d*e*x^
2)/2 + (a^2*e^2*x^3)/3 + (2*b^2*e^2*x^3)/27 - (2*a*b*d^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/c - (4*a*b*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (a*b*d*e
*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (2*a*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])/(9*c) - (b*(-6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + b*Sqrt[-1 + c*x
```

```
]*Sqrt[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)))*ArcCosh[c*x])
/(9*c^3) + (b^2*((-3*d*e)/c^2 + 2*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcCosh[c*
x]^2)/6 - (a*b*d*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/c^2
```

Maple [B] time = 0.052, size = 517, normalized size = 2.

$$-\frac{b^2 \operatorname{arccosh}(cx)\, xde}{c}\sqrt{cx-1}\sqrt{cx+1}-\frac{abedx}{c}\sqrt{cx-1}\sqrt{cx+1}-\frac{2\,abd^3}{3\,e}\sqrt{cx-1}\sqrt{cx+1}\ln\left(cx+\sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{c^2x^2-1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e-1/c*a*b*e*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*d*x-2/3*a*b/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1
/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))+1/3*a^2*e^2*x^3+a^2*x*d^2+2*b^2*d^2*x-1/2
/c^2*b^2*arccosh(c*x)^2*d*e+2/27*b^2*e^2*x^3-2/c*a*b*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*d^2+1/3*a^2/e*d^3+1/3*b^2*arccosh(c*x)^2*x^3*e^2+b^2*arccosh(c*x)^2*x
*d^2+a^2*e*x^2*d-1/c^2*a*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*
d*ln(c*x+(c^2*x^2-1)^(1/2))+b^2*arccosh(c*x)^2*x^2*d*e+2*a*b*arccosh(c*x)*x
*d^2+2/3*a*b/e*arccosh(c*x)*d^3+2/3*a*b*e^2*arccosh(c*x)*x^3-2/9/c*a*b*e^2*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-2/9/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*x^2*e^2-4/9/c^3*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2-2/
c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-4/9/c^3*a*b*e^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)+2*a*b*e*arccosh(c*x)*x^2*d+4/9*b^2*e^2*x/c^2+1/2*b^2*d*
e*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a^2e^2x^3+b^2d^2x\,\operatorname{arcosh}(cx)^2+a^2dex^2+\left(2\,x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)abde+\frac{2}{9}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*e^2*x^3 + b^2*d^2*x*arccosh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arccosh(c
*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c
```

```
^2))/(sqrt(c^2)*c^2)))*a*b*d*e + 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2
- 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*e^2 + 2*b^2*d^2*(x - sqrt(c^2*
x^2 - 1)*arccosh(c*x)/c) + a^2*d^2*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 -
 1))*a*b*d^2/c + 1/3*(b^2*e^2*x^3 + 3*b^2*d*e*x^2)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2 - integrate(2/3*(b^2*c^3*e^2*x^5 + 3*b^2*c^3*d*e*x^4 - b^2
*c*e^2*x^3 - 3*b^2*c*d*e*x^2 + (b^2*c^2*e^2*x^4 + 3*b^2*c^2*d*e*x^3)*sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c
^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A] time = 2.33087, size = 687, normalized size = 2.65

$$\frac{2\left(9a^2+2b^2\right)c^3e^2x^3+27\left(2a^2+b^2\right)c^3dex^2+9\left(2b^2c^3e^2x^3+6b^2c^3dex^2+6b^2c^3d^2x-3b^2cde\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*b
^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x - 3*b^2*c*d*e)*log(c*x
 + sqrt(c^2*x^2 - 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 + 4*b^2*c*e^2)*x + 6*(
6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x - 9*a*b*c*d*e - (
2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 + 4*b^2*e^2)*sqrt(c^2*
x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b*c^2*d
*e*x + 18*a*b*c^2*d^2 + 4*a*b*e^2)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] time = 2.45324, size = 461, normalized size = 1.78

$$\begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\operatorname{acosh}(cx) + 2abdex^2\operatorname{acosh}(cx) + \frac{2abe^2x^3\operatorname{acosh}(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2-1}}{c} - \frac{abdex\sqrt{c^2x^2-1}}{c} - \cdots \\ \left(a+\frac{i\pi b}{2}\right)^2\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*ac
osh(c*x) + 2*a*b*d*e*x**2*acosh(c*x) + 2*a*b*e**2*x**3*acosh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 - 1)/c - a*b*d*e*x*sqrt(c**2*x**2 - 1)/c - 2*a*b*e**2*
```

```
x**2*sqrt(c**2*x**2 - 1)/(9*c) - a*b*d*e*acosh(c*x)/c**2 - 4*a*b*e**2*sqrt(
c**2*x**2 - 1)/(9*c**3) + b**2*d**2*x*acosh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*acosh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*acosh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 -
1)*acosh(c*x)/(9*c) - b**2*d*e*acosh(c*x)**2/(2*c**2) + 4*b**2*e**2*x/(9*c*
*2) - 4*b**2*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3), Ne(c, 0)), ((a +
 I*pi*b/2)**2*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2(b\operatorname{arcosh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arccosh(c*x) + a)^2, x)
```

3.23 $\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=150

$$-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx\right.}{c}$$

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2
*c) - (d^2*(a + b*ArcCosh[c*x])^2)/(2*e) - (e*(a + b*ArcCosh[c*x])^2)/(4*c^
2) + ((d + e*x)^2*(a + b*ArcCosh[c*x])^2)/(2*e)
```

Rubi [A] time = 0.756242, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(c\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCosh[c*x])^2,x]
```


```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2
*c) - (d^2*(a + b*ArcCosh[c*x])^2)/(2*e) - (e*(a + b*ArcCosh[c*x])^2)/(4*c^
2) + ((d + e*x)^2*(a + b*ArcCosh[c*x])^2)/(2*e)
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
```

```
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{e}$$

$$= \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int\left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2dex\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{e^2x^2(a}{\sqrt{-}}\right.}{e}$$

$$= \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - (2bcd)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}dx - \frac{\left(bcd^2\right)\int \frac{a+b\,\mathrm{co}}{\sqrt{-1+c}}}{e}$$

$$= -\frac{2bd\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(c\right.}{2c}$$

$$= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{-1+cx}\sqrt{1+c}}{2c}$$

Mathematica [A] time = 0.389524, size = 174, normalized size = 1.16

$$\frac{c\left(2a^2cx(2d+ex)-2ab\sqrt{cx-1}\sqrt{cx+1}(4d+ex)+b^2cx(8d+ex)\right)-2bc\cosh^{-1}(cx)\left(b\sqrt{cx-1}\sqrt{cx+1}(4d+ex)-2acx(\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) - 2*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d + e*x)
+ b^2*c*x*(8*d + e*x)) - 2*b*c*(-2*a*c*x*(2*d + e*x) + b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*d + e*x))*ArcCosh[c*x] + b^2*(4*c^2*d*x + e*(-1 + 2*c^2*x^2))
*ArcCosh[c*x]^2 - 2*a*b*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(4*c^2)
```

Maple [A] time = 0.043, size = 245, normalized size = 1.6

$$\frac{a^2x^2e}{2} + a^2dx + \frac{b^2(\mathrm{arccosh}(cx))^2x^2e}{2} - \frac{b^2\mathrm{arccosh}(cx)\,xe}{2c}\sqrt{cx-1}\sqrt{cx+1} + \frac{b^2ex^2}{4} - \frac{b^2(\mathrm{arccosh}(cx))^2e}{4c^2} + b^2(\mathrm{arccos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2*e+a^2*d*x+1/2*b^2*arccosh(c*x)^2*x^2*e-1/2/c*b^2*arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e+1/4*b^2*e*x^2-1/4/c^2*b^2*arccosh(c*x)^2*e+b
^2*arccosh(c*x)^2*x*d-2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d+2*
b^2*d*x+a*b*arccosh(c*x)*x^2*e+2*a*b*arccosh(c*x)*x*d-1/2/c*a*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*e*x-2/c*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d-1/2/c^2*a*b*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2dx\,\mathrm{arcosh}\,(cx)^2+\frac{1}{2}\,a^2ex^2+\frac{1}{2}\left(2\,x^2\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}c^2}\right)\right)abe+\frac{1}{2}\left(x^2\log\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d*x*arccosh(c*x)^2 + 1/2*a^2*e*x^2 + 1/2*(2*x^2*arccosh(c*x) - c*(sqrt(
c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2
)*c^2)))*a*b*e + 1/2*(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*inte
grate((c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*c^2*x^3 - c*x^2)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x -
 1) - c*x), x))*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^
2*d*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d/c
```

Fricas [A] time = 2.30077, size = 406, normalized size = 2.71

$$\frac{\left(2\,a^2+b^2\right)c^2ex^2+4\left(a^2+2\,b^2\right)c^2dx+\left(2\,b^2c^2ex^2+4\,b^2c^2dx-b^2e\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\left(2\,abc^2ex^2+4\,abc^2dx-abe\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2 +
 4*b^2*c^2*d*x - b^2*e)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(2*a*b*c^2*e*x^2
 + 4*a*b*c^2*d*x - a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 - 1))*log(c
*x + sqrt(c^2*x^2 - 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 - 1))/c^2
```

Sympy [A] time = 1.02839, size = 240, normalized size = 1.6

$$\begin{cases} a^2dx + \frac{a^2ex^2}{2} + 2abdx\,\mathrm{acosh}\,(cx) + abex^2\,\mathrm{acosh}\,(cx) - \frac{2abd\sqrt{c^2x^2-1}}{c} - \frac{abex\sqrt{c^2x^2-1}}{2c} - \frac{abe\,\mathrm{acosh}\,(cx)}{2c^2} + b^2dx\,\mathrm{acosh}^2\,(cx) + 2b^2d \\ \left(a + \frac{i\pi b}{2}\right)^2\left(dx + \frac{ex^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*acosh(c*x) + a*b*e*x**2*aco
sh(c*x) - 2*a*b*d*sqrt(c**2*x**2 - 1)/c - a*b*e*x*sqrt(c**2*x**2 - 1)/(2*c)
 - a*b*e*acosh(c*x)/(2*c**2) + b**2*d*x*acosh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*acosh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 - 1)*acosh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(2*c) - b**2*e*acosh(c*x)*
*2/(4*c**2), Ne(c, 0)), ((a + I*pi*b/2)**2*(d*x + e*x**2/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(b\,\mathrm{arcosh}\,(cx) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arccosh(c*x) + a)^2, x)
```

3.24 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=303

$$\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}-\frac{2b^2\text{PolyLog}\left(3,-\right.}{e}$$

```
[Out] -(a + b*ArcCosh[c*x])^3/(3*b*e) + ((a + b*ArcCosh[c*x])^2*Log[1 + (e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])^2*Log[1 +
 (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (2*b*(a + b*ArcCosh[c
*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*
b*(a + b*ArcCosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 -
 e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 -
e^2]))])/e
```

Rubi [A] time = 0.456348, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5800, 5562, 2190, 2531, 2282, 6589}

$$\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}-\frac{2b^2\text{PolyLog}\left(3,-\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x),x]
```

```
[Out] -(a + b*ArcCosh[c*x])^3/(3*b*e) + ((a + b*ArcCosh[c*x])^2*Log[1 + (e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])^2*Log[1 +
 (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (2*b*(a + b*ArcCosh[c
*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*
b*(a + b*ArcCosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 -
 e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 -
e^2]))])/e
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd}\right.$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log}{e}$$

Mathematica [A] time = 0.241815, size = 285, normalized size = 0.94

$$\frac{6b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right)+6b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)-6b^2\text{PolyLog}\left(3,\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])^3/b) + 3*(a + b*ArcCosh[c*x])^2*Log[1 + (e*E^ArcCos
h[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])] + 3*(a + b*ArcCosh[c*x])^2*Log[1 + (e*
E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])] + 6*b*(a + b*ArcCosh[c*x])*Pol
yLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^2 - e^2])] + 6*b*(a + b*Arc
Cosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^2 - e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(3*e)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(a + b\mathrm{arccosh}\,(cx))^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log (ex + d)}{e} + \int \frac{b^2 \log \left(cx + \sqrt{cx + 1}\sqrt{cx - 1}\right)^2}{ex + d} + \frac{2\,ab \log \left(cx + \sqrt{cx + 1}\sqrt{cx - 1}\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2
/(e*x + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2 \operatorname{arcosh}\,(cx)^2 + 2\,ab \operatorname{arcosh}\,(cx) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d), x)
```

3.25 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=279

$$\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2bc\left(\right.}{}$$

```
[Out] -((a + b*ArcCosh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcCosh[c*x])*Log[1
 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2])
 - (2*b*c*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d
^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2
- e^2])
```

Rubi [A] time = 0.610123, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5802, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2bc\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x)^2,x]
```



```
[Out] -((a + b*ArcCosh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcCosh[c*x])*Log[1
 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2])
 - (2*b*c*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d
^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2
- e^2])
```


Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
```

```
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\,\mathrm{Subst}\left(\int \frac{a+bx}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\mathrm{Subst}\left(\int \frac{e^x(a+bx)}{e+2cde^x+ee^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\mathrm{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2-e^2}+2ee^x}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{c^2d^2-e^2}} - \frac{(4bc)\,\mathrm{Subst}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2bc\left(a+b\cosh^{-1}(c\right.}{e\sqrt{c}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2bc\left(a+b\cosh^{-1}(c\right.}{e\sqrt{c}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2bc\left(a+b\cosh^{-1}(c\right.}{e\sqrt{c}}$$

Mathematica [C] time = 4.46334, size = 959, normalized size = 3.44

$$-\frac{a^2}{e(d+ex)} + 2bc\left(\frac{2\tan^{-1}\left(\frac{\sqrt{e(e-cd)}\sqrt{\frac{cx-1}{cx+1}}}{\sqrt{e(cd+e)}}\right)}{\sqrt{e(e-cd)}\sqrt{e(cd+e)}} - \frac{\cosh^{-1}(cx)}{e(cd+cex)}\right)a - \frac{b^2c\left(\frac{\cosh^{-1}(cx)^2}{cd+cex} + \frac{2\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right)-2i\,\mathrm{c}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -(a^2/(e*(d + e*x))) + 2*a*b*c*(-(ArcCosh[c*x]/(e*(c*d + c*e*x))) + (2*ArcT
an[(Sqrt[e*(-(c*d) + e)]*Sqrt[(-1 + c*x)/(1 + c*x)])/Sqrt[e*(c*d + e)]])/(S
qrt[e*(-(c*d) + e)]*Sqrt[e*(c*d + e)])) - (b^2*c*(ArcCosh[c*x]^2/(c*d + c*e
*x) + (2*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2
*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[
c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)] + 2*(ArcTan[((c*d +
e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tan
h[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-(c^2*d^2) + e^2]/(Sq
rt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d + c*e*x])] + (ArcCos[-((c*d)/e)]
- 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcT
an[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[(Sqrt[
-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*Sqrt[c*d + c*e*x])]
- (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-
(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c^2*d^2) + e^2])*(-1
+ Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCos
h[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*
x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d) + e + I*Sqrt[-(c^2*d
^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e
^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*Sqrt[-(c^2*d^2) + e^2
])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e +
 I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*d + I*Sq
rt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]
/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqr
t[-(c^2*d^2) + e^2]))/e
```

Maple [A] time = 0.063, size = 536, normalized size = 1.9

$$-\frac{ca^2}{(cxe+cd)\,e}-\frac{cb^2\left(\operatorname{arccosh}(cx)\right)^2}{(cxe+cd)\,e}+2\,\frac{cb^2\operatorname{arccosh}(cx)}{e\sqrt{c^2d^2-e^2}}\ln\left(\frac{-\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)-2\,\frac{cb^2\operatorname{arccos}}{e\sqrt{c^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d)^2,x)
```

```
[Out] -c*a^2/(c*e*x+c*d)/e-c*b^2*arccosh(c*x)^2/e/(c*e*x+c*d)+2*c*b^2/e*arccosh(c
*x)/(c^2*d^2-e^2)^(1/2)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d
^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))-2*c*b^2/e*arccosh(c*x)/(c^2*d^2-
e^2)^(1/2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))
/(c*d+(c^2*d^2-e^2)^(1/2)))+2*c*b^2/e/(c^2*d^2-e^2)^(1/2)*dilog((-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/
2)))-2*c*b^2/e/(c^2*d^2-e^2)^(1/2)*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

```
*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))-2*c*a*b/(c*e*x+c*d)/
e*arccosh(c*x)-2*c*a*b/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-2*(c^2*d*x-(c^2*
x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))/((c^2*d^2-e^2)/e^2
)^(1/2)/(c^2*x^2-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arcosh}\,(cx)^2 + 2\,ab\,\text{arcosh}\,(cx) + a^2}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e^2*x^2 + 2*d*e*x
 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acosh}\,(cx))^2}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{(ex + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d)^2, x)
```

3.26 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=380

$$\frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc\sqrt{-\frac{1-cx}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\left(a+\right.}{}$$

```
[Out] -((b*c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*(a + b*ArcCosh[c*x]))/((c^2*d
^2 - e^2)*(d + e*x))) - (a + b*ArcCosh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d
*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]
)])/(e*(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*(a + b*ArcCosh[c*x])*Log[1 + (e*E^
ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (b^
2*c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2)) - (b^2*c
^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*(c^2
*d^2 - e^2)^(3/2))
```

Rubi [A] time = 0.746648, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5802, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc\sqrt{-\frac{1-cx}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*(a + b*ArcCosh[c*x]))/((c^2*d
^2 - e^2)*(d + e*x))) - (a + b*ArcCosh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d
*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]
)])/(e*(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*(a + b*ArcCosh[c*x])*Log[1 + (e*E^
ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (b^
2*c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2)) - (b^2*c
^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*(c^2
```

```
*d^2 - e^2)^(3/2))
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{a+bx}{(cd+e\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(b^2c^2\right)\text{Subst}\left(\int \frac{\sinh(x)}{cd+e\cosh(x)}\right.}{c^2d^2-e^2}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{cd+x}\,dx, x,\right.}{e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)} + \frac{\left(2bc^3d\right)}{}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log}{e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log}{e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log}{e\left(c^2d^2-e^2\right)}$$

Mathematica [C] time = 8.40222, size = 1089, normalized size = 2.87

$$-\frac{a^2}{2e(d+ex)^2} + bc^2\left(-\frac{\cosh^{-1}(cx)}{e(cd+cex)^2} - \frac{2cd\tan^{-1}\left(\frac{\sqrt{e-cd}\sqrt{\frac{cx-1}{cx+1}}}{\sqrt{cd+e}}\right)}{e(e-cd)^{3/2}(cd+e)^{3/2}} + \frac{\sqrt{cx-1}\sqrt{cx+1}}{(e-cd)(cd+e)(cd+cex)}\right)a + b^2c^2\left(-\frac{\cosh^{-1}(cx)^2}{2e(cd+cex)^2} - \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -a^2/(2*e*(d + e*x)^2) + a*b*c^2*((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((-(c*d) +
 e)*(c*d + e)*(c*d + c*e*x)) - ArcCosh[c*x]/(e*(c*d + c*e*x)^2) - (2*c*d*Ar
cTan[(Sqrt[-(c*d) + e]*Sqrt[(-1 + c*x)/(1 + c*x)])/Sqrt[c*d + e]])/(e*(-(c*
d) + e)^(3/2)*(c*d + e)^(3/2))) + b^2*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*ArcCosh[c*x])/((c*d - e)*(c*d + e)*(c*d + c*e*x))) - ArcCosh[c*x]^2
/(2*e*(c*d + c*e*x)^2) + Log[1 + (e*x)/d]/(c^2*d^2*e - e^3) + (c*d*(2*ArcCo
sh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (
2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c
^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c
*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2]
)/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(
ArcCosh[c*x]/2)*Sqrt[c*(d + e*x)])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d
 + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*
Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]
*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*Sqrt[c*(d + e*x)])] - (ArcCos[-((c*d)
/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]
)*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*
x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (A
rcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2
*d^2) + e^2]])*Log[((c*d + e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1 +
Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[
c*x]/2]))] + I*(PolyLog[2, ((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*S
qrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2
) + e^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e
^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e
 + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))])))/(e*(-(c^2*d^2) + e^2
)^(3/2)))
```

Maple [B] time = 0.112, size = 1170, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a^2/(c*e*x+c*d)^2/e-1/2*c^4*b^2*arccosh(c*x)^2/e/(c*e*x+c*d)^2/(c^
2*d^2-e^2)*d^2-c^3*b^2*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x-c^3*b^2*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d^2-e^2)*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*d+c^4*b^2*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)
*x^2+2*c^4*b^2*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d^2-e^2)*x*d+c^4*b^2*arccosh
(c*x)/e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*d^2+1/2*c^2*b^2*arccosh(c*x)^2*e/(c*e*x
```

```
+c*d)^2/(c^2*d^2-e^2)+c^3*b^2/e/(c^2*d^2-e^2)^(3/2)*arccosh(c*x)*ln((-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)
^(1/2)))*d-c^3*b^2/e/(c^2*d^2-e^2)^(3/2)*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))*d+c^
3*b^2/e/(c^2*d^2-e^2)^(3/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d
+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))*d-c^3*b^2/e/(c^2*d^2-e^2)
^(3/2)*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/
(c*d+(c^2*d^2-e^2)^(1/2)))*d+c^2*b^2/e/(c^2*d^2-e^2)*ln(2*c*d*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+e)-2*c^2*b^2/e/(
c^2*d^2-e^2)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-c^2*a*b/(c*e*x+c*d)^2/e*ar
ccosh(c*x)-c^4*a*b/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/
(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1
/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d-c^4*a*b/e^2*(c*x+1)^(1/
2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/((c^2*d^2-e^2)/e^2)^(1/2
)/(c*e*x+c*d)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+
e)/(c*e*x+c*d))*d^2-c^2*a*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c*
e*x+c*d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acosh}(cx))^2}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d)^3, x)
```

3.27 $\int \frac{(d+ex)^3}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=394

$$-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{3de^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right.}{4bc^3}$$

```
[Out] -((d^3*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (3*d*e^2*CoshIn
tegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*
a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (e^3*CoshIntegral[(2*a)/b
 + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (3*d*e^2*CoshIntegral[(3*a)/b
 + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) - (e^3*CoshIntegral[(4*a)/b + 4
*ArcCosh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) + (d^3*Cosh[a/b]*SinhIntegral[a/b +
 ArcCosh[c*x]])/(b*c) + (3*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]]
)/(2*b*c^2) + (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(4
*b*c^4) + (3*d*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4
*b*c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c
^4)
```

Rubi [A] time = 1.17025, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5806, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{3de^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right.}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^3*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (3*d*e^2*CoshIn
tegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*
a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (e^3*CoshIntegral[(2*a)/b
 + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (3*d*e^2*CoshIntegral[(3*a)/b
 + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) - (e^3*CoshIntegral[(4*a)/b + 4
*ArcCosh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) + (d^3*Cosh[a/b]*SinhIntegral[a/b +
 ArcCosh[c*x]])/(b*c) + (3*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]]
)/(2*b*c^2) + (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(4
```

```
*b*c^4) + (3*d*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4
*b*c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c
^4)
```

Rule 5806

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*(c*d + e*Cosh[x])^m*Sin
h[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b\cosh^{-1}(cx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{(cd+e\cosh(x))^3\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\mathrm{Subst}\left(\int \left(\frac{c^3d^3\sinh(x)}{a+bx} + \frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx} + \frac{3cde^2\cosh^2(x)\sinh(x)}{a+bx} + \frac{e^3\cosh^3(x)\sinh(x)}{a+bx}\right)dx, x, \cosh^{-1}\right.}{c^4}$$

$$= \frac{d^3\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{\left(3d^2e\right)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{(3d}{}$$

$$= \frac{\left(3d^2e\right)\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{\left(3de^2\right)\mathrm{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}\right.}{c^3}$$

$$= -\frac{d^3\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^3\cosh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{\left(3d^2e\right)\mathrm{Subst}\left(\int \frac{\sinh}{a+}\right.}{2}$$

$$= -\frac{d^3\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^3\cosh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{\left(3de^2\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subs}}{}$$

$$= -\frac{d^3\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{3de^2\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{3d^2e\mathbf{Chi}\left(\frac{2a}{b}+2\,\mathrm{co}\right.}{2b}$$

Mathematica [A] time = 0.567684, size = 287, normalized size = 0.73

$$\frac{-2cd\sinh\left(\frac{a}{b}\right)\left(4c^2d^2+3e^2\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - 2e\sinh\left(\frac{2a}{b}\right)\left(6c^2d^2+e^2\right)\mathbf{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 12c^2d^2e\cosh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*c*d*(4*c^2*d^2 + 3*e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 2*
e*(6*c^2*d^2 + e^2)*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - 6*
c*d*e^2*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - e^3*CoshIntegr
al[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + 8*c^3*d^3*Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c*x]] + 6*c*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
```

```
 + 12*c^2*d^2*e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*e^3*
Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*c*d*e^2*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + e^3*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])])/(8*b*c^4)
```

Maple [A] time = 0.141, size = 394, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e^3}{16\,c^3b}\mathrm{e}^{-4\,\frac{a}{b}}\mathrm{Ei}\left(1,-4\,\mathrm{arccosh}\,(cx)-4\,\frac{a}{b}\right)+\frac{e^3}{16\,c^3b}\mathrm{e}^{4\,\frac{a}{b}}\mathrm{Ei}\left(1,4\,\mathrm{arccosh}\,(cx)+4\,\frac{a}{b}\right)+\frac{3\,ed^2}{4\,bc}\mathrm{e}^{2\,\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{arccosh}\,(cx)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/16/c^3*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)+1/16/c^3*e^3/b
*exp(4*a/b)*Ei(1,4*arccosh(c*x)+4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arccosh(
c*x)+2*a/b)*d^2+1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b)-3/4/c*e
/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)*d^2-1/8/c^3*e^3/b*exp(-2*a/b)*Ei
(1,-2*arccosh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-
3*a/b)+1/2*d^3/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)+3/8/c^2*d/b*exp(a/b)*Ei(1,
arccosh(c*x)+a/b)*e^2-1/2*d^3/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)-3/8/c^2*d
/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e^2+3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*
arccosh(c*x)+3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(b*arccosh(c*x) + a), x)
```

3.28 $\int \frac{(d+ex)^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$-\frac{de\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{bc^2}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\cdots$$

```
[Out] -((d^2*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e^2*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2
*ArcCosh[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (e^2*CoshIntegral[(3*a)/b + 3*ArcCo
sh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) + (d^2*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]])/(b*c) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*c^3
) + (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b*c^2) + (e
^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^3)
```

Rubi [A] time = 0.704273, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5806, 6742, 3303, 3298, 3301, 5448}

$$-\frac{de\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{bc^2}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^2*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e^2*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2
*ArcCosh[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (e^2*CoshIntegral[(3*a)/b + 3*ArcCo
sh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) + (d^2*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]])/(b*c) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*c^3
) + (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b*c^2) + (e
^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^3)
```

Rule 5806

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*(c*d + e*Cosh[x])^m*Sin
h[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
```

```
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))^2\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^2d^2\sinh(x)}{a+bx} + \frac{e^2\cosh^2(x)\sinh(x)}{a+bx} + \frac{cde\sinh(2x)}{a+bx}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{(de)\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{e^2\,\text{Subst}}{}$$

$$= \frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, c\right.}{c}$$

$$= -\frac{d^2\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{de\textbf{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} + \frac{d^2\cosh\left(\frac{a}{b}\right)\textbf{Shi}}{}$$

$$= -\frac{d^2\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{de\textbf{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} + \frac{d^2\cosh\left(\frac{a}{b}\right)\textbf{Shi}}{}$$

$$= -\frac{d^2\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e^2\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{de\textbf{Chi}\left(\frac{2a}{b}+2\cosh\right.}{bc^2}$$

Mathematica [A] time = 0.317103, size = 187, normalized size = 0.76

$$\frac{-\sinh\left(\frac{a}{b}\right)\left(4c^2d^2+e^2\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 4c^2d^2\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - 4cde\sinh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\cosh\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-((4*c^2*d^2 + e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b]) - 4*c*d*e*
CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - e^2*CoshIntegral[3*(a/
b + ArcCosh[c*x])]*Sinh[(3*a)/b] + 4*c^2*d^2*Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 4*c*d*e*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + e^2*Cosh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcCosh[c*x])])/(4*b*c^3)
```

Maple [A] time = 0.1, size = 254, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e^2}{8c^2b}\mathrm{e}^{-3\frac{a}{b}}\mathrm{Ei}\left(1,-3\operatorname{arccosh}(cx)-3\frac{a}{b}\right)+\frac{e^2}{8c^2b}\mathrm{e}^{3\frac{a}{b}}\mathrm{Ei}\left(1,3\operatorname{arccosh}(cx)+3\frac{a}{b}\right)+\frac{d^2}{2b}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/8/c^2*e^2/b*e
xp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
cosh(c*x)-a/b)*d^2-1/8/c^2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e^2-1/2/c*d*
e/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)+1/2/c*d*e/b*exp(2*a/b)*Ei(1,2*a
rccosh(c*x)+2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{b\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a), x)
```

3.29 $\int \frac{d+ex}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=116

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{d\cosh}{}$$

```
[Out] -((d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e*CoshIntegral[(
2*a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (d*Cosh[a/b]*SinhIntegr
al[a/b + ArcCosh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*A
rcCosh[c*x]])/(2*b*c^2)
```

Rubi [A] time = 0.337515, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5806, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{d\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e*CoshIntegral[(
2*a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (d*Cosh[a/b]*SinhIntegr
al[a/b + ArcCosh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*A
rcCosh[c*x]])/(2*b*c^2)
```

Rule 5806

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*(c*d + e*Cosh[x])^m*Sin
h[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{d+ex}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{cd\sinh(x)}{a+bx}+\frac{e\cosh(x)\sinh(x)}{a+bx}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2}$$
$$= \frac{d\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2}$$
$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} - \cdots$$
$$= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, \cdots\right.}{2c^2}$$
$$= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int\cdots\right.}{\cdots}$$
$$= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cdots\right.}{bc}$$

Mathematica [A] time = 0.137259, size = 98, normalized size = 0.84

$$\frac{-2cd\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 2cd\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + e\cosh\cdots}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*c*d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - e*CoshIntegral[2*(a/b
+ ArcCosh[c*x])]*Sinh[(2*a)/b] + 2*c*d*Cosh[a/b]*SinhIntegral[a/b + ArcCosh
[c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])])/(2*b*c^2)
```

Maple [A] time = 0.066, size = 120, normalized size = 1.

$$\frac{1}{c}\left(\frac{d}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right) - \frac{d}{2b}e^{-\frac{a}{b}}\text{Ei}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right) - \frac{e}{4bc}e^{-2\frac{a}{b}}\text{Ei}\left(1,-2\,\text{arccosh}(cx)-2\frac{a}{b}\right) + \frac{e}{4bc}e^{2\frac{a}{b}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/2/b*exp(-a/b)*Ei(1,-arccosh(
c*x)-a/b)*d-1/4/c*e/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)+1/4/c*e/b*exp
(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex+d}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b*arccosh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)/(a + b*acosh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a), x)
```

3.30 $\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.0336285, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcCosh[c*x])),x]

[Out] Defer[Int][1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.206075, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)\,(a+b\mathrm{arccosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{aex+ad+(bex+bd)\,\mathrm{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\text{acosh}\,(cx))\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex + d)(b\,\text{arcosh}\,(cx) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)), x)
```

3.31 $\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi [A] time = 0.031335, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.375632, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.221, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+d\right)^{2} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(ex+d\right)}^{2}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{ae^{2}x^{2}+2\,adex+ad^{2}+\left(be^{2}x^{2}+2\,bdex+bd^{2}\right)\mathrm{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2)
*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{acosh}(cx))(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)), x)
```

3.32 $\int \frac{(d+ex)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=374

$$\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^2}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{2de\sinh}{}$$

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshInt
egral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (e^2*Cosh[a/b]*CoshIntegral[(a + b
*ArcCosh[c*x])/b])/(4*b^2*c^3) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a +
b*ArcCosh[c*x]))/b])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
b*ArcCosh[c*x]))/b])/(4*b^2*c^3) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c*x])/b])/(b^2*c) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])
/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b
])/(b^2*c^2) - (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b
])/(4*b^2*c^3)
```

Rubi [A] time = 0.750857, antiderivative size = 366, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5804, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4b^2c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshInt
egral[a/b + ArcCosh[c*x]])/(b^2*c) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcC
osh[c*x]])/(4*b^2*c^3) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcC
osh[c*x]])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCos
h[c*x]])/(4*b^2*c^3) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(b^
2*c) - (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b^2*c^3) - (2*d*
```

```
e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c^2) - (3*e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*c^3)
```

Rule 5804

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcCosh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d^2}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{2dex}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{e^2x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx$$

$$= d^2\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+(2de)\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+e^2\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(cd^2\right)\int \frac{}{\sqrt{-1+c}}}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{2de\cosh\left(\frac{2a}{b}\right)}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{C}}{}$$

Mathematica [A] time = 2.30647, size = 304, normalized size = 0.81

$$\frac{4c^2d^2\left(\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\frac{4bc^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)(d+ex)^2}{a+b\cosh^{-1}(cx)}+8cde\left(\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((-4*b*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x)^2)/(a + b*ArcCosh
[c*x]) - 8*c*d*e*Log[a + b*ArcCosh[c*x]] + 4*c^2*d^2*(Cosh[a/b]*CoshIntegra
l[a/b + ArcCosh[c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]) - 8*e^2
*(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
 ArcCosh[c*x]]) + 8*c*d*e*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x]
)] + Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[
c*x])]) + 3*e^2*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + Ar
cCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c
^3)
```

Maple [A] time = 0.174, size = 649, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c^3*x^3-3*c*x)*e^2/c^2/b/(a+b*arccosh(c*x))-3/8*e^2/c^2/b^2*exp(3*a/b)*
Ei(1,3*arccosh(c*x)+3*a/b)-1/8*e^2/c^2/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-3/8*e^
2/c^2/b^2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+
1)^(1/2)+c*x)*d^2/b/(a+b*arccosh(c*x))-1/2*d^2/b^2*exp(a/b)*Ei(1,arccosh(c*
x)+a/b)+1/8*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e^2/c^2/b/(a+b*arccosh(c*x))
-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*d^2*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arccosh(c*
x)-a/b)-1/8/c^2/b*e^2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-
1/8/c^2/b^2*e^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)+1/2*(-2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+2*c^2*x^2-1)*d*e/c/(a+b*arccosh(c*x))/b-d/c*e/b^2*exp(2*a/b)
*Ei(1,2*arccosh(c*x)+2*a/b)-1/2*d/c*e/b*(2*c^2*x^2-1+2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c)/(a+b*arccosh(c*x))-d/c*e/b^2*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-
2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3e^2x^5 + 2\,c^3dex^4 - 2\,cdex^2 - cd^2x + \left(c^3d^2 - ce^2\right)x^3 + \left(c^2e^2x^4 + 2\,c^2dex^3 - 2\,dex + \left(c^2d^2 - e^2\right)x^2 - d^2\right)\sqrt{cx+1}\sqrt{cx-1}}{abc^3x^2 + \sqrt{cx+1}\sqrt{cx-1}abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\sqrt{cx-1}b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^5 + 2*c^3*d*e*x^4 - 2*c*d*e*x^2 - c*d^2*x + (c^3*d^2 - c*e^2)*x
^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 - 2*d*e*x + (c^2*d^2 - e^2)*x^2 - d^2)*sq
rt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c
^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*c^5*e^2*x^6 + 4*c^
5*d*e*x^5 - 8*c^3*d*e*x^3 + (c^5*d^2 - 6*c^3*e^2)*x^4 + 4*c*d*e*x + (3*c^3*
e^2*x^4 + 4*c^3*d*e*x^3 + c*d^2 + (c^3*d^2 - c*e^2)*x^2)*(c*x + 1)*(c*x - 1
) + c*d^2 - (2*c^3*d^2 - 3*c*e^2)*x^2 + (6*c^4*e^2*x^5 + 8*c^4*d*e*x^4 - 8*
c^2*d*e*x^2 + (2*c^4*d^2 - 7*c^2*e^2)*x^3 + 2*d*e - (c^2*d^2 - 2*e^2)*x)*sq
rt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 -
 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x
 - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^
2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{2} x^{2} + 2\, dex + d^{2}}{b^{2} \operatorname{arcosh}\left(cx\right)^{2} + 2\, ab \operatorname{arcosh}\left(cx\right) + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x)
 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex\right)^{2}}{\left(a + b \operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**2/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a)^2, x)
```

3.33 $\int \frac{d+ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=190

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}}{b^2}$$

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshInte
gral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a
 + b*ArcCosh[c*x]))/b])/(b^2*c^2) - (d*Sinh[a/b]*SinhIntegral[(a + b*ArcCos
h[c*x])/b])/(b^2*c) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x])
)/b])/(b^2*c^2)
```

Rubi [A] time = 0.474596, antiderivative size = 186, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5804, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-d\sinh$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshInte
gral[a/b + ArcCosh[c*x]])/(b^2*c) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b +
 2*ArcCosh[c*x]])/(b^2*c^2) - (d*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(b^2*c) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c
^2)
```

Rule 5804

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcCosh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx + e\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{b} + \frac{e\,S}{}$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc} +$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\mathbf{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2} - \frac{e}{}$$

$$= -\frac{d\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{d\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c} + \frac{e\cos}{}$$

Mathematica [A] time = 2.43025, size = 157, normalized size = 0.83

$$\frac{cd\left(\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+e\left(\cosh\left(\frac{2a}{b}\right)\mathbf{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{2}{}\right.\right.}{b^2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-((b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x))/(a + b*ArcCosh[c*x]
)) - e*Log[a + b*ArcCosh[c*x]] + c*d*(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[
c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]) + e*(Cosh[(2*a)/b]*Cosh
Integral[2*(a/b + ArcCosh[c*x])] + Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*
SinhIntegral[2*(a/b + ArcCosh[c*x])]))/(b^2*c^2)
```

Maple [A] time = 0.106, size = 285, normalized size = 1.5

$$\frac{1}{c}\left(\frac{d}{2b\left(a+b\operatorname{arccosh}(cx)\right)}\left(-\sqrt{cx-1}\sqrt{cx+1}+cx\right)-\frac{d}{2b^2}e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)-\frac{d}{2b\left(a+b\operatorname{arccosh}(cx)\right)}\left(cx+\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d/b/(a+b*arccosh(c*x))-1/2/b^2*
exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(
a+b*arccosh(c*x))*d-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d+1/4*(-2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c+2*c^2*x^2-1)*e/c/(a+b*arccosh(c*x))/b-1/2/c*e/
b^2*exp(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b)-1/4/c*e/b*(2*c^2*x^2-1+2*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c)/(a+b*arccosh(c*x))-1/2/c*e/b^2*exp(-2*a/b)*Ei(1,-2
*arccosh(c*x)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3ex^4 + c^3dx^3 - cex^2 - cdx + \left(c^2ex^3 + c^2dx^2 - ex - d\right)\sqrt{cx+1}\sqrt{cx-1}}{abc^3x^2 + \sqrt{cx+1}\sqrt{cx-1}abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\sqrt{cx-1}b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)} + \int \frac{}{abc^5x^4 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^4 + c^3*d*x^3 - c*e*x^2 - c*d*x + (c^2*e*x^3 + c^2*d*x^2 - e*x -
d)*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*
a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x -
b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((2*c^5*e*x^5 + c
^5*d*x^4 - 4*c^3*e*x^3 - 2*c^3*d*x^2 + (2*c^3*e*x^3 + c^3*d*x^2 + c*d)*(c*x
 + 1)*(c*x - 1) + 2*c*e*x + (4*c^4*e*x^4 + 2*c^4*d*x^3 - 4*c^2*e*x^2 - c^2*
d*x + e)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^4 + (c*x + 1)*(c*x -
 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(
c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2
*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex+d}{b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)/(a + b*acosh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a)^2, x)
```

3.34 $\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.0314219, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 9.89005, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)\,(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\sqrt{cx-1}-cx}{abc^3ex^3+abc^3dx^2-abcex-abcd+\left(abc^2ex^2+abc^2dx\right)\sqrt{cx+1}\sqrt{cx-1}+\left(b^2c^3ex^3+b^2c^3dx^2-b^2cex-b^2cd+\left(b^2c^2e\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e*x^3
 + a*b*c^3*d*x^2 - a*b*c*e*x - a*b*c*d + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt
(c*x + 1)*sqrt(c*x - 1) + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 - b^2*c*e*x - b^2*
c*d + (b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((c^5*d*x^4 - 2*c^3*d*x^2 + (c^3*d
*x^2 + 2*c*e*x + c*d)*(c*x + 1)*(c*x - 1) + (2*c^4*d*x^3 + 2*c^2*e*x^2 - c^
2*d*x - e)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*e^2*x^6 + 2*a*b*c^5*
d*e*x^5 - 4*a*b*c^3*d*e*x^3 + (c^5*d^2 - 2*c^3*e^2)*a*b*x^4 + 2*a*b*c*d*e*x
 + a*b*c*d^2 - (2*c^3*d^2 - c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^4 + 2*a*b*c^3*d
*e*x^3 + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*e^2*x^5 + 2*a*b*
c^4*d*e*x^4 - 2*a*b*c^2*d*e*x^2 - a*b*c^2*d^2*x + (c^4*d^2 - c^2*e^2)*a*b*x
^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^2*x^6 + 2*b^2*c^5*d*e*x^5 - 4*
b^2*c^3*d*e*x^3 + (c^5*d^2 - 2*c^3*e^2)*b^2*x^4 + 2*b^2*c*d*e*x + b^2*c*d^2
 - (2*c^3*d^2 - c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4 + 2*b^2*c^3*d*e*x^3 + b^2
*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^2*x^5 + 2*b^2*c^4*d*e*x^4
- 2*b^2*c^2*d*e*x^2 - b^2*c^2*d^2*x + (c^4*d^2 - c^2*e^2)*b^2*x^3)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2ex + a^2d + \left(b^2ex + b^2d\right)\operatorname{arcosh}(cx)^2 + 2\,(abex + abd)\operatorname{arcosh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arccosh(c*x)^2 + 2*(a*b*e*x
 + a*b*d)*arccosh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(cx))^2\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex + d)(b\operatorname{arcosh}(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)^2), x)
```

3.35 $\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi [A] time = 0.0315763, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 98.7401, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2 (a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e^2*x
^4 + 2*a*b*c^3*d*e*x^3 - 2*a*b*c*d*e*x - a*b*c*d^2 + (c^3*d^2 - c*e^2)*a*b*
x^2 + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x)*sqrt(c*x + 1)*s
qrt(c*x - 1) + (b^2*c^3*e^2*x^4 + 2*b^2*c^3*d*e*x^3 - 2*b^2*c*d*e*x - b^2*c
*d^2 + (c^3*d^2 - c*e^2)*b^2*x^2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b
^2*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1))) - integrate((c^5*e*x^5 - c^5*d*x^4 - 2*c^3*e*x^3 + 2*c^3*d*x^2 + (c
^3*e*x^3 - c^3*d*x^2 - 3*c*e*x - c*d)*(c*x + 1)*(c*x - 1) + c*e*x + (2*c^4*
e*x^4 - 2*c^4*d*x^3 - 5*c^2*e*x^2 + c^2*d*x + 2*e)*sqrt(c*x + 1)*sqrt(c*x -
 1) - c*d)/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*d^2*e - 2*c^3*e^
3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 - 6*c^3*d*e^2)*a*b*x^4 + a*b*c*d^3
- (6*c^3*d^2*e - c*e^3)*a*b*x^3 - (2*c^3*d^3 - 3*c*d*e^2)*a*b*x^2 + (a*b*c^
3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 + a*b*c^3*d^3*x^2)*(c
*x + 1)*(c*x - 1) + 2*(a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 - 3*a*b*c^2*d^
2*e*x^2 - a*b*c^2*d^3*x + (3*c^4*d^2*e - c^2*e^3)*a*b*x^4 + (c^4*d^3 - 3*c^
2*d*e^2)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^
5*d*e^2*x^6 + (3*c^5*d^2*e - 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^
3 - 6*c^3*d*e^2)*b^2*x^4 + b^2*c*d^3 - (6*c^3*d^2*e - c*e^3)*b^2*x^3 - (2*c
^3*d^3 - 3*c*d*e^2)*b^2*x^2 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^
```

```
2*c^3*d^2*e*x^3 + b^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^3*x^6
 + 3*b^2*c^4*d*e^2*x^5 - 3*b^2*c^2*d^2*e*x^2 - b^2*c^2*d^3*x + (3*c^4*d^2*e
 - c^2*e^3)*b^2*x^4 + (c^4*d^3 - 3*c^2*d*e^2)*b^2*x^3)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2e^2x^2 + 2\,a^2dex + a^2d^2 + \left(b^2e^2x^2 + 2\,b^2dex + b^2d^2\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abe^2x^2 + 2\,abdex + abd^2\right)\operatorname{arcosh}\left(cx\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*
x + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arcco
sh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{acosh}\left(cx\right)\right)^2\left(d + ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + d\right)^2\left(b\operatorname{arcosh}\left(cx\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)^2), x)
```

3.36 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=81

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)^3}{e(m+1)}-\frac{3bc\text{Unintegrable}\left(\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}},x\right)}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^3)/(e*(1 + m)) - (3*b*c*Unintegrabl
e[((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
, x])/(e*(1 + m))
```

Rubi [A] time = 0.383444, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x])^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^3)/(e*(1 + m)) - (3*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
x])/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx=\frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^3}{e(1+m)}-\frac{(3bc)\int \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} \, dx}{e(1+m)}$$

Mathematica [A] time = 6.27858, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^3,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^3, x]
```

Maple [A] time = 3.002, size = 0, normalized size = 0.

$$\int (ex+d)^m (a+b\operatorname{arccosh}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x))^3,x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3 \operatorname{arcosh}(cx)^3 + 3\,ab^2 \operatorname{arcosh}(cx)^2 + 3\,a^2b \operatorname{arcosh}(cx) + a^3\right)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c*x
) + a^3)*(e*x + d)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acosh}(cx))^3 (d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x))**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))**3*(d + e*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(cx) + a)^3 (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(e*x + d)^m, x)
```

3.37 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=79

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)^2}{e(m+1)}-\frac{2bc\text{Unintegrable}\left(\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}},x\right)}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(e*(1 + m)) - (2*b*c*Unintegrabl
e[((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
x])/(e*(1 + m))
```

Rubi [A] time = 0.321708, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
)/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{e(1+m)}-\frac{(2bc)\int \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e(1+m)}$$

Mathematica [A] time = 0.184084, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 3.13, size = 0, normalized size = 0.

$$\int (ex+d)^m (a+b\operatorname{arccosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*(e*x + d)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{acosh}\,(cx))^2\,(d + ex)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*(d + e*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arcosh}\,(cx) + a)^2 (ex + d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(e*x + d)^m, x)
```

3.38 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=125

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{e(m+1)}-\frac{\sqrt{2}b\sqrt{cx-1}(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(m+1)}$$

```
[Out] -((Sqrt[2]*b*(c*d + e)*Sqrt[-1 + c*x]*(d + e*x)^m*AppellF1[1/2, 1/2, -1 - m
, 3/2, (1 - c*x)/2, (e*(1 - c*x))/(c*d + e)])/(c*e*(1 + m)*((c*(d + e*x))/(
c*d + e))^m)) + ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(e*(1 + m))
```

Rubi [A] time = 0.0811056, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5802, 139, 138}

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{e(m+1)}-\frac{\sqrt{2}b\sqrt{cx-1}(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((Sqrt[2]*b*(c*d + e)*Sqrt[-1 + c*x]*(d + e*x)^m*AppellF1[1/2, 1/2, -1 - m
, 3/2, (1 - c*x)/2, (e*(1 - c*x))/(c*d + e)])/(c*e*(1 + m)*((c*(d + e*x))/(
c*d + e))^m)) + ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(e*(1 + m))
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{e(1+m)} - \frac{\left(b(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}\right)\int \frac{\left(\frac{cd}{cd+e}+\frac{cex}{cd+e}\right)^{1+m}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e(1+m)}$$

$$= -\frac{\sqrt{2}b(cd+e)\sqrt{-1+cx}(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(1+m)} +$$

Mathematica [A] time = 0.223621, size = 177, normalized size = 1.42

$$\frac{(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}\left(c(d+ex)\left(a+b\cosh^{-1}(cx)\right)\left(\frac{c(d+ex)}{cd+e}\right)^m - 2be\sqrt{2cx-2}F_1\left(\frac{1}{2};-\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}-\frac{cx}{2},\frac{e-cex}{cd+e}\right) + b\sqrt{2cx-2}(e\right.}{ce(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] ((d + e*x)^m*(-2*b*e*Sqrt[-2 + 2*c*x]*AppellF1[1/2, -1/2, -m, 3/2, 1/2 - (c
*x)/2, (e - c*e*x)/(c*d + e)] + b*(-(c*d) + e)*Sqrt[-2 + 2*c*x]*AppellF1[1/
2, 1/2, -m, 3/2, 1/2 - (c*x)/2, (e - c*e*x)/(c*d + e)] + c*(d + e*x)*((c*(d
 + e*x))/(c*d + e))^m*(a + b*ArcCosh[c*x])))/(c*e*(1 + m)*((c*(d + e*x))/(c
```

```
*d + e))^m)
```

Maple [F] time = 3.666, size = 0, normalized size = 0.

$$\int (ex+d)^m (a+b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b\operatorname{arcosh}(cx)+a)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\operatorname{acosh}(cx))\,(d+ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(cx) + a)(ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(e*x + d)^m, x)
```

3.39 $\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(d+ex)^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

Rubi [A] time = 0.0300302, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.359003, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

Maple [A] time = 1.266, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^m}{b\,\mathrm{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*acosh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b \operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

3.40 $\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

Rubi [A] time = 0.0286618, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.775035, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A] time = 1.105, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^2-1\right)\sqrt{cx+1}\sqrt{cx-1}(ex+d)^m+\left(c^3x^3-cx\right)(ex+d)^m}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}+\int\frac{}{abc^5ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1)*(e*x + d)^m + (c^3*x^3 - c*x)*(
e*x + d)^m)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c +
(b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*e*(m + 1)*x^3 + c^3*d*x^2 - c*
e*(m - 1)*x + c*d)*(c*x + 1)*(c*x - 1)*(e*x + d)^m + (2*c^4*e*(m + 1)*x^4 +
 2*c^4*d*x^3 - c^2*e*(3*m + 1)*x^2 - c^2*d*x + e*m)*sqrt(c*x + 1)*sqrt(c*x
- 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 - 2*c^3*e*(m + 1)*x^3 - 2
*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5 + a*b*c^5*d*x
^4 - 2*a*b*c^3*e*x^3 - 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d + (a*b*c^3*e*x
^3 + a*b*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*e*x^4 + a*b*c^4*d*x^3
- a*b*c^2*e*x^2 - a*b*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e*x^5
 + b^2*c^5*d*x^4 - 2*b^2*c^3*e*x^3 - 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d
+ (b^2*c^3*e*x^3 + b^2*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e*x^4 +
b^2*c^4*d*x^3 - b^2*c^2*e*x^2 - b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**m/(a + b*acosh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(b \operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a)^2, x)
```

3.41 $\int \left(c+dx^2\right)^4 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=370

$$\frac{2d^2\left(1-a^2x^2\right)^3\left(63a^4c^2+90a^2cd+35d^2\right)}{525a^9\sqrt{ax-1}\sqrt{ax+1}}-\frac{4d\left(1-a^2x^2\right)^2\left(189a^4c^2d+105a^6c^3+135a^2cd^2+35d^3\right)}{945a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(378\right.}{}$$

```
[Out] ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*(
1 - a^2*x^2))/(315*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d*(105*a^6*c^3 +
189*a^4*c^2*d + 135*a^2*c*d^2 + 35*d^3)*(1 - a^2*x^2)^2)/(945*a^9*Sqrt[-1 +
 a*x]*Sqrt[1 + a*x]) + (2*d^2*(63*a^4*c^2 + 90*a^2*c*d + 35*d^2)*(1 - a^2*x
^2)^3)/(525*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d^3*(9*a^2*c + 7*d)*(1 -
 a^2*x^2)^4)/(441*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (d^4*(1 - a^2*x^2)^5)
/(81*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^4*x*ArcCosh[a*x] + (4*c^3*d*x^3*
ArcCosh[a*x])/3 + (6*c^2*d^2*x^5*ArcCosh[a*x])/5 + (4*c*d^3*x^7*ArcCosh[a*x
])/7 + (d^4*x^9*ArcCosh[a*x])/9
```

Rubi [A] time = 0.459296, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$\frac{2d^2\left(1-a^2x^2\right)^3\left(63a^4c^2+90a^2cd+35d^2\right)}{525a^9\sqrt{ax-1}\sqrt{ax+1}}-\frac{4d\left(1-a^2x^2\right)^2\left(189a^4c^2d+105a^6c^3+135a^2cd^2+35d^3\right)}{945a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(378\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcCosh[a*x],x]
```

```
[Out] ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*(
1 - a^2*x^2))/(315*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d*(105*a^6*c^3 +
189*a^4*c^2*d + 135*a^2*c*d^2 + 35*d^3)*(1 - a^2*x^2)^2)/(945*a^9*Sqrt[-1 +
 a*x]*Sqrt[1 + a*x]) + (2*d^2*(63*a^4*c^2 + 90*a^2*c*d + 35*d^2)*(1 - a^2*x
^2)^3)/(525*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d^3*(9*a^2*c + 7*d)*(1 -
 a^2*x^2)^4)/(441*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (d^4*(1 - a^2*x^2)^5)
/(81*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^4*x*ArcCosh[a*x] + (4*c^3*d*x^3*
ArcCosh[a*x])/3 + (6*c^2*d^2*x^5*ArcCosh[a*x])/5 + (4*c*d^3*x^7*ArcCosh[a*x
])/7 + (d^4*x^9*ArcCosh[a*x])/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \cosh^{-1}(ax)\,dx = c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(ax)+\frac{1}{9}d^4x$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(ax)+\frac{1}{9}d^4x$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(ax)+\frac{1}{9}d^4x$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(ax)+\frac{1}{9}d^4x$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(ax)+\frac{1}{9}d^4x$$

$$= \frac{\left(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4\right)\left(1-a^2x^2\right)}{315a^9\sqrt{-1+ax}\sqrt{1+ax}} - \frac{4d\left(105a^6c^3+189a^4\right.}{945a}$$

Mathematica [A] time = 0.281725, size = 216, normalized size = 0.58

$$\frac{1}{315}x\cosh^{-1}(ax)\left(378c^2d^2x^4+420c^3dx^2+315c^4+180cd^3x^6+35d^4x^8\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^8\left(23814c^2d^2x^4+44100c^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCosh[a*x],x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(4480*d^4 + 320*a^2*d^3*(81*c + 7*d*x^2) + 4
8*a^4*d^2*(1323*c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 8*a^6*d*(11025*c^3 + 3969
*c^2*d*x^2 + 1215*c*d^2*x^4 + 175*d^3*x^6) + a^8*(99225*c^4 + 44100*c^3*d*x
^2 + 23814*c^2*d^2*x^4 + 8100*c*d^3*x^6 + 1225*d^4*x^8)))/(99225*a^9) + (x*
(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*d^3*x^6 + 35*d^4*x^8)*Ar
cCosh[a*x])/315
```

Maple [A] time = 0.03, size = 255, normalized size = 0.7

$$\frac{1}{a}\left(\frac{a\operatorname{arccosh}(ax)\,d^4x^9}{9}+\frac{4\,a\operatorname{arccosh}(ax)\,cd^3x^7}{7}+\frac{6\,a\operatorname{arccosh}(ax)\,c^2d^2x^5}{5}+\frac{4\,a\operatorname{arccosh}(ax)\,c^3dx^3}{3}+\operatorname{arccosh}(ax)\,c^4ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arccosh(a*x),x)
```

```
[Out] 1/a*(1/9*a*arccosh(a*x)*d^4*x^9+4/7*a*arccosh(a*x)*c*d^3*x^7+6/5*a*arccosh(
a*x)*c^2*d^2*x^5+4/3*a*arccosh(a*x)*c^3*d*x^3+arccosh(a*x)*c^4*a*x-1/99225/
a^8*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(1225*a^8*d^4*x^8+8100*a^8*c*d^3*x^6+23814*
a^8*c^2*d^2*x^4+1400*a^6*d^4*x^6+44100*a^8*c^3*d*x^2+9720*a^6*c*d^3*x^4+992
25*a^8*c^4+31752*a^6*c^2*d^2*x^2+1680*a^4*d^4*x^4+88200*a^6*c^3*d+12960*a^4
*c*d^3*x^2+63504*a^4*c^2*d^2+2240*a^2*d^4*x^2+25920*a^2*c*d^3+4480*d^4))
```

Maxima [A] time = 1.13737, size = 520, normalized size = 1.41

$$-\frac{1}{99225}\left(\frac{1225\sqrt{a^2x^2-1}d^4x^8}{a^2}+\frac{8100\sqrt{a^2x^2-1}cd^3x^6}{a^2}+\frac{23814\sqrt{a^2x^2-1}c^2d^2x^4}{a^2}+\frac{1400\sqrt{a^2x^2-1}d^4x^6}{a^4}+\frac{44100\sqrt{a^2x^2-}}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/99225*(1225*sqrt(a^2*x^2 - 1)*d^4*x^8/a^2 + 8100*sqrt(a^2*x^2 - 1)*c*d^3
*x^6/a^2 + 23814*sqrt(a^2*x^2 - 1)*c^2*d^2*x^4/a^2 + 1400*sqrt(a^2*x^2 - 1)
*d^4*x^6/a^4 + 44100*sqrt(a^2*x^2 - 1)*c^3*d*x^2/a^2 + 9720*sqrt(a^2*x^2 -
1)*c*d^3*x^4/a^4 + 99225*sqrt(a^2*x^2 - 1)*c^4/a^2 + 31752*sqrt(a^2*x^2 - 1
)*c^2*d^2*x^2/a^4 + 1680*sqrt(a^2*x^2 - 1)*d^4*x^4/a^6 + 88200*sqrt(a^2*x^2
 - 1)*c^3*d/a^4 + 12960*sqrt(a^2*x^2 - 1)*c*d^3*x^2/a^6 + 63504*sqrt(a^2*x^
2 - 1)*c^2*d^2/a^6 + 2240*sqrt(a^2*x^2 - 1)*d^4*x^2/a^8 + 25920*sqrt(a^2*x^
2 - 1)*c*d^3/a^8 + 4480*sqrt(a^2*x^2 - 1)*d^4/a^10)*a + 1/315*(35*d^4*x^9 +
 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)*arccosh(a*x)
```

Fricas [A] time = 2.33466, size = 590, normalized size = 1.59

$$\frac{315\left(35\,a^9d^4x^9+180\,a^9cd^3x^7+378\,a^9c^2d^2x^5+420\,a^9c^3dx^3+315\,a^9c^4x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(1225\,a^8d^4x^8+99225\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/99225*(315*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 42
0*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (1225*a^8*d
^4*x^8 + 99225*a^8*c^4 + 88200*a^6*c^3*d + 63504*a^4*c^2*d^2 + 100*(81*a^8*
c*d^3 + 14*a^6*d^4)*x^6 + 25920*a^2*c*d^3 + 6*(3969*a^8*c^2*d^2 + 1620*a^6*
c*d^3 + 280*a^4*d^4)*x^4 + 4480*d^4 + 4*(11025*a^8*c^3*d + 7938*a^6*c^2*d^2
 + 3240*a^4*c*d^3 + 560*a^2*d^4)*x^2)*sqrt(a^2*x^2 - 1))/a^9
```

Sympy [A] time = 25.8003, size = 503, normalized size = 1.36

$$\begin{cases} c^4 x \operatorname{acosh}(ax) + \frac{4c^3dx^3 \operatorname{acosh}(ax)}{3} + \frac{6c^2d^2x^5 \operatorname{acosh}(ax)}{5} + \frac{4cd^3x^7 \operatorname{acosh}(ax)}{7} + \frac{d^4x^9 \operatorname{acosh}(ax)}{9} - \frac{c^4\sqrt{a^2x^2-1}}{a} - \frac{4c^3dx^2\sqrt{a^2x^2-1}}{9a} - \frac{6c^2d^2x^4\sqrt{}}{25a} \\ \frac{i\pi\left(c^4x + \frac{4c^3dx^3}{3} + \frac{6c^2d^2x^5}{5} + \frac{4cd^3x^7}{7} + \frac{d^4x^9}{9}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acosh(a*x),x)
```

```
[Out] Piecewise((c**4*x*acosh(a*x) + 4*c**3*d*x**3*acosh(a*x)/3 + 6*c**2*d**2*x**
5*acosh(a*x)/5 + 4*c*d**3*x**7*acosh(a*x)/7 + d**4*x**9*acosh(a*x)/9 - c**4
*sqrt(a**2*x**2 - 1)/a - 4*c**3*d*x**2*sqrt(a**2*x**2 - 1)/(9*a) - 6*c**2*d
**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*c*d**3*x**6*sqrt(a**2*x**2 - 1)/(49
*a) - d**4*x**8*sqrt(a**2*x**2 - 1)/(81*a) - 8*c**3*d*sqrt(a**2*x**2 - 1)/(
9*a**3) - 8*c**2*d**2*x**2*sqrt(a**2*x**2 - 1)/(25*a**3) - 24*c*d**3*x**4*s
qrt(a**2*x**2 - 1)/(245*a**3) - 8*d**4*x**6*sqrt(a**2*x**2 - 1)/(567*a**3)
- 16*c**2*d**2*sqrt(a**2*x**2 - 1)/(25*a**5) - 32*c*d**3*x**2*sqrt(a**2*x**
2 - 1)/(245*a**5) - 16*d**4*x**4*sqrt(a**2*x**2 - 1)/(945*a**5) - 64*c*d**3
*sqrt(a**2*x**2 - 1)/(245*a**7) - 64*d**4*x**2*sqrt(a**2*x**2 - 1)/(2835*a*
*7) - 128*d**4*sqrt(a**2*x**2 - 1)/(2835*a**9), Ne(a, 0)), (I*pi*(c**4*x +
4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x**7/7 + d**4*x**9/9)/2, Tr
ue))
```

Giac [A] time = 1.15932, size = 478, normalized size = 1.29

$$\frac{1}{315}\left(35\,d^4x^9 + 180\,cd^3x^7 + 378\,c^2d^2x^5 + 420\,c^3dx^3 + 315\,c^4x\right)\log\left(ax + \sqrt{a^2x^2-1}\right) - \frac{99225\,\sqrt{a^2x^2-1}a^8c^4 + 44100\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/99225*(99225*sqrt(a^2*x^2 - 1)*a^8*c
^4 + 44100*(a^2*x^2 - 1)^(3/2)*a^6*c^3*d + 132300*sqrt(a^2*x^2 - 1)*a^6*c^3
*d + 23814*(a^2*x^2 - 1)^(5/2)*a^4*c^2*d^2 + 79380*(a^2*x^2 - 1)^(3/2)*a^4*
c^2*d^2 + 8100*(a^2*x^2 - 1)^(7/2)*a^2*c*d^3 + 119070*sqrt(a^2*x^2 - 1)*a^4
*c^2*d^2 + 34020*(a^2*x^2 - 1)^(5/2)*a^2*c*d^3 + 1225*(a^2*x^2 - 1)^(9/2)*d
^4 + 56700*(a^2*x^2 - 1)^(3/2)*a^2*c*d^3 + 6300*(a^2*x^2 - 1)^(7/2)*d^4 + 5
6700*sqrt(a^2*x^2 - 1)*a^2*c*d^3 + 13230*(a^2*x^2 - 1)^(5/2)*d^4 + 14700*(a
^2*x^2 - 1)^(3/2)*d^4 + 11025*sqrt(a^2*x^2 - 1)*d^4)/a^9
```

3.42 $\int \left(c+dx^2\right)^3 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=267

$$-\frac{d\left(1-a^2x^2\right)^2\left(35a^4c^2+42a^2cd+15d^2\right)}{105a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)}{35a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{3d^2\left(1-a^2x^2\right)^3\left(7a^2c+5\right.}{175a^7\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] ((35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*(1 - a^2*x^2))/(35*a^7*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d*(35*a^4*c^2 + 42*a^2*c*d + 15*d^2)*(1 -
a^2*x^2)^2)/(105*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*d^2*(7*a^2*c + 5*d)
*(1 - a^2*x^2)^3)/(175*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d^3*(1 - a^2*x^
2)^4)/(49*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^3*x*ArcCosh[a*x] + c^2*d*x^
3*ArcCosh[a*x] + (3*c*d^2*x^5*ArcCosh[a*x])/5 + (d^3*x^7*ArcCosh[a*x])/7
```

Rubi [A] time = 0.350182, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$-\frac{d\left(1-a^2x^2\right)^2\left(35a^4c^2+42a^2cd+15d^2\right)}{105a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)}{35a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{3d^2\left(1-a^2x^2\right)^3\left(7a^2c+5\right.}{175a^7\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCosh[a*x],x]
```


```
[Out] ((35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*(1 - a^2*x^2))/(35*a^7*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d*(35*a^4*c^2 + 42*a^2*c*d + 15*d^2)*(1 -
a^2*x^2)^2)/(105*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*d^2*(7*a^2*c + 5*d)
*(1 - a^2*x^2)^3)/(175*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d^3*(1 - a^2*x^
2)^4)/(49*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^3*x*ArcCosh[a*x] + c^2*d*x^
3*ArcCosh[a*x] + (3*c*d^2*x^5*ArcCosh[a*x])/5 + (d^3*x^7*ArcCosh[a*x])/7
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \cosh^{-1}(ax)\,dx = c^3x\cosh^{-1}(ax)+c^2dx^3\cosh^{-1}(ax)+\frac{3}{5}cd^2x^5\cosh^{-1}(ax)+\frac{1}{7}d^3x^7\cosh^{-1}(ax)-a\int \frac{x\left(3\right.}{}$$

$$= c^3x\cosh^{-1}(ax)+c^2dx^3\cosh^{-1}(ax)+\frac{3}{5}cd^2x^5\cosh^{-1}(ax)+\frac{1}{7}d^3x^7\cosh^{-1}(ax)-\frac{1}{35}a\int \frac{x}{}$$

$$= c^3x\cosh^{-1}(ax)+c^2dx^3\cosh^{-1}(ax)+\frac{3}{5}cd^2x^5\cosh^{-1}(ax)+\frac{1}{7}d^3x^7\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+\right.}{}$$

$$= c^3x\cosh^{-1}(ax)+c^2dx^3\cosh^{-1}(ax)+\frac{3}{5}cd^2x^5\cosh^{-1}(ax)+\frac{1}{7}d^3x^7\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+\right.}{}$$

$$= c^3x\cosh^{-1}(ax)+c^2dx^3\cosh^{-1}(ax)+\frac{3}{5}cd^2x^5\cosh^{-1}(ax)+\frac{1}{7}d^3x^7\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+\right.}{}$$

$$= \frac{\left(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3\right)\left(1-a^2x^2\right)}{35a^7\sqrt{-1+ax}\sqrt{1+ax}}-\frac{d\left(35a^4c^2+42a^2cd+15d^2\right)\left(1-a^2x^2\right)}{105a^7\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.195587, size = 154, normalized size = 0.58

$$\frac{1}{35}x\cosh^{-1}(ax)\left(35c^2dx^2+35c^3+21cd^2x^4+5d^3x^6\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^6\left(1225c^2dx^2+3675c^3+441cd^2x^4+75d^3x^6\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCosh[a*x],x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(240*d^3 + 24*a^2*d^2*(49*c + 5*d*x^2) + 2*a
^4*d*(1225*c^2 + 294*c*d*x^2 + 45*d^2*x^4) + a^6*(3675*c^3 + 1225*c^2*d*x^2
 + 441*c*d^2*x^4 + 75*d^3*x^6)))/(3675*a^7) + (x*(35*c^3 + 35*c^2*d*x^2 + 2
1*c*d^2*x^4 + 5*d^3*x^6)*ArcCosh[a*x])/35
```

Maple [A] time = 0.012, size = 176, normalized size = 0.7

$$\frac{1}{a}\left(\frac{a\operatorname{arccosh}(ax)\,d^3x^7}{7}+\frac{3\,a\operatorname{arccosh}(ax)\,cd^2x^5}{5}+a\operatorname{arccosh}(ax)\,c^2dx^3+\operatorname{arccosh}(ax)\,c^3ax-\frac{75\,a^6d^3x^6+441\,a^6cd^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arccosh(a*x),x)
```

```
[Out] 1/a*(1/7*a*arccosh(a*x)*d^3*x^7+3/5*a*arccosh(a*x)*c*d^2*x^5+a*arccosh(a*x)
*c^2*d*x^3+arccosh(a*x)*c^3*a*x-1/3675/a^6*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(75*
a^6*d^3*x^6+441*a^6*c*d^2*x^4+1225*a^6*c^2*d*x^2+90*a^4*d^3*x^4+3675*a^6*c^
3+588*a^4*c*d^2*x^2+2450*a^4*c^2*d+120*a^2*d^3*x^2+1176*a^2*c*d^2+240*d^3))
```

Maxima [A] time = 1.11726, size = 347, normalized size = 1.3

$$-\frac{1}{3675}\left(\frac{75\sqrt{a^2x^2-1}d^3x^6}{a^2}+\frac{441\sqrt{a^2x^2-1}cd^2x^4}{a^2}+\frac{1225\sqrt{a^2x^2-1}c^2dx^2}{a^2}+\frac{90\sqrt{a^2x^2-1}d^3x^4}{a^4}+\frac{3675\sqrt{a^2x^2-1}c^3}{a^2}+\frac{58\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/3675*(75*sqrt(a^2*x^2 - 1)*d^3*x^6/a^2 + 441*sqrt(a^2*x^2 - 1)*c*d^2*x^4
/a^2 + 1225*sqrt(a^2*x^2 - 1)*c^2*d*x^2/a^2 + 90*sqrt(a^2*x^2 - 1)*d^3*x^4/
a^4 + 3675*sqrt(a^2*x^2 - 1)*c^3/a^2 + 588*sqrt(a^2*x^2 - 1)*c*d^2*x^2/a^4
+ 2450*sqrt(a^2*x^2 - 1)*c^2*d/a^4 + 120*sqrt(a^2*x^2 - 1)*d^3*x^2/a^6 + 11
76*sqrt(a^2*x^2 - 1)*c*d^2/a^6 + 240*sqrt(a^2*x^2 - 1)*d^3/a^8)*a + 1/35*(5
*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*arccosh(a*x)
```

Fricas [A] time = 2.22104, size = 406, normalized size = 1.52

$$\frac{105\left(5a^7d^3x^7+21a^7cd^2x^5+35a^7c^2dx^3+35a^7c^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(75a^6d^3x^6+3675a^6c^3+2450a^4c^2d+1176\cdots\right.}{3675a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7*c^2*d*x^3 + 35*a^7*c
^3*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (75*a^6*d^3*x^6 + 3675*a^6*c^3 + 2450*
a^4*c^2*d + 1176*a^2*c*d^2 + 9*(49*a^6*c*d^2 + 10*a^4*d^3)*x^4 + 240*d^3 +
(1225*a^6*c^2*d + 588*a^4*c*d^2 + 120*a^2*d^3)*x^2)*sqrt(a^2*x^2 - 1))/a^7
```

Sympy [A] time = 9.39132, size = 328, normalized size = 1.23

$$\begin{cases} c^3x\operatorname{acosh}(ax) + c^2dx^3\operatorname{acosh}(ax) + \frac{3cd^2x^5\operatorname{acosh}(ax)}{5} + \frac{d^3x^7\operatorname{acosh}(ax)}{7} - \frac{c^3\sqrt{a^2x^2-1}}{a} - \frac{c^2dx^2\sqrt{a^2x^2-1}}{3a} - \frac{3cd^2x^4\sqrt{a^2x^2-1}}{25a} - \frac{d^3x^6\sqrt{a^2x^2 \\ \frac{i\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*acosh(a*x),x)
```

```
[Out] Piecewise((c**3*x*acosh(a*x) + c**2*d*x**3*acosh(a*x) + 3*c*d**2*x**5*acosh
(a*x)/5 + d**3*x**7*acosh(a*x)/7 - c**3*sqrt(a**2*x**2 - 1)/a - c**2*d*x**2
*sqrt(a**2*x**2 - 1)/(3*a) - 3*c*d**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - d**
3*x**6*sqrt(a**2*x**2 - 1)/(49*a) - 2*c**2*d*sqrt(a**2*x**2 - 1)/(3*a**3) -
 4*c*d**2*x**2*sqrt(a**2*x**2 - 1)/(25*a**3) - 6*d**3*x**4*sqrt(a**2*x**2 -
 1)/(245*a**3) - 8*c*d**2*sqrt(a**2*x**2 - 1)/(25*a**5) - 8*d**3*x**2*sqrt(
a**2*x**2 - 1)/(245*a**5) - 16*d**3*sqrt(a**2*x**2 - 1)/(245*a**7), Ne(a, 0
)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7)/2, True))
```

Giac [A] time = 1.13379, size = 325, normalized size = 1.22

$$\frac{1}{35}\left(5\,d^3x^7 + 21\,cd^2x^5 + 35\,c^2dx^3 + 35\,c^3x\right)\log\left(ax + \sqrt{a^2x^2-1}\right) - \frac{3675\,\sqrt{a^2x^2-1}a^6c^3 + 1225\left(a^2x^2-1\right)^{\frac{3}{2}}a^4c^2d + 3675$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*log(a*x + sqrt(a^
2*x^2 - 1)) - 1/3675*(3675*sqrt(a^2*x^2 - 1)*a^6*c^3 + 1225*(a^2*x^2 - 1)^(
3/2)*a^4*c^2*d + 3675*sqrt(a^2*x^2 - 1)*a^4*c^2*d + 441*(a^2*x^2 - 1)^(5/2)
*a^2*c*d^2 + 1470*(a^2*x^2 - 1)^(3/2)*a^2*c*d^2 + 75*(a^2*x^2 - 1)^(7/2)*d^
3 + 2205*sqrt(a^2*x^2 - 1)*a^2*c*d^2 + 315*(a^2*x^2 - 1)^(5/2)*d^3 + 525*(a
^2*x^2 - 1)^(3/2)*d^3 + 525*sqrt(a^2*x^2 - 1)*d^3)/a^7
```

3.43 $\int (c+dx^2)^2 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=181

$$\frac{(1-a^2x^2)(15a^4c^2+10a^2cd+3d^2)}{15a^5\sqrt{ax-1}\sqrt{ax+1}}-\frac{2d(1-a^2x^2)^2(5a^2c+3d)}{45a^5\sqrt{ax-1}\sqrt{ax+1}}+\frac{d^2(1-a^2x^2)^3}{25a^5\sqrt{ax-1}\sqrt{ax+1}}+c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh$$

```
[Out] ((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*(1 - a^2*x^2))/(15*a^5*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) - (2*d*(5*a^2*c + 3*d)*(1 - a^2*x^2)^2)/(45*a^5*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]) + (d^2*(1 - a^2*x^2)^3)/(25*a^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]) + c^2*x*ArcCosh[a*x] + (2*c*d*x^3*ArcCosh[a*x])/3 + (d^2*x^5*ArcCosh[a*x
])/5
```

Rubi [A] time = 0.188672, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {194, 5705, 12, 520, 1247, 698}

$$\frac{(1-a^2x^2)(15a^4c^2+10a^2cd+3d^2)}{15a^5\sqrt{ax-1}\sqrt{ax+1}}-\frac{2d(1-a^2x^2)^2(5a^2c+3d)}{45a^5\sqrt{ax-1}\sqrt{ax+1}}+\frac{d^2(1-a^2x^2)^3}{25a^5\sqrt{ax-1}\sqrt{ax+1}}+c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcCosh[a*x],x]
```



```
[Out] ((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*(1 - a^2*x^2))/(15*a^5*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) - (2*d*(5*a^2*c + 3*d)*(1 - a^2*x^2)^2)/(45*a^5*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]) + (d^2*(1 - a^2*x^2)^3)/(25*a^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]) + c^2*x*ArcCosh[a*x] + (2*c*d*x^3*ArcCosh[a*x])/3 + (d^2*x^5*ArcCosh[a*x
])/5
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
```

```
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int\left(c+dx^2\right)^2\cosh^{-1}(ax)\,dx=c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-a\int\frac{x\left(15c^2+10cdx^2+3d^2x^4\right)}{15\sqrt{-1+ax}\sqrt{1+ax}}$$

$$=c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{1}{15}a\int\frac{x\left(15c^2+10cdx^2+3d^2x\right.}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$=c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)\int\frac{x\left(15c^2+10cdx^2+3\right.}{\sqrt{-1+a^2x^2}}}{15\sqrt{-1+ax}\sqrt{1+ax}}$$

$$=c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int\frac{15c^2+1}{\sqrt{}}\right.}{30\sqrt{-1+ax}\sqrt{1}}$$

$$=c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int\left(\frac{15a^4c}{a}\right.\right.}{}$$

$$=\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\left(1-a^2x^2\right)}{15a^5\sqrt{-1+ax}\sqrt{1+ax}}-\frac{2d\left(5a^2c+3d\right)\left(1-a^2x^2\right)^2}{45a^5\sqrt{-1+ax}\sqrt{1+ax}}+\frac{d^2\left(1-a^2x^2\right)^3}{25a^5\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.145899, size = 103, normalized size = 0.57

$$\cosh^{-1}(ax)\left(c^2x+\frac{2}{3}cdx^3+\frac{d^2x^5}{5}\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^4\left(225c^2+50cdx^2+9d^2x^4\right)+4a^2d\left(25c+3dx^2\right)+24d^2\right)}{225a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcCosh[a*x],x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(24*d^2 + 4*a^2*d*(25*c + 3*d*x^2) + a^4*(22
5*c^2 + 50*c*d*x^2 + 9*d^2*x^4)))/(225*a^5) + (c^2*x + (2*c*d*x^3)/3 + (d^2
*x^5)/5)*ArcCosh[a*x]
```

Maple [A] time = 0.011, size = 113, normalized size = 0.6

$$\frac{1}{a}\left(\frac{a\text{arccosh}\,(ax)\,x^5d^2}{5}+\frac{2\,a\text{arccosh}\,(ax)\,cdx^3}{3}+\text{arccosh}\,(ax)\,c^2ax-\frac{9\,a^4d^2x^4+50\,a^4cdx^2+225\,a^4c^2+12\,a^2d^2x^2+100}{225\,a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arccosh(a*x),x)
```

```
[Out] 1/a*(1/5*a*arccosh(a*x)*x^5*d^2+2/3*a*arccosh(a*x)*c*d*x^3+arccosh(a*x)*c^2
*a*x-1/225/a^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(9*a^4*d^2*x^4+50*a^4*c*d*x^2+22
5*a^4*c^2+12*a^2*d^2*x^2+100*a^2*c*d+24*d^2))
```

Maxima [A] time = 1.12145, size = 208, normalized size = 1.15

$$-\frac{1}{225}\left(\frac{9\sqrt{a^2x^2-1}d^2x^4}{a^2}+\frac{50\sqrt{a^2x^2-1}cdx^2}{a^2}+\frac{225\sqrt{a^2x^2-1}c^2}{a^2}+\frac{12\sqrt{a^2x^2-1}d^2x^2}{a^4}+\frac{100\sqrt{a^2x^2-1}cd}{a^4}+\frac{24\sqrt{a^2x^2-\cdots}}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/225*(9*sqrt(a^2*x^2 - 1)*d^2*x^4/a^2 + 50*sqrt(a^2*x^2 - 1)*c*d*x^2/a^2
+ 225*sqrt(a^2*x^2 - 1)*c^2/a^2 + 12*sqrt(a^2*x^2 - 1)*d^2*x^2/a^4 + 100*sq
rt(a^2*x^2 - 1)*c*d/a^4 + 24*sqrt(a^2*x^2 - 1)*d^2/a^6)*a + 1/15*(3*d^2*x^5
 + 10*c*d*x^3 + 15*c^2*x)*arccosh(a*x)
```

Fricas [A] time = 2.1763, size = 269, normalized size = 1.49

$$\frac{15\left(3a^5d^2x^5+10a^5cdx^3+15a^5c^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(9a^4d^2x^4+225a^4c^2+100a^2cd+2\left(25a^4cd+6a^2d^2\right)x^2+\cdots\right.}{225a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a^5*c^2*x)*log(a*x + sqrt(a^
2*x^2 - 1)) - (9*a^4*d^2*x^4 + 225*a^4*c^2 + 100*a^2*c*d + 2*(25*a^4*c*d +
6*a^2*d^2)*x^2 + 24*d^2)*sqrt(a^2*x^2 - 1))/a^5
```

Sympy [A] time = 2.93482, size = 199, normalized size = 1.1

$$\begin{cases} c^2x\operatorname{acosh}(ax)+\frac{2cdx^3\operatorname{acosh}(ax)}{3}+\frac{d^2x^5\operatorname{acosh}(ax)}{5}-\frac{c^2\sqrt{a^2x^2-1}}{a}-\frac{2cdx^2\sqrt{a^2x^2-1}}{9a}-\frac{d^2x^4\sqrt{a^2x^2-1}}{25a}-\frac{4cd\sqrt{a^2x^2-1}}{9a^3}-\frac{4d^2x^2\sqrt{a^2x^2-1}}{75a^3}-\frac{8d^2\cdots}{\cdots} \\ \frac{i\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*acosh(a*x),x)
```

```
[Out] Piecewise((c**2*x*acosh(a*x) + 2*c*d*x**3*acosh(a*x)/3 + d**2*x**5*acosh(a*
x)/5 - c**2*sqrt(a**2*x**2 - 1)/a - 2*c*d*x**2*sqrt(a**2*x**2 - 1)/(9*a) -
d**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*c*d*sqrt(a**2*x**2 - 1)/(9*a**3) -
 4*d**2*x**2*sqrt(a**2*x**2 - 1)/(75*a**3) - 8*d**2*sqrt(a**2*x**2 - 1)/(75
*a**5), Ne(a, 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))
```

Giac [A] time = 1.11683, size = 203, normalized size = 1.12

$$\frac{1}{15}\left(3\,d^2x^5+10\,cdx^3+15\,c^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{225\,\sqrt{a^2x^2-1}a^4c^2+50\left(a^2x^2-1\right)^{\frac{3}{2}}a^2cd+150\,\sqrt{a^2x^2-1}a^2cd+9}{225\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/2
25*(225*sqrt(a^2*x^2 - 1)*a^4*c^2 + 50*(a^2*x^2 - 1)^(3/2)*a^2*c*d + 150*sq
rt(a^2*x^2 - 1)*a^2*c*d + 9*(a^2*x^2 - 1)^(5/2)*d^2 + 30*(a^2*x^2 - 1)^(3/2
)*d^2 + 45*sqrt(a^2*x^2 - 1)*d^2)/a^5
```

3.44 $\int \left(c+dx^2\right)\cosh^{-1}(ax)\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(9a^2c+2d\right)}{9a^3}+cx\cosh^{-1}(ax)-\frac{dx^2\sqrt{ax-1}\sqrt{ax+1}}{9a}+\frac{1}{3}dx^3\cosh^{-1}(ax)$$

```
[Out] -((9*a^2*c + 2*d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) - (d*x^2*Sqrt[-1 +
a*x]*Sqrt[1 + a*x])/(9*a) + c*x*ArcCosh[a*x] + (d*x^3*ArcCosh[a*x])/3
```

Rubi [A] time = 0.0710306, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5705, 460, 74}

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(9a^2c+2d\right)}{9a^3}+cx\cosh^{-1}(ax)-\frac{dx^2\sqrt{ax-1}\sqrt{ax+1}}{9a}+\frac{1}{3}dx^3\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcCosh[a*x],x]
```

```
[Out] -((9*a^2*c + 2*d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) - (d*x^2*Sqrt[-1 +
a*x]*Sqrt[1 + a*x])/(9*a) + c*x*ArcCosh[a*x] + (d*x^3*ArcCosh[a*x])/3
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \left(c+dx^2\right)\cosh^{-1}(ax)\,dx = cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)-a\int \frac{x\left(c+\frac{dx^2}{3}\right)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx$$

$$= -\frac{dx^2\sqrt{-1+ax}\sqrt{1+ax}}{9a}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)+\frac{1}{9}\left(a\left(-9c-\frac{2d}{a^2}\right)\right)\int \frac{}{\sqrt{-1+}}$$

$$= -\frac{\left(9a^2c+2d\right)\sqrt{-1+ax}\sqrt{1+ax}}{9a^3}-\frac{dx^2\sqrt{-1+ax}\sqrt{1+ax}}{9a}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(a$$

Mathematica [A] time = 0.0596275, size = 60, normalized size = 0.71

$$\cosh^{-1}(ax)\left(cx+\frac{dx^3}{3}\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^2\left(9c+dx^2\right)+2d\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCosh[a*x],x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2*d + a^2*(9*c + d*x^2)))/(9*a^3) + (c*x +
(d*x^3)/3)*ArcCosh[a*x]
```

Maple [A] time = 0.009, size = 62, normalized size = 0.7

$$\frac{1}{a}\left(\frac{a\mathrm{arccosh}\,(ax)\,dx^3}{3}+\mathrm{arccosh}\,(ax)\,cax-\frac{a^2dx^2+9\,a^2c+2\,d}{9\,a^2}\sqrt{ax-1}\sqrt{ax+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arccosh(a*x),x)
```

```
[Out] 1/a*(1/3*a*arccosh(a*x)*d*x^3+arccosh(a*x)*c*a*x-1/9/a^2*(a*x-1)^(1/2)*(a*x
+1)^(1/2)*(a^2*d*x^2+9*a^2*c+2*d))
```

Maxima [A] time = 1.14981, size = 100, normalized size = 1.19

$$-\frac{1}{9}\left(\frac{\sqrt{a^2x^2-1}dx^2}{a^2}+\frac{9\sqrt{a^2x^2-1}c}{a^2}+\frac{2\sqrt{a^2x^2-1}d}{a^4}\right)a+\frac{1}{3}\left(dx^3+3\,cx\right)\operatorname{arcosh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/9*(sqrt(a^2*x^2 - 1)*d*x^2/a^2 + 9*sqrt(a^2*x^2 - 1)*c/a^2 + 2*sqrt(a^2*
x^2 - 1)*d/a^4)*a + 1/3*(d*x^3 + 3*c*x)*arccosh(a*x)
```

Fricas [A] time = 2.35689, size = 154, normalized size = 1.83

$$\frac{3\left(a^3dx^3+3\,a^3cx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(a^2dx^2+9\,a^2c+2\,d\right)\sqrt{a^2x^2-1}}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(a^3*d*x^3 + 3*a^3*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (a^2*d*x^2 +
9*a^2*c + 2*d)*sqrt(a^2*x^2 - 1))/a^3
```

Sympy [A] time = 0.742982, size = 90, normalized size = 1.07

$$\begin{cases} cx\operatorname{acosh}\left(ax\right)+\frac{dx^3\operatorname{acosh}\left(ax\right)}{3}-\frac{c\sqrt{a^2x^2-1}}{a}-\frac{dx^2\sqrt{a^2x^2-1}}{9a}-\frac{2d\sqrt{a^2x^2-1}}{9a^3} & \text{for } a\neq 0\\ \frac{i\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acosh(a*x),x)
```

```
[Out] Piecewise((c*x*acosh(a*x) + d*x**3*acosh(a*x)/3 - c*sqrt(a**2*x**2 - 1)/a -
 d*x**2*sqrt(a**2*x**2 - 1)/(9*a) - 2*d*sqrt(a**2*x**2 - 1)/(9*a**3), Ne(a,
 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

Giac [A] time = 1.13231, size = 107, normalized size = 1.27

$$\frac{1}{3}\left(dx^3+3\,cx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{9\,\sqrt{a^2x^2-1}a^2c+\left(a^2x^2-1\right)^{\frac{3}{2}}d+3\,\sqrt{a^2x^2-1}d}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*(d*x^3 + 3*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/9*(9*sqrt(a^2*x^2 - 1)
*a^2*c + (a^2*x^2 - 1)^(3/2)*d + 3*sqrt(a^2*x^2 - 1)*d)/a^3
```

3.45 $\int \frac{\cosh^{-1}(ax)}{c+dx^2}\,dx$

Optimal. Leaf size=481

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cos}}{\sqrt{a^2(-c)-}}\right.}{2\sqrt{-c}\sqrt{d}}$$

```
[Out] (ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c)
- d])])/(2*Sqrt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x
])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) + (ArcCosh[a*x]
*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sq
rt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c
] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^Ar
cCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + Poly
Log[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(2*Sqrt[
-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a
^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a
*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(2*Sqrt[-c]*Sqrt[d])
```

Rubi [A] time = 0.757693, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cos}}{\sqrt{a^2(-c)-}}\right.}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2),x]
```

```
[Out] (ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c)
- d])])/(2*Sqrt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x
])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) + (ArcCosh[a*x]
*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sq
rt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c
] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^Ar
cCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + Poly
Log[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(2*Sqrt[
-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a
```

```
^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a
*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(2*Sqrt[-c]*Sqrt[d])
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\cosh^{-1}(ax)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\sqrt{-c}\cosh^{-1}(ax)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\sinh(x)}{a\sqrt{-c}-\sqrt{d}\cosh(x)}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}} - \frac{\text{Subst}\left(\int \frac{x\sinh(x)}{a\sqrt{-c}+\sqrt{d}\cosh(x)}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x x}{a\sqrt{-c}-\sqrt{-a^2c-d}-\sqrt{d}e^x}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}} - \frac{\text{Subst}\left(\int \frac{e^x x}{a\sqrt{-c}+\sqrt{-a^2c-d}-\sqrt{d}e^x}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e}{a\sqrt{-}}\right.}{2\sqrt{-c}\sqrt{d}}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e}{a\sqrt{-}}\right.}{2\sqrt{-c}\sqrt{d}}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e}{a\sqrt{-}}\right.}{2\sqrt{-c}\sqrt{d}}$$

Mathematica [A] time = 0.37311, size = 375, normalized size = 0.78

$$\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}-a\sqrt{-c}}\right)-\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax}}{\sqrt{a^2(-c)-d}+a\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c
) - d])]) + ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(-(a*Sqrt[-c]) +
Sqrt[-(a^2*c) - d])] + ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqr
t[-c] + Sqrt[-(a^2*c) - d])] - ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x]
)/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])] + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])] - PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(-
```

```
(a*Sqrt[-c]) + Sqrt[-(a^2*c) - d])] - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])
/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d]))] + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d])
```

Maple [C] time = 0.623, size = 214, normalized size = 0.4

$$\frac{a}{2}\sum_{\_R1=\mathrm{RootOf}\left(d\_Z^4+\left(4a^2c+2d\right)\_Z^2+d\right)}\frac{\_R1}{\_R1^2d+2a^2c+d}\left(\mathrm{arccosh}\,(ax)\ln\left(\frac{1}{\_R1}\left(\_R1-ax-\sqrt{ax-1}\sqrt{ax+1}\right)\right)+\mathrm{dilog}\left(\frac{1}{\_R1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c),x)
```

```
[Out] 1/2*a*sum(_R1/(_R1^2*d+2*a^2*c+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(
a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=Ro
otOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))-1/2*a*sum(1/_R1/(_R1^2*d+2*a^2*c+d)*(arc
cosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcosh}\,(ax)}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(d*x^2 + c), x)
```

3.46 $\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=774

$$\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(}}{\sqrt{a^2(-c)-d}+a}\right.}{4(-c)^{3/2}\sqrt{d}}$$

```
[Out] -ArcCosh[a*x]/(4*c*Sqrt[d]*(Sqrt[-c] - Sqrt[d]*x)) + ArcCosh[a*x]/(4*c*Sqrt
[d]*(Sqrt[-c] + Sqrt[d]*x)) + (a*ArcTanh[(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[1
 + a*x])/(Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[-1 + a*x])])/(2*c*Sqrt[a*Sqrt[-c]
 - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (a*ArcTanh[(Sqrt[a*Sqrt[-
c] + Sqrt[d]]*Sqrt[1 + a*x])/(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[-1 + a*x])])/
(2*c*Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (ArcC
osh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])
])/(4*(-c)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) - (ArcCosh[a*x]*
Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(4*(-c
)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-
c] + Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E
^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) -
 PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(4*
(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + S
qrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) - PolyLog[2, (Sqrt[d]*E^ArcCosh
[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(4*(-c)^(3/2)*Sqrt[d])
```

Rubi [A] time = 1.1014, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(}}{\sqrt{a^2(-c)-d}+a}\right.}{4(-c)^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -ArcCosh[a*x]/(4*c*Sqrt[d]*(Sqrt[-c] - Sqrt[d]*x)) + ArcCosh[a*x]/(4*c*Sqrt
[d]*(Sqrt[-c] + Sqrt[d]*x)) + (a*ArcTanh[(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[1
 + a*x])/(Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[-1 + a*x])])/(2*c*Sqrt[a*Sqrt[-c]
 - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (a*ArcTanh[(Sqrt[a*Sqrt[-
c] + Sqrt[d]]*Sqrt[1 + a*x])/(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[-1 + a*x])])/
(2*c*Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (ArcC
osh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])
])/(4*(-c)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) - (ArcCosh[a*x]*
Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(4*(-c
)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-
c] + Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E
^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) -
 PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(4*
(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + S
qrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) - PolyLog[2, (Sqrt[d]*E^ArcCosh
[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(4*(-c)^(3/2)*Sqrt[d])
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \int \left(-\frac{d\cosh^{-1}(ax)}{4c\left(\sqrt{-c}\sqrt{d}-dx\right)^2} - \frac{d\cosh^{-1}(ax)}{4c\left(\sqrt{-c}\sqrt{d}+dx\right)^2} - \frac{d\cosh^{-1}(ax)}{2c\left(-cd-d^2x^2\right)}\right)dx$$

$$= -\frac{d\int \frac{\cosh^{-1}(ax)}{\left(\sqrt{-c}\sqrt{d}-dx\right)^2}\,dx}{4c} - \frac{d\int \frac{\cosh^{-1}(ax)}{\left(\sqrt{-c}\sqrt{d}+dx\right)^2}\,dx}{4c} - \frac{d\int \frac{\cosh^{-1}(ax)}{-cd-d^2x^2}\,dx}{2c}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\left(\sqrt{-c}\sqrt{d}-dx\right)}\,dx}{4c} - \frac{a\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\left(\sqrt{-c\cdots}\right.}}{4c}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4(-c)^{3/2}} + \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4(-c)^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{\cdots}{a\sqrt{-c}\cdots}\right.}{\cdots}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a\cdots}}\right.}{2c\sqrt{a\sqrt{-c}-\sqrt{d}\cdots}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a\cdots}}\right.}{2c\sqrt{a\sqrt{-c}-\sqrt{d}\cdots}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a\cdots}}\right.}{2c\sqrt{a\sqrt{-c}-\sqrt{d}\cdots}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a\cdots}}\right.}{2c\sqrt{a\sqrt{-c}-\sqrt{d}\cdots}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a\cdots}}\right.}{2c\sqrt{a\sqrt{-c}-\sqrt{d}\cdots}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{a\cdots}}\right.}{2c\sqrt{a\sqrt{-c}-\sqrt{d}\cdots}}$$

Mathematica [C] time = 1.22343, size = 687, normalized size = 0.89

$$i\left(2\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}-ia\sqrt{c}}\right)+2\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+ia\sqrt{c}}\right)+\cosh^{-1}(ax)\left(-\cosh^{-1}(ax)+2\left(\log\left(1+\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{-\sqrt{a^2(-c)-d}+ia\sqrt{}}\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*(ArcCosh[a*x]/((-I)*Sqrt[c] + Sqrt[d]*x) + (a*Log[(2*d*(I*Sqrt[d
] + a^2*Sqrt[c]*x - I*Sqrt[-(a^2*c) - d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(a*
Sqrt[-(a^2*c) - d]*(Sqrt[c] + I*Sqrt[d]*x))])/Sqrt[-(a^2*c) - d]) - 2*Sqrt[
c]*(-(ArcCosh[a*x]/(I*Sqrt[c] + Sqrt[d]*x)) - (a*Log[(2*d*(-Sqrt[d] - I*a^2
*Sqrt[c]*x + Sqrt[-(a^2*c) - d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(a*Sqrt[-(a^
2*c) - d]*(I*Sqrt[c] + Sqrt[d]*x))])/Sqrt[-(a^2*c) - d]) + I*(ArcCosh[a*x]*
(-ArcCosh[a*x] + 2*(Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] - Sqrt[-(
a^2*c) - d])] + Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*
c) - d])])) + 2*PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[
-(a^2*c) - d])] + 2*PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sq
rt[-(a^2*c) - d]))]) - I*(ArcCosh[a*x]*(-ArcCosh[a*x] + 2*(Log[1 + (Sqrt[d]
*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[-(a^2*c) - d])] + Log[1 - (Sqrt[d]*
E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*c) - d])])) + 2*PolyLog[2, -((Sqr
t[d]*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[-(a^2*c) - d]))] + 2*PolyLog[2,
 (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*c) - d])]))/(8*c^(3/2)*
Sqrt[d])
```

Maple [C] time = 1.255, size = 1632, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*a^2*arccosh(a*x)*x/c/(a^2*d*x^2+a^2*c)+1/4*a/c*sum(_R1/(_R1^2*d+2*a^2*c
+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-
a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2
```

```
+d))+a^5*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2)
)*c/(a^2*c+d)/d^3+a^3*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arct
anh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2
)-d)*d)^(1/2))/(a^2*c+d)/d^3*(a^2*c*(a^2*c+d))^(1/2)+a^3*(-(2*a^2*c-2*(a^2*
c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/
((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))/(a^2*c+d)/d^2+1/2*a*(-(2*
a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))/c/(a^2*c+d)/
d^2*(a^2*c*(a^2*c+d))^(1/2)-a^3*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^
(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*
c+d))^(1/2)-d)*d)^(1/2))/d^3-a*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(
1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c
+d))^(1/2)-d)*d)^(1/2))/c/d^3*(a^2*c*(a^2*c+d))^(1/2)-1/2*a*(-(2*a^2*c-2*(a
^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2
))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))/c/d^2+a^5*((2*a^2*c+2*
(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))*c/(a^2*c+d)/d^3-a^3*((
2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))/(a^2*c+d)/d^
3*(a^2*c*(a^2*c+d))^(1/2)+a^3*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/
2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))
^(1/2)+d)*d)^(1/2))/(a^2*c+d)/d^2-1/2*a*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)
+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*
(a^2*c+d))^(1/2)+d)*d)^(1/2))/c/(a^2*c+d)/d^2*(a^2*c*(a^2*c+d))^(1/2)-a^3*(
(2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*
(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))/d^3+a*((2*a
^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x
+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))/c/d^3*(a^2*c*(a
^2*c+d))^(1/2)-1/2*a*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan
(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)
*d)^(1/2))/c/d^2-1/4*a/c*sum(1/_R1/(_R1^2*d+2*a^2*c+d)*(arccosh(a*x)*ln((_R
1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x-1)^(1/2)*(a*x+1
)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(d*x^2 + c)^2, x)
```

3.47 $\int \sqrt{c+dx^2}\cosh^{-1}(ax)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\cosh^{-1}(ax)\sqrt{c+dx^2},x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

Rubi [A] time = 0.0313553, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+dx^2}\cosh^{-1}(ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcCosh[a*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2}\cosh^{-1}(ax)\,dx=\int \sqrt{c+dx^2}\cosh^{-1}(ax)\,dx$$

Mathematica [A] time = 3.04199, size = 0, normalized size = 0.

$$\int \sqrt{c+dx^2}\cosh^{-1}(ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCosh[a*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

Maple [A] time = 0.256, size = 0, normalized size = 0.

$$\int \sqrt{d x^2 + c}\operatorname{arccosh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

```
[Out] int((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d x^2 + c}\operatorname{arcosh}(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + d x^2}\operatorname{acosh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acosh(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*acosh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + c}\,\operatorname{arcosh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

3.48 $\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable[ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi [A] time = 0.0266791, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[Int][ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}} dx = \int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Mathematica [A] time = 1.89639, size = 0, normalized size = 0.

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Integrate[ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A] time = 0.224, size = 0, normalized size = 0.

$$\int \operatorname{arccosh}(ax) \frac{1}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/sqrt(c + d*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

3.49 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] (x*ArcCosh[a*x])/(c*Sqrt[c + d*x^2]) - (Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]
*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(c*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1
 + a*x])
```

Rubi [A] time = 0.193077, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {191, 5705, 12, 519, 444, 63, 217, 206}

$$\frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x])/(c*Sqrt[c + d*x^2]) - (Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]
*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(c*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1
 + a*x])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
```

```
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}}\,dx \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x}{\sqrt{-1+a^2x^2}\sqrt{c+dx^2}}\,dx}{c\sqrt{-1+ax}\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+a^2x}\sqrt{c+dx}}\,dx, x, x^2\right)}{2c\sqrt{-1+ax}\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{d}{a^2}+\frac{dx^2}{a^2}}}\,dx, x, \sqrt{-1+a^2x^2}\right)}{ac\sqrt{-1+ax}\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a^2}}\,dx, x, \frac{\sqrt{-1+a^2x^2}}{\sqrt{c+dx^2}}\right)}{ac\sqrt{-1+ax}\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{-1+a^2x^2}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\sqrt{-1+ax}\sqrt{1+ax}}
\end{aligned}
$$

<u>Mathematica</u> [C] time = 2.9659, size = 551, normalized size = 5.74

$$
\frac{x\cosh^{-1}(ax)+\frac{2(ax-1)^{3/2}\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(\frac{a\left(\sqrt{d}-ia\sqrt{c}\right)\left(\sqrt{d}x+i\sqrt{c}\right)\sqrt{\frac{\frac{ia\sqrt{c}}{\sqrt{d}}+a(-x)+\frac{i\sqrt{d}x}{\sqrt{c}}+1}{1-ax}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{2-2ax}}\right),\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)}{ax-1}+a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right.\cdots\right)}{a\sqrt{ax+1}\left(a^2c+d\right)\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{1-ax}}\cdots}}{c\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x] + (2*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1 + a*
x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*((a*((-I)*a*Sqrt[c] + Sqrt[d])*(I
```

```
*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*x + (I*Sqrt[d]*x)
/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] +
a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*S
qrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c]) + I*Sqrt[d])*S
qrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((-1 + (I*Sqrt[d]*x
)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*EllipticPi[(2*a*Sqrt[c
])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*(
(I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt
[c] + I*Sqrt[d])^2]))/(a*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-((-1 + (I*Sqrt[d]*
x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(c*Sqrt[c + d*x^2])
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \operatorname{arccosh}(ax)\left(dx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48004, size = 649, normalized size = 6.76

$$\left[\frac{4\sqrt{dx^2+c}dx\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(dx^2+c\right)\sqrt{d}\log\left(8a^4d^2x^4+a^4c^2-6a^2cd+8\left(a^4cd-a^2d^2\right)x^2-4\left(2a^3dx^2+a^3c-\right.\right.}{4\left(cd^2x^2+c^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(d*x^2 + c)*d*x*log(a*x + sqrt(a^2*x^2 - 1)) + (d*x^2 + c)*sqrt
(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2)*x^2 - 4
*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(d) + d^
2))/(c*d^2*x^2 + c^2*d), 1/2*(2*sqrt(d*x^2 + c)*d*x*log(a*x + sqrt(a^2*x^2
- 1)) + (d*x^2 + c)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c - d)*sqrt(a^2*
x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c*d - a*d^2)*
x^2)))/(c*d^2*x^2 + c^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [A] time = 1.1966, size = 111, normalized size = 1.16

$$\frac{x\log\left(ax+\sqrt{a^2x^2-1}\right)}{\sqrt{dx^2+c}c}+\frac{a\log\left(\left|-\sqrt{a^2x^2-1}\sqrt{d}+\sqrt{a^2c+\left(a^2x^2-1\right)d+d}\right|\right)}{c\sqrt{d}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1))/(sqrt(d*x^2 + c)*c) + a*log(abs(-sqrt(a^2*x^
2 - 1)*sqrt(d) + sqrt(a^2*c + (a^2*x^2 - 1)*d + d)))/(c*sqrt(d)*abs(a))
```

3.50 $\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{2\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{3c^2\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{3c\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

```
[Out] (a*(1 - a^2*x^2))/(3*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*
x^2]) + (x*ArcCosh[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcCosh[a*x])/(3*c^
2*Sqrt[c + d*x^2]) - (2*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x
^2])/(a*Sqrt[c + d*x^2])])/(3*c^2*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 0.175823, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {192, 191, 5705, 12, 519, 571, 78, 63, 217, 206}

$$-\frac{2\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{3c^2\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{3c\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(3*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*
x^2]) + (x*ArcCosh[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcCosh[a*x])/(3*c^
2*Sqrt[c + d*x^2]) - (2*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x
^2])/(a*Sqrt[c + d*x^2])])/(3*c^2*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x\left(3c+2dx^2\right)}{3c^2\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}}\,dx$$

$$= \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(3c+2dx^2\right)}{\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x\left(3c+2dx^2\right)}{\sqrt{-1+a^2x^2}\left(c+dx^2\right)^{3/2}}\,dx}{3c^2\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int \frac{3c+2dx}{\sqrt{-1+a^2x}(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subs}\cdots}{3c^2\sqrt{-\cdots}}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(2\sqrt{-1+a^2x^2}\right)\text{Subs}\cdots}{3ac\cdots}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(2\sqrt{-1+a^2x^2}\right)\text{Subs}\cdots}{3ac^2\sqrt{-\cdots}}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{2\sqrt{-1+a^2x^2}\tanh^{-\cdots}}{3c^2\sqrt{d}\sqrt{-1+a\cdots}}$$

Mathematica [C] time = 1.97651, size = 609, normalized size = 3.38

$$\frac{\dfrac{4(ax-1)^{3/2}\left(c+dx^2\right)\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(\dfrac{a\left(\sqrt{d}-ia\sqrt{c}\right)\left(\sqrt{d}x+i\sqrt{c}\right)\sqrt{\frac{\frac{ia\sqrt{c}}{\sqrt{d}}+a(-x)+\frac{i\sqrt{d}x}{\sqrt{c}}+1}{1-ax}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{2-2ax}}\right),\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)}{ax-1}+a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{\left(a^2c+d\right)(c+\cdots}{cd(ax-1\cdots}}\cdots\right)}{a\sqrt{ax+1}\left(a^2c+d\right)\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{1-ax}}}\cdots}{3c^2\left(c+dx^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] (-((a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2))/(a^2*c + d)) + x*(3*c + 2
*d*x^2)*ArcCosh[a*x] + (4*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1
 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*(c + d*x^2)*((a*((-I)*a*Sqrt
[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*
x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt
[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt
[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c
]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((
-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*Ellip
ticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]
*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]
*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(a*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-(
(-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(3
*c^2*(c + d*x^2)^(3/2))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \operatorname{arccosh}(ax)\left(dx^2+c\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73017, size = 1270, normalized size = 7.06

$$\left[\frac{\left(a^2c^3 + \left(a^2cd^2 + d^3\right)x^4 + c^2d + 2\left(a^2c^2d + cd^2\right)x^2\right)\sqrt{d}\log\left(8\,a^4d^2x^4 + a^4c^2 - 6\,a^2cd + 8\left(a^4cd - a^2d^2\right)x^2 - 4\left(2\,a^3dx^2 + \right.\right.}{6\left(a^2c^5d + c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((a^2*c^3 + (a^2*c*d^2 + d^3)*x^4 + c^2*d + 2*(a^2*c^2*d + c*d^2)*x^2)
*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2)*x^
2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(d)
 + d^2) + 2*(2*(a^2*c*d^2 + d^3)*x^3 + 3*(a^2*c^2*d + c*d^2)*x)*sqrt(d*x^2
+ c)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a*c*d^2*x^2 + a*c^2*d)*sqrt(a^2*x^2
- 1)*sqrt(d*x^2 + c))/(a^2*c^5*d + c^4*d^2 + (a^2*c^3*d^3 + c^2*d^4)*x^4 +
2*(a^2*c^4*d^2 + c^3*d^3)*x^2), 1/3*((a^2*c^3 + (a^2*c*d^2 + d^3)*x^4 + c^2
*d + 2*(a^2*c^2*d + c*d^2)*x^2)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c -
d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c
*d - a*d^2)*x^2)) + (2*(a^2*c*d^2 + d^3)*x^3 + 3*(a^2*c^2*d + c*d^2)*x)*sqr
t(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (a*c*d^2*x^2 + a*c^2*d)*sqrt(a^
2*x^2 - 1)*sqrt(d*x^2 + c))/(a^2*c^5*d + c^4*d^2 + (a^2*c^3*d^3 + c^2*d^4)*
x^4 + 2*(a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax)}{\left(c + dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.51 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{7/2}}\, dx$

Optimal. Leaf size=269

$$\frac{2a\left(1-a^2x^2\right)\left(3a^2c+2d\right)}{15c^2\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^2\sqrt{c+dx^2}}-\frac{8\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{15c^3\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{15c\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}$$

```
[Out] (a*(1 - a^2*x^2))/(15*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(3/2)) + (2*a*(3*a^2*c + 2*d)*(1 - a^2*x^2))/(15*c^2*(a^2*c + d)^2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(5*c*(c + d*x^2)
^(5/2)) + (4*x*ArcCosh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCosh[a*x]
)/(15*c^3*Sqrt[c + d*x^2]) - (8*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1
 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(15*c^3*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])
```

Rubi [A] time = 0.805397, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {192, 191, 5705, 12, 519, 6715, 949, 78, 63, 217, 206}

$$\frac{2a\left(1-a^2x^2\right)\left(3a^2c+2d\right)}{15c^2\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^2\sqrt{c+dx^2}}-\frac{8\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{15c^3\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{15c\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(15*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(3/2)) + (2*a*(3*a^2*c + 2*d)*(1 - a^2*x^2))/(15*c^2*(a^2*c + d)^2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(5*c*(c + d*x^2)
^(5/2)) + (4*x*ArcCosh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCosh[a*x]
)/(15*c^3*Sqrt[c + d*x^2]) - (8*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1
 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(15*c^3*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
```

```
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx = \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{15c^3\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}}\,dx$$

$$= \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{\sqrt{-1+a^2x^2}\left(c+dx^2\right)^{5/2}}\,dx}{15c^3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{\sqrt{-1+a^2x}(c+dx)^{5/2}}\,dx,\right.}{30c^3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh}{5c\,(c+}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh}{5c\,(c+}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh}{5c\,(c+}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+dx^2}} + \frac{x\cosh}{5c\,(c+}$$

Mathematica [C] time = 3.21406, size = 655, normalized size = 2.43

$$\frac{16(ax-1)^{3/2}\left(c+dx^2\right)^2\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(\frac{a\left(\sqrt{d}-ia\sqrt{c}\right)\left(\sqrt{d}x+i\sqrt{c}\right)\sqrt{\frac{\frac{ia\sqrt{c}}{\sqrt{d}}+a(-x)+\frac{i\sqrt{d}x}{\sqrt{c}}+1}{1-ax}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{2-2ax}}\right),\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)}{ax-1}+a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{\left(a^2c+d\right)(}{cd(ax}}\right.}{ac^3\sqrt{ax+1}\left(a^2c+d\right)\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{1-ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (-((a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(
7*c + 6*d*x^2)))/(c^2*(a^2*c + d)^2)) + (x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4
)*ArcCosh[a*x])/c^3 + (16*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1
 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*(c + d*x^2)^2*((a*((-I)*a*Sq
rt[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] -
a*x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sq
rt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sq
rt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt
[c]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-
((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*Ell
ipticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[
d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[
c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(a*c^3*(a^2*c + d)*Sqrt[1 + a*x]*S
qrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))
]))/(15*(c + d*x^2)^(5/2))
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int \operatorname{arccosh}(ax)\left(dx^2+c\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.53707, size = 2221, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(a^4*c^5 + 2*a^2*c^4*d + (a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^6 + c
^3*d^2 + 3*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^4 + 3*(a^4*c^4*d + 2*a^2
*c^3*d^2 + c^2*d^3)*x^2)*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d +
8*(a^4*c*d - a^2*d^2)*x^2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)
*sqrt(d*x^2 + c)*sqrt(d) + d^2) + (8*(a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^5
+ 20*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^4*c^4*d + 2*a^2*c^3*
d^2 + c^2*d^3)*x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (7*a^3*c^4
*d + 5*a*c^3*d^2 + 2*(3*a^3*c^2*d^3 + 2*a*c*d^4)*x^4 + (13*a^3*c^3*d^2 + 9*
a*c^2*d^3)*x^2)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^4*c^8*d + 2*a^2*c^7*d
^2 + c^6*d^3 + (a^4*c^5*d^4 + 2*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^4*c^6*d^3
 + 2*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^4*c^7*d^2 + 2*a^2*c^6*d^3 + c^5*d^4)
*x^2), 1/15*(4*(a^4*c^5 + 2*a^2*c^4*d + (a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x
^6 + c^3*d^2 + 3*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^4 + 3*(a^4*c^4*d +
 2*a^2*c^3*d^2 + c^2*d^3)*x^2)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c - d
)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c*
d - a*d^2)*x^2)) + (8*(a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^5 + 20*(a^4*c^3*d
^2 + 2*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^4*c^4*d + 2*a^2*c^3*d^2 + c^2*d^3)*
x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (7*a^3*c^4*d + 5*a*c^3*d^
2 + 2*(3*a^3*c^2*d^3 + 2*a*c*d^4)*x^4 + (13*a^3*c^3*d^2 + 9*a*c^2*d^3)*x^2)
*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^4*c^8*d + 2*a^2*c^7*d^2 + c^6*d^3 +
```

```
(a^4*c^5*d^4 + 2*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^4*c^6*d^3 + 2*a^2*c^5*d^
4 + c^4*d^5)*x^4 + 3*(a^4*c^7*d^2 + 2*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.52 $\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=369

$$\frac{4a\left(1-a^2x^2\right)\left(11a^4c^2+15a^2cd+6d^2\right)}{105c^3\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^3\sqrt{c+dx^2}}+\frac{2a\left(1-a^2x^2\right)\left(5a^2c+3d\right)}{105c^2\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}-\frac{16\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2}}{a\sqrt{c+}}\right.}{35c^4\sqrt{d}\sqrt{ax-1}\sqrt{ax+}}$$

```
[Out] (a*(1 - a^2*x^2))/(35*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(5/2)) + (2*a*(5*a^2*c + 3*d)*(1 - a^2*x^2))/(105*c^2*(a^2*c + d)^2*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)^(3/2)) + (4*a*(11*a^4*c^2 + 15*a^2*c*d
+ 6*d^2)*(1 - a^2*x^2))/(105*c^3*(a^2*c + d)^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcCosh
[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCosh[a*x])/(35*c^3*(c + d*x^2)^
(3/2)) + (16*x*ArcCosh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (16*Sqrt[-1 + a^2*x
^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(35*c^4*Sqrt
[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rubi [A] time = 1.01382, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {192, 191, 5705, 12, 519, 6715, 1622, 949, 78, 63, 217, 206}

$$\frac{4a\left(1-a^2x^2\right)\left(11a^4c^2+15a^2cd+6d^2\right)}{105c^3\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^3\sqrt{c+dx^2}}+\frac{2a\left(1-a^2x^2\right)\left(5a^2c+3d\right)}{105c^2\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}-\frac{16\sqrt{a^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2}}{a\sqrt{c+}}\right.}{35c^4\sqrt{d}\sqrt{ax-1}\sqrt{ax+}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(9/2),x]
```


```
[Out] (a*(1 - a^2*x^2))/(35*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(5/2)) + (2*a*(5*a^2*c + 3*d)*(1 - a^2*x^2))/(105*c^2*(a^2*c + d)^2*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)^(3/2)) + (4*a*(11*a^4*c^2 + 15*a^2*c*d
+ 6*d^2)*(1 - a^2*x^2))/(105*c^3*(a^2*c + d)^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcCosh
[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCosh[a*x])/(35*c^3*(c + d*x^2)^
(3/2)) + (16*x*ArcCosh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (16*Sqrt[-1 + a^2*x
^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(35*c^4*Sqrt
[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 1622

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
```

```
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && GtQ[Expon[Px, x], 2
]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{9/2}}\,dx = \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x\left(35c^3+70c^2dx^2\cdots\right.}{35c^4\sqrt{-1+ax}\cdots}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(35c^3+70c^2dx^2+56cd^2\cdots\right.}{\sqrt{-1+ax}\sqrt{1+ax}\left(c+\cdots\right.}}{35c^4}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x\left(35c\cdots\right.}{\cdots}}{35c^4\sqrt{-1\cdots}}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\cdots}{70c\cdots}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}} + \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(a\cdots}{35c^3\left(c+dx^2\cdots\right.}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \cdots$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \cdots$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \cdots$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \cdots$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \cdots$$

Mathematica [C] time = 6.17026, size = 723, normalized size = 1.96

$$\frac{32(ax-1)^{3/2}\left(c+dx^2\right)^3\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(\frac{a\left(\sqrt{d}-ia\sqrt{c}\right)\left(\sqrt{d}x+i\sqrt{c}\right)\sqrt{\frac{\frac{ia\sqrt{c}}{\sqrt{d}}+a(-x)+\frac{i\sqrt{d}x}{\sqrt{c}}+1}{1-ax}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{2-2ax}}\right),\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)}{ax-1}+a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{\left(a^2c+d\right)(c+\cdots}{cd(ax-1\cdots}}\cdots\right.}{ac^4\sqrt{ax+1}\left(a^2c+d\right)\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{1-ax}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (-(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)*(3*d^2*(11*c^2 + 18*c*d*x^2 +
 8*d^2*x^4) + 2*a^2*c*d*(41*c^2 + 68*c*d*x^2 + 30*d^2*x^4) + a^4*c^2*(57*c^
2 + 98*c*d*x^2 + 44*d^2*x^4)))/(3*c^3*(a^2*c + d)^3) + (x*(35*c^3 + 70*c^2*
d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*ArcCosh[a*x])/c^4 + (32*(-1 + a*x)^(3/2)
*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*
x))]*(c + d*x^2)^3*((a*((-I)*a*Sqrt[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*S
qrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*El
lipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d]
+ x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])
/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d
*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[
c])/Sqrt[d] + x))/(1 - a*x))]*EllipticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[
d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x
))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(
a*c^4*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*
Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(35*(c + d*x^2)^(7/2))
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \operatorname{arccosh}(ax)\left(dx^2+c\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.56558, size = 3546, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/105*(12*(a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + (a^6*c^3*d^4 + 3*a^4*c^
2*d^5 + 3*a^2*c*d^6 + d^7)*x^8 + c^4*d^3 + 4*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 +
 3*a^2*c^2*d^5 + c*d^6)*x^6 + 6*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^
4 + c^2*d^5)*x^4 + 4*(a^6*c^6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*
x^2)*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2
)*x^2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqr
t(d) + d^2) + 3*(16*(a^6*c^3*d^4 + 3*a^4*c^2*d^5 + 3*a^2*c*d^6 + d^7)*x^7 +
 56*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^6*c^5
*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^6*c^6*d + 3*a^4
*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x
^2 - 1)) - (57*a^5*c^6*d + 82*a^3*c^5*d^2 + 33*a*c^4*d^3 + 4*(11*a^5*c^3*d^
4 + 15*a^3*c^2*d^5 + 6*a*c*d^6)*x^6 + 2*(71*a^5*c^4*d^3 + 98*a^3*c^3*d^4 +
39*a*c^2*d^5)*x^4 + (155*a^5*c^5*d^2 + 218*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2)
*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^6*c^11*d + 3*a^4*c^10*d^2 + 3*a^2*c^
9*d^3 + c^8*d^4 + (a^6*c^7*d^5 + 3*a^4*c^6*d^6 + 3*a^2*c^5*d^7 + c^4*d^8)*x
^8 + 4*(a^6*c^8*d^4 + 3*a^4*c^7*d^5 + 3*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^6
*c^9*d^3 + 3*a^4*c^8*d^4 + 3*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^6*c^10*d^2 +
 3*a^4*c^9*d^3 + 3*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/105*(24*(a^6*c^7 + 3*a^4*
c^6*d + 3*a^2*c^5*d^2 + (a^6*c^3*d^4 + 3*a^4*c^2*d^5 + 3*a^2*c*d^6 + d^7)*x
^8 + c^4*d^3 + 4*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5 + c*d^6)*x^6
```

```
+ 6*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x^4 + 4*(a^6*c^
6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x^2)*sqrt(-d)*arctan(1/2*(2*
a^2*d*x^2 + a^2*c - d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*
x^4 - a*c*d + (a^3*c*d - a*d^2)*x^2)) + 3*(16*(a^6*c^3*d^4 + 3*a^4*c^2*d^5
+ 3*a^2*c*d^6 + d^7)*x^7 + 56*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5
+ c*d^6)*x^5 + 70*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x
^3 + 35*(a^6*c^6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x)*sqrt(d*x^2
 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (57*a^5*c^6*d + 82*a^3*c^5*d^2 + 33*a*
c^4*d^3 + 4*(11*a^5*c^3*d^4 + 15*a^3*c^2*d^5 + 6*a*c*d^6)*x^6 + 2*(71*a^5*c
^4*d^3 + 98*a^3*c^3*d^4 + 39*a*c^2*d^5)*x^4 + (155*a^5*c^5*d^2 + 218*a^3*c^
4*d^3 + 87*a*c^3*d^4)*x^2)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^6*c^11*d +
 3*a^4*c^10*d^2 + 3*a^2*c^9*d^3 + c^8*d^4 + (a^6*c^7*d^5 + 3*a^4*c^6*d^6 +
3*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^6*c^8*d^4 + 3*a^4*c^7*d^5 + 3*a^2*c^6*d
^6 + c^5*d^7)*x^6 + 6*(a^6*c^9*d^3 + 3*a^4*c^8*d^4 + 3*a^2*c^7*d^5 + c^6*d^
6)*x^4 + 4*(a^6*c^10*d^2 + 3*a^4*c^9*d^3 + 3*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(9/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")

[Out] Timed out
```

3.53 $\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=713

$$-\frac{f^2g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{4bc\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*g^3
*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^3*x^
2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*f*g^2*x^2*Sq
rt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^2*g*x^3*Sqr
t[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^3*x^3*Sqrt[d - c^
2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*f*g^2*x^4*Sqrt[d - c
^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^3*x^5*Sqrt[d - c^2*d*
x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8
*c^2) + (3*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (f^2*g*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/c^2 - (2*g^3*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*c^4) - (g^
3*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
 - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.48386, antiderivative size = 713, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5836, 5822, 5683, 5676, 30, 5718, 5743, 5759, 100, 12, 74, 5733}

$$-\frac{f^2g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{4bc\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*g^3
*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^3*x^
2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*f*g^2*x^2*Sq
rt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^2*g*x^3*Sqr
t[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^3*x^3*Sqrt[d - c^
2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*f*g^2*x^4*Sqrt[d - c
```

```
^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^3*x^5*Sqrt[d - c^2*d*
x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8
*c^2) + (3*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (f^2*g*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/c^2 - (2*g^3*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*c^4) - (g^
3*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
 - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
```

```
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int\left(f^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)+3f^2gx\sqrt{-1+c\right.}{}$$

$$= \frac{\left(f^3\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3f^2g\sqrt{d\right.}{}$$

$$= \frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.$$

$$= \frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2gx^3\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bcf}{16\sqrt{}}$$

$$= \frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3}{1}$$

<u>Mathematica</u> [A] time = 1.99707, size = 491, normalized size = 0.69

$$\frac{240a\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(6c^4x\left(20f^2gx+10f^3+15fg^2x^2+4g^3x^3\right)-c^2g\left(120f^2+45fgx+8g^2x^2\right)-16g^3\right)-36}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (240*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-16*g^3 -
c^2*g*(120*f^2 + 45*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*
f*g^2*x^2 + 4*g^3*x^3)) - 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 + 3*g^2)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
c^2*x^2))] + 2400*b*c^2*f^2*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(
1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 3600*b*c
^3*f^3*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[
c*x] - Sinh[2*ArcCosh[c*x]])) - 675*b*c*f*g^2*Sqrt[d - c^2*d*x^2]*(8*ArcCos
h[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + 8*
b*g^3*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*Arc
Cosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(28
```

```
800*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.544, size = 1213, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-2*b*(-
d*(c^2*x^2-1))^(1/2)*g/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*f^2-3/16*b*(-d*(c^2
*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*f*arccosh(c*x)^2*g^2+1/5*b*(
-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^6-1/15*b*(-d*(
c^2*x^2-1))^(1/2)*g^3/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^2+b*(-d*(c^2*x^2-1
))^(1/2)*g/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*f^2+1/2*b*(-d*(c^2*x^2-1))^(1/2
)*f^3/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3-3/16*b*(-d*(c^2*x^2-1))^(1/2)*f*
g^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+3/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(
c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2-1/3*b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)^(1/
2)*c/(c*x-1)^(1/2)*x^3*f^2+b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)^(1/2)/c/(c*x-
1)^(1/2)*x*f^2+1/2*a*f^3*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f^3*d/(c^2*d)^(1/2)*a
rctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/4*b*(-d*(c^2*x^2-1))^(1/2)*f*
g^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2
/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)*c^2/
(c*x-1)*arccosh(c*x)*x^4*f^2-2/15*a*g^3/d/c^4*(-c^2*d*x^2+d)^(3/2)+1/8*b*(-
d*(c^2*x^2-1))^(1/2)*f^3/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)-1/5*a*g^3*x^2*(-c^2*
d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2)-a*f^2*g/c^2/d*(
-c^2*d*x^2+d)^(3/2)-3/4*a*f*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^
2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/4*b*(-d*(c
^2*x^2-1))^(1/2)*f^3/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2-1/25*b*(-d*(c^2*x^2-
1))^(1/2)*g^3/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^5+1/45*b*(-d*(c^2*x^2-1))^(1/
2)*g^3/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^3+2/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/
(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*x-3/128*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x
+1)^(1/2)/c^3/(c*x-1)^(1/2)-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)/c*f^3*arccosh(c*x)^2+2/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)/c^
4/(c*x-1)*arccosh(c*x)-4/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)/(c*x-1)*ar
ccosh(c*x)*x^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3/(c*x+1)/(c*x-1)*arccosh(c*x
)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a + b\operatorname{acosh}\left(cx\right)\right)\left(f + gx\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.54 $\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=479

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{2fg(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{3c^2}$$

```
[Out] (2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f
^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^2*x^2*S
qrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*f*g*x^3*Sq
rt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^2*x^4*Sqrt[d -
 c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (2*f*g*(1
 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2) - (f^2*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.17579, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {5836, 5822, 5683, 5676, 30, 5718, 5743, 5759}

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{2fg(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f
^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^2*x^2*S
qrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*f*g*x^3*Sq
rt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^2*x^4*Sqrt[d -
 c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (2*f*g*(1
 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2) - (f^2*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)+2fgx\sqrt{-1+cx}\right.}{\sqrt{-1+}}$$

$$= \frac{\left(f^2\sqrt{d-c^2dx^2}\right)\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(2fg\sqrt{d-\right.}{}$$

$$= \frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.$$

$$= \frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcg}{16\sqrt{}}$$

$$= \frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bc}{9\sqrt{}}$$

<u>Mathematica</u> [A] time = 1.20764, size = 356, normalized size = 0.74

$$\frac{48ac\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(12c^2f^2x+16fg\left(c^2x^2-1\right)+3g^2x\left(2c^2x^2-1\right)\right)-144a\sqrt{d}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(4c^2f^2+g^2\right)\tan}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(12*c^2*f^
2*x + 16*f*g*(-1 + c^2*x^2) + 3*g^2*x*(-1 + 2*c^2*x^2)) - 144*a*Sqrt[d]*(4*
c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*c*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x
 + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCos
h[c*x]]) - 144*b*c^2*f^2*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcC
osh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) - 9*b*g^2*Sqrt[d - c^2*d*x^
2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh
[c*x]]))/(1152*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] time = 0.425, size = 855, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2
)+1/8*a*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)-2/3*a*f*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f^2*x*(-c^2*d*x^2+d)^(1/2)+1/2
*a*f^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/4*b*(
-d*(c^2*x^2-1))^(1/2)*f^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2+2/3*b*(-d*(c^2*
x^2-1))^(1/2)*f*g/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-4/3*b*(-d*(c^2*x^2-1
))^(1/2)*f*g/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2-1/16*b*(-d*(c^2*x^2-1))^(1/2)
*g^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/16*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c
*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2-2/9*b*(-d*(c^2*x^2-1))^(1/2)*f*g/(c*x+1)^(1
/2)*c/(c*x-1)^(1/2)*x^3+2/3*b*(-d*(c^2*x^2-1))^(1/2)*f*g/(c*x+1)^(1/2)/c/(c
*x-1)^(1/2)*x-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ar
ccosh(c*x)^2*f^2-1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
c^3*arccosh(c*x)^2*g^2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)*c^2/(c*x-1)
*arccosh(c*x)*x^5-3/8*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)/(c*x-1)*arccosh(
c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*
x-1/128*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)+2/3*b*
(-d*(c^2*x^2-1))^(1/2)*f*g/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)+1/2*b*(-d*(c^2*
x^2-1))^(1/2)*f^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1
))^(1/2)*f^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^
2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arcosh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccosh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\,\text{acosh}\left(cx\right)\right)\left(f+gx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.55 $\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=255

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}$$

```
[Out] (b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f*x^2
*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g*x^3*Sqrt[d
- c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x]))/2 - (g*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(3*c^2) - (f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*
b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.545669, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5836, 5822, 5683, 5676, 30, 5718}

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```


```
[Out] (b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f*x^2
*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g*x^3*Sqrt[d
- c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x]))/2 - (g*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(3*c^2) - (f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*
b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```


Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}(f+gx)\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)+gx\sqrt{-1+cx}\sqrt{1-}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(f\sqrt{d-c^2dx^2}\right)\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(g\sqrt{d-c^2d\right.}{}$$

$$= \frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{g(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\right.}{3c^2}$$

$$= \frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}} + \frac{1}{2}fx$$

Mathematica [A] time = 1.05511, size = 251, normalized size = 0.98

$$\frac{12a\sqrt{d-c^2dx^2}\left(3c^2fx+2g\left(c^2x^2-1\right)\right)-36ac\sqrt{d}f\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-\frac{9bcf\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}}{72c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (12*a*Sqrt[d - c^2*d*x^2]*(3*c^2*f*x + 2*g*(-1 + c^2*x^2)) - 36*a*c*Sqrt[d]
*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*b*g*Sqrt
[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCos
h[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (9
*b*c*f*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*Arc
Cosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/(7
2*c^2)
```

Maple [B] time = 0.389, size = 461, normalized size = 1.8

$$-\frac{ag}{3c^2d}\left(-c^2dx^2+d\right)^{\frac{3}{2}}+\frac{afx}{2}\sqrt{-c^2dx^2+d}+\frac{afd}{2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}+\frac{bc^2\operatorname{arccosh}(cx)\,x^4}{(3cx+3)(cx-1)}\sqrt{-d\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f*
d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b*(-d*(c^2
*x^2-1))^(1/2)*g/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-2/3*b*(-d*(c^2*x^2-1)
)^(1/2)*g/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f/(
c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f/(c*x+1)/
(c*x-1)*arccosh(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)*f/(c*x+1)^(1/2)/(c*x-1)
^(1/2)/c-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1
/9*b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+1/3*b*(-d*(
c^2*x^2-1))^(1/2)*g/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x-1/4*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f*arccosh(c*x)^2+1/3*b*(-d*(c^2*x^2-1))
^(1/2)*g/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\text{arcosh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x)), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.56 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=785

$$\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}\left(\cdots\right.}{2bc\sqrt{\cdots}}$$

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (a*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2])/(g*(1 - c*x)*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^
2]*ArcCosh[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*
g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - ((
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2]*Sq
rt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*
x^2])])/(g^2*(1 - c*x)*(1 + c*x)) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*Pol
yLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 3.37884, antiderivative size = 785, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.71, Rules used = {5836, 5824, 683, 5816, 6742, 93, 208, 1610, 1654, 12, 725, 206, 5860, 5858, 5718, 8, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}\left(\cdots\right.}{2bc\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (a*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2])/(g*(1 - c*x)*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^
```

```
2]*ArcCosh[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*
g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - ((
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2]*Sq
rt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*
x^2])])/(g^2*(1 - c*x)*(1 + c*x)) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*Pol
yLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5824

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[((f + g*
x)^m*(d1*d2 + e1*e2*x^2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*
(n + 1)), x] - Dist[1/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 5816

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
```

```
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*
Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[
n, 0] && IGtQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5860

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5858

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)} - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(g+2c^2fx+c^2gx^2\right)\left(a+b\cosh\right.}{(f+gx)^2}}{2bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)}$$

$$= \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2}}{2bc\sqrt{-1+c}}$$

$$= \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{2bg\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - cx\sqrt{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - cx\sqrt{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - cx\sqrt{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - cx\sqrt{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - cx\sqrt{}$$

Mathematica [C] time = 4.05017, size = 1121, normalized size = 1.43

$$2a\sqrt{d-c^2dx^2}g-2ac\sqrt{d}f\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)+2a\sqrt{d}\sqrt{g^2-c^2f^2}\log(f+gx)-2a\sqrt{d}\sqrt{g^2-c^2f^2}\log\left(d\left(fxc^2+g\right)+\sqrt{d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (2*a*g*Sqrt[d - c^2*d*x^2] - 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[f + g
*x] - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt
[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*((2*c*g*x*S
qrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + 2*g*ArcCosh[c*x] + (c*f*Sqrt[(-1 + c
*x)/(1 + c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + (2*(-(c*f) + g)*(c*f + g)*(2*Arc
Cosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] -
 (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-
(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCos
h[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c
*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g
)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*S
qrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*
f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2
]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[
c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] -
(ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1
+ Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCos
h[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I
*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f
^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) +
 g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f +
 g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(Sqrt[-(c^2*f^2) +
 g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(2*g^2)
```

Maple [A] time = 0.347, size = 1072, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x)
```

```
[Out] a/g*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a/g^2*
c^2*d*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*
(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^3*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln
((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))
*c^2*f^2-a/g*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^
2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/
g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(-d*(c^2*x^2-1))^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*c/g^2+b*(-d*(c^2*x^2-1))^(1/2
)/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/g*c*x-b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)/(c*x+1)/g*arccosh
(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)/g^2*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g
^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(
-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*d
ilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(
c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/g^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*
f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c^2dx^2+d}(b\,\text{arcosh}\,(cx)+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g*x + f), x)
```

3.57 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=918

$$\frac{bf^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}+\frac{af^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}-\frac{2af\sqrt{d-c^2dx^2}\tanh^{-1}\left(\frac{\sqrt{cf+g}\sqrt{cx+1}}{\sqrt{cf-g}\sqrt{cx-1}}\right)c^2}{\sqrt{cf-g}g^2\sqrt{cf+g}\sqrt{cx-1}\sqrt{cx+1}}-\frac{bf\sqrt{d-c^2}\cdots}{g\cdots}$$

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) + (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x])/(g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-
((1 - c*x)/(1 + c*x))]*Sqrt[1 + c*x]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(g*S
qrt[-1 + c*x]*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2)/(
2*g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((g + c^2*f*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(f + g*x)^2) - ((1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)^2) - (2*a*c^2
*f*Sqrt[d - c^2*d*x^2]*ArcTanh[(Sqrt[c*f + g]*Sqrt[1 + c*x])/(Sqrt[c*f - g]
*Sqrt[-1 + c*x])])/(Sqrt[c*f - g]*g^2*Sqrt[c*f + g]*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rubi [A] time = 3.56108, antiderivative size = 918, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.71, Rules used = {5836, 5824, 37, 5814, 12, 180, 52, 96, 93, 208, 5860, 5858, 5676, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bf^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}+\frac{af^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}-\frac{2af\sqrt{d-c^2dx^2}\tanh^{-1}\left(\frac{\sqrt{cf+g}\sqrt{cx+1}}{\sqrt{cf-g}\sqrt{cx-1}}\right)c^2}{\sqrt{cf-g}g^2\sqrt{cf+g}\sqrt{cx-1}\sqrt{cx+1}}-\frac{bf\sqrt{d-c^2}\cdots}{g\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) + (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x])/(g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-
((1 - c*x)/(1 + c*x))]*Sqrt[1 + c*x]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(g*S
qrt[-1 + c*x]*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2)/(
2*g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((g + c^2*f*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(f + g*x)^2) - ((1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)^2) - (2*a*c^2
*f*Sqrt[d - c^2*d*x^2]*ArcTanh[(Sqrt[c*f + g]*Sqrt[1 + c*x])/(Sqrt[c*f - g]
*Sqrt[-1 + c*x])])/(Sqrt[c*f - g]*g^2*Sqrt[c*f + g]*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5824

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[((f + g*
x)^m*(d1*d2 + e1*e2*x^2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*
(n + 1)), x] - Dist[1/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5814

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[(u*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x
], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && IL
tQ[m, 0] && LtQ[m + p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5860

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5858

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(2g+2c^2fx\right)\left(a+b\cosh^{-1}(cx\right.}{(f+gx)^3}}{2bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+g}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+g}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+g}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+g}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+g}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+g}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(c}{2g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+cx}\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}(f+gx}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+cx}\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}(f+gx}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+cx}\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}(f+gx}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+cx}\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}(f+gx}$$

Mathematica [C] time = 7.0517, size = 1139, normalized size = 1.24

$$\frac{2a\sqrt{d}f\log(f+gx)c^2}{\sqrt{g^2-c^2f^2}} - \frac{2a\sqrt{d}f\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)c^2}{\sqrt{g^2-c^2f^2}} + 2a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)c + b\sqrt{d-c^2dx^2}\left(\frac{\cosh^{-1}(cx)^2}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)} - \frac{2g\,\mathrm{co}}{cf}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x)^2,x]
```

```
[Out] ((-2*a*g*Sqrt[d - c^2*d*x^2])/(f + g*x) + 2*a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d
- c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*a*c^2*Sqrt[d]*f*Log[f + g*x])/
Sqrt[-(c^2*f^2) + g^2] - (2*a*c^2*Sqrt[d]*f*Log[d*(g + c^2*f*x) + Sqrt[d]*S
qrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[-(c^2*f^2) + g^2] + b*c*Sq
rt[d - c^2*d*x^2]*((-2*g*ArcCosh[c*x])/(c*f + c*g*x) + ArcCosh[c*x]^2/(Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (2*Log[1 + (g*x)/f])/(Sqrt[(-1 + c*x)/(
1 + c*x)]*(1 + c*x)) + (2*c*f*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCos
h[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*
f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)]
 + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + Arc
Tan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[
-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] +
 (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2)
 + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]
*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[Ar
cCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(
c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^
2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f)
 + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g +
I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*
Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - Po
lyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g
^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcC
osh[c*x]/2]))])))/(Sqrt[-(c^2*f^2) + g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x))))/(2*g^2)
```

Maple [B] time = 0.309, size = 1956, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2-g^2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-a/g^2*c^4*f^2/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*a
rctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)
/g^2)^(1/2))-a/g^3*c^4*f^3/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln(
(-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+
a/g*c^2*f/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^
2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d
+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/(c^2*f^2-g^
2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+a*c^2
/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c/g^2+b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-1
)/(c*x+1)/g^2/(g*x+f)*x^3*c^4*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)/g/(g*x+f)*x*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/
(c*x-1)/(c*x+1)/g/(g*x+f)*x^2*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(g*x+f)*c*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c
*x)/(c*x-1)/(c*x+1)/g^2/(g*x+f)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*
x)/(c*x-1)/(c*x+1)/g/(g*x+f)+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*c^2*f*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c
^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*c^2*f*
arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/
2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/g^2/(c^2*f^2-g^2)*c^3*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2+b
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)*c^3*l
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))+g)*f^2-b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f
^2-g^2)^(1/2)*c^2*f*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^
2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*c^2*f*dilog(((c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+2*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(c^2*f^2-g^2)*c*ln(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))-b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
```

```
1/2)/(c^2*f^2-g^2)*c*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))+g)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}(b\,\mathrm{arcosh}\,(cx)+a)}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}{\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/(f + g*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g*x + f)^2, x)
```

3.58 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=1029

$$\frac{bc^3dg^3\sqrt{d-c^2dx^2}x^7}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dfg^2\sqrt{d-c^2dx^2}x^6}{12\sqrt{cx-1}\sqrt{cx+1}}-\frac{8bcdg^3\sqrt{d-c^2dx^2}x^5}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3df^2g\sqrt{d-c^2dx^2}x^5}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3df^3\sqrt{d-c^2dx^2}x^4}{16\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
*b*d*g^3*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*
b*c*d*f^3*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*
b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^
3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c
*d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*
c^3*d*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8
*b*c*d*g^3*x^5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3
*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d*f*g^2*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (3*d*f*g^2*x^3*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/4 + (d*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (3*d*f^2*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (2*d*g^3*(1 - c*x)^2*(1 + c
*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (d*g^3*x^2*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (3*d
*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 2.05705, antiderivative size = 1029, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 5718, 194, 5745, 5743, 5759, 100, 12, 74, 5733, 373}

$$\frac{bc^3dg^3\sqrt{d-c^2dx^2}x^7}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dfg^2\sqrt{d-c^2dx^2}x^6}{12\sqrt{cx-1}\sqrt{cx+1}}-\frac{8bcdg^3\sqrt{d-c^2dx^2}x^5}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3df^2g\sqrt{d-c^2dx^2}x^5}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3df^3\sqrt{d-c^2dx^2}x^4}{16\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
*b*d*g^3*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*
b*c*d*f^3*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*
b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^
3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c
*d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*
c^3*d*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8
*b*c*d*g^3*x^5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3
*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d*f*g^2*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (3*d*f*g^2*x^3*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/4 + (d*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (3*d*f^2*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (2*d*g^3*(1 - c*x)^2*(1 + c
*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (d*g^3*x^2*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (3*d
*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int (-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)^3 \left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int \left(f^3(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\cosh^{-1}(cx)\right) + 3f^2\right.}{}$$

$$= -\frac{\left(df^3\sqrt{d-c^2dx^2}\right) \int (-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}} - $$

$$= \frac{1}{4}df^3x(1-cx)(1+cx)\sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) + \frac{1}{2}dfg^2x^3(1-c$$

$$= \frac{3}{8}df^3x\sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) + \frac{3}{8}dfg^2x^3\sqrt{d-c^2dx^2} \left(a+b\,\mathrm{co}\right.$$

$$= \frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcdf^2gx^3\sqrt{d-c^2dx^2}}{5\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 4.53608, size = 901, normalized size = 0.88

$$\frac{-352800bc^3d\sqrt{d-c^2dx^2} \left(\cosh\left(2\cosh^{-1}(cx)\right) + 2\cosh^{-1}(cx) \left(\cosh^{-1}(cx) - \sinh\left(2\cosh^{-1}(cx)\right)\right)\right) f^3 + 22050bc^3d\sqrt{d-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5040*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(32*g^3
 + c^2*g*(336*f^2 + 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*
x + 70*f*g^2*x^2 + 20*g^3*x^3) - 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2
*x^2 + 64*g^3*x^3)) - 529200*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*
x^2))] + 235200*b*c^2*d*f^2*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(
1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 352800*b
*c^3*d*f^3*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcC
```

```
osh[c*x] - Sinh[2*ArcCosh[c*x]]]) + 22050*b*c^3*d*f^3*Sqrt[d - c^2*d*x^2]*(
8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x
]]) - 66150*b*c*d*f*g^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcC
osh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 2352*b*c^2*d*f^2*g*Sqrt[
d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*S
inh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]) + 784*b*d*g^3*S
qrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*
x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]) - 3675*b*c*
d*f*g^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] -
 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh
[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) - 4*b*d*
g^3*Sqrt[d - c^2*d*x^2]*(55125*c*x - 55125*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x] - 1225*Cosh[3*ArcCosh[c*x]] - 1323*Cosh[5*ArcCosh[c*x]] -
 225*Cosh[7*ArcCosh[c*x]] + 3675*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 6615*A
rcCosh[c*x]*Sinh[5*ArcCosh[c*x]] + 1575*ArcCosh[c*x]*Sinh[7*ArcCosh[c*x]]))
/(2822400*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.52, size = 1638, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/2*a*f*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+3/16*a*f*g^2/c^2*d*x*(-c^2*d*x^2+
d)^(1/2)+3/16*a*f*g^2/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*
x^2+d)^(1/2))+17/128*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)^(1/2)/c/(c*x-1)
^(1/2)+13/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x
)*x^6-1/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*
x^2-17/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3
-9/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*f^2-2/5*
b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3*f^2+3/5*b*(-
d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x*f^2+1/12*b*(-d*(c^
2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-7/32*b*(-d*(c^2
*x^2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+3/32*b*(-d*(c^2*x^
2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+3/25*b*(-d*(c^2*x^2-1
))^(1/2)*g*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5*f^2+3/5*b*(-d*(c^2*x^2-1))
^(1/2)*g*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*f^2-1/4*b*(-d*(c^2*x^2-1))^(1/2
)*f^3*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^5+7/8*b*(-d*(c^2*x^2-1))^(1/2)*f
```

```
^3*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3-3/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/c^3*f*arccosh(c*x)^2*d*g^2-1/7*b*(-d*(c^2*x^2-1))^
(1/2)*g^3*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^8-2/35*a*g^3/d/c^4*(-c^2*d*x
^2+d)^(5/2)+1/105*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)^(1/2)/c/(c*x-1)^(1
/2)*x^3+2/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)
*x+7/768*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)-5
/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x-9/35*b*(-d
*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4-3/16*b*(-d*(c^2*
x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f^3*arccosh(c*x)^2*d+2/35*b*(-d
*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)+1/16*b*(-d*(c^2*
x^2-1))^(1/2)*f^3*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^4-5/16*b*(-d*(c^2*x^2
-1))^(1/2)*f^3*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2+1/49*b*(-d*(c^2*x^2-1))^
(1/2)*g^3*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^7-8/175*b*(-d*(c^2*x^2-1))^(1
/2)*g^3*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^5+3/8*a*f^3*d^2/(c^2*d)^(1/2)*arc
tan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+11/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g
^2*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+3/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g
^2*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x-3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c
*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6*f^2+9/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*
x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4*f^2+3/8*a*f^3*d*x*(-c^2*d*x^2+d)^(1/2)-3/
5*a*f^2*g/c^2/d*(-c^2*d*x^2+d)^(5/2)-1/7*a*g^3*x^2*(-c^2*d*x^2+d)^(5/2)/c^2
/d+1/8*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/4*a*f^3*x*(-c^2*d*x^2+d)^(3/2)-
1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(-\big(ac^2dg^3x^5+3\,ac^2dfg^2x^4-3\,adf^2gx-adf^3+\big(3\,ac^2df^2g-adg^3\big)x^3+\big(ac^2df^3-3\,adfg^2\big)x^2+\big(bc^2dg^3x^5+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 - 3*a*d*f^2*g*x - a*d*f^3
+ (3*a*c^2*d*f^2*g - a*d*g^3)*x^3 + (a*c^2*d*f^3 - 3*a*d*f*g^2)*x^2 + (b*c^
2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x - b*d*f^3 + (3*b*c^2*d*f^
2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2)*arccosh(c*x))*sqrt(-c
^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.59 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=725

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}df^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{16bc\sqrt{cx-1}\sqrt{cx+}}$$

```
[Out] (2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b
*c*d*f^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d
*f*g*x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*d*g
^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*
f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g
^2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f^2*x*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/8 + (d*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/4 + (d*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/6 - (2*d*f*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/(5*c^2) - (3*d*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*g^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 1.6929, antiderivative size = 725, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 5718, 194, 5745, 5743, 5759}

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}df^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{16bc\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b
*c*d*f^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d
*f*g*x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*d*g
```

```
^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*
f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g
^2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f^2*x*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/8 + (d*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/4 + (d*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/6 - (2*d*f*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/(5*c^2) - (3*d*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*g^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
```

```
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int (f+gx)^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+2fg\right.}{}$$

$$= -\frac{\left(df^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} - $$

$$= \frac{1}{4}df^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}dg^2x^3(1-cx$$

$$= \frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.$$

$$= \frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 2.86543, size = 623, normalized size = 0.86

$$\frac{-3600ad^{3/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(6c^2f^2+g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-240acd\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(30c^2f^2x\left(2c^2x^2-5\right)+96f\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-240*a*c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(96*f*
g*(-1 + c^2*x^2)^2 + 30*c^2*f^2*x*(-5 + 2*c^2*x^2) + 5*g^2*x*(3 - 14*c^2*x^
2 + 8*c^4*x^4)) - 3600*a*d^(3/2)*(6*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3
200*b*c*d*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*
(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 7200*b*c^2*d*f^2*Sqrt[d
- c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*
ArcCosh[c*x]])) + 450*b*c^2*d*f^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + C
osh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 450*b*d*g^2*Sq
```

```
rt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]
*Sinh[4*ArcCosh[c*x]]) - 32*b*c*d*f*g*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] -
 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh
[c*x]*Sinh[5*ArcCosh[c*x]]) - 25*b*d*g^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c
*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh
[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]]
+ Sinh[6*ArcCosh[c*x]])))/(57600*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.44, size = 1177, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -3/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2
*d*f^2-1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccos
h(c*x)^2*d*g^2-17/48*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)/(c*x-1)*arccosh
(c*x)*x^3-5/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x*f^2
+1/16*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^4*f^2-5/
16*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2*f^2+1/36*b*
(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-7/96*b*(-d
*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/32*b*(-d*(c^2
*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+7/2304*b*(-d*(c^2*x^
2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)+17/128*b*(-d*(c^2*x^2-1))
^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*f^2+1/24*a*g^2/c^2*x*(-c^2*d*x^2+d)^
(3/2)+3/8*a*f^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f^2*d^2/(c^2*d)^(1/2)*arctan
((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(
c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6+6/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(c*x
+1)*c^2/(c*x-1)*arccosh(c*x)*x^4+1/4*a*f^2*x*(-c^2*d*x^2+d)^(3/2)+1/16*b*(-
d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+2/5*b*(-d*(c^
2*x^2-1))^(1/2)*f*g*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)-1/4*b*(-d*(c^2*x^2-1
))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^5*f^2+7/8*b*(-d*(c^2*x^2-1))^
(1/2)*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3*f^2-6/5*b*(-d*(c^2*x^2-1))^(1/
2)*f*g*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+2/25*b*(-d*(c^2*x^2-1))^(1/2)*f*g
*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5-4/15*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/
(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(c*x+1
)^(1/2)/c/(c*x-1)^(1/2)*x-1/6*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)*c^4/(c
*x-1)*arccosh(c*x)*x^7+11/24*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)*c^2/(c*
x-1)*arccosh(c*x)*x^5-1/6*a*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/16*a*g^2/c^2
```

```
*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a*g^2/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(
1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/5*a*f*g/c^2/d*(-c^2*d*x^2+d)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(-\Big(ac^2dg^2x^4 + 2\,ac^2dfgx^3 - 2\,adfgx - adf^2 + \big(ac^2df^2 - adg^2\big)x^2 + \big(bc^2dg^2x^4 + 2\,bc^2dfgx^3 - 2\,bdfgx - bdf^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 - 2*a*d*f*g*x - a*d*f^2 + (a
*c^2*d*f^2 - a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*
g*x - b*d*f^2 + (b*c^2*d*f^2 - b*d*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2
+ d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.60 $\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=398

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dfx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\cosh^{-}\right.}{16bc\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d
*f*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*g*
x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*f*x^4
*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f*x*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/4 - (d*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 0.756826, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 5718, 194}

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dfx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\cosh^{-}\right.}{16bc\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d
*f*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*g*
x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*f*x^4
*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f*x*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/4 - (d*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+gx(\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(df\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(}{}$$

$$= \frac{1}{4}dfx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{dg(1-cx)^2(1+}{}$$

$$= \frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{4}dfx(1-cx)(1+cx)\sqrt{d-c^2d}$$

$$= \frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 1.70124, size = 432, normalized size = 1.09

$$\frac{-10800acd^{3/2}f\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 720ad\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(5c^2fx\left(2c^2x^2-5\right)+8g\left(c^2x^2-1\right)^2\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-720*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(8*g*(-1
 + c^2*x^2)^2 + 5*c^2*f*x*(-5 + 2*c^2*x^2)) - 10800*a*c*d^(3/2)*f*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
 c^2*x^2))] + 800*b*d*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*
x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 3600*b*c*d*f*S
qrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - S
inh[2*ArcCosh[c*x]])) + 225*b*c*d*f*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 +
 Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 8*b*d*g*Sqrt
[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh
[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*
Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(28800*c^2*Sq
```

```
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.373, size = 656, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/5*a*g/c^2/d*(-c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f*
d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))-1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)*c^4/(c*x-1)*a
rccosh(c*x)*x^6+3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)*c^2/(c*x-1)*arccos
h(c*x)*x^4-3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^
2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+7/8
*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-5/8*b*(-
d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x+17/128*b*(-d*(c^2*x
^2-1))^(1/2)*f*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c-3/16*b*(-d*(c^2*x^2-1))^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f*arccosh(c*x)^2*d+1/25*b*(-d*(c^2*x^2-1))^
(1/2)*g*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)
*g*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*
x+1)^(1/2)/c/(c*x-1)^(1/2)*x+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)^(1/2
)/(c*x-1)^(1/2)*c^3*x^4-5/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c*x^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)/c^2/(c*x-1)*arcco
sh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac^2dgx^3+ac^2dfx^2-adgx-adf+\left(bc^2dgx^3+bc^2dfx^2-bdgx-bdf\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*g*x^3 + a*c^2*d*f*x^2 - a*d*g*x - a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.61 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1270

result too large to display

```
[Out] -((a*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2])/g^3) + (b*c*d*(c*f - g)*(c*
f + g)*x*Sqrt[d - c^2*d*x^2])/(g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^2*d
*(c*f - g)*x^2*Sqrt[d - c^2*d*x^2])/(4*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(a*d*(2 + 3*c*x - 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(6*g) + (b*c*d*x*(-12 - 9
*c*x + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(36*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/g^3 - (a*d*Sqr
t[d - c^2*d*x^2]*ArcCosh[c*x])/(2*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(2
 + 3*c*x - 2*c^2*x^2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(6*g) - (b*d*Sqrt[d
 - c^2*d*x^2]*ArcCosh[c*x]^2)/(4*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d*(c*
f - g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*g^2) - (d*(c*f - g)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*g^2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + (c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/(2*b*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d*(c*f - g)^2*(c*f + g)^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(f + g*x)) + (d*(c*f - g)*(c*f + g)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)
) - (2*a*d*(c*f - g)^(3/2)*(c*f + g)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcTanh[(Sqr
t[c*f + g]*Sqrt[1 + c*x])/(Sqrt[c*f - g]*Sqrt[-1 + c*x])])/(g^4*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d -
c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^
2])])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt[c^
2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c
*f + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d*(c*f
- g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcC
osh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyL
og[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^4*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rubi [F] time = 3.8533, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2])/(g^3*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (b*c^2*d*(c*f - g)*x^2*Sqrt[d - c^2*d*x^2])/(4*g^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (a*d*(c*f - g)*(c*f + g)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x
^2])/(g^3*(1 - c*x)*(1 + c*x)) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^
2]*ArcCosh[c*x])/g^3 + (c*d*(c*f - g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(2*g^2) - (d*(c*f - g)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
4*b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d*(c*f - g)*(c*f + g)*x*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (d*(c*f - g)^2*(c*f + g)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2
*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) + (d*(c*f - g)*(c*f + g)*(
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(f + g*x)) + (a*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 -
g^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2
*f^2 - g^2]*Sqrt[-1 + c^2*x^2])])/(g^4*(1 - c*x)*(1 + c*x)) - (b*d*(c*f - g
)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E
^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^4*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]
*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 -
g^2]))])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt
[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f +
Sqrt[c^2*f^2 - g^2]))])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d*Sqrt[d -
c^2*d*x^2]*Defer[Int][(-1 + c*x)^(3/2)*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]),
x])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(-\frac{c(cf-g)\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{g^2} + \frac{c(-1+cx)^{3/2}\sqrt{1+cx}\left(a+b\cosh^{-1}\right.}{g}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(cd(cf-g)\sqrt{d-c^2dx^2}\right)\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{g^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(cd\sqrt{d-\right.}{}$$

$$= \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^2} + \frac{d(cf-g)(cf+g)\left(1-c^2x^2\right)\sqrt{d-}}{2bcg^2\sqrt{-1+cx}\sqrt{1-}}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^2} - \frac{d(c}{}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^2} - \frac{d(c}{}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^2} - \frac{d(c}{}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^2} - \frac{d(c}{}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ad(cf-g)(cf+g)\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g^3(1-cx)(1+cx)} + \frac{cd(c}{}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ad(cf-g)(cf+g)\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g^3(1-cx)(1+cx)} - \frac{bd(c}{}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ad(cf-g)(cf}{g^3(}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ad(cf-g)(cf}{g^3(}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{ad(cf-g)(cf}{g^3(}$$

Mathematica [C] time = 11.717, size = 3068, normalized size = 2.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((a*d*(-3*c^2*f^2 + 4*g^2))/(3*g^3) + (a*c^2*d*f*
x)/(2*g^2) - (a*c^2*d*x^2)/(3*g)) + (a*c*d^(3/2)*f*(2*c^2*f^2 - 3*g^2)*ArcT
an[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*g^4) + (a*
d^(3/2)*(-(c^2*f^2) + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*d^(3/2)*(-(c^2*f^2)
 + g^2)^(3/2)*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d
*(-1 + c^2*x^2))]])/g^4 + (b*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*((-2*c*g*x)/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + 2*g*ArcCosh[c*x] - (c*f*ArcCosh[c*
x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (2*(-(c*f) + g)*(c*f + g)*(2
*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2
]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c
*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(Ar
cCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan
[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/
2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-
((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) +
 g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcC
osh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))
] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt
[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])
*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[Ar
cCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g
 - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c
^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^
2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c
*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(Sqrt[-(c^2*f^
2) + g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(2*g^2) - (b*d*Sqrt[-(d*(
-1 + c*x)*(1 + c*x))]*((-9*(-2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[
c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (ArcCos[-((c*f)/g)] +
 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTa
n[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(
c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] - (
ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*
```

```
f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) +
 g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*S
qrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcC
osh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^
2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
 + g^2]*Tanh[ArcCosh[c*x]/2]))] + (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) +
 g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) +
 g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*
Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - I*(PolyLog[2, ((c*f - I*Sq
rt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]
/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - Poly
Log[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2
]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCos
h[c*x]/2]))])))/Sqrt[-(c^2*f^2) + g^2] - (-18*c*g*(-4*c^2*f^2 + g^2)*x + 18
*g*(-4*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 1
8*c*f*(2*c^2*f^2 - g^2)*ArcCosh[c*x]^2 - 9*c*f*g^2*Cosh[2*ArcCosh[c*x]] + 2
*g^3*Cosh[3*ArcCosh[c*x]] + (9*(8*c^4*f^4 - 8*c^2*f^2*g^2 + g^4)*(2*ArcCosh
[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2*
I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2
*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*
x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f +
 g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Ta
nh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[
-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-((c*f)/g
)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*
Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]
/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (Arc
Cos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f
^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Ta
nh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqr
t[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
+ g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2
])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g +
 I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[-(c^2*f^2) + g^2]
 + 18*c*f*g^2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 6*g^3*ArcCosh[c*x]*Sinh[3
*ArcCosh[c*x]])/g^4))/(72*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.27, size = 1965, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x)
```

```
[Out] 1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*x^3*c^3-4/3*b*
(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*c*x+1/8*b*(-d*(c^2*x
^2-1))^(1/2)*f*c*d/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2+b*(c^2*f^2-g^2)^(3/2)*d*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*dilog(((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-
b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/
g^4*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-
c*f+(c^2*f^2-g^2)^(1/2)))-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*c^3*d/(c*x-1)^(1/2
)/(c*x+1)^(1/2)/g^2*x^2+b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)^(1/2)/(c*x+1)^(1
/2)/g^3*x*c^3*f^2-b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g
-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(c^2*f^2-g^2)^(3/2)
*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln((
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-
g^2)^(1/2)))+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^3*a
rccosh(c*x)^2*c^3*d/g^4-3/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*f*arccosh(c*x)^2*c*d/g^2-a/g^4*d^2*c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*
d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)
)-a/g^5*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d
*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(
x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4-4/3*b*(-d*(c^2*x^2-1))^
(1/2)*d/(c*x-1)/(c*x+1)/g*arccosh(c*x)+2*a/g^3*d^2/(-d*(c^2*f^2-g^2)/g^2)^(
1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2
)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(
x+f/g))*c^2*f^2+1/2*a/g^2*c^2*d*f*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(
c^2*f^2-g^2)/g^2)^(1/2)*x+3/2*a/g^2*c^2*d^2*f/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a
/g*d*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+1/3*a
/g*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)-1/3*b*(
-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c
^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^2*c^2+b*(-d*(c^2*x^2-1)
)^(1/2)*d/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*c^2*f^2+1/2*b*(-d*(c^2*x^2-1))^(
1/2)*f*c^4*d/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1
/2)*f*c^2*d/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x-b*(-d*(c^2*x^2-1))^(1/2)*d/(
c*x-1)/(c*x+1)/g^3*arccosh(c*x)*x^2*c^4*f^2-a/g^3*d*(-(x+f/g)^2*c^2*d+2*c^2
*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^2*f^2-a/g*d^2/(-d*(c^2*f^2-g^2)
/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g
^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(
1/2))/(x+f/g))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}\left(a + b\,\text{acosh}\left(cx\right)\right)}{f + gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="gia
c")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)/(g*x + f), x)
```

3.62 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=1385

result too large to display

```
[Out] (3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*d^2*g^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (25*b*c*d^2*f^3*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (15*b*d^2*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (17*b*c^3*d^2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^2*g^3*x^7*Sqrt[d - c^2*d*x^2])
/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d - c^2*d
*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*
d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f^3*(1 - c^2*x^2)^3*Sqrt
[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^3*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 - (15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(128*c^2) + (15*d^2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/64 + (5*d^2*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/24 + (5*d^2*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (d^2*f^3*x*(1 - c*x)^2*(1 + c*x)^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 + (3*d^2*f*g^2*x^3*(1 - c*x)^2*(
1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d^2*f^2*g*(1 -
c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (2*d
^2*g^3*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(6
3*c^4) - (d^2*g^3*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/(9*c^2) - (5*d^2*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*d^2*f*g^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 2.5179, antiderivative size = 1385, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 20, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.645, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 261, 5718, 194, 5745, 5743, 5759, 266, 43, 100, 12, 74, 5733, 373}

$$-\frac{bc^5d^2g^3\sqrt{d-c^2dx^2}x^9}{81\sqrt{cx-1}\sqrt{cx+1}}-\frac{3bc^5d^2fg^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{cx-1}\sqrt{cx+1}}+\frac{19bc^3d^2g^3\sqrt{d-c^2dx^2}x^7}{441\sqrt{cx-1}\sqrt{cx+1}}-\frac{3bc^5d^2f^2g\sqrt{d-c^2dx^2}x^7}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{17bc^3d^2fg^2}{96\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*d^2*g^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (25*b*c*d^2*f^3*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (15*b*d^2*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (17*b*c^3*d^2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^2*g^3*x^7*Sqrt[d - c^2*d*x^2])
/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d - c^2*d
*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*
d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f^3*(1 - c^2*x^2)^3*Sqrt
[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^3*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 - (15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(128*c^2) + (15*d^2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/64 + (5*d^2*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/24 + (5*d^2*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (d^2*f^3*x*(1 - c*x)^2*(1 + c*x)^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 + (3*d^2*f*g^2*x^3*(1 - c*x)^2*(
1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d^2*f^2*g*(1 -
c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (2*d
^2*g^3*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(6
3*c^4) - (d^2*g^3*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/(9*c^2) - (5*d^2*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*d^2*f*g^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
```

```
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
```

```
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(f^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+3f^2}{}$$

$$= \frac{\left(d^2f^3\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= \frac{1}{6}d^2f^3x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}d^2fg^2x^3($$

$$= \frac{bd^2f^3\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{24}d^2f^3x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,($$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bcd^2f^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{9bc^3d^2f^2gx^5\sqrt{d-}}{35\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^3x^2\sqrt{d-c^2}}{96\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^3x^2\sqrt{d-c^2}}{96\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 7.97355, size = 1802, normalized size = 1.3

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-(a*d^2*g*(27*c^2*f^2 + 2*g^2))/(63*c^4) + (a*d^
2*f*(88*c^2*f^2 - 15*g^2)*x)/(128*c^2) - (a*d^2*g*(-81*c^2*f^2 + g^2)*x^2)/
(63*c^2) - (a*d^2*f*(104*c^2*f^2 - 177*g^2)*x^3)/192 + (a*d^2*g*(-27*c^2*f^
2 + 5*g^2)*x^4)/21 + (a*c^2*d^2*f*(8*c^2*f^2 - 51*g^2)*x^5)/48 - (a*c^2*d^2
*g*(-27*c^2*f^2 + 19*g^2)*x^6)/63 + (3*a*c^4*d^2*f*g^2*x^7)/8 + (a*c^4*d^2*
g^3*x^8)/9) - (5*a*d^(5/2)*f*(8*c^2*f^2 + 3*g^2)*ArcTan[(c*x*Sqrt[-(d*(-1 +
 c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(128*c^3) - (b*d^2*f^2*g*Sqrt[-(d*(
-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3
```

```
*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]))/(12*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)) - (b*d^2*f^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(Cosh[2*ArcCosh[c*x]
] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])))/(8*c*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(8*
ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]
))/(64*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (3*b*d^2*f*g^2*Sqrt[-(d*(-
1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c
*x]*Sinh[4*ArcCosh[c*x]]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
 (b*d^2*f^2*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*Cosh[5*A
rcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*x]*Sinh[
5*ArcCosh[c*x]]))/(600*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*g
^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*Cosh[5*ArcCosh[c*x]
] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c
*x]]))/(3600*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^3*Sqrt[-(
d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]]
- 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcC
osh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*Ar
cCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f*g^2
*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
h[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sin
h[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*S
inh[6*ArcCosh[c*x]])))/(384*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*
d^2*f^2*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-55125*c*x + 1225*Cosh[3*ArcCosh
[c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 441*C
osh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c*x]] - 1225*ArcCosh[c*x]*Sinh[3*Ar
cCosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] - 525*ArcCosh[c*x]*Sin
h[7*ArcCosh[c*x]])))/(235200*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b
*d^2*g^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-55125*c*x + 1225*Cosh[3*ArcCosh[
c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 441*Co
sh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c*x]] - 1225*ArcCosh[c*x]*Sinh[3*Arc
Cosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] - 525*ArcCosh[c*x]*Sinh
[7*ArcCosh[c*x]])))/(352800*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*
d^2*f*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(1440*ArcCosh[c*x]^2 - 576*Cosh[2
*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*ArcCosh[c*x]] + 9*Cos
h[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 576*ArcCosh[c*
x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] - 72*ArcCos
h[c*x]*Sinh[8*ArcCosh[c*x]]))/(24576*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)) - (b*d^2*g^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-1389150*c*x + 31752*Cosh
[5*ArcCosh[c*x]] + 5*(2025*Cosh[7*ArcCosh[c*x]] + 245*Cosh[9*ArcCosh[c*x]]
- 63*ArcCosh[c*x]*(-4410*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 504*Sinh[5*
ArcCosh[c*x]] + 225*Sinh[7*ArcCosh[c*x]] + 35*Sinh[9*ArcCosh[c*x]]))))/(254
01600*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.622, size = 2116, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 5/16*a*f^3*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f^3*d^3/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/63*a*g^3/d/c^4*(-c^2*d*x^2+d)^(7/2)+
5/64*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+15/128*a*f*g^2/c^2*d^2*x*(-c^2*d*
x^2+d)^(1/2)+15/128*a*f*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2))-3/8*a*f*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+299/2304*b*(-d
*(c^2*x^2-1))^(1/2)*f^3*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)+19/441*b*(-d*(c^2
*x^2-1))^(1/2)*g^3*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^7-1/21*b*(-d*(c^2*
x^2-1))^(1/2)*g^3*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^5+1/189*b*(-d*(c^2*x^
2-1))^(1/2)*g^3*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^3+2/63*b*(-d*(c^2*x^2-1
))^(1/2)*g^3*d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*x-11/16*b*(-d*(c^2*x^2-1))
^(1/2)*f^3*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x-16/63*b*(-d*(c^2*x^2-1))^(1/2
)*g^3*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4-5/32*b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)/c*f^3*arccosh(c*x)^2*d^2+2/63*b*(-d*(c^2*x^2-1))^
(1/2)*g^3*d^2/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)+359/24576*b*(-d*(c^2*x^2-1))
^(1/2)*f*g^2*d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)-1/36*b*(-d*(c^2*x^2-1))^(1
/2)*f^3*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^6+13/96*b*(-d*(c^2*x^2-1))^(1
/2)*f^3*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^4-11/32*b*(-d*(c^2*x^2-1))^(1
/2)*f^3*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2-1/81*b*(-d*(c^2*x^2-1))^(1/2)
*g^3*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^9-15/256*b*(-d*(c^2*x^2-1))^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*f*arccosh(c*x)^2*d^2*g^2-12/7*b*(-d*(c^2*
x^2-1))^(1/2)*g*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*f^2-133/128*b*(-d*(c^2
*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/9*b*(-d*(c^2*x^
2-1))^(1/2)*g^3*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^10-26/63*b*(-d*(c^2*
x^2-1))^(1/2)*g^3*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^8-3/64*b*(-d*(c^2*
x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^8-3/49*b*(-d*(c^2
*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^7*f^2+17/96*b*(-d*(c
^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-59/256*b*(-d
*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+15/256*b*(-
d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+9/35*b*(-d
*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5*f^2-3/7*b*(-d
*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3*f^2+3/7*b*(-d*(
c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x*f^2-1/9*a*g^3*x^2*(
-c^2*d*x^2+d)^(7/2)/c^2/d+1/16*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)-3/7*a*f^2
*g/c^2/d*(-c^2*d*x^2+d)^(7/2)+34/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x+1
)*c^2/(c*x-1)*arccosh(c*x)*x^6-1/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x+1
```

```
)/c^2/(c*x-1)*arccosh(c*x)*x^2+3/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)/c
^2/(c*x-1)*arccosh(c*x)*f^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c*x+1)*c^
6/(c*x-1)*arccosh(c*x)*x^7-17/24*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c*x+1)*c
^4/(c*x-1)*arccosh(c*x)*x^5+59/48*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c*x+1)*
c^2/(c*x-1)*arccosh(c*x)*x^3+5/24*a*f^3*d*x*(-c^2*d*x^2+d)^(3/2)+1/6*a*f^3*
x*(-c^2*d*x^2+d)^(5/2)+3/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)*c^6/(c*x-
1)*arccosh(c*x)*x^8*f^2-12/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)*c^4/(c*
x-1)*arccosh(c*x)*x^6*f^2+18/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)*c^2/(
c*x-1)*arccosh(c*x)*x^4*f^2+3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)*
c^6/(c*x-1)*arccosh(c*x)*x^9-23/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+
1)*c^4/(c*x-1)*arccosh(c*x)*x^7+127/64*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(
c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+15/128*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d
^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\big(ac^4d^2g^3x^7 + 3\,ac^4d^2fg^2x^6 + 3\,ad^2f^2gx + ad^2f^3 + \big(3\,ac^4d^2f^2g - 2\,ac^2d^2g^3\big)x^5 + \big(ac^4d^2f^3 - 6\,ac^2d^2fg^2\big)x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g - 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 - 6*a*c
^2*d^2*f*g^2)*x^4 - (6*a*c^2*d^2*f^2*g - a*d^2*g^3)*x^3 - (2*a*c^2*d^2*f^3
- 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
```

```
^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3 - (2*
b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.63 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)\, dx$

Optimal. Leaf size=1015

$$-\frac{bc^5d^2g^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc^5d^2fg\sqrt{d-c^2dx^2}x^7}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{17bc^3d^2g^2\sqrt{d-c^2dx^2}x^6}{288\sqrt{cx-1}\sqrt{cx+1}}+\frac{6bc^3d^2fg\sqrt{d-c^2dx^2}x^5}{35\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bc^3d^2f^2\sqrt{d-}}{96\sqrt{cx-1}}$$

```
[Out] (2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2
5*b*c*d^2*f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*b*d^2*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (2*b*c*d^2*f*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*d^2*f^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128
*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d^
2*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 +
(5*d^2*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
)/48 + (d^2*f^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/6 + (d^2*g^2*x^3*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/8 - (2*d^2*f*g*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(7*c^2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*g^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi [A] time = 2.10583, antiderivative size = 1015, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 261, 5718, 194, 5745, 5743, 5759, 266, 43}

$$-\frac{bc^5d^2g^2\sqrt{d-c^2dx^2}x^8}{64\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc^5d^2fg\sqrt{d-c^2dx^2}x^7}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{17bc^3d^2g^2\sqrt{d-c^2dx^2}x^6}{288\sqrt{cx-1}\sqrt{cx+1}}+\frac{6bc^3d^2fg\sqrt{d-c^2dx^2}x^5}{35\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bc^3d^2f^2\sqrt{d-}}{96\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2
5*b*c*d^2*f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*b*d^2*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (2*b*c*d^2*f*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*d^2*f^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128
*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d^
2*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 +
(5*d^2*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
)/48 + (d^2*f^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/6 + (d^2*g^2*x^3*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/8 - (2*d^2*f*g*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(7*c^2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*g^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
```

```
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
```

```
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)^2 \left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+2fg\right.}{}$$

$$= \frac{\left(d^2f^2\sqrt{d-c^2dx^2}\right)\int (-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} + $$

$$= \frac{1}{6}d^2f^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}d^2g^2x^3(1$$

$$= \frac{bd^2f^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{24}d^2f^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,($$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{6bc^3d^2fgx^5\sqrt{d-c^2d}}{35\sqrt{-1+cx}\sqrt{1+c}}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2fgx^3\sqrt{d-c^2d}}{7\sqrt{-1+cx}\sqrt{1+c}}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5bd^2g^2x^2\sqrt{d-c^2d}}{256c\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 7.34846, size = 1282, normalized size = 1.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-2*a*d^2*f*g)/(7*c^2) + (a*d^2*(88*c^2*f^2 - 5*
g^2)*x)/(128*c^2) + (6*a*d^2*f*g*x^2)/7 + (a*d^2*(-104*c^2*f^2 + 59*g^2)*x^
3)/192 - (6*a*c^2*d^2*f*g*x^4)/7 + (a*c^2*d^2*(8*c^2*f^2 - 17*g^2)*x^5)/48
+ (2*a*c^4*d^2*f*g*x^6)/7 + (a*c^4*d^2*g^2*x^7)/8) - (5*a*d^(5/2)*(8*c^2*f^
2 + g^2)*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/
(128*c^3) - (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]))/(18
*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f^2*Sqrt[-(d*(-1 + c*x)
```

```
*(1 + c*x))]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*
ArcCosh[c*x]])))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^2*Sq
rt[-(d*(-1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(64*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[
4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(128*c^3*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-
450*c*x + 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3
*ArcCosh[c*x]] + 9*Cosh[5*ArcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*
x]] - 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(900*c^2*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)) + (b*d^2*f^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*Ar
cCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCos
h[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*Arc
Cosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cos
h[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*
ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh
[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(1152*c^3*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
*(-55125*c*x + 1225*Cosh[3*ArcCosh[c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcCosh[c*x] + 441*Cosh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c
*x]] - 1225*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*Ar
cCosh[c*x]] - 525*ArcCosh[c*x]*Sinh[7*ArcCosh[c*x]])))/(352800*c^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*
(1440*ArcCosh[c*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]]
+ 64*Cosh[6*ArcCosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh
[2*ArcCosh[c*x]] - 576*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]
*Sinh[6*ArcCosh[c*x]] - 72*ArcCosh[c*x]*Sinh[8*ArcCosh[c*x]]))/(73728*c^3*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.526, size = 1540, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -133/384*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-
11/16*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x*f^2-5/32*
b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d^2*f
^2-5/256*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c
```

```
*x)^2*d^2*g^2+13/96*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^
(1/2)*x^4*f^2-11/32*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1
/2)*x^2*f^2-1/64*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)
^(1/2)*x^8+17/288*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)^(1/2)*c^3/(c*x-1
)^(1/2)*x^6-59/768*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)^(1/2)*c/(c*x-1)
^(1/2)*x^4+5/256*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(
1/2)*x^2-1/36*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*
x^6*f^2-1/8*a*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+5/192*a*g^2/c^2*d*x*(-c^2*d*
x^2+d)^(3/2)+5/128*a*g^2/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/128*a*g^2/c^2*d^3
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/7*a*f*g/c^2/d
*(-c^2*d*x^2+d)^(7/2)+1/48*a*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/16*a*f^2*d^3/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+5/24*a*f^2*d*x*(
-c^2*d*x^2+d)^(3/2)+5/16*a*f^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-2/49*b*(-d*(c^2*x
^2-1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^7+6/35*b*(-d*(c^2*x^
2-1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5-2/7*b*(-d*(c^2*x^2-
1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+2/7*b*(-d*(c^2*x^2-1))^
(1/2)*f*g*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x-8/7*b*(-d*(c^2*x^2-1))^(1/2)*
f*g*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+1/8*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d
^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-23/48*b*(-d*(c^2*x^2-1))^(1/2)*g^2*
d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+127/192*b*(-d*(c^2*x^2-1))^(1/2)*g
^2*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+5/128*b*(-d*(c^2*x^2-1))^(1/2)*
g^2*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g
*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)+1/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x
+1)*c^6/(c*x-1)*arccosh(c*x)*x^7*f^2-17/24*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*
x+1)*c^4/(c*x-1)*arccosh(c*x)*x^5*f^2+59/48*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c
*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3*f^2+1/6*a*f^2*x*(-c^2*d*x^2+d)^(5/2)+2/7
*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^8-8/7*
b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6+12/7*
b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4+359/7
3728*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)+299/2
304*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2g^2x^6 + 2\,ac^4d^2fgx^5 - 4\,ac^2d^2fgx^3 + 2\,ad^2fgx + ad^2f^2 + \left(ac^4d^2f^2 - 2\,ac^2d^2g^2\right)x^4 - \left(2\,ac^2d^2f^2 - ad^2g^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c^2
*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*
g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2
 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.64 $\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=568

$$\frac{1}{6}d^2fx(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2fx(1-cx)(cx+1)$$

```
[Out] (b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (25*b*
c*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d
^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c^3*d
^2*f*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*
d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f*
(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d^2*f*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 + (d^2*f*x*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (d^2*g*(1
- c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (5
*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rubi [A] time = 0.904908, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 261, 5718, 194}

$$\frac{1}{6}d^2fx(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2fx(1-cx)(cx+1)$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```



```
[Out] (b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (25*b*
c*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d
^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c^3*d
^2*f*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*
d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f*
(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d^2*f*x*(1 - c
```

```
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 + (d^2*f*x*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (d^2*g*(1
- c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (5
*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
```

```
&& GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)\left(a+b\cosh^{-1}(cx)\right)d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(f(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+gx(-\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2f\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(d}{}$$

$$= \frac{1}{6}d^2fx(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2g(1-cx)^3}{}$$

$$= \frac{bd^2f\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5}{24}d^2fx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a\right.$$

$$= \frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3d^2gx^5\sqrt{d-c^2dx^2}}{35\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 6.30286, size = 644, normalized size = 1.13

$$\frac{d^2\left(8400a\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(7c^2fx\left(8c^4x^4-26c^2x^2+33\right)+48g\left(c^2x^2-1\right)^3\right)-882000ac\sqrt{d}f\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(8400*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(48*g
*(-1 + c^2*x^2)^3 + 7*c^2*f*x*(33 - 26*c^2*x^2 + 8*c^4*x^4)) - 882000*a*c*S
qrt[d]*f*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + 78400*b*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]
) - 352800*b*c*f*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]
*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 44100*b*c*f*Sqrt[d - c^2*d*x^2]*(
8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x
]]) - 1568*b*g*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)
```

```
]*(1 + c*x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]]
 + 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*
x]]) + 1225*b*c*f*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCo
sh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x
]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])
) + 4*b*g*Sqrt[d - c^2*d*x^2]*(55125*c*x - 55125*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*ArcCosh[c*x] - 1225*Cosh[3*ArcCosh[c*x]] - 1323*Cosh[5*ArcCosh[c
*x]] - 225*Cosh[7*ArcCosh[c*x]] + 3675*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] +
6615*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] + 1575*ArcCosh[c*x]*Sinh[7*ArcCosh[c
*x]])))/(2822400*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [A] time = 0.465, size = 877, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/7*a*g/c^2/d*(-c^2*d*x^2+d)^(7/2)+1/6*a*f*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f
*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/7*b*(-d*(c^2*x
^2-1))^(1/2)*g*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^8-4/7*b*(-d*(c^2*x^2-
1))^(1/2)*g*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6+6/7*b*(-d*(c^2*x^2-1))
^(1/2)*g*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-4/7*b*(-d*(c^2*x^2-1))^(1
/2)*g*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)*f*d
^2/(c*x+1)/(c*x-1)*c^6*arccosh(c*x)*x^7-17/24*b*(-d*(c^2*x^2-1))^(1/2)*f*d^
2/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+59/48*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2
/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-11/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/
(c*x+1)/(c*x-1)*arccosh(c*x)*x+299/2304*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x
+1)^(1/2)/(c*x-1)^(1/2)/c-5/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)/c*f*arccosh(c*x)^2*d^2+1/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)/
c^2/(c*x-1)*arccosh(c*x)-1/49*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*
c^5/(c*x-1)^(1/2)*x^7+3/35*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*c^3
/(c*x-1)^(1/2)*x^5-1/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*c/(c*x-
1)^(1/2)*x^3+1/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/
2)*x-1/36*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5*x^
6+13/96*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*x^4-
11/32*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2gx^5 + ac^4d^2fx^4 - 2\,ac^2d^2gx^3 - 2\,ac^2d^2fx^2 + ad^2gx + ad^2f + \left(bc^4d^2gx^5 + bc^4d^2fx^4 - 2\,bc^2d^2gx^3 - 2\,bc\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b*c
^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arccosh(c*x))*sqrt(
-c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.65 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1744

result too large to display

```
[Out] (2*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2])/(3*g^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2])/(g^5*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d - c^2*d
*x^2])/(4*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^3*Sqrt[d - c^2*d
*x^2])/(45*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x
^3*Sqrt[d - c^2*d*x^2])/(9*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^5*d^2*f
*x^4*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a*d^2*(c^
2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(g^5*(1 - c*x)*(1 + c*x))
 + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/g^5 + (c^2*d^
2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*g^2) - (c^2*d^2*f*(c^2*f
^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*g^4) - (c^4*d^2*
f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*g^2) - (2*d^2*(1 - c*x)*
(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*g) - (d^2*(c^2*f^2
- 2*g^2)*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*g
^3) - (c^2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(5*g) + (c*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*g^
2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d
^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(c^2*f^2 - g^2)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x))
 - (d^2*(c^2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(2*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*d^2*(c^2*f
^2 - g^2)^(5/2)*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2])]])/(g^6*(1 - c*x)*(1 + c*x)) + (b
*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcC
osh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^A
rcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
```

```
c*x])
```

Rubi [A] time = 4.77926, antiderivative size = 1744, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 31, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5836, 5826, 5683, 5676, 30, 5718, 5743, 5759, 100, 12, 74, 5733, 5824, 683, 5816, 6742, 93, 208, 1610, 1654, 725, 206, 5860, 5858, 8, 5832, 3320, 2264, 2190, 2279, 2391}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (2*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2])/(3*g^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2])/(g^5*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d - c^2*d
*x^2])/(4*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^3*Sqrt[d - c^2*d
*x^2])/(45*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x
^3*Sqrt[d - c^2*d*x^2])/(9*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^5*d^2*f
*x^4*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a*d^2*(c^
2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(g^5*(1 - c*x)*(1 + c*x))
 + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/g^5 + (c^2*d^
2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*g^2) - (c^2*d^2*f*(c^2*f
^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*g^4) - (c^4*d^2*
f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*g^2) - (2*d^2*(1 - c*x)*
(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*g) - (d^2*(c^2*f^2
- 2*g^2)*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*g
^3) - (c^2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(5*g) + (c*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*g^
2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d
^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(c^2*f^2 - g^2)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x))
 - (d^2*(c^2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(2*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*d^2*(c^2*f
^2 - g^2)^(5/2)*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2])]])/(g^6*(1 - c*x)*(1 + c*x)) + (b
*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcC
```

```
osh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^A
rcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5826

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n, (f + g*x)
^m*(d1 + e1*x)^(p - 1/2)*(d2 + e2*x)^(p - 1/2), x], x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Integ
erQ[m] && IGtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5824

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[((f + g*
x)^m*(d1*d2 + e1*e2*x^2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*
(n + 1)), x] - Dist[1/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 5816

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*
Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[
n, 0] && IGtQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
```

```
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5860

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5858

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(-\frac{c^2f\left(c^2f^2-2g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{g^4}+\frac{c^2\left(c^2f^2-2g^2\right)x\sqrt{-1+c\cdots}}{\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{\left(c^4d^2f\sqrt{d-c^2dx^2}\right)\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\,dx}{g^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(c^4d^2\sqrt{d\cdots}\right.}{\cdots}$$

$$= -\frac{c^2d^2f\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^4}-\frac{c^4d^2fx^3\sqrt{d-c^2dx^2}\left(a\cdots\right.}{4g^2}$$

$$= \frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d^2f\left(c^2f^2-2g^2\right)x^2\sqrt{d-c^2dx^2}}{4g^4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2\left(\cdots\right.}{9\cdots}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2fx^2\sqrt{d-c^2d\cdots}}{16g^2\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2fx^2\sqrt{d-c^2d\cdots}}{16g^2\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2fx^2\sqrt{d-c^2d\cdots}}{16g^2\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2fx^2\sqrt{d-c^2d\cdots}}{16g^2\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2fx^2\sqrt{d-c^2d\cdots}}{16g^2\sqrt{-1+cx}\sqrt{1+\cdots}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2-g^2\right)^2x\cdots}{g^5\sqrt{-1+cx}\cdots}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2-g^2\right)^2x\cdots}{g^5\sqrt{-1+cx}\cdots}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2-g^2\right)^2x\cdots}{g^5\sqrt{-1+cx}\cdots}$$

Mathematica [C] time = 20.71, size = 6244, normalized size = 3.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.368, size = 4234, normalized size = 2.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x)
```

```
[Out] b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)/g^2*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/
(-c*f+(c^2*f^2-g^2)^(1/2)))-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-1/25*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*x^5*c^5+11/45*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*x^3*c^3-23/15*b*(-d*(c^2*x^2-1))^(
1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*c*x+33/128*b*(-d*(c^2*x^2-1))^(1/2)*
f*d^2*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2-1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^
2*c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^
6/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x^5+11/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*
c^4/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x^3-9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2
*c^2/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x+1/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c
*x-1)/(c*x+1)/g^3*arccosh(c*x)*x^4*c^6*f^2-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*x^2*c^4*f^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*
f^3*d^2*c^6/(c*x-1)/(c*x+1)/g^4*arccosh(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(1/
2)*f^3*d^2*c^4/(c*x-1)/(c*x+1)/g^4*arccosh(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(c*x-1)/(c*x+1)/g^5*arccosh(c*x)*x^2*c^6*f^4+b*d^2*(-d*(c^2*x^2-1))^(1/
2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*dilog((-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2))
)*c^4*f^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/g^6*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2
)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*
(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*dilog((-(c*x+(c*x-1)^(1
```

```
/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*c
^2*f^2+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/g^4*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)
^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^2+1/5*a/g*(-(x+f/g)^2*c^2*d+2*c^2*
d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(5/2)+1/3*a/g*d*(-(x+f/g)^2*c^2*d+2*c^2*
d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)+a/g*d^2*(-(x+f/g)^2*c^2*d+2*c^2*d*
f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)
/(c*x+1)/g^5*arccosh(c*x)*c^4*f^4-15/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*d^2*c/g^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^4*c^4+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d^
2*c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*x^4-9/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d
^2*c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*x^2-1/9*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^3*x^3*c^5*f^2+7/3*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^3*x*c^3*f^2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*f
^3*d^2*c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*x^2-b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^5*x*c^5*f^4+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^
2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*arccosh(c*x)*ln((-(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1
/2)))*c^4*f^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)/g^6*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*
f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-2*b*d^2*(-d*(c^2*
x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c
*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f
+(c^2*f^2-g^2)^(1/2)))*c^2*f^2+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^
2-a/g*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f
/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+
f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-1/3*a/g^3*d*(-(x+f/g)^2*c^2*d+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*c^2*f^2+a/g^5*d^2*(-(x+f/g)^2*c
^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^4*f^4-2*a/g^3*d^2*(-(
x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^2*f^2+34/15
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^2*c^2+7/3*b*
(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*c^2*f^2-1/2*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^5*arccosh(c*x)^2*d^2*c^
5/g^6+5/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^3*arccosh(
c*x)^2*d^2*c^3/g^4+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-b*d^2*(-d*(c^2*x^2
-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)
*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2
*f^2-g^2)^(1/2)))+a/g^7*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-
g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2
*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^6*f^6-3*a/g^5
```

```
*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x
+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-
d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4+3*a/g^3*d^3/(-d*(c^2*f^2-g^2)/
g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^
2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1
/2))/(x+f/g))*c^2*f^2+15/8*a/g^2*c^2*d^3*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/
2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-1/2*
a/g^4*d^2*c^4*f^3*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2
)^(1/2)*x-5/2*a/g^4*d^3*c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f
/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^6*d^3*c^6*f
^5/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+1/4*a/g^2*c^2*d*f*(-(x+f/g)^2*c^2*d+2*c^2*d*
f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*x+7/8*a/g^2*c^2*d^2*f*(-(x+f/g)^2*c^
2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x-23/15*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(c*x-1)/(c*x+1)/g*arccosh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arcosh}\,(cx)+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="gia
c")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)/(g*x + f), x)
```

3.66 $\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=478

$$-\frac{3f^2g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{f^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}+\frac{3fg^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}\right.}{4bc^3\sqrt{d-c^2dx^2}}$$

```
[Out] (-3*b*f^2*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*
g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^
2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (3*f^2*g*(1 - c*x
)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*g^3*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^4*Sqrt[d - c^2*d*x^2]) - (3*f*g^2*
x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) - (
g^3*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^2*Sqrt[d - c^2*d*x^2
]) + (f^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[
d - c^2*d*x^2]) + (3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.2883, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {5836, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{3f^2g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{f^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}+\frac{3fg^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}\right.}{4bc^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```



```
[Out] (-3*b*f^2*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*
g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^
2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (3*f^2*g*(1 - c*x
)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*g^3*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^4*Sqrt[d - c^2*d*x^2]) - (3*f*g^2*
x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) - (
g^3*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^2*Sqrt[d - c^2*d*x^2
]) + (f^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[
```

```
d - c^2*d*x^2]) + (3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(\frac{f^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3f^2gx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3fg^2x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} +\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(3f^2g\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\sqrt{1+cx}}}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{3f^2g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{3fg^2x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3x^3\sqrt{-1+cx}\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 2.05386, size = 405, normalized size = 0.85

$$-\frac{12a\sqrt{d-c^2dx^2}\left(c^2g\left(18f^2+9fgx+2g^2x^2\right)+4g^3\right)}{d} - \frac{36acf\left(2c^2f^2+3g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} + \frac{216bc^2f^2g\sqrt{d-c^2dx^2}\left(cx-\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)\right)}{d\sqrt{cx-1}\sqrt{cx+1}} + \frac{36bc^3f^3\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-12*a*Sqrt[d - c^2*d*x^2]*(4*g^3 + c^2*g*(18*f^2 + 9*f*g*x + 2*g^2*x^2)))
/d + (36*b*c^3*f^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/Sqr
t[d - c^2*d*x^2] + (216*b*c^2*f^2*g*Sqrt[d - c^2*d*x^2]*(c*x - Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*ArcCosh[c*x]))/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*g^3
*Sqrt[d - c^2*d*x^2]*(c*x*(6 + c^2*x^2) - 3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2
 + c^2*x^2)*ArcCosh[c*x]))/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (36*a*c*f*(2*
c^2*f^2 + 3*g^2)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]
)/Sqrt[d] + (27*b*c*f*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*Arc
Cosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d
- c^2*d*x^2])/(72*c^4)
```

Maple [B] time = 0.323, size = 859, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a*g^3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(-c^2*d*x^2+d)^(1
/2)-3/2*a*f*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+3/2*a*f*g^2/c^2/(c^2*d)^(1/2)*
arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3*a*f^2*g/c^2/d*(-c^2*d*x^2+d)
^(1/2)+a*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3
*b*(-d*(c^2*x^2-1))^(1/2)*g^3/d/(c^2*x^2-1)*arccosh(c*x)*x^4-1/3*b*(-d*(c^2
*x^2-1))^(1/2)*g^3/c^2/d/(c^2*x^2-1)*arccosh(c*x)*x^2+3*b*(-d*(c^2*x^2-1))^
(1/2)*g/c^2/d/(c^2*x^2-1)*arccosh(c*x)*f^2-3/4*b*(-d*(c^2*x^2-1))^(1/2)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*f*arccosh(c*x)^2*g^2+3/4*b*(-d*(
c^2*x^2-1))^(1/2)*f*g^2/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+3*b
*(-d*(c^2*x^2-1))^(1/2)*g/c/d/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*f^2
-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c/(c^2*x^2-1)*f
^3*arccosh(c*x)^2+1/9*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c/d/(c^2*x^2-1)*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3+2/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^3/d/(c^2*x^2-1)*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-3/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/(c^2*x^2
-1)*arccosh(c*x)*x^3+3/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/c^2/(c^2*x^2-1)*a
rccosh(c*x)*x-3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/c^3/(c^2*x^2-1)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)-3*b*(-d*(c^2*x^2-1))^(1/2)*g/d/(c^2*x^2-1)*arccosh(c*x)
*x^2*f^2+2/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^4/d/(c^2*x^2-1)*arccosh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(ag^3x^3+3\,afg^2x^2+3\,af^2gx+af^3+\left(bg^3x^3+3\,bfg^2x^2+3\,bf^2gx+bf^3\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2+d}}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(c^2
*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\text{acosh}\,(cx))\left(f+gx\right)^3}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**3/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^3(b\,\mathrm{arcosh}\,(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^3*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.67 $\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=288

$$\frac{f^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}$$

```
[Out] (-2*b*f*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 -
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) + (f^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^
2]) + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sq
rt[d - c^2*d*x^2])
```

Rubi [A] time = 0.914242, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {5836, 5822, 5676, 5718, 8, 5759, 30}

$$\frac{f^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 -
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) + (f^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^
2]) + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sq
rt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
```

```
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
```

```
x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(\frac{f^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2fgx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{g^2x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\right) dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(f^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(2fg\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}}{\sqrt{d-c^2dx^2}}$$
$$= -\frac{2fg(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} +$$
$$= -\frac{2bfgx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{bg^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{2fg(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.58204, size = 267, normalized size = 0.93

$$\frac{-\frac{4a\left(2c^2f^2+g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} - \frac{4acg\sqrt{d-c^2dx^2}(4f+gx)}{d} + \frac{4bc^2f^2\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)^2}{\sqrt{d-c^2dx^2}} + \frac{16bcfg\sqrt{d-c^2dx^2}\left(\frac{cx}{\sqrt{cx-1}\sqrt{cx+1}}-\cosh^{-1}(cx)\right)}{d} + bg^2\sqrt{\cdots}}{8c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-4*a*c*g*(4*f + g*x)*Sqrt[d - c^2*d*x^2])/d + (16*b*c*f*g*Sqrt[d - c^2*d*
x^2]*((c*x)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ArcCosh[c*x]))/d + (4*b*c^2*f^
2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/Sqrt[d - c^2*d*x^2]
- (4*a*(2*c^2*f^2 + g^2)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^
```

```
2*x^2))])/Sqrt[d] + (b*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*Ar
cCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d
 - c^2*d*x^2])/(8*c^3)
```

Maple [B] time = 0.258, size = 559, normalized size = 1.9

$$-\frac{ag^2x}{2\,c^2d}\sqrt{-c^2dx^2+d}+\frac{ag^2}{2\,c^2}\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}}-2\,\frac{afg\sqrt{-c^2dx^2+d}}{c^2d}+af^2\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a*g^2/c^2/(c^2*d)^(1/2)*arctan(
(c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2*a*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*
f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+2*b*(-d*(c^2
*x^2-1))^(1/2)*f*g/c/(c^2*x^2-1)/d*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/4*b*(-d*
(c^2*x^2-1))^(1/2)*g^2/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-2*b*
(-d*(c^2*x^2-1))^(1/2)*f*g/(c^2*x^2-1)/d*arccosh(c*x)*x^2-1/2*b*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c/(c^2*x^2-1)*arccosh(c*x)^2*f^2-
1/4*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*
arccosh(c*x)^2*g^2-1/8*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/c^3/(c^2*x^2-1)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)+2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/c^2/(c^2*x^2-1)/d*arc
cosh(c*x)-1/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/(c^2*x^2-1)*arccosh(c*x)*x^3+1
/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/c^2/(c^2*x^2-1)*arccosh(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arcosh}\left(cx\right)\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2
+ 2*b*f*g*x + b*f^2)*arccosh(c*x))/(c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\text{acosh}\left(cx\right)\right)\left(f+gx\right)^2}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx+f\right)^2\left(b\,\text{arcosh}\left(cx\right)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^2*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.68 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{f\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

```
[Out] -((b*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) + (f*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.472078, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5836, 5822, 5676, 5718, 8}

$$\frac{f\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) + (f*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
```

```
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(\frac{f\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{gx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(g\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} + \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}} -$$

$$= -\frac{bgx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} + \frac{f\sqrt{-1+cx}\sqrt{1+c}}{2bc\sqrt{}}$$

Mathematica [A] time = 0.667384, size = 172, normalized size = 1.26

$$\frac{-\frac{2acf\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)}{\sqrt{d}} - \frac{2ag\sqrt{d-c^2dx^2}}{d} + \frac{bcf\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)^2}{\sqrt{d-c^2dx^2}} + \frac{2bg\sqrt{d-c^2dx^2}\left(\frac{cx}{\sqrt{cx-1}\sqrt{cx+1}}-\cosh^{-1}(cx)\right)}{d}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-2*a*g*Sqrt[d - c^2*d*x^2])/d + (2*b*g*Sqrt[d - c^2*d*x^2]*((c*x)/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - ArcCosh[c*x]))/d + (b*c*f*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcCosh[c*x]^2)/Sqrt[d - c^2*d*x^2] - (2*a*c*f*ArcTan[(c*x*Sq
rt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d])/(2*c^2)
```

Maple [A] time = 0.225, size = 239, normalized size = 1.8

$$-\frac{ag}{c^2d}\sqrt{-c^2dx^2+d} + af\arctan\left(x\sqrt{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}}\right)\frac{1}{\sqrt{c^2d}} - \frac{bf(\operatorname{arccosh}(cx))^2}{2cd(c^2x^2-1)}\sqrt{-d(c^2x^2-1)}\sqrt{cx-1}\sqrt{cx+1} - \frac{bga}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/c/(c^2*x^2-1)*f*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*g/d/(c^2*x^2-1)
*arccosh(c*x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)*g/c/d/(c^2*x^2-1)*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x+b*(-d*(c^2*x^2-1))^(1/2)*g/c^2/d/(c^2*x^2-1)*arccosh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\text{arcosh}\left(cx\right)\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x))/(
c^2*d*x^2 - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\text{acosh}\,(cx))\left(f+gx\right)}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(b \operatorname{arcosh}(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.69 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=365

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cos\right.}{\sqrt{d-c^2d}}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
- (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x
]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f -
Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 0.702993, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {5836, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cos\right.}{\sqrt{d-c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```


```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
- (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x
]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f -
Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
```


Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(f+gx)}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\cosh(x)}\,dx,x,\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf+g+e^{2x}g}\,dx,x,\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(2g\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2-g^2}}\,dx,x,\cosh^{-1}(cx)\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{\left(2g\sqrt{-1+cx}\sqrt{1+c\ldots}\right.}{}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-}\ldots\right.}{\sqrt{c^2f^2-g^2}\ldots}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-}\ldots\right.}{\sqrt{c^2f^2-g^2}\ldots}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-}\ldots\right.}{\sqrt{c^2f^2-g^2}\ldots}$$

Mathematica [C] time = 1.86594, size = 932, normalized size = 2.55

$$\frac{\frac{a\log(f+gx)}{\sqrt{d}} - \frac{a\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cf+g)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{g^2-c^2f^2}}\right)-2i\cos^{-1}\left(-\frac{cf}{g}\right)\tan^{-1}\left(\frac{(g-cf)\tanh\ldots}{\sqrt{g^2\ldots}}\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2
) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^
2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*
Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[A
rcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2
*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[
((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCo
sh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (
ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + T
anh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c
*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2)
 + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]
*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*
(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*
Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[
-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]
))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[d
 - c^2*d*x^2])/Sqrt[-(c^2*f^2) + g^2]
```

Maple [A] time = 0.153, size = 754, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/g/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x
+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-
d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)/d/(c^4*f^2*x^2-c^2*g^2*x^2-
c^2*f^2+g^2)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/
2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f
^2+g^2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c
```

```
*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f^2+g^2)*dilog((-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2
)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f^2+g^2)*dilog(((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx) + a}{\sqrt{-c^2dx^2 + d}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2 + d}(b\operatorname{arcosh}(cx) + a)}{c^2dgx^3 + c^2dfx^2 - dgx - df}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*
x^2 - d*g*x - d*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acosh}(cx)}{\sqrt{-d(cx-1)(cx+1)}\left(f + gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\sqrt{-c^2dx^2 + d}(gx + f)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

3.70 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=523

$$\frac{bc^2f\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{bc^2f\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{g\sqrt{cx-1}\sqrt{-\frac{1-cx}{cx+1}}(c}{\sqrt{d-c^2dx^2}\,(}$$

```
[Out] -((g*Sqrt[-1 + c*x]*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)^(3/2)*(a + b*Arc
Cosh[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c^2*d*x^2])) + (c^2*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (c
^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*
x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[
d - c^2*d*x^2]) - (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^Arc
Cosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d -
 c^2*d*x^2])
```

Rubi [A] time = 0.836335, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {5836, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bc^2f\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{bc^2f\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{g\sqrt{cx-1}\sqrt{-\frac{1-cx}{cx+1}}(c}{\sqrt{d-c^2dx^2}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((g*Sqrt[-1 + c*x]*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)^(3/2)*(a + b*Arc
Cosh[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c^2*d*x^2])) + (c^2*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (c
^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*
```

```
x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[
d - c^2*d*x^2]) - (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^Arc
Cosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d -
 c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(c\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(c^2f\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{}{cf+}\right.}{\left(c^2f^2-g^2\right)\sqrt{d-}}$$

$$= -\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{cf+x}\right.}{\left(c^2f^2-g^2\right)\sqrt{d-c^2dx^2}}$$

$$= -\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}\log(f+gx)}{\left(c^2f^2-g^2\right)\sqrt{d-c^2dx^2}} + -$$

$$= -\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(\right.}{\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-}}$$

$$= -\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(\right.}{\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-}}$$

$$= -\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(\right.}{\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-}}$$

Mathematica [C] time = 5.80011, size = 1115, normalized size = 2.13

$$-\frac{af\log(f+gx)c^2}{\sqrt{d}\left(g^2-c^2f^2\right)^{3/2}} - \frac{af\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)c^2}{\sqrt{d}(cf-g)(cf+g)\sqrt{g^2-c^2f^2}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-\frac{g\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)}{(cf-g)(cf+g)(cf+cgx)} + \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*Lo
g[f + g*x])/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*f*
x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f - g
)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) + (b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*(-((g*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/((c*f - g)*(c
*f + g)*(c*f + c*g*x))) + Log[1 + (g*x)/f]/(c^2*f^2 - g^2) + (c*f*(2*ArcCos
h[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2
*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*
x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])
/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c
*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f
+ g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*T
anh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt
[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/
g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])
*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x
]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (Ar
cCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*
f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + T
anh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c
*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sq
rt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
 + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^
2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g
+ I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))]))))/(-(c^2*f^2) + g^2)^(3
/2)))/Sqrt[d - c^2*d*x^2]
```

Maple [B] time = 0.223, size = 1978, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2
*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(
x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(
```

```
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x
+1)*(c*x-1)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^
2*f^2-g^2)/(g*x+f)*x^3*c^4*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x
^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c*g+b*(-d*(c^2*x^
2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-b*(-
d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2
-1)/(c^2*f^2-g^2)/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(
c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*g-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2
*g^2-g^4)*c^2*f*arccosh(c*x)*(c^2*f^2-g^2)^(1/2)*ln((-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^6*f^4*x^2-2*c^4*f^2*g^2*
x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*c^2*f*arccosh(c*x)*(c^2*f^2-g^2)
^(1/2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*
f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*c
^3*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))+g)*f^2+2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c
^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*c^3*ln(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f
^2*g^2-g^4)*c^2*f*(c^2*f^2-g^2)^(1/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*
f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*c^2*f*(c^2*f^2-g^2)^(1/2)*dilog(((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1
/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^6*f^4*x^2-2
*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*c*ln((c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+g)*g^2-2*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^6*f^4*x^2-2*c^4*f^2
*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*c*ln(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*g^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}(b\operatorname{arcosh}(cx)+a)}{c^2dg^2x^4+2\,c^2dfgx^3-2\,dfgx-df^2+\left(c^2df^2-dg^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*
d*f*g*x^3 - 2*d*f*g*x - d*f^2 + (c^2*d*f^2 - d*g^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{acosh}(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.71 $\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=549

$$-\frac{3fg^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^3\left(a+b\cosh^{-1}(cx\right.}{2c^4d\sqrt{d-c^2dx^2}}$$

```
[Out] (b*g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - ((c*f
- g)^3*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + ((c*
f + g)^3*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + (g
^3*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^4*d*Sqrt[d - c^2*d*x^2]) -
(3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sq
rt[d - c^2*d*x^2]) + (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*
x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]
*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*
Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(
1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sq
rt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
+ c*x)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.59949, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {5836, 5834, 37, 5848, 12, 6719, 260, 266, 36, 31, 29, 5676, 5718, 8}

$$-\frac{3fg^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^3\left(a+b\cosh^{-1}(cx\right.}{2c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```


```
[Out] (b*g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - ((c*f
- g)^3*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + ((c*
f + g)^3*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + (g
^3*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^4*d*Sqrt[d - c^2*d*x^2]) -
(3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sq
rt[d - c^2*d*x^2]) + (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*
```

```
x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]]])/(c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]
*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*
Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(
1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sq
rt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
+ c*x)*Sqrt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5834

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5848

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[(b*c*Sqrt[1 - c^2*x^2])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
```

```
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(f+gx)^3 \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(-\frac{(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^3\sqrt{-1+cx}(1+cx)^{3/2}} + \frac{(cf+g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^3(-1+cx)^{3/2}\sqrt{1+cx}} + \frac{3fg^2\left(a+b\cos\right.}{c^2\sqrt{-1+cx}}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left((cf-g)^3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}(1+cx)^{3/2}}\,dx}{2c^3d\sqrt{d-c^2dx^2}} - \frac{\left(3fg^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b}{\sqrt{-}}}{c^2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{g^3}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)}{2c^4d\sqrt{}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)}{2c^4d\sqrt{}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)}{2c^4d\sqrt{}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)}{2c^4d\sqrt{}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)}{2c^4d\sqrt{}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)}{2c^4d\sqrt{}}$$

Mathematica [A] time = 1.85331, size = 353, normalized size = 0.64

$$\frac{-2\sqrt{d}\left(-a\left(c^2g\left(3f^2+3fgx-g^2x^2\right)+c^4f^3x+2g^3\right)+bcf\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(c^2f^2+3g^2\right)\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)+bg\sqrt{\frac{cx-1}{cx+1}}(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*b*c*Sqrt[d]*f*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 +
 6*a*c*f*g^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*
(-1 + c^2*x^2))] + b*Sqrt[d]*ArcCosh[c*x]*(3*g^3 + 2*c^4*f^3*x + 6*c^2*f*g*
(f + g*x) - g^3*Cosh[2*ArcCosh[c*x]]) - 2*Sqrt[d]*(-(a*(2*g^3 + c^4*f^3*x +
 c^2*g*(3*f^2 + 3*f*g*x - g^2*x^2))) + b*c*f*(c^2*f^2 + 3*g^2)*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + b*g*(3
*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/
2]]) + b*Sqrt[d]*g^3*Sinh[2*ArcCosh[c*x]])/(2*c^4*d^(3/2)*Sqrt[d - c^2*d*x^
2])
```

Maple [B] time = 0.319, size = 1238, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*g^3*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a*g^3/d/c^4/(-c^2*d*x^2+d)^(1/2)+3*
a*f*g^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-3*a*f*g^2/c^2/d/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3*a*f^2*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a
*f^3/d*x/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^
2*x^2-1)*x*f^3-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(
c^2*x^2-1)*f^3*arccosh(c*x)-3*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^
2*x^2-1)*x^2*f^2*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^4/d^2/(c^2*x^2-1
)*(c*x+1)*(c*x-1)*g^3-3*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2/d^2/(c^2*
x^2-1)*x*f*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(
c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f^3+b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)-1)*g^3+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2/(c^2*x^2-1)*f^3-b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4
/d^2/(c^2*x^2-1)*g^3-b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^3/d^2/(c^2*x^2-1)*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x+3/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2*f*g^2-b*(-d*(c^2*x^2-1))^(1/2)*g^
3/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)-3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*f*arccosh(c*x)*g^2+3*b*(-d*(c^2*x^2-1))^
(1/2)*arccosh(c*x)/c^2/d^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*f^2*g+3*b*(-d*(c^2*x
```

```
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)-1)*f^2*g+3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^3/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f*g^2-3
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/c^2/d^2/(c^2*x^2-1)*f^2*g+3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2/(c^2*x
^2-1)*f*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2 + d}}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(c^4*
d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acosh}\,(cx))\left(f + gx\right)^3}{\left(-d\,(cx - 1)\,(cx + 1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**3/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^3(b\operatorname{arcosh}(cx)+a)}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^3*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

3.72 $\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$-\frac{(1-cx)(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}$$

```
[Out] -((c*f - g)^2*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2])
 + ((c*f + g)^2*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) - (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*S
qrt[d - c^2*d*x^2]) + (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2
*x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))
]*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^2*Sqrt[(1 - c*x)*(1 + c*x)]
*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*
(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*S
qrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
 + c*x)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.26664, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5836, 5834, 37, 5848, 12, 6719, 260, 266, 36, 31, 29, 5676}

$$-\frac{(1-cx)(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((c*f - g)^2*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2])
 + ((c*f + g)^2*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) - (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*S
qrt[d - c^2*d*x^2]) + (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2
*x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))
]*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^2*Sqrt[(1 - c*x)*(1 + c*x)]
*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*
(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*S
```

```
qrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
 + c*x)*Sqrt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5834

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5848

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[(b*c*Sqrt[1 - c^2*x^2])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
```

```
[v, x] &&  !FreeQ[w, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\left(-\frac{(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{-1+cx}(1+cx)^{3/2}}+\frac{(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2(-1+cx)^{3/2}\sqrt{1+cx}}+\frac{g^2\left(a+b\cosh^{-1}(\right.}{c^2\sqrt{-1+cx}\sqrt{1+}}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left((cf-g)^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}(1+cx)^{3/2}}\,dx}{2c^2d\sqrt{d-c^2dx^2}} - \frac{\left(g^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh}{\sqrt{-1+cx}\sqrt{}}}{c^2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{}}{}$$

Mathematica [A] time = 1.07785, size = 281, normalized size = 0.61

$$\frac{2\sqrt{d}\left(ac\left(c^2f^2x+2fg+g^2x\right)-b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(c^2f^2+g^2\right)\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)-2bcfg\sqrt{\frac{cx-1}{cx+1}}(cx+1)\log\left(\tanh\left(\frac{1}{2}\cosh\right.\right.\right.}{2c^3d^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*b*c*Sqrt[d]*(2*f*g + c^2*f^2*x + g^2*x)*ArcCosh[c*x] - b*Sqrt[d]*g^2*Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 + 2*a*g^2*Sqrt[d - c^2*d*x
^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*Sqrt[d]*
(a*c*(2*f*g + c^2*f^2*x + g^2*x) - b*(c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] - 2*b*c*f*g*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]]))/(2*c^3*d^(3/2)*S
qrt[d - c^2*d*x^2])
```

Maple [B] time = 0.276, size = 879, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*g^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a*g^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+2*a*f*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a*f^2/d*x
/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d^2/c^3/(c^2*x^2-1)*g^2*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*f^2-b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*g^2+2*b*
(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*(c*x-1)*(c*x+1)*f*g
-2*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*f*g-b*(-d*(c^2
*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f^2-b*(-d*(c^2*x^2-1))^(1/2)*
arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f^
2+2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^2/(c^2*x^2-1
)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)-1)*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^
2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2-2*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*f*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/
(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arcosh}\left(cx\right)\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccosh(c*x))/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\text{acosh}\,(cx))\left(f+gx\right)^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^2 (b\,\mathrm{arcosh}\,(cx)+a)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^2*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

3.73 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{\left(c^2fx+g\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}(cf-g)\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{cx-1}\sqrt{cx+1}\log(1-cx)}{cd\sqrt{d-c^2dx^2}}$$

```
[Out] ((g + c^2*f*x)*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (b*(c*f
- g)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
 - (b*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c*x])/(c*d*Sqrt[d - c^2*d*x^2]
)
```

Rubi [A] time = 0.306586, antiderivative size = 178, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5836, 78, 37, 5820, 35, 206}

$$-\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}(cf-g)\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{cx-1}\sqrt{cx+}}{cd\sqrt{d-c}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -(((c*f - g)*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2])) + (f*(1 + c
*x)*(a + b*ArcCosh[c*x]))/(c*d*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (b*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c*x])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5820

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d1_) + (e1_.)*(x_))^(p_)*((d2_) +
 (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHid
e[(f + g*x)^m*(d1 + e1*x)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u,
 x] - Dist[b*c, Int[Dist[1/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), u, x], x], x]] /
; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2
+ c*d2, 0] && IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] &&
 (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right.}{d}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} - \frac{bf\sqrt{-1+cx}\sqrt{1+cx}}{cd\sqrt{d-c^2dx}}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} - \frac{bf\sqrt{-1+cx}\sqrt{1+cx}}{cd\sqrt{d-c^2dx}}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} - \frac{b(cf-g)\sqrt{-1+cx}}{c^2d\sqrt{d-}}$$

Mathematica [A] time = 0.375863, size = 123, normalized size = 0.87

$$\frac{b\sqrt{d-c^2dx^2}((cf+g)\log(1-cx)+(cf-g)\log(cx+1))}{2c^2d^2\sqrt{cx-1}\sqrt{cx+1}} - \frac{\sqrt{d-c^2dx^2}\left(c^2fx+g\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -(((g + c^2*f*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d^2*(-1 + c
^2*x^2))) + (b*Sqrt[d - c^2*d*x^2]*((c*f + g)*Log[1 - c*x] + (c*f - g)*Log[
1 + c*x]))/(2*c^2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [B] time = 0.227, size = 498, normalized size = 3.5

$$\frac{ag}{c^2d}\frac{1}{\sqrt{-c^2dx^2+d}} + \frac{afx}{d}\frac{1}{\sqrt{-c^2dx^2+d}} - \frac{bf\operatorname{arccosh}(cx)}{cd^2\left(c^2x^2-1\right)}\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1} + \frac{b\operatorname{arccosh}(cx)(cx+1)(cx-1)}{c^2d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a*f/d*x/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*f*arccosh(c*x)+b*(-
d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2/d^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*g-b*(
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*g-b*(-d*(c^2*x^2-1))
^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2/(c^2*x^2-1)*f
-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/c^2/d^2/(c^2*x^2-1)*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f+
b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcf\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{2\,d}+bg\left(\frac{\frac{\left(c\sqrt{d}x+\sqrt{cx+1}\sqrt{cx-1}\sqrt{d}\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{-cx+1}}+\frac{\sqrt{cx+1}\sqrt{cx-1}\sqrt{d}}{\sqrt{-cx+1}}}{\sqrt{cx+1}c^3d^2x+(cx+1)\sqrt{cx-1}c^2d^2}-\int\frac{}{\sqrt{-cx+1}\left(\left(c^2d^{\frac{3}{2}}x^2-d^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/2*b*c*f*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*g*(((c*sqrt(d)*x + sqrt(
c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt
(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/sqrt(-c*x + 1))/(sqrt(c*x
+ 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2) - integrate((c^2*x^3 + c*
x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x)/(sqrt(-c*x + 1)*((c^2*d^(3
/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 2*(c^3*d^(3/2)*x^3
 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1)) + (c^4*d^(3/2)*x^4 - c^
2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + b*f*x*arccosh(c*x)/(sqrt(-c^2*d*x^2 +
 d)*d) + a*f*x/(sqrt(-c^2*d*x^2 + d)*d) + a*g/(sqrt(-c^2*d*x^2 + d)*c^2*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\operatorname{arcosh}\left(cx\right)\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x))/(c
^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(f + gx\right)}{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx + f\right)(b\,\mathrm{arcosh}\,(cx) + a)}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

3.74 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=773

$$-\frac{bg^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{d\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}+\frac{bg^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{d\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\cdots}{d\sqrt{\cdots}}$$

```
[Out] -((1 - c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f - g)*Sqrt[d - c^2*d*x^2]) + ((1
 + c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f + g)*Sqrt[d - c^2*d*x^2]) - (g^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
 + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c
^2*d*x^2]) + (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[Sqrt[-((1 -
 c*x)/(1 + c*x))]])/(d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqr
t[d - c^2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1
 + c*x)])/(2*d*(c*f - g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^
2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)]
)/(2*d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^2*d*x^2]
) - (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) +
 (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f +
 Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] time = 1.85733, antiderivative size = 773, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {5836, 5834, 37, 5848, 12, 6719, 260, 266, 36, 31, 29, 5832, 3320, 2264, 2190, 2279, 2391}

$$-\frac{bg^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{d\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}+\frac{bg^2\sqrt{cx-1}\sqrt{cx+1}\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{d\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\cdots}{d\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((1 - c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f - g)*Sqrt[d - c^2*d*x^2]) + ((1
 + c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f + g)*Sqrt[d - c^2*d*x^2]) - (g^2*Sq
```

```
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
 + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c
^2*d*x^2]) + (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[Sqrt[-((1 -
 c*x)/(1 + c*x))]])/(d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqr
t[d - c^2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1
 + c*x)])/(2*d*(c*f - g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^
2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)]
)/(2*d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^2*d*x^2]
) - (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) +
 (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f +
 Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5834

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5848

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[(b*c*Sqrt[1 - c^2*x^2])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
```

```
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
```

```
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\left(-\frac{c\left(a+b\cosh^{-1}(cx)\right)}{2(cf-g)\sqrt{-1+cx}(1+cx)^{3/2}} + \frac{c\left(a+b\cosh^{-1}(cx)\right)}{2(cf+g)(-1+cx)^{3/2}\sqrt{1+cx}} + \frac{g^2\left(a+b\cosh\right.}{(cf-g)(cf+g)\sqrt{-1+c}}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(c\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}(1+cx)^{3/2}}\,dx}{2d(cf-g)\sqrt{d-c^2dx^2}} - \frac{\left(c\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}\sqrt{1+cx}}\,dx}{2d(cf+g)\sqrt{d-c^2dx^2}} - \ldots$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{\left(g^2\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Sub}}{d\left(c^2f^2\right.}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{\left(2g^2\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Su}}{d\left(c^2f^2\right.}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{\left(2g^3\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Su}}{d\left(c^2\right.}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{d\left(c^2f^2-\right.}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{d\left(c^2f^2-\right.}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{d\left(c^2f^2-\right.}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{d\left(c^2f^2-\right.}$$

Mathematica [C] time = 9.71484, size = 1203, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((-(a*g) + a*c^2*f*x)*Sqrt[-(d*(-1 + c^2*x^2))])/(d^2*(-(c^2*f^2) + g^2)*(-
1 + c^2*x^2)) + (a*g^2*Log[f + g*x])/(d^(3/2)*(-(c*f) + g)*(c*f + g)*Sqrt[-
(c^2*f^2) + g^2]) - (a*g^2*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) +
g^2]*Sqrt[-(d*(-1 + c^2*x^2))]])/(d^(3/2)*(-(c*f) + g)*(c*f + g)*Sqrt[-(c^2
*f^2) + g^2]) - (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((ArcCosh[c*x]*Co
th[ArcCosh[c*x]/2])/(c*f + g)) + (2*c*f*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)])/(c^2*f^2 - g^2) + (2*g*Log[Tanh[ArcCosh[c*x]/2]])/(-(c^2*f^2) + g^2
) + (2*g^2*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCos
h[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f
+ g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*T
anh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(
Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)
] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + Ar
cTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])
] - (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt
[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-
1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcC
osh[c*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[
c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2
*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) +
 g^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g
^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g
 + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*
Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2])])))/(
(-(c*f) + g)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (ArcCosh[c*x]*Tanh[ArcCosh
[c*x]/2])/(c*f - g)))/(2*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Maple [B] time = 0.255, size = 2484, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x)

[Out] -a*g/d/(c^2*f^2-g^2)/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a*f/(c^2*f^2-g^2)/d/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^2*x+a*g/d/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*(c*x-1)*(c*x+1)*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*x^2*c^2*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*x*c^2*f-2*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3*f^3+b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^3*f^3+b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3*f^3-b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2*f^2*g+b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^2*f^2*g+b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g^2-b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g^2+2*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c*f*g^2-b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c*f*g^2-b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c*f*g^2+b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g^3-b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g^3+b*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1

```
/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g^2-b*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f
^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2 + d}(b\operatorname{arcosh}(cx) + a)}{c^4d^2gx^5 + c^4d^2fx^4 - 2\,c^2d^2gx^3 - 2\,c^2d^2fx^2 + d^2gx + d^2f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*g*x^5 + c^4*d^2
*f*x^4 - 2*c^2*d^2*g*x^3 - 2*c^2*d^2*f*x^2 + d^2*g*x + d^2*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{acosh}(cx)}{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}\left(f + gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/((-d*(c*x - 1)*(c*x + 1))**(3/2)*(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}}(gx + f)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*(g*x + f)), x)
```

3.75 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=239

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}-\frac{ge^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\right.}{}$$

```
[Out] (f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*
Sqrt[1 - c^2*x^2]) + (g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n
*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c^2*x^2]*
(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 -
c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.470914, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5836, 5832, 3317, 3307, 2181}

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}-\frac{ge^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*
Sqrt[1 - c^2*x^2]) + (g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n
*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c^2*x^2]*
(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 -
c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx\right.}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-x}(a+\right.}{2c^2\sqrt{1-c^2x^2}}$$

$$= \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{e^{-\frac{a}{b}}g\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(c\right.}{2c}$$

Mathematica [A] time = 0.632461, size = 204, normalized size = 0.85

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(bg(n+1)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right.}{2bc^2(n+1)\sqrt{1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2*c*E^(a/b)*f
*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n - b*E^((2*a)/b)*g*(
1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] + b*g
*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(
2*b*c^2*E^(a/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n
)
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int \left(gx+f\right)\left(a+b\text{arccosh}\left(cx\right)\right)^n\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}(gx+f)(b\,\mathrm{arcosh}\,(cx)+a)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(g*x + f)*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^n\,(f+gx)}{\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] sage0*x
```

3.76 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=200

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{ge^{a/b}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n}{}$$

```
[Out] (f*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*x])
 + (g*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c
*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^
(a/b)*g*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c
*x])/b])/(2*c^2*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rubi [A] time = 0.625353, antiderivative size = 242, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5837, 5832, 3317, 3307, 2181}

$$\frac{ge^{-\frac{a}{b}}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}\sqrt{cx+1}}-\frac{ge^{a/b}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (f*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*
x]*Sqrt[1 + c*x]) + (g*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c*x]*Sqrt[1 + c*x]*(-
((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 - c*x]*Sqrt[1 +
 c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5837

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[((-(d
1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(1 - c^2
*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[
c*x])^n, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[e1 -
```

```
 c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !(Gt
Q[d1, 0] && LtQ[d2, 0])
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\sqrt{1+cx}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int (a+bx)^n\cosh(x)\,dx,\right.}{c^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\sqrt{1+cx}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \text{co}\right.}{2c^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\sqrt{1+cx}} + \frac{e^{-\frac{a}{b}}g\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh}{b}\right.}{2c^2\sqrt{1-cx}\sqrt{1-}}$$

Mathematica [A] time = 0.13196, size = 204, normalized size = 1.02

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(bg(n+1)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right.\right.}{2bc^2(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2*c*E^(a/b)*f
*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n - b*E^((2*a)/b)*g*(
1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] + b*g
*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(
2*b*c^2*E^(a/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n
)
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \left(gx+f\right)\left(a+b\text{arccosh}\left(cx\right)\right)^n\frac{1}{\sqrt{-cx+1}}\frac{1}{\sqrt{cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{cx+1}\sqrt{-cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*x + 1)*sqrt(-c*x + 1)),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{cx+1}\sqrt{-cx+1}(gx+f)(b\operatorname{arcosh}(cx)+a)^n}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(-sqrt(c*x + 1)*sqrt(-c*x + 1)*(g*x + f)*(b*arccosh(c*x) + a)^n/(c^
2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(-c*x+1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{\sqrt{cx+1}\,\sqrt{-cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*x + 1)*sqrt(-c*x + 1)),
x)
```

3.77 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{\mathbf{d1}+c\mathbf{d1}x}\sqrt{\mathbf{d2}-c\mathbf{d2}x}}\,dx$

Optimal. Leaf size=260

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{cd1x+d1}\sqrt{d2-cd2x}}-\frac{ge^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\right.}{}$$

```
[Out] (f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*
Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*
Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/
b)*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a +
b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((a + b*ArcC
osh[c*x])/b)^n)
```

Rubi [A] time = 0.694553, antiderivative size = 248, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {5837, 5832, 3317, 3307, 2181}

$$\frac{ge^{-\frac{a}{b}}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{cd1x+d1}\sqrt{d2-cd2x}}-\frac{ge^{a/b}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right.}{2c^2\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]
),x]
```

```
[Out] (f*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[d1 + c
*d1*x]*Sqrt[d2 - c*d2*x]) + (g*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[d1 + c*d1*x]*Sqrt
[d2 - c*d2*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[1 - c^2*x^2]
*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d
1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 5837

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[((-(d
```

```
1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(1 - c^2
*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[
c*x])^n, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[e1 -
 c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !(Gt
Q[d1, 0] && LtQ[d2, 0])
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d1+cd1x}\sqrt{d2-cd2x}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$
$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d1+cd1x}\sqrt{d2-cd2x}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int (a+bx)^n\cosh(x)\,dx,x\right.}{c^2\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$
$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d1+cd1x}\sqrt{d2-cd2x}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int e^{-x}(a+bx)^n\,dx,x,\cos\right.}{2c^2\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$
$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d1+cd1x}\sqrt{d2-cd2x}} + \frac{e^{-\frac{a}{b}}g\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}}{b}\right.}{2c^2\sqrt{d1+cd1x}\sqrt{d2}}$$

Mathematica [A] time = 2.06034, size = 219, normalized size = 0.84

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}\sqrt{cd1x+d1}\sqrt{d2-cd2x}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(-bg(n+1)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\mathrm{Gamma}\left(n+\right.\right.}{2bc^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcC
osh[c*x])^n*(-2*c*E^(a/b)*f*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/
b^2))^n + b*E^((2*a)/b)*g*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n
, a/b + ArcCosh[c*x]] - b*g*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((
a + b*ArcCosh[c*x])/b)]))/(2*b*c^2*d1*d2*E^(a/b)*(1 + n)*(-1 + c*x)*(-((a +
 b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F] time = 0.439, size = 0, normalized size = 0.

$$\int \left(gx+f\right)\left(a+b\operatorname{arccosh}(cx)\right)^n\frac{1}{\sqrt{cd1\,x+d1}}\frac{1}{\sqrt{-cd2\,x+d2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x
+ d2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}(gx+f)(b\operatorname{arcosh}(cx)+a)^n}{c^2d_1d_2x^2-d_1d_2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(g*x + f)*(b*arccosh(c*x) +
a)^n/(c^2*d1*d2*x^2 - d1*d2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x
+ d2)), x)
```

3.78 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{cx-1}\sqrt{cx+1}},x\right)}{\sqrt{1-c^2x^2}}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Unintegrable[((a + b*ArcCosh[c*x])^n*Log[h*(f
 + g*x)^m])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi [A] time = 0.955479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((a + b*ArcCosh[c*x])^n*Log[h*(f +
 g*x)^m])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.169582, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [A] time = 1.755, size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{arccosh}\,(cx))^{n}\ln \left(h\left(gx+f\right)^{m}\right)\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n}\log \left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}(b\,\mathrm{arcosh}\,(cx)+a)^{n}\log \left(\left(gx+f\right)^{m}h\right)}{c^{2}x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*log((g*x + f)^m*h)/(c^2
*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

3.79 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2 \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}} dx$

Optimal. Leaf size=774

$$-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2,\right.}{c\sqrt{1-c^2x^2}}$$

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^4)/(12*b^2*c*Sqrt[1 -
c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 +
(E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2])
- (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c*Sqr
t[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Po
lyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^
2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*PolyLog[2,
 -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2])
+ (2*b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (2*b
*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
 - c^2*x^2])
```

Rubi [A] time = 1.59875, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {5713, 5676, 5841, 5839, 5800, 5562, 2190, 2531, 6609, 2282, 6589}

$$-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2,\right.}{c\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^4)/(12*b^2*c*Sqrt[1 -
c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 +
(E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2])
- (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c*Sqr
t[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Po
lyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^
2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*PolyLog[2,
 -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2])
+ (2*b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (2*b
*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
 - c^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5841

```
Int[Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_.
))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d
+ e*x^2)^FracPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[L
og[h*(f + g*x)^m]*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ
[p - 1/2]
```

Rule 5839

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_
.))^(n_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol]
 :> Simp[(Log[h*(f + g*x)^m]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d
2)]*(n + 1)), x] - Dist[(g*m)/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(a + b*ArcC
osh[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g
, h, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-1+cx}\sqrt{1\right.}{3b}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-1+cx}\sqrt{1\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}\right.}{3bc\sqrt{1-c^2}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{3bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{3bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{3bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{3bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{3bc\sqrt{1-}}$$

Mathematica [F] time = 4.24758, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [F] time = 1.726, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))^2 \ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2 \log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2
)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 \log\left(\left(gx + f\right)^m h\right)}{\sqrt{-c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.80 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=600

$$-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\right.}{c\sqrt{1-c^2x^2}}$$

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b^2*c*Sqrt[1 - c
^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) -
 (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (E^ArcCosh[
c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c*Sqrt
[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyL
og[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x
^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, -((E
^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*
m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[
c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
- c^2*x^2])
```

Rubi [A] time = 1.12927, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {5713, 5676, 5841, 5839, 5800, 5562, 2190, 2531, 2282, 6589}

$$-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\right.}{c\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```



```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b^2*c*Sqrt[1 - c
^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) -
 (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (E^ArcCosh[
c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c*Sqrt
[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyL
og[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x
^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, -((E
^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*
m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[
c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
- c^2*x^2])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5841

```
Int[Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_.
))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d
+ e*x^2)^FracPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[L
og[h*(f + g*x)^m]*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ
[p - 1/2]
```

Rule 5839

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_
.))^(n_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol]
 :> Simp[(Log[h*(f + g*x)^m]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d
2)]*(n + 1)), x] - Dist[(g*m)/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(a + b*ArcC
osh[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g
, h, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-1+cx}\sqrt{1\right.}{2b}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc\sqrt{1-c^2x^2}} - \left(gm\sqrt{-1+cx}\sqrt{1\right.$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(\right.}{2bc\sqrt{1-c^2}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2bc\sqrt{1-}}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\right.}{2bc\sqrt{1-}}$$

Mathematica [F] time = 1.84202, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [F] time = 1.412, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(cx))\ln\left(h\left(gx+f\right)^{m}\right)\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(cx)+a)\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}(b\operatorname{arcosh}(cx)+a)\log\left(\left(gx+f\right)^{m}h\right)}{c^{2}x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*log((g*x + f)^m*h)/(c^2*x
^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\log\left(h\left(f + gx\right)^{m}\right)}{\sqrt{-\left(cx - 1\right)\left(cx + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)\log\left(\left(gx + f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2} + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.81 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=237

$$\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\right.}{c}$$

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```

Rubi [A] time = 0.345612, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {216, 2404, 4741, 4519, 2190, 2279, 2391}

$$\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```


```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```


Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
```

```
b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sin^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{x\cos(x)}{c^2f+cg\sin(x)}\,dx, x, \sin^{-1}(cx)\right)$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} + \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{e^{ix}x}{c^2f-ice^{ix}g-c\sqrt{c^2f^2-g^2}}\,dx,\right.$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\sin^{-}}{}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\sin^{-}}{}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\sin^{-}}{}$$

Mathematica [A] time = 0.0204454, size = 246, normalized size = 1.04

$$\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{icge^{i\sin^{-1}(cx)}}{c^2f-c\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*ArcSin[c*x])*g
)/(c^2*f - c*Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*Ar
cSin[c*x])*g)/(c^2*f + c*Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f +
 g*x)^m])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 -
 g^2])])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 -
g^2])])/c
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(h\left(f+gx\right)^m\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left((gx+f)^m h \right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.82 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\text{Unintegrable}\left(\frac{\log(h(f+gx)^m)}{\sqrt{cx-1}\sqrt{cx+1}(a+b\cosh^{-1}(cx))},x\right)}{\sqrt{1-c^2x^2}}$$

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Unintegrable[Log[h*(f + g*x)^m]/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi [A] time = 1.06267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\log(h(f+gx)^m)}{\sqrt{-1+cx}\sqrt{1+cx}(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.279179, size = 0, normalized size = 0.

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A] time = 0.925, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{a+b\mathrm{arccosh}\left(cx\right)}\frac{1}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}(b\,\mathrm{arcosh}\left(cx\right)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c^{2}x^{2}+1}\log\left(\left(gx+f\right)^{m}h\right)}{ac^{2}x^{2}+\left(bc^{2}x^{2}-b\right)\mathrm{arcosh}\left(cx\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 - b
)*arccosh(c*x) - a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(h \left(f+gx\right)^{m}\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))
), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

3.83 $\int x^3 \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(4a\left(19a^2+16\right)-\left(26a^2+9\right)(a+bx)\right)}{96b^4}-\frac{\left(8a^4+24a^2+3\right)\cosh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{a+bx-1}}{48b^2}$$

```
[Out] (7*a*x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(48*b^2) - (x^3*Sqrt[-1 + a
+ b*x]*Sqrt[1 + a + b*x])/(16*b) + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4
*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x)))/(96*b^4) - ((3 + 24*a^2 + 8*a^4
)*ArcCosh[a + b*x])/(32*b^4) + (x^4*ArcCosh[a + b*x])/4
```

Rubi [A] time = 0.186638, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5866, 5802, 100, 153, 147, 52}

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(4a\left(19a^2+16\right)-\left(26a^2+9\right)(a+bx)\right)}{96b^4}-\frac{\left(8a^4+24a^2+3\right)\cosh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{a+bx-1}}{48b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(48*b^2) - (x^3*Sqrt[-1 + a
+ b*x]*Sqrt[1 + a + b*x])/(16*b) + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4
*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x)))/(96*b^4) - ((3 + 24*a^2 + 8*a^4
)*ArcCosh[a + b*x])/(32*b^4) + (x^4*ArcCosh[a + b*x])/4
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x^3\cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^3\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{4}x^4\cosh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{-1+x}\sqrt{1+x}}dx,x,a+bx\right)$$
$$= -\frac{x^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}{16b}+\frac{1}{4}x^4\cosh^{-1}(a+bx)-\frac{1}{16}\text{Subst}\left(\int\frac{\left(\frac{3+4a^2}{b^2}-\frac{7ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\right.$$
$$= \frac{7ax^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}{16b}+\frac{1}{4}x^4\cosh^{-1}(a+bx)-\frac{1}{48}\text{Su}$$
$$= \frac{7ax^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}{16b}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\,(}{}$$
$$= \frac{7ax^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}{16b}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\,(}{}$$

Mathematica [A] time = 0.17599, size = 121, normalized size = 0.8

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-26a^2bx+50a^3+14ab^2x^2+55a-6b^3x^3-9bx\right)-3\left(8a^4+24a^2+3\right)\log\left(\sqrt{a+bx-1}\sqrt{a+b}\right.}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a + b*x],x]
```

```
[Out] (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x +
 14*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCosh[a + b*x] - 3*(3 + 24*a^2 +
8*a^4)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(96*b^4)
```

Maple [B] time = 0.033, size = 308, normalized size = 2.

$$\frac{x^4\text{arccosh}\,(bx+a)}{4}-\frac{x^3}{16\,b}\sqrt{bx+a-1}\sqrt{bx+a+1}+\frac{7\,ax^2}{48\,b^2}\sqrt{bx+a-1}\sqrt{bx+a+1}-\frac{a^4}{4\,b^4}\sqrt{bx+a-1}\sqrt{bx+a+1}\ln\Big(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(b*x+a),x)
```

```
[Out] 1/4*x^4*arccosh(b*x+a)-1/16*x^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b+7/48*a*x^
2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/4/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/
2)/((b*x+a)^2-1)^(1/2)*a^4*ln(b*x+a+((b*x+a)^2-1)^(1/2))-13/48/b^3*(b*x+a-1
)^(1/2)*(b*x+a+1)^(1/2)*x*a^2+25/48/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*a^3
-3/4/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2-1)^(1/2)*a^2*ln(b*x+a+(
(b*x+a)^2-1)^(1/2))-3/32/b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*x+55/96/b^4*(b
*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*a-3/32/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((
b*x+a)^2-1)^(1/2)*ln(b*x+a+((b*x+a)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27272, size = 262, normalized size = 1.72

$$\frac{3\left(8\,b^4x^4-8\,a^4-24\,a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(6\,b^3x^3-14\,ab^2x^2-50\,a^3+\left(26\,a^2+9\right)bx-55\,a\right)\sqrt{}}{96\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x - 55
*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^4
```

Sympy [A] time = 1.93184, size = 255, normalized size = 1.68

$$\begin{cases} -\frac{a^4\operatorname{acosh}(a+bx)}{4b^4}+\frac{25a^3\sqrt{a^2+2abx+b^2x^2-1}}{48b^4}-\frac{13a^2x\sqrt{a^2+2abx+b^2x^2-1}}{48b^3}-\frac{3a^2\operatorname{acosh}(a+bx)}{4b^4}+\frac{7ax^2\sqrt{a^2+2abx+b^2x^2-1}}{48b^2}+\frac{55a\sqrt{a^2+2abx+b^2x^2-1}}{96b^4}+ x \\ \frac{x^4\operatorname{acosh}(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acosh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(48
*b**3) - 3*a**2*acosh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/(48*b**2) + 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(96*b**4
) + x**4*acosh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(16*b
) - 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(32*b**3) - 3*acosh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*acosh(a)/4, True))
```

Giac [A] time = 1.20281, size = 220, normalized size = 1.45

$$\frac{1}{4}x^4\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\frac{1}{96}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(\left(2\,x\left(\frac{3\,x}{b^2}-\frac{7\,a}{b^3}\right)+\frac{26\,a^2b^3+9\,b^3}{b^7}\right)x-\frac{5\left(10\,a^3b^2+11\right.}{b^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 + 9*b^3)/b^7)*x - 5*(10
*a^3*b^2 + 11*a*b^2)/b^7) - 3*(8*a^4 + 24*a^2 + 3)*log(abs(-a*b - (x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^4*abs(b)))*b
```

3.84 $\int x^2 \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(11a^2-5abx+4\right)}{18b^3}+\frac{a\left(2a^2+3\right)\cosh^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{9b}+\frac{1}{3}x^3\cosh^{-1}(a$$

```
[Out] -(x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(9*b) - (Sqrt[-1 + a + b*x]*Sqr
t[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x))/(18*b^3) + (a*(3 + 2*a^2)*ArcCosh[a
+ b*x])/(6*b^3) + (x^3*ArcCosh[a + b*x])/3
```

Rubi [A] time = 0.119097, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5866, 5802, 100, 147, 52}

$$-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(11a^2-5abx+4\right)}{18b^3}+\frac{a\left(2a^2+3\right)\cosh^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{9b}+\frac{1}{3}x^3\cosh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a + b*x],x]
```


```
[Out] -(x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(9*b) - (Sqrt[-1 + a + b*x]*Sqr
t[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x))/(18*b^3) + (a*(3 + 2*a^2)*ArcCosh[a
+ b*x])/(6*b^3) + (x^3*ArcCosh[a + b*x])/3
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x^2\cosh^{-1}(a+bx)\,dx=\frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$=\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx\right)$$

$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}+\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{1}{9}\text{Subst}\left(\int\frac{\left(\frac{2+3a^2}{b^2}-\frac{5ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)}{\sqrt{-1+x}\sqrt{1+x}}\,d\right.$$

$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\left(4+11a^2-5abx\right)}{18b^3}+\frac{1}{3}x^3\cosh^{-1}($$

$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\left(4+11a^2-5abx\right)}{18b^3}+\frac{a\left(3+2a^2\right)\text{c}}{6}$$

Mathematica [A] time = 0.114483, size = 101, normalized size = 0.97

$$\frac{-\sqrt{a+bx-1}\sqrt{a+bx+1}\left(11a^2-5abx+2b^2x^2+4\right)+\left(6a^3+9a\right)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)+6b^3x^3\cosh^{-1}}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a + b*x],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2))
 + 6*b^3*x^3*ArcCosh[a + b*x] + (9*a + 6*a^3)*Log[a + b*x + Sqrt[-1 + a + b
*x]*Sqrt[1 + a + b*x]])/(18*b^3)
```

Maple [B] time = 0.01, size = 207, normalized size = 2.

$$\frac{x^3\mathrm{arccosh}\,(bx+a)}{3}-\frac{x^2}{9\,b}\sqrt{bx+a-1}\sqrt{bx+a+1}+\frac{a^3}{3\,b^3}\sqrt{bx+a-1}\sqrt{bx+a+1}\ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)\frac{1}{\sqrt{(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(b*x+a),x)
```

```
[Out] 1/3*x^3*arccosh(b*x+a)-1/9*x^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b+1/3/b^3*(b
*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2-1)^(1/2)*a^3*ln(b*x+a+((b*x+a)^2-1
)^(1/2))+5/18/b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*x*a-11/18/b^3*(b*x+a-1)^(
1/2)*(b*x+a+1)^(1/2)*a^2+1/2/b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2
-1)^(1/2)*a*ln(b*x+a+((b*x+a)^2-1)^(1/2))-2/9/b^3*(b*x+a-1)^(1/2)*(b*x+a+1)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.34271, size = 216, normalized size = 2.08

$$\frac{3\left(2\,b^3x^3+2\,a^3+3\,a\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(2\,b^2x^2-5\,abx+11\,a^2+4\right)\sqrt{b^2x^2+2\,abx+a^2-1}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 - 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 + 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))/b^3
```

Sympy [A] time = 0.887139, size = 170, normalized size = 1.63

$$\begin{cases}\frac{a^3\operatorname{acosh}(a+bx)}{3b^3}-\frac{11a^2\sqrt{a^2+2abx+b^2x^2-1}}{18b^3}+\frac{5ax\sqrt{a^2+2abx+b^2x^2-1}}{18b^2}+\frac{a\operatorname{acosh}(a+bx)}{2b^3}+\frac{x^3\operatorname{acosh}(a+bx)}{3}-\frac{x^2\sqrt{a^2+2abx+b^2x^2-1}}{9b}-\frac{2\sqrt{a^2+2abx+b^2}}{9b^3}\\ \frac{x^3\operatorname{acosh}(a)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(b*x+a),x)
```

```
[Out] Piecewise((a**3*acosh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 - 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(18*b**2
) + a*acosh(a + b*x)/(2*b**3) + x**3*acosh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 - 1)/(9*b) - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(9*b*
*3), Ne(b, 0)), (x**3*acosh(a)/3, True))
```

Giac [A] time = 1.20715, size = 178, normalized size = 1.71

$$\frac{1}{3}\,x^3\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b+4\,b}{b^5}\right)+\frac{3\left(2\,a^3+3\,a\right)\log\left(\left|-a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b + 4*b)/b^5) + 3*(2*a^3 + 3*a
)*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b
^3*abs(b)))*b
```

3.85 $\int x \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=90

$$-\frac{(2a^2+1)\cosh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{4b^2}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{x\sqrt{a+bx-1}\sqrt{a+bx+1}}{4b}$$

```
[Out] (3*a*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*b^2) - (x*Sqrt[-1 + a + b*x]*
Sqrt[1 + a + b*x])/(4*b) - ((1 + 2*a^2)*ArcCosh[a + b*x])/(4*b^2) + (x^2*Ar
cCosh[a + b*x])/2
```

Rubi [A] time = 0.0638357, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5866, 5802, 90, 80, 52}

$$-\frac{(2a^2+1)\cosh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{a+bx-1}\sqrt{a+bx+1}}{4b^2}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{x\sqrt{a+bx-1}\sqrt{a+bx+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*b^2) - (x*Sqrt[-1 + a + b*x]*
Sqrt[1 + a + b*x])/(4*b) - ((1 + 2*a^2)*ArcCosh[a + b*x])/(4*b^2) + (x^2*Ar
cCosh[a + b*x])/2
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx\right)\\ &= -\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\frac{1+2a^2}{b^2}-\frac{3ax}{b^2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx\right)\\ &= \frac{3a\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b^2}-\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{\left(1+2a^2\right)\text{S}}{}\\ &= \frac{3a\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b^2}-\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}-\frac{\left(1+2a^2\right)\cosh^{-1}(a+bx)}{4b^2}+\frac{1}{2}x^2\text{c}\end{aligned}$$

Mathematica [A] time = 0.0730523, size = 87, normalized size = 0.97

$$\frac{-\left(2a^2+1\right)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)+2b^2x^2\cosh^{-1}(a+bx)+(3a-bx)\sqrt{a+bx-1}\sqrt{a+bx+1}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*b^2*x^2*ArcCosh[a + b
*x] - (1 + 2*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(4*b
^2)
```

Maple [A] time = 0.01, size = 120, normalized size = 1.3

$$\frac{x^2 \operatorname{arccosh}(bx+a)}{2} - \frac{\operatorname{arccosh}(bx+a)\,a^2}{2\,b^2} - \frac{x}{4\,b}\sqrt{bx+a-1}\sqrt{bx+a+1} + \frac{3\,a}{4\,b^2}\sqrt{bx+a-1}\sqrt{bx+a+1} - \frac{1}{4\,b^2}\sqrt{bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(b*x+a),x)
```

```
[Out] 1/2*x^2*arccosh(b*x+a)-1/2/b^2*arccosh(b*x+a)*a^2-1/4*x*(b*x+a-1)^(1/2)*(b*
x+a+1)^(1/2)/b+3/4*a*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/4/b^2*(b*x+a-1)^
(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2-1)^(1/2)*ln(b*x+a+((b*x+a)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26021, size = 178, normalized size = 1.98

$$\frac{\left(2\,b^2x^2 - 2\,a^2 - 1\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 - 1}\right) - \sqrt{b^2x^2 + 2\,abx + a^2 - 1}(bx - 3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 3*a))/b^2
```

Sympy [A] time = 0.384441, size = 104, normalized size = 1.16

$$\begin{cases} -\frac{a^2 \operatorname{acosh}(a+bx)}{2b^2} + \frac{3a\sqrt{a^2+2abx+b^2x^2-1}}{4b^2} + \frac{x^2 \operatorname{acosh}(a+bx)}{2} - \frac{x\sqrt{a^2+2abx+b^2x^2-1}}{4b} - \frac{\operatorname{acosh}(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acosh}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acosh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 - 1)/(4*b**2) + x**2*acosh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 - 1)/(4*b) - acosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*acosh(a)/2, True)
)
```

Giac [A] time = 1.20757, size = 151, normalized size = 1.68

$$\frac{1}{2}\,x^2 \log\left(bx + a + \sqrt{(bx+a)^2 - 1}\right) - \frac{1}{4}\left(\sqrt{b^2x^2 + 2\,abx + a^2 - 1}\left(\frac{x}{b^2} - \frac{3\,a}{b^3}\right) - \frac{\left(2\,a^2 + 1\right)\log\left(\left|-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx\right.\right.\right.}{b^2|b|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*(x/b^2 - 3*a/b^3) - (2*a^2 + 1)*log(abs(-a*b - (x*abs(b) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^2*abs(b)))*b
```

3.86 $\int \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{(a+bx)\cosh^{-1}(a+bx)}{b}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{b}$$

```
[Out] -((Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/b) + ((a + b*x)*ArcCosh[a + b*x])/
b
```

Rubi [A] time = 0.0158476, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5864, 5654, 74}

$$\frac{(a+bx)\cosh^{-1}(a+bx)}{b}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x],x]
```

```
[Out] -((Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/b) + ((a + b*x)*ArcCosh[a + b*x])/
b
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^{-1}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \cosh^{-1}(x)\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\cosh^{-1}(a+bx)}{b} - \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{b} + \frac{(a+bx)\cosh^{-1}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0362489, size = 56, normalized size = 1.37

$$x\cosh^{-1}(a+bx) - \frac{\sqrt{a+bx-1}\sqrt{a+bx+1} - 2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x],x]
```

```
[Out] x*ArcCosh[a + b*x] - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] - 2*a*ArcSinh[Sq
rt[-1 + a + b*x]/Sqrt[2]])/b
```

Maple [A] time = 0.001, size = 36, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\operatorname{arccosh}(bx+a) - \sqrt{bx+a-1}\sqrt{bx+a+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*arccosh(b*x+a)-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))
```

Maxima [A] time = 1.17374, size = 41, normalized size = 1.

$$\frac{(bx+a)\operatorname{arcosh}(bx+a)-\sqrt{(bx+a)^2-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arccosh(b*x + a) - sqrt((b*x + a)^2 - 1))/b
```

Fricas [A] time = 2.31516, size = 135, normalized size = 3.29

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1))/b
```

Sympy [A] time = 0.22608, size = 46, normalized size = 1.12

$$\begin{cases} \frac{a\operatorname{acosh}(a+bx)}{b}+x\operatorname{acosh}(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2-1}}{b} & \text{for } b\neq 0\\ x\operatorname{acosh}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a),x)
```

```
[Out] Piecewise((a*acosh(a + b*x)/b + x*acosh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/b, Ne(b, 0)), (x*acosh(a), True))
```

Giac [B] time = 1.17564, size = 126, normalized size = 3.07

$$-b\left(\frac{a\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2-1}}{b^2}\right)+x\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b))
)/(b*abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)/b^2) + x*log(b*x + a + sqr
t((b*x + a)^2 - 1))
```

3.87 $\int \frac{\cosh^{-1}(a+bx)}{x} dx$

Optimal. Leaf size=131

$$\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}+a}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(1\right.$$

```
[Out] -ArcCosh[a + b*x]^2/2 + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a - Sq
rt[-1 + a^2])] + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a + Sqrt[-1 +
 a^2])] + PolyLog[2, E^ArcCosh[a + b*x]/(a - Sqrt[-1 + a^2])] + PolyLog[2,
E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Rubi [A] time = 0.249189, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5866, 5800, 5562, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}+a}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x,x]
```

```
[Out] -ArcCosh[a + b*x]^2/2 + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a - Sq
rt[-1 + a^2])] + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a + Sqrt[-1 +
 a^2])] + PolyLog[2, E^ArcCosh[a + b*x]/(a - Sqrt[-1 + a^2])] + PolyLog[2,
E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x\sinh(x)}{-\frac{a}{b}+\frac{\cosh(x)}{b}}\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{-1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \cosh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}+\frac{\sqrt{-1+a^2}}{b}+\frac{e^x}{b}}\right.}{b}$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh}}{a+\sqrt{}}\right.$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh}}{a+\sqrt{}}\right.$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh}}{a+\sqrt{}}\right.$$

Mathematica [A] time = 0.0131778, size = 153, normalized size = 1.17

$$\text{PolyLog}\left(2,-\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}-a}\right)+\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}+a}\right)+\cosh^{-1}(a+bx)\log\left(\frac{e^{\cosh^{-1}(a+bx)}}{b\left(-\frac{\sqrt{a^2-1}}{b}-\frac{a}{b}\right)}+1\right)+\cosh^{-1}(a+bx)\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x,x]
```

```
[Out] -ArcCosh[a + b*x]^2/2 + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/((-(a/b
) - Sqrt[-1 + a^2]/b)*b)] + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/((-
(a/b) + Sqrt[-1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcCosh[a + b*x]/(-a + Sqrt[
-1 + a^2]))] + PolyLog[2, E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Maple [B] time = 0.081, size = 436, normalized size = 3.3

$$-\frac{(\text{arccosh}(bx+a))^2}{2}+a\text{arccosh}(bx+a)\ln\left(\left(\sqrt{a^2-1}-bx-\sqrt{bx+a-1}\sqrt{bx+a+1}\right)\left(a+\sqrt{a^2-1}\right)^{-1}\right)\frac{1}{\sqrt{a^2-1}}-a\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x,x)
```

```
[Out] -1/2*arccosh(b*x+a)^2+a*arccosh(b*x+a)/(a^2-1)^(1/2)*ln(((a^2-1)^(1/2)-b*x-
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))-a*arccosh(b*x+a)/(a^2-1
)^(1/2)*ln(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^
(1/2)))-(a^2-1+a*(a^2-1)^(1/2))*arccosh(b*x+a)*(2*ln(((a^2-1)^(1/2)-b*x-(b*
x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))*a^2-ln(((a^2-1)^(1/2)-b*x-
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))-ln(((a^2-1)^(1/2)+b*x+(
b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^(1/2)))-2*a*(a^2-1)^(1/2)*ln(((
a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2))))/(a^2-
1)+dilog(((a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/
2)))+dilog(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^
(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcosh}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(b*x + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x,x)
```

```
[Out] Integral(acosh(a + b*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(b*x + a)/x, x)
```

3.88 $\int \frac{\cosh^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=64

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\cosh^{-1}(a+bx)}{x}$$

```
[Out] -(ArcCosh[a + b*x]/x) - (2*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
 + a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2]
```

Rubi [A] time = 0.0828594, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5866, 5802, 93, 205}

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\cosh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(ArcCosh[a + b*x]/x) - (2*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
 + a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(a+bx)}{x^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cosh^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx\right)\\
&= -\frac{\cosh^{-1}(a+bx)}{x} + 2\,\text{Subst}\left(\int \frac{1}{-\frac{1}{b}-\frac{a}{b}-\left(\frac{1}{b}-\frac{a}{b}\right)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{-1+a+bx}}\right)\\
&= -\frac{\cosh^{-1}(a+bx)}{x} - \frac{2b\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [C] time = 0.107167, size = 83, normalized size = 1.3

$$-\frac{\cosh^{-1}(a+bx)}{x} - \frac{ib\log\left(\frac{2\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+\frac{i\left(a^2+abx-1\right)}{\sqrt{1-a^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(ArcCosh[a + b*x]/x) - (I*b*Log[(2*(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] +
 (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(b*x)])/Sqrt[1 - a^2]
```

Maple [A] time = 0.014, size = 97, normalized size = 1.5

$$-\frac{\operatorname{arccosh}(bx+a)}{x}-\frac{b}{(a-1)(1+a)}\sqrt{bx+a-1}\sqrt{bx+a+1}\sqrt{a^2-1}\ln\left(2\,\frac{\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+a(bx+a)-1}{bx}\right)\frac{1}{\sqrt{(bx+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^2,x)
```

```
[Out] -arccosh(b*x+a)/x-b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(a^2-1)^(1/2)*ln(2*((a^
2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)/((b*x+a)^2-1)^(1/2)/(a-1)/
(1+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6116, size = 771, normalized size = 12.05

$$\left[\frac{\sqrt{a^2-1}bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)+\left(a^2-1\right)x\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\cdots\right.}{\left(a^2-1\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*
(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (a^
```

```
2 - 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 -
1)*x - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x),
(2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + (a^2 - 1)*x*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 - 1)*x - 1)*log(b*x + a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**2,x)
```

```
[Out] Integral(acosh(a + b*x)/x**2, x)
```

Giac [A] time = 1.20085, size = 99, normalized size = 1.55

$$\frac{2\,b\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}}-\frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1))/
sqrt(-a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 - 1))/x
```

3.89 $\int \frac{\cosh^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=106

$$-\frac{ab^2 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}$$

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x) - ArcCosh[a + b*x]
/(2*x^2) - (a*b^2*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
-1 + a + b*x])])/(1 - a^2)^(3/2)
```

Rubi [A] time = 0.102059, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5866, 5802, 96, 93, 205}

$$-\frac{ab^2 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x) - ArcCosh[a + b*x]
/(2*x^2) - (a*b^2*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
-1 + a + b*x])])/(1 - a^2)^(3/2)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cosh^{-1}(a+bx)}{2x^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x} - \frac{\cosh^{-1}(a+bx)}{2x^2} + \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx\right)}{2\left(1-a^2\right)}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x} - \frac{\cosh^{-1}(a+bx)}{2x^2} + \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{b}-\frac{a}{b}-\left(\frac{1}{b}-\frac{a}{b}\right)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{-1+a+bx}}\right)}{1-a^2}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x} - \frac{\cosh^{-1}(a+bx)}{2x^2} - \frac{ab^2\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\left(1-a^2\right)^{3/2}}$$

Mathematica [C] time = 0.263804, size = 136, normalized size = 1.28

$$\frac{-\cosh^{-1}(a+bx) + \frac{bx\left(-\sqrt{a+bx-1}\sqrt{a+bx+1} + \frac{iabx\log\left(\frac{4i\sqrt{1-a^2}\left(-i\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+a^2+abx-1\right)}{ab^2x}\right)}{\sqrt{1-a^2}}\right)}{a^2-1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (-ArcCosh[a + b*x] + (b*x*(-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) + (I*a*b
*x*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 + a
 + b*x]*Sqrt[1 + a + b*x]))/(a*b^2*x)])/Sqrt[1 - a^2]))/(-1 + a^2))/(2*x^2)
```

Maple [B] time = 0.02, size = 181, normalized size = 1.7

$$-\frac{\operatorname{arccosh}(bx+a)}{2\,x^2} + \frac{b^2a}{(2+2\,a)(a-1)}\sqrt{bx+a-1}\sqrt{bx+a+1}\ln\left(2\,\frac{\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+a(bx+a)-1}{bx}\right)\frac{1}{\sqrt{a^2-1}}\frac{}{\sqrt{(b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^3,x)
```

```
[Out] -1/2*arccosh(b*x+a)/x^2+1/2*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(a^2-1)^(1/
2)/(1+a)/(a-1)/((b*x+a)^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+
a*(b*x+a)-1)/b/x)*a-1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)/(1+a)/(
a-1)*a^2+1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)/(1+a)/(a-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.67325, size = 1091, normalized size = 10.29

$$\left[\frac{\sqrt{a^2-1}ab^2x^2\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2+\sqrt{a^2-1}a-1\right)+\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-\left(a^2-1\right)b^2x^2+\left(a^4-2\,a^2+1\right)x^2\log\left(-bx-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - 1)*a*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a
)/x) - (a^2 - 1)*b^2*x^2 + (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*b*x -
 (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*a*b^2*x^2
*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2
+ 1))/(a^2 - 1)) + (a^2 - 1)*b^2*x^2 - (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a +
```

```
 sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^
2 - 1)*b*x + (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**3,x)
```

```
[Out] Integral(acosh(a + b*x)/x**3, x)
```

Giac [A] time = 1.19093, size = 230, normalized size = 2.17

$$-\left(\frac{ab\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\left(a^2-1\right)\sqrt{-a^2+1}}-\frac{\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)ab+a^2|b|-|b|}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)^2-a^2+1\right)\left(a^2-1\right)}\right)b-\frac{\log\left(bx+a+\sqrt{(bx+a)^2}\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -(a*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1)
)/((a^2 - 1)*sqrt(-a^2 + 1)) - ((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
1))*a*b + a^2*abs(b) - abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))^2 - a^2 + 1)*(a^2 - 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 - 1))/
x^2
```

3.90 $\int \frac{\cosh^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$\frac{ab^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)^2 x}-\frac{\left(2a^2+1\right)b^3\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{3\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{6\left(1-a^2\right)x^2}-\frac{\cosh^{-1}(a+bx)}{3x^3}$$

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[-1
 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)^2*x) - ArcCosh[a + b*x]/(3*x^3)
 - ((1 + 2*a^2)*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqr
t[-1 + a + b*x])])/(3*(1 - a^2)^(5/2))
```

Rubi [A] time = 0.171842, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5866, 5802, 103, 151, 12, 93, 205}

$$\frac{ab^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)^2 x}-\frac{\left(2a^2+1\right)b^3\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{3\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{6\left(1-a^2\right)x^2}-\frac{\cosh^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[-1
 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)^2*x) - ArcCosh[a + b*x]/(3*x^3)
 - ((1 + 2*a^2)*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqr
t[-1 + a + b*x])])/(3*(1 - a^2)^(5/2))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
```

```
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6(1-a^2)x^2} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{b^2\,\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{6(1-a^2)}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{b^4\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{\left((1+2a^2)b^2\right)}{}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{\left((1+2a^2)b^2\right)}{}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx)}{3x^3} - \frac{(1+2a^2)b^3\,\text{ta}}{3(}$$

Mathematica [C] time = 0.296328, size = 162, normalized size = 1.05

$$\frac{1}{6}\left(-\frac{i(2a^2+1)b^3\log\left(\frac{12(1-a^2)^{3/2}\left(\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+ia^2+iabx-i\right)}{b^3(2a^2x+x)}\right)}{(1-a^2)^{5/2}} + \frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-a^2+3abx+1\right)}{(a^2-1)^2x^2} - \frac{2\cos}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^4,x]
```

```
[Out] ((b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(1 - a^2 + 3*a*b*x))/((-1 + a^2)^2
*x^2) - (2*ArcCosh[a + b*x])/x^3 - (I*(1 + 2*a^2)*b^3*Log[(12*(1 - a^2)^(3/
```

```
2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*
x]))/(b^3*(x + 2*a^2*x))])/(1 - a^2)^(5/2))/6
```

Maple [B] time = 0.02, size = 397, normalized size = 2.6

$$-\frac{\operatorname{arccosh}(bx+a)}{3x^3}-\frac{b^3a^2}{(3+3a)(a-1)}\sqrt{bx+a-1}\sqrt{bx+a+1}\ln\left(2\,\frac{\sqrt{a^2-1}\sqrt{(bx+a)^2-1}+a(bx+a)-1}{bx}\right)\left(a^2-1\right)^{-\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^4,x)
```

```
[Out] -1/3*arccosh(b*x+a)/x^3-1/3*b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(a^2-1)^(3/
2)/(1+a)/(a-1)/((b*x+a)^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+
a*(b*x+a)-1)/b/x)*a^2-1/6*b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(a^2-1)^(3/2)
/(1+a)/(a-1)/((b*x+a)^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*
(b*x+a)-1)/b/x)+1/2*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)^2/(1+a)/(
a-1)*a^3-1/6*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)*a^
4-1/2*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)^2/(1+a)/(a-1)*a+1/3*b*(
b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)*a^2-1/6*b*(b*x+a-1
)^(1/2)*(b*x+a+1)^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.71583, size = 1305, normalized size = 8.47

$$\left[\frac{\left(2\,a^2+1\right)\sqrt{a^2-1}b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}a-1\right)-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)+3\left(a^3-a\right)b^3x^3+2\left(a^6-3\,a^4+3\,a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*a^2 + 1)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^
2 - 1) - a)/x) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*a^4 - (a^6 - 3*a
^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), 1/6*(2*(2*a^2 + 1)*sqrt(-a^
2 + 1)*b^3*x^3*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a
^2 - 1)*x^3*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*
a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}\left(a+bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**4,x)
```

```
[Out] Integral(acosh(a + b*x)/x**4, x)
```

Giac [B] time = 1.19736, size = 459, normalized size = 2.98

$$\frac{1}{3}\,b\left(\frac{\left(2\,a^{2}b^{2}+b^{2}\right)\arctan\left(-\frac{x|b|-\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}}{\sqrt{-a^{2}+1}}\right)}{\left(a^{4}-2\,a^{2}+1\right)\sqrt{-a^{2}+1}}-\frac{2\left(x|b|-\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\right)^{3}a^{2}b^{2}-6\left(x|b|-\sqrt{b^{2}x^{2}+2\,abx+a^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b*((2*a^2*b^2 + b^2)*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))/sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - (2*(x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 - 1))*a^4*b^2 - 4*a^5*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1))^3*b^2 + 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*
a^2*b^2 + 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*b
^2 - 4*a*b*abs(b))/((a^4 - 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1))^2 - a^2 + 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 - 1))/x^3
```

3.91 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rubi [A] time = 0.135609, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5864, 5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}\right.}{2bd}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}}\right.}{bd}$$
$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Mathematica [A] time = 0.16071, size = 110, normalized size = 1.2

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]
] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x
])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arcosh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.92 $\int \frac{1}{\sqrt{a-b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rubi [A] time = 0.128634, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5864, 5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c\right.}{2bd}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a-b\cosh^{-1}(\right.}}{bd}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Mathematica [A] time = 0.100241, size = 111, normalized size = 1.18

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}-\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}-\cosh^{-1}(c+dx)\right)+\sqrt{\cosh^{-1}(c+dx)-\frac{a}{b}}\,\text{Gamma}\left(\frac{1}{2},\cosh^{-1}(c+dx)-\frac{a}{b}\right)\right)}{2d\sqrt{a-b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b - ArcCosh[c + d*x]]*Gamma[1/2, a/b - ArcCosh[c + d*x]
] + Sqrt[-(a/b) + ArcCosh[c + d*x]]*Gamma[1/2, -(a/b) + ArcCosh[c + d*x]])/
(2*d*E^(a/b)*Sqrt[a - b*ArcCosh[c + d*x]])
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-b\,\text{arccosh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b \operatorname{arcosh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-b\operatorname{acosh}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a - b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.93 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=135

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}-\frac{be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{25d}-\frac{4be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{75d}-\frac{8b}{}$$

```
[Out] (-8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(75*d) - (4*b*e^4*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(75*d) - (b*e^4*Sqrt[-1 + c + d*x]
*(c + d*x)^4*Sqrt[1 + c + d*x])/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcCosh[c
+ d*x]))/(5*d)
```

Rubi [A] time = 0.0855618, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 100, 74}

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}-\frac{be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{25d}-\frac{4be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{75d}-\frac{8b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(75*d) - (4*b*e^4*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(75*d) - (b*e^4*Sqrt[-1 + c + d*x]
*(c + d*x)^4*Sqrt[1 + c + d*x])/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcCosh[c
+ d*x]))/(5*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int (ce+dex)^4\left(a+b\cosh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right.}{5d}$$
$$= -\frac{be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}$$
$$= -\frac{be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}$$
$$= -\frac{4be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c}}{25d}$$
$$= -\frac{4be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c}}{25d}$$
$$= -\frac{8be^4\sqrt{-1+c+dx}\sqrt{1+c+dx}}{75d} - \frac{4be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{75d} -$$

Mathematica [A] time = 0.11841, size = 103, normalized size = 0.76

$$\frac{e^4\left((c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right) - \frac{4}{15}b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(c^2+2cdx+d^2x^2+2\right) - \frac{1}{5}b\sqrt{c+dx-1}\sqrt{c+dx+1}(\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^4*(-(b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/5 - (4*b*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^2*x^2))/15 + (c + d*x
)^5*(a + b*ArcCosh[c + d*x])))/(5*d)
```

Maple [A] time = 0.013, size = 78, normalized size = 0.6

$$\frac{1}{d}\left(\frac{(dx+c)^5e^4a}{5} + e^4b\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)}{5} - \frac{3\,(dx+c)^4+4\,(dx+c)^2+8}{75}\sqrt{dx+c-1}\sqrt{dx+c+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a+e^4*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46632, size = 608, normalized size = 4.5

$$\frac{15\,ad^5e^4x^5 + 75\,acd^4e^4x^4 + 150\,ac^2d^3e^4x^3 + 150\,ac^3d^2e^4x^2 + 75\,ac^4de^4x + 15\left(bd^5e^4x^5 + 5\,bcd^4e^4x^4 + 10\,bc^2d^3e^4x^3 + 10\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*a
*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x^4
 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5*e^
4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (3*b*d^4*e^4*x^4 + 12
*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 + 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 + 2*b*c)*d*e^
4*x + (3*b*c^4 + 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 4.6963, size = 527, normalized size = 3.9

$$\begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4\operatorname{acosh}(c+dx)}{5d} + bc^4e^4x\operatorname{acosh}(c+dx) - \frac{bc^4e^4\sqrt{c^2+2cdx+d^2x^2-1}}{25d} + \\ c^4e^4x\left(a + b\operatorname{acosh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*acosh(c + d*x)/(5*d)
 + b*c**4*e**4*x*acosh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)/(25*d) + 2*b*c**3*d*e**4*x**2*acosh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 2*b*c**2*d**2*e**4*x**3*acosh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(75*d) + b*c*d**3*e**4*x**4*acos
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 -
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 + b*d**4*e**4*x**5*ac
osh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 -
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(
c)), True))
```

Giac [B] time = 2.8434, size = 1110, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/600*(120*a*d^4*x^5 + 600*a*c*d^3*x^4 + 1200*a*c^2*d^2*x^3 + 1200*a*c^3*d*
x^2 - 600*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^4 + 600*(2*x^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x
/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*c^3*d + 200*(6*x^3*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*
(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c
*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*
d)*b*c^2*d^2 + 25*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^
3 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2 + 3)
*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^
4*abs(d)))*d)*b*c*d^3 + (120*x^5*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*(3*x*(4*x/d^2 - 9*c/d^3) +
```

```
(47*c^2*d^5 + 16*d^5)/d^9)*x - 7*(22*c^3*d^4 + 23*c*d^4)/d^9)*x + (274*c^4*
d^3 + 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c^5 + 40*c^3 + 15*c)*log(abs(-c*d
- (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^5*abs(d)))*d)*
b*d^4 + 600*a*c^4*x)*e^4
```

3.94 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=119

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{16d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{32d}-\frac{3be$$

```
[Out] (-3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(32*d) - (b*e^3*S
qrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(16*d) - (3*b*e^3*ArcCosh[
c + d*x])/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(4*d)
```

Rubi [A] time = 0.0705448, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 100, 90, 52}

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{16d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{32d}-\frac{3be$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(32*d) - (b*e^3*S
qrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(16*d) - (3*b*e^3*ArcCosh[
c + d*x])/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(4*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\cosh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^3 \,\text{Subst}\left(\int x^3 \left(a+b\cosh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right) \text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c+dx\right.}{4d}$$
$$= -\frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{16d} + \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$
$$= -\frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{16d} + \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$
$$= -\frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c-}}{16d}$$
$$= -\frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c-}}{16d}$$

Mathematica [A] time = 0.124771, size = 115, normalized size = 0.97

$$\frac{e^3 \left((c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right) - \frac{1}{4}b\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3 - \frac{3}{8}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)+2\,\text{ta}\right.\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^3*(-(b*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/4 + (c + d*x)^4
*(a + b*ArcCosh[c + d*x]) - (3*b*(Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c +
 d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/8))/(4*d)
```

Maple [B] time = 0.004, size = 359, normalized size = 3.

$$\frac{d^3x^4ae^3}{4} + d^2x^3ace^3 + \frac{3\,dx^2ac^2e^3}{2} + xac^3e^3 + \frac{ac^4e^3}{4\,d} + \frac{d^3\text{arccosh}\,(dx+c)\,x^4be^3}{4} + d^2\text{arccosh}\,(dx+c)\,x^3bce^3 + \frac{3\,d\text{arccc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/4*d^3*x^4*a*e^3+d^2*x^3*a*c*e^3+3/2*d*x^2*a*c^2*e^3+x*a*c^3*e^3+1/4/d*a*c
^4*e^3+1/4*d^3*arccosh(d*x+c)*x^4*b*e^3+d^2*arccosh(d*x+c)*x^3*b*c*e^3+3/2*
d*arccosh(d*x+c)*x^2*b*c^2*e^3+arccosh(d*x+c)*x*b*c^3*e^3+1/4/d*arccosh(d*x
+c)*b*c^4*e^3-1/16*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*b*e^3-3/16*d*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*b*c*e^3-3/16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x*b*c^2*e^3-1/16/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*b*c^3*e^3-3/32*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*x*b*e^3-3/32/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*b*
c*e^3-3/32/d*e^3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d
*x+c+((d*x+c)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42512, size = 495, normalized size = 4.16

$$\frac{8\,ad^4e^3x^4 + 32\,acd^3e^3x^3 + 48\,ac^2d^2e^3x^2 + 32\,ac^3de^3x + \left(8\,bd^4e^3x^4 + 32\,bcd^3e^3x^3 + 48\,bc^2d^2e^3x^2 + 32\,bc^3de^3x + \left(8\,bc^4 - 3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c^
3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2 +
32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 + b)*d*e^
3*x + (2*b*c^3 + 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 2.35689, size = 394, normalized size = 3.31

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3 \operatorname{acosh}(c+dx)}{4d} + bc^3e^3x \operatorname{acosh}(c+dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2-1}}{16d} + \frac{3bc^2de^3x^2 \operatorname{acosh}(}{2} \\ c^3e^3x(a + b \operatorname{acosh}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*acosh(c + d*x)/(4*d) + b*c**3*e**3*x*acosh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*acosh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)/16 + b*c*d**2*e**3*x**3*acosh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)/(32*d) + b*d**3*e**3*x**4*acosh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)/32 - 3*b*e**3*acosh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*acosh(c)), True))
```

Giac [B] time = 2.4579, size = 807, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(24*a*d^3*x^4 + 96*a*c*d^2*x^3 + 144*a*c^2*d*x^2 - 96*(d*(c*log(abs(-c
*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) + s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)))*b*c^3 + 72*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 +
1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(
d^2*abs(d)))*d)*b*c^2*d + 16*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11
*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*d)*b*c*d^2 + (24*x^4*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2
 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2 + 3)*log(abs(-c*d - (x*abs(d) - sqrt(
```

```
d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^4*abs(d)))*d)*b*d^3 + 96*a*c^3*x)
*e^3
```

3.95 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}-\frac{be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{9d}-\frac{2be^2\sqrt{c+dx-1}\sqrt{c+dx+1}}{9d}$$

```
[Out] (-2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(9*d) - (b*e^2*Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcCo
sh[c + d*x]))/(3*d)
```

Rubi [A] time = 0.0630765, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 100, 74}

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}-\frac{be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{9d}-\frac{2be^2\sqrt{c+dx-1}\sqrt{c+dx+1}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(9*d) - (b*e^2*Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcCo
sh[c + d*x]))/(3*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)dx &= \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}\\ &= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}\\ &= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}-\frac{\left(be^2\right)\text{Subst}\left(\int\frac{x^3}{\sqrt{-1+x}\sqrt{1+x}}dx,x,c+dx\right)}{3d}\\ &= -\frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{9d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}\\ &= -\frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{9d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}\\ &= -\frac{2be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{9d}-\frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{9d}+\end{aligned}$$

Mathematica [A] time = 0.0715758, size = 71, normalized size = 0.73

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)-\frac{1}{9}b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(c^2+2cdx+d^2x^2+2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^2*(-(b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^2*x^2
))/9 + ((c + d*x)^3*(a + b*ArcCosh[c + d*x]))/3))/d
```

Maple [A] time = 0.007, size = 67, normalized size = 0.7

$$\frac{1}{d}\left(\frac{(dx+c)^3 e^2 a}{3}+e^2 b\left(\frac{\operatorname{arccosh}(dx+c)(dx+c)^3}{3}-\frac{(dx+c)^2+2}{9}\sqrt{dx+c-1}\sqrt{dx+c+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a+e^2*b*(1/3*arccosh(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42013, size = 365, normalized size = 3.76

$$\frac{3\,ad^3e^2x^3+9\,acd^2e^2x^2+9\,ac^2de^2x+3\left(bd^3e^2x^3+3\,bcd^2e^2x^2+3\,bc^2de^2x+bc^3e^2\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*x
^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 + 2*b
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 1.10139, size = 258, normalized size = 2.66

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{acosh}(c+dx)}{3d} + bc^2e^2x\operatorname{acosh}(c+dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2-1}}{9d} + bcde^2x^2\operatorname{acosh}(c+dx) - \frac{2b}{} \\ c^2e^2x\left(a + b\operatorname{acosh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*acosh(c + d*x)/(3*d) + b*c**2*e**2*x*acosh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + b*c*d*e**2*x**2*acosh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 + b*d**2*e**2*x**3*acosh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*acosh(c)), True))
```

Giac [B] time = 2.13946, size = 548, normalized size = 5.65

$$\frac{1}{18}\left(6\,ad^2x^3 + 18\,acdx^2 - 18\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}}{d^2}\right) - x\log\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/18*(6*a*d^2*x^3 + 18*a*c*d*x^2 - 18*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^
2 + 9*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x
```

```
^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*a
bs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*c*d
+ (6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c
^3 + 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs
(d)))/(d^3*abs(d)))*d)*b*d^2 + 18*a*c^2*x)*e^2
```

3.96 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=75

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{4d}-\frac{be\cosh^{-1}(c+dx)}{4d}$$

```
[Out] -(b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(4*d) - (b*e*ArcCosh[
c + d*x])/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(2*d)
```

Rubi [A] time = 0.0389998, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5866, 12, 5662, 90, 52}

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{4d}-\frac{be\cosh^{-1}(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(4*d) - (b*e*ArcCosh[
c + d*x])/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}}dx, x, c+dx\right)}{2d}$$
$$= -\frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{4d} + \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d} - $$
$$= -\frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{4d} - \frac{be\cosh^{-1}(c+dx)}{4d} + \frac{e(c+dx)^2\left(a+b\right.}{2}$$

Mathematica [A] time = 0.154843, size = 81, normalized size = 1.08

$$\frac{e\left(\frac{1}{2}(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)-\frac{1}{4}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)+2\tanh^{-1}\left(\sqrt{\frac{c+dx-1}{c+dx+1}}\right)\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x]))/2 - (b*(Sqrt[-1 + c + d*x]*(c +
d*x)*Sqrt[1 + c + d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/4)
)/d
```

Maple [B] time = 0.004, size = 162, normalized size = 2.2

$$\frac{ax^2de}{2} + xace + \frac{ac^2e}{2d} + \frac{d\operatorname{arccosh}(dx+c)\,x^2be}{2} + \operatorname{arccosh}(dx+c)\,xbce + \frac{b\operatorname{arccosh}(dx+c)\,c^2e}{2d} - \frac{bxe}{4}\sqrt{dx+c-1}\sqrt{dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/2*a*x^2*d*e+x*a*c*e+1/2/d*a*c^2*e+1/2*d*arccosh(d*x+c)*x^2*b*e+arccosh(d*
x+c)*x*b*c*e+1/2/d*arccosh(d*x+c)*b*c^2*e-1/4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x*b*e-1/4/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*b*c*e-1/4/d*e*b*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4183, size = 257, normalized size = 3.43

$$\frac{2\,ad^2ex^2 + 4\,acdex + \left(2\,bd^2ex^2 + 4\,bcdex + \left(2\,bc^2 - b\right)e\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - (bdex + bce)\sqrt{d^2x^2 + 2\,c}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
- b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (b*d*e*x + b*c*e
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 0.454334, size = 148, normalized size = 1.97

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{acosh}(c+dx)}{2d} + bcex\operatorname{acosh}(c+dx) - \frac{bce\sqrt{c^2+2cdx+d^2x^2-1}}{4d} + \frac{bdex^2\operatorname{acosh}(c+dx)}{2} - \frac{bex\sqrt{c^2+2cdx+d^2x^2-1}}{4} - \frac{be\operatorname{acosh}}{4d} \\ cex\left(a + b\operatorname{acosh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*acosh(c + d*x)/(2*d) + b*c*e*x
*acosh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(4*d) + b*d*e*
x**2*acosh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 - b*e*
acosh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c)), True))
```

Giac [B] time = 1.80385, size = 331, normalized size = 4.41

$$\frac{1}{4}\left(2\,adx^2 - 4\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{d^2x^2 \cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x^2 - 4*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) -
x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c + (2*x^2*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*d + 4*a*c*x)*e
```

3.97 $\int \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$ax-\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

```
[Out] a*x - (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (b*(c + d*x)*ArcCosh[c +
 d*x])/d
```

Rubi [A] time = 0.0235829, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5864, 5654, 74}

$$ax-\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (b*(c + d*x)*ArcCosh[c +
 d*x])/d
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}(c+dx)\right)dx &= ax+b\int \cosh^{-1}(c+dx)\,dx\\ &= ax+\frac{b\,\mathrm{Subst}\left(\int \cosh^{-1}(x)\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{b\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{d}\\ &= ax-\frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0492835, size = 61, normalized size = 1.33

$$ax-\frac{b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}-2c\sinh^{-1}\left(\frac{\sqrt{c+dx-1}}{\sqrt{2}}\right)\right)}{d}+bx\cosh^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcCosh[c + d*x] - (b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*c
*ArcSinh[Sqrt[-1 + c + d*x]/Sqrt[2]]))/d
```

Maple [A] time = 0.003, size = 41, normalized size = 0.9

$$ax+\frac{b}{d}\left((dx+c)\,\mathrm{arccosh}\,(dx+c)-\sqrt{dx+c-1}\sqrt{dx+c+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x+c),x)
```

```
[Out] a*x+b/d*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
```

Maxima [A] time = 1.22302, size = 47, normalized size = 1.02

$$ax + \frac{\left((dx+c)\operatorname{arcosh}(dx+c) - \sqrt{(dx+c)^2 - 1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b/d
```

Fricas [A] time = 2.16048, size = 154, normalized size = 3.35

$$\frac{adx + (bdx + bc)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b)/d
```

Sympy [A] time = 0.1967, size = 51, normalized size = 1.11

$$ax + b\left(\begin{cases} \frac{c\operatorname{acosh}(c+dx)}{d} + x\operatorname{acosh}(c+dx) - \frac{\sqrt{c^2+2cdx+d^2x^2-1}}{d} & \text{for}\ d \neq 0 \\ x\operatorname{acosh}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*acosh(c + d*x)/d + x*acosh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)/d, Ne(d, 0)), (x*acosh(c), True))
```

Giac [B] time = 1.14232, size = 135, normalized size = 2.93

$$-\left(d\left(\frac{c\log\left(\left|-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2-1}\right)|d|\right|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}}{d^2}\right)-x\log\left(dx+c+\sqrt{(dx+c)^2-1}\right)\right)b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)
))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sq
rt((d*x + c)^2 - 1)))*b + a*x
```

3.98 $\int \frac{a+b\cosh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=81

$$-\frac{b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}$$

```
[Out] (a + b*ArcCosh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcCosh[c + d*x])*Log[1 + E^
(-2*ArcCosh[c + d*x])])/(d*e) - (b*PolyLog[2, -E^(-2*ArcCosh[c + d*x])])/(2
*d*e)
```

Rubi [A] time = 0.107032, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5866, 12, 5660, 3718, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(e^{2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcCosh[c + d*x])*Log[1 + E
^(2*ArcCosh[c + d*x])])/(d*e) + (b*PolyLog[2, -E^(2*ArcCosh[c + d*x])])/(2*
d*e)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{ex}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x}\,dx, x, c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} - \frac{b\,\text{Subst}\left(\int \text{lo}\right.}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} - \frac{b\,\text{Subst}\left(\int \frac{\text{lo}}{}\right.}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b\text{Li}_2\left(-e^{2\cosh}\right.}{2de}$$

Mathematica [A] time = 0.0615755, size = 69, normalized size = 0.85

$$\frac{-b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)+2a\log(c+dx)+b\cosh^{-1}(c+dx)^2+2b\cosh^{-1}(c+dx)\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)}{2de}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (b*ArcCosh[c + d*x]^2 + 2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x]
)] + 2*a*Log[c + d*x] - b*PolyLog[2, -E^(-2*ArcCosh[c + d*x])])/(2*d*e)
```

Maple [A] time = 0.031, size = 111, normalized size = 1.4

$$\frac{a\ln(dx+c)}{de} - \frac{b\left(\text{arccosh}(dx+c)\right)^2}{2\,de} + \frac{b\text{arccosh}(dx+c)}{de}\ln\left(\left(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1}\right)^2+1\right) + \frac{b}{2\,de}\text{polylog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] 1/d*a/e*ln(d*x+c)-1/2/d*b/e*arccosh(d*x+c)^2+1/d*b/e*arccosh(d*x+c)*ln((d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+1/2/d*b/e*polylog(2,-(d*x+c+(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arcosh}(dx + c) + a}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c+dx}\, dx + \int \frac{b \operatorname{acosh}(c+dx)}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(dx + c) + a}{dex + ce} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.99 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=56

$$\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)}{de^2}-\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}$$

```
[Out] -((a + b*ArcCosh[c + d*x])/(d*e^2*(c + d*x))) + (b*ArcTan[Sqrt[-1 + c + d*x
]*Sqrt[1 + c + d*x]])/(d*e^2)
```

Rubi [A] time = 0.0516872, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 92, 203}

$$\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)}{de^2}-\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])/(d*e^2*(c + d*x))) + (b*ArcTan[Sqrt[-1 + c + d*x
]*Sqrt[1 + c + d*x]])/(d*e^2)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^2x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^2}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{de^2}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\tan^{-1}\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{de^2}
\end{aligned}
$$

Mathematica [A] time = 0.108186, size = 76, normalized size = 1.36

$$
\frac{\frac{b\sqrt{(c+dx)^2-1}\tan^{-1}\left(\sqrt{(c+dx)^2-1}\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}-\frac{a+b\cosh^{-1}(c+dx)}{c+dx}}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] (-((a + b*ArcCosh[c + d*x])/(c + d*x)) + (b*Sqrt[-1 + (c + d*x)^2]*ArcTan[S
qrt[-1 + (c + d*x)^2]])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(d*e^2)
```

Maple [A] time = 0.006, size = 88, normalized size = 1.6

$$-\frac{a}{de^{2}(dx+c)}-\frac{b\operatorname{arccosh}(dx+c)}{de^{2}(dx+c)}-\frac{b}{de^{2}}\sqrt{dx+c-1}\sqrt{dx+c+1}\arctan\left(\frac{1}{\sqrt{(dx+c)^{2}-1}}\right)\frac{1}{\sqrt{(dx+c)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a/e^2/(d*x+c)-1/d*b/e^2/(d*x+c)*arccosh(d*x+c)-1/d*b/e^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53022, size = 312, normalized size = 5.57

$$\frac{bdx\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}-1}\right)-ac+2\left(bcdx+bc^{2}\right)\arctan\left(-dx-c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}-1}\right)+(bdx+bc}{cd^{2}e^{2}x+c^{2}de^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - a*c + 2*(b*c*d*x
+ b*c^2)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (b*d*x + b*
```

```
c)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))/(c*d^2*e^2*x + c^2*d*
e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*acosh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.100 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=66

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*d*e^3*(c + d*x)) - (a + b*ArcCo
sh[c + d*x])/(2*d*e^3*(c + d*x)^2)
```

Rubi [A] time = 0.0542246, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5866, 12, 5662, 95}

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*d*e^3*(c + d*x)) - (a + b*ArcCo
sh[c + d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx,x,c+dx\right)}{2de^3}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2de^3(c+dx)} - \frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Mathematica [A] time = 0.0561951, size = 55, normalized size = 0.83

$$-\frac{a-b\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -(a - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] + b*ArcCosh[c + d*x]
)/(2*d*e^3*(c + d*x)^2)
```

Maple [A] time = 0.011, size = 65, normalized size = 1.

$$\frac{1}{d}\left(-\frac{a}{2\,e^3\,(dx+c)^2}+\frac{b}{e^3}\left(-\frac{\text{arccosh}\,(dx+c)}{2\,(dx+c)^2}+\frac{1}{2\,dx+2\,c}\sqrt{dx+c-1}\sqrt{dx+c+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Maxima [B] time = 1.76058, size = 159, normalized size = 2.41

$$\frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}d}{d^3e^3x+cd^2e^3}-\frac{\operatorname{arcosh}\left(dx+c\right)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)-\frac{a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*d^2*e^3) - arccos
h(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x^2
+ 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [B] time = 2.41292, size = 257, normalized size = 3.89

$$\frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\left(bc^2dx+bc^3\right)\sqrt{d^2x^2+2\,cdx+c^2-1}}{2\left(c^2d^3e^3x^2+2\,c^3d^2e^3x+c^4de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) + (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^2*d^3*
e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{b\operatorname{acosh}\left(c+dx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(dx + c) + a}{(dex + ce)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

3.101 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^4}\,dx$

Optimal. Leaf size=99

$$-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{6de^4(c+dx)^2}+\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)}{6de^4}$$

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^4*(c + d*x)^2) - (a + b*Arc
Cosh[c + d*x])/(3*d*e^4*(c + d*x)^3) + (b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(6*d*e^4)
```

Rubi [A] time = 0.0650914, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 103, 92, 203}

$$-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{6de^4(c+dx)^2}+\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^4*(c + d*x)^2) - (a + b*Arc
Cosh[c + d*x])/(3*d*e^4*(c + d*x)^3) + (b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(6*d*e^4)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^4x^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{de^4}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{3de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx, x, c+dx\right)}{6de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+c+dx}\right)}{6de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\tan^{-1}\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)}{6de^4}$$

Mathematica [A] time = 0.206815, size = 101, normalized size = 1.02

$$\frac{\frac{b\left(\frac{(c+dx-1)(c+dx+1)}{(c+dx)^2}+\sqrt{(c+dx)^2-1}\tan^{-1}\left(\sqrt{(c+dx)^2-1}\right)\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{(c+dx)^3}}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*(a + b*ArcCosh[c + d*x]))/(c + d*x)^3 + (b*(((-1 + c + d*x)*(1 + c + d
*x))/(c + d*x)^2 + Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c + d*x)^2]]))/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(6*d*e^4)
```

Maple [A] time = 0.004, size = 120, normalized size = 1.2

$$-\frac{a}{3\,de^4\,(dx+c)^3} - \frac{b\mathrm{arccosh}\,(dx+c)}{3\,de^4\,(dx+c)^3} - \frac{b}{6\,de^4}\sqrt{dx+c-1}\sqrt{dx+c+1}\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)\frac{1}{\sqrt{(dx+c)^2-1}} + \frac{}{6\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a/e^4/(d*x+c)^3-1/3/d*b/e^4/(d*x+c)^3*arccosh(d*x+c)-1/6/d*b/e^4*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/
2))+1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,b\left(\frac{2\,d^2x^2+4\,cdx+2\,c^2-\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\log\left(dx+c+1\right)+\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\log\left(dx+c-\right.}{d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*((2*d^2*x^2 + 4*c*d*x + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)*log(d*x + c + 1) + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*log(d*x +
 c - 1) - 2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^4*e^4*x^
3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 6*integrate(1/3/(d^6*e
^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x
^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*
x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x + (d^5*e^4*x^5 + 5*c*d^4*e^4*x^4 + c^
5*e^4 - c^3*e^4 + (10*c^2*d^3*e^4 - d^3*e^4)*x^3 + (10*c^3*d^2*e^4 - 3*c*d^
2*e^4)*x^2 + (5*c^4*d*e^4 - 3*c^2*d*e^4)*x)*e^(1/2*log(d*x + c + 1) + 1/2*l
og(d*x + c - 1))), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e
^4*x + c^3*d*e^4)
```

Fricas [B] time = 2.78078, size = 610, normalized size = 6.16

$$-\frac{2\,ac^3-2\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\,bc^5dx+bc^6\right)\arctan\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x + b*c^6)*a
rctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2
*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)) - (b*c^3*d*x + b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))/(c^3*d^4*e^4*x^3 + 3*c^4*d^3*e^4*x^2 + 3*c^5*d^2*e^4*x + c^6*d*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4), x) + Integral(b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\operatorname{arcosh}(dx+c)+a}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

3.102 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^5}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{6de^5(c+dx)}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{12de^5(c+dx)^3}$$

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^5*(c + d*x)) - (a + b*ArcCosh[c + d*
x])/(4*d*e^5*(c + d*x)^4)
```

Rubi [A] time = 0.0694655, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 103, 95}

$$-\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{6de^5(c+dx)}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{12de^5(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```


```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^5*(c + d*x)) - (a + b*ArcCosh[c + d*
x])/(4*d*e^5*(c + d*x)^4)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^5}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^5x^5}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^5}\,dx, x, c+dx\right)}{de^5} \\
&= -\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x^4\sqrt{1+x}}\,dx, x, c+dx\right)}{4de^5} \\
&= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{12de^5(c+dx)^3} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx, x, c+d\right.}{12de^5} \\
&= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{12de^5(c+dx)^3} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx, x, c+d\right.}{6de^5} \\
&= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{12de^5(c+dx)^3} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{6de^5(c+dx)} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}
\end{aligned}$$

Mathematica [A] time = 0.0798584, size = 86, normalized size = 0.83

$$\frac{-3a+b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(6c^2dx+2c^3+6cd^2x^2+c+2d^3x^3+dx\right)-3b\cosh^{-1}(c+dx)}{12de^5(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] (-3*a + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(c + 2*c^3 + d*x + 6*c^2*d*x
 + 6*c*d^2*x^2 + 2*d^3*x^3) - 3*b*ArcCosh[c + d*x])/(12*d*e^5*(c + d*x)^4)
```

Maple [A] time = 0.004, size = 76, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{a}{4e^5(dx+c)^4}+\frac{b}{e^5}\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{2(dx+c)^2+1}{12(dx+c)^3}\sqrt{dx+c-1}\sqrt{dx+c+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arccosh(d*x+c)+1/12*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^2+1)/(d*x+c)^3))
```

Maxima [B] time = 1.31417, size = 351, normalized size = 3.38

$$\frac{1}{12}b\left(\frac{\left(2d^4x^4+8cd^3x^3+2c^4+\left(12c^2d^2-d^2\right)x^2-c^2+2\left(4c^3d-cd\right)x-1\right)d}{\left(d^5e^5x^3+3cd^4e^5x^2+3c^2d^3e^5x+c^3d^2e^5\right)\sqrt{dx+c+1}\sqrt{dx+c-1}}-\frac{3\operatorname{arcosh}(dx+c)}{d^5e^5x^4+4cd^4e^5x^3+6c^2d^3e^5x^2+4c^3d^2e^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 - d^2)*x^2 - c^2 + 2
*(4*c^3*d - c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x
 + c^3*d^2*e^5)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)) - 3*arccosh(d*x + c)/(
d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d
*e^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d
```

```
^2*e^5*x + c^4*d*e^5)
```

Fricas [B] time = 2.59492, size = 448, normalized size = 4.31

$$\frac{3\,ad^4x^4 + 12\,acd^3x^3 + 18\,ac^2d^2x^2 + 12\,ac^3dx - 3\,bc^4\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) + \left(2\,bc^4d^3x^3 + 6\,bc^5d^2x^2 + 2\right.}{12\left(c^4d^5e^5x^4 + 4\,c^5d^4e^5x^3 + 6\,c^6d^3e^5x^2 + 4\,c^7d^2e^5x + c^8de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (2*b*c^4*d^3*x^3 + 6
*b*c^5*d^2*x^2 + 2*b*c^7 + b*c^5 + (6*b*c^6 + b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*x^2
+ 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*acosh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.103 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^6}\,dx$

Optimal. Leaf size=137

$$-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}}{40de^6(c+dx)^2}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{20de^6(c+dx)^4}+\frac{3b\tan^{-1}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)}{40de^6}$$

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcCosh[c
 + d*x])/(5*d*e^6*(c + d*x)^5) + (3*b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]])/(40*d*e^6)
```

Rubi [A] time = 0.0866639, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 103, 92, 203}

$$-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}}{40de^6(c+dx)^2}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{20de^6(c+dx)^4}+\frac{3b\tan^{-1}\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right)}{40de^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```


```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcCosh[c
 + d*x])/(5*d*e^6*(c + d*x)^5) + (3*b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]])/(40*d*e^6)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^6}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^6x^6}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^6}\,dx, x, c+dx\right)}{de^6}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x^5\sqrt{1+x}}\,dx, x, c+dx\right)}{5de^6}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{20de^6(c+dx)^4} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{b\,\text{Subst}\left(\int \frac{3}{\sqrt{-1+x}x^3\sqrt{1+x}}\,dx, x, c+d\right.}{20de^6}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{20de^6(c+dx)^4} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x^3\sqrt{1+x}}\,dx, x, c\right.}{20de^6}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)}{}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)}{}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{3b\,\mathrm{t}}{}$$

Mathematica [A] time = 0.265301, size = 136, normalized size = 0.99

$$\frac{-\frac{a+b\cosh^{-1}(c+dx)}{(c+dx)^5} + \frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}}{4(c+dx)^4} + \frac{3b\left(\frac{(c+dx-1)(c+dx+1)}{(c+dx)^2} + \sqrt{(c+dx)^2-1}\tan^{-1}\left(\sqrt{(c+dx)^2-1}\right)\right)}{8\sqrt{c+dx-1}\sqrt{c+dx+1}}}{5de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] ((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(4*(c + d*x)^4) - (a + b*ArcCosh[
c + d*x])/(c + d*x)^5 + (3*b*(((-1 + c + d*x)*(1 + c + d*x))/(c + d*x)^2 +
Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c + d*x)^2]]))/(8*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]))/(5*d*e^6)
```

Maple [A] time = 0.005, size = 152, normalized size = 1.1

$$-\frac{a}{5\,de^{6}\left(dx+c\right)^{5}}-\frac{b\operatorname{arccosh}\left(dx+c\right)}{5\,de^{6}\left(dx+c\right)^{5}}-\frac{3\,b}{40\,de^{6}}\sqrt{dx+c-1}\sqrt{dx+c+1}\arctan\left(\frac{1}{\sqrt{\left(dx+c\right)^{2}-1}}\right)\frac{1}{\sqrt{\left(dx+c\right)^{2}-1}}+\frac{}{40\,de^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x)
```

```
[Out] -1/5/d*a/e^6/(d*x+c)^5-1/5/d*b/e^6/(d*x+c)^5*arccosh(d*x+c)-3/40/d*b/e^6*(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1
/2))+3/40*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^2+1/20*b*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] 1/30*b*((6*d^4*x^4 + 24*c*d^3*x^3 + 6*c^4 + 2*(18*c^2*d^2 + d^2)*x^2 + 2*c^
2 + 4*(6*c^3*d + c*d)*x - 3*(d^5*x^5 + 5*c*d^4*x^4 + 10*c^2*d^3*x^3 + 10*c^
3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c + 1) + 3*(d^5*x^5 + 5*c*d^4*x^4 +
10*c^2*d^3*x^3 + 10*c^3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c - 1) - 6*log
(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^6*e^6*x^5 + 5*c*d^5*e^6
*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^
6) - 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*e^6*x^7 + c^8*e^6 - c^6*e^6 +
(28*c^2*d^6*e^6 - d^6*e^6)*x^6 + 2*(28*c^3*d^5*e^6 - 3*c*d^5*e^6)*x^5 + 5*(
14*c^4*d^4*e^6 - 3*c^2*d^4*e^6)*x^4 + 4*(14*c^5*d^3*e^6 - 5*c^3*d^3*e^6)*x^
3 + (28*c^6*d^2*e^6 - 15*c^4*d^2*e^6)*x^2 + 2*(4*c^7*d*e^6 - 3*c^5*d*e^6)*x
 + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e^6 - c^5*e^6 + (21*c^2*d^5*e^6 - d
^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 - c*d^4*e^6)*x^4 + 5*(7*c^4*d^3*e^6 - 2*c^2*
d^3*e^6)*x^3 + (21*c^5*d^2*e^6 - 10*c^3*d^2*e^6)*x^2 + (7*c^6*d*e^6 - 5*c^4
*d*e^6)*x)*e^(1/2*log(d*x + c + 1) + 1/2*log(d*x + c - 1))), x)) - 1/5*a/(d
^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 + 5*
c^4*d^2*e^6*x + c^5*d*e^6)
```

Fricas [B] time = 3.22602, size = 909, normalized size = 6.64

$$-\frac{8\,ac^{5} - 6\left(bc^{5}d^{5}x^{5} + 5\,bc^{6}d^{4}x^{4} + 10\,bc^{7}d^{3}x^{3} + 10\,bc^{8}d^{2}x^{2} + 5\,bc^{9}dx + bc^{10}\right)\arctan\left(-dx - c + \sqrt{d^{2}x^{2} + 2\,cdx + c^{2} - 1}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 6*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10
*b*c^8*d^2*x^2 + 5*b*c^9*d*x + b*c^10)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)) - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c
^3*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*
c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (3*b*c
^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 + 2*b*c^6 + (9*b*c^7 + 2*b*c^5)*d*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^5*d^6*e^6*x^5 + 5*c^6*d^5*e^6*x^4 +
10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x + c^10*d*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^{6}+6c^{5}dx+15c^{4}d^{2}x^{2}+20c^{3}d^{3}x^{3}+15c^{2}d^{4}x^{4}+6cd^{5}x^{5}+d^{6}x^{6}}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^{6}+6c^{5}dx+15c^{4}d^{2}x^{2}+20c^{3}d^{3}x^{3}+15c^{2}d^{4}x^{4}+6cd^{5}x^{5}+d^{6}x^{6}}\,dx}{e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*acosh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(dx + c) + a}{(dex + ce)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

3.104 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=218

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{25d}-\frac{8be^4\sqrt{c+dx-1}}{}$$

```
[Out] (16*b^2*e^4*x)/75 + (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x]))/(75*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*
x]*(a + b*ArcCosh[c + d*x]))/(75*d) - (2*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)
^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(25*d) + (e^4*(c + d*x)^5*(a
 + b*ArcCosh[c + d*x])^2)/(5*d)
```

Rubi [A] time = 0.485118, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5662, 5759, 5718, 8, 30}

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{25d}-\frac{8be^4\sqrt{c+dx-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```


```
[Out] (16*b^2*e^4*x)/75 + (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x]))/(75*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*
x]*(a + b*ArcCosh[c + d*x]))/(75*d) - (2*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)
^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(25*d) + (e^4*(c + d*x)^5*(a
 + b*ArcCosh[c + d*x])^2)/(5*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d} - \frac{\left(2be^4\right)\text{Subst}\left(\int \frac{x^5\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}} dx, x,\right.}{5d}$$

$$= -\frac{2be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{25d} + \frac{e^4(c+dx}{}$$

$$= \frac{2b^2e^4(c+dx)^5}{125d} - \frac{8be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{75d}$$

$$= \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\,\text{co}\right.}{75d}$$

$$= \frac{16}{75}b^2e^4x + \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{-1+c+dx}\sqrt{1+c+}}{75d}$$

Mathematica [A] time = 0.32343, size = 220, normalized size = 1.01

$$\frac{e^4\left(9\left(25a^2+2b^2\right)(c+dx)^5+30ab\sqrt{c+dx-1}\sqrt{c+dx+1}\left(-3(c+dx)^4-4(c+dx)^2-8\right)+30b\cosh^{-1}(c+dx)\left(15a(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^4*(240*b^2*(c + d*x) + 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8 - 4*(c + d*x)^2 - 3*(c
 + d*x)^4) + 30*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d
*x] - 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 3*b*Sqrt[-1 +
c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 225*b^2*(c + d*x
)^5*ArcCosh[c + d*x]^2))/(1125*d)
```

Maple [A] time = 0.044, size = 294, normalized size = 1.4

$$\frac{1}{d}\left(\frac{(dx+c)^5e^4a^2}{5} + e^4b^2\left(\frac{(dx+c)^3(\operatorname{arccosh}(dx+c))^2(dx+c-1)(dx+c+1)}{5} + \frac{(\operatorname{arccosh}(dx+c))^2(dx+c-1)(dx+}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^2+e^4*b^2*(1/5*(d*x+c)^3*arccosh(d*x+c)^2*(d*x+c-1
)*(d*x+c+1)+1/5*arccosh(d*x+c)^2*(d*x+c-1)*(d*x+c+1)*(d*x+c)+1/5*arccosh(d*
x+c)^2*(d*x+c)-2/25*(d*x+c)^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)-16/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-8/75*arccosh(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+2/125*(d*x+c-1)*(d*x+c+1)*(d*x+c)
^3+58/1125*(d*x+c-1)*(d*x+c+1)*(d*x+c)+298/1125*d*x+298/1125*c)+2*e^4*a*b*(
1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c
)^4+4*(d*x+c)^2+8)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63977, size = 1331, normalized size = 6.11

$$\frac{9\left(25\,a^2+2\,b^2\right)d^5e^4x^5+45\left(25\,a^2+2\,b^2\right)cd^4e^4x^4+10\left(9\left(25\,a^2+2\,b^2\right)c^2+4\,b^2\right)d^3e^4x^3+30\left(3\left(25\,a^2+2\,b^2\right)c^3+4\,b^2c\right)d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
+ 10*(9*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2)*
c^3 + 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16*b^
2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^3*e^
4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 30*(15*a*b*d^5*e^4*x^5 + 75*a*b
*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 + 75*a*b
```

```
*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e^4*x^3 +
 2*(9*b^2*c^2 + 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*e^4*x + (3*b
^2*c^4 + 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 30*(3*a*b*d^4*e^4*x^4 + 12*a*b*
c*d^3*e^4*x^3 + 2*(9*a*b*c^2 + 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^3 + 2*a*b*c)
*d*e^4*x + (3*a*b*c^4 + 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))/d
```

Sympy [A] time = 10.2041, size = 1268, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
*4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
cosh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*acosh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*acosh(c
+ d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*acosh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/25 - 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*acosh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/75 + 2*a*b*d**4*e**4*x**5*acosh(c + d*x)/5 - 2*a*b*d
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 - 1)/(75*d) + b**2*c**5*e**4*acosh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*acosh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*acosh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*acosh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*b**2*c**2
*e**4*x/75 - 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/(75*d) + b**2*c*d**3*e**4*x**4*acosh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*ac
osh(c + d*x)/25 + 8*b**2*c*d*e**4*x**2/75 - 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/75 + b**2*d**4*e**4*x**5*acosh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*b**2*d**2*e**4*x**3/225 - 8*b**
```

```
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(c))**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^2, x)
```

3.105 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=186

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{8d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c}}{}$$

```
[Out] (3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) - (3*b*e^3*Sq
rt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(16*
d) - (b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh
[c + d*x]))/(8*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^2)/(32*d) + (e^3*(c + d
*x)^4*(a + b*ArcCosh[c + d*x])^2)/(4*d)
```

Rubi [A] time = 0.435838, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{8d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) - (3*b*e^3*Sq
rt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(16*
d) - (b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh
[c + d*x]))/(8*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^2)/(32*d) + (e^3*(c + d
*x)^4*(a + b*ArcCosh[c + d*x])^2)/(4*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c\right.}{2d}$$

$$= -\frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{8d} + \frac{e^3(c+dx)}{}$$

$$= \frac{b^2e^3(c+dx)^4}{32d} - \frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx\right.}{16d}$$

$$= \frac{3b^2e^3(c+dx)^2}{32d} + \frac{b^2e^3(c+dx)^4}{32d} - \frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a\right.}{16d}$$

Mathematica [A] time = 0.266461, size = 212, normalized size = 1.14

$$\frac{e^3\left(\left(8a^2+b^2\right)(c+dx)^4+2ab\sqrt{c+dx-1}\sqrt{c+dx+1}\left(-2(c+dx)^2-3\right)(c+dx)-6ab\log\left(\sqrt{c+dx-1}\sqrt{c+dx+1}+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^3*(3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*Sqrt[-1 + c + d
*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3 - 2*(c + d*x)^2) + 2*b*(c + d*x)*(8*a*(
c + d*x)^3 - 3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-3 + 8*(c + d*x)
^4)*ArcCosh[c + d*x]^2 - 6*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]]))/(32*d)
```

Maple [B] time = 0.042, size = 822, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 2*arccosh(d*x+c)*x*a*b*c^3*e^3-3/8*d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*a*
b*c*e^3-3/8*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2+1/
32/d*e^3*b^2*c^4-3/16/d*e^3*a*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-
1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))-3/8*d*e^3*b^2*arccosh(d*x+c)*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c+1/2*d^3*arccosh(d*x+c)*x^4*a*b*e^3+3/32/d*e^
3*b^2*c^2+1/4/d*a^2*c^4*e^3+1/8*e^3*b^2*x*c^3+1/32*d^3*e^3*b^2*x^4+3/16*e^3
*b^2*x*c+x*a^2*c^3*e^3-3/16/d*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)*c-1/8/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a*b*c^3*e^3-3/8*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*x*a*b*c^2*e^3+3*d*arccosh(d*x+c)*x^2*a*b*c^2*e^3+3/1
6*d*e^3*b^2*x^2*c^2+1/4*d^3*x^4*a^2*e^3+3/32*d*e^3*b^2*x^2+1/8*d^2*e^3*b^2*
x^3*c+d^2*x^3*a^2*c*e^3+3/2*d*x^2*a^2*c^2*e^3-3/32/d*e^3*b^2*arccosh(d*x+c)
^2+e^3*b^2*arccosh(d*x+c)^2*x*c^3+1/4/d*e^3*b^2*arccosh(d*x+c)^2*c^4+1/4*d^
3*e^3*b^2*arccosh(d*x+c)^2*x^4-3/16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a*b*e
^3+1/2/d*arccosh(d*x+c)*a*b*c^4*e^3+d^2*e^3*b^2*arccosh(d*x+c)^2*x^3*c-3/16
*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+2*d^2*arccosh(d*x
+c)*x^3*a*b*c*e^3-1/8/d*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)*c^3-1/8*d^2*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-
1/8*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*a*b*e^3+3/2*d*e^3*b^2*arccosh(d
*x+c)^2*x^2*c^2-3/16/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a*b*c*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50646, size = 1034, normalized size = 5.56

$$\frac{\left(8\,a^2+b^2\right)d^4e^3x^4+4\left(8\,a^2+b^2\right)cd^3e^3x^3+3\left(2\left(8\,a^2+b^2\right)c^2+b^2\right)d^2e^3x^2+2\left(2\left(8\,a^2+b^2\right)c^3+3\,b^2c\right)de^3x+\left(8\,b^2d^4e^3x^4+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*a
^2 + b^2)*c^2 + b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*e^3*
x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 + 32
*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48*a*b*
c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b^2*d^3
*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 + b^2)*d*e^3*x + (2*b^2*c^3 +
 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x^2 + 3*(2
*a*b*c^2 + a*b)*d*e^3*x + (2*a*b*c^3 + 3*a*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1))/d
```

Sympy [A] time = 6.3001, size = 916, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*acosh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*acosh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*acosh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)/8 + 2*a*b*c*d**2*e**3*x**3*acosh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(16*d) + a*b*d**3*e**3*x**4*acosh(c +
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*a*b*e**3*acosh(c + d*x)/
(16*d) + b**2*c**4*e**3*acosh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*acosh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)*acosh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)*acosh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 +
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)*acosh(c + d*x)/8 + 3*b**2*c*e**3*x/16 - 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*acosh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/8 + 3*b**2*d*e**3*x**2/32 - 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/16 - 3*b**2*e**3*a
```

```
cosh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*acosh(c))**2, True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^2, x)
```

3.106 $\int (ce + dex)^2 \left(a + b\cosh^{-1}(c + dx)\right)^2 dx$

Optimal. Leaf size=150

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{4be^2\sqrt{c+dx-1}\sqrt{}}{}$$

```
[Out] (4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) - (4*b*e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 +
c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) + (e
^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(3*d)
```

Rubi [A] time = 0.323398, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5662, 5759, 5718, 8, 30}

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{4be^2\sqrt{c+dx-1}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```


```
[Out] (4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) - (4*b*e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 +
c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) + (e
^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(3*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d} - \frac{\left(2be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x,\right.}{3d}$$

$$= -\frac{2be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{9d} + \frac{e^2(c+dx}{}$$

$$= \frac{2b^2e^2(c+dx)^3}{27d} - \frac{4be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{9d} - \frac{2l}{}$$

$$= \frac{4}{9}b^2e^2x + \frac{2b^2e^2(c+dx)^3}{27d} - \frac{4be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{9d}$$

Mathematica [A] time = 0.227183, size = 168, normalized size = 1.12

$$\frac{e^2\left(\left(9a^2+2b^2\right)(c+dx)^3+6ab\sqrt{c+dx-1}\sqrt{c+dx+1}\left(-(c+dx)^2-2\right)+6b\cosh^{-1}(c+dx)\left(3a(c+dx)^3-b\sqrt{c+dx-}\right.\right.}{27d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^2*(12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(-2 - (c + d*x)^2) + 6*b*(3*a*(c + d*x)^3 - 2*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x] - b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
+ c + d*x])*ArcCosh[c + d*x] + 9*b^2*(c + d*x)^3*ArcCosh[c + d*x]^2))/(27*d
)
```

Maple [A] time = 0.038, size = 202, normalized size = 1.4

$$\frac{1}{d}\left(\frac{(dx+c)^3e^2a^2}{3} + e^2b^2\left(\frac{(\text{arccosh}(dx+c))^2(dx+c-1)(dx+c+1)(dx+c)}{3} + \frac{(\text{arccosh}(dx+c))^2(dx+c)}{3} - \frac{2\,\text{arcc}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^2+e^2*b^2*(1/3*arccosh(d*x+c)^2*(d*x+c-1)*(d*x+c+1
)*(d*x+c)+1/3*arccosh(d*x+c)^2*(d*x+c)-2/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*(d*x+c)^2-4/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
+2/27*(d*x+c-1)*(d*x+c+1)*(d*x+c)+14/27*d*x+14/27*c)+2*e^2*a*b*(1/3*arccosh
(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34188, size = 767, normalized size = 5.11

$$\frac{\left(9\,a^2+2\,b^2\right)d^3e^2x^3+3\left(9\,a^2+2\,b^2\right)cd^2e^2x^2+3\left(\left(9\,a^2+2\,b^2\right)c^2+4\,b^2\right)de^2x+9\left(b^2d^3e^2x^3+3\,b^2cd^2e^2x^2+3\,b^2c^2de^2x+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((9
*a^2 + 2*b^2)*c^2 + 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2*x
^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*d*e^
2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 + 2*b^2
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 + 2*a*b)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 2.66837, size = 610, normalized size = 4.07

$$\begin{cases} a^2c^2e^2x + a^2cde^2x^2 + \frac{a^2d^2e^2x^3}{3} + \frac{2abc^3e^2\operatorname{acosh}(c+dx)}{3d} + 2abc^2e^2x\operatorname{acosh}(c+dx) - \frac{2abc^2e^2\sqrt{c^2+2cdx+d^2x^2-1}}{9d} + 2abcde^2x^2\operatorname{acosh} \\ c^2e^2x(a+b\operatorname{acosh}(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
2*a*b*c**3*e**2*acosh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*acosh(c + d*x) - 2
*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + 2*a*b*c*d*e**2*
x**2*acosh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9
 + 2*a*b*d**2*e**2*x**3*acosh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/9 - 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/
(9*d) + b**2*c**3*e**2*acosh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*acosh(c +
 d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*acosh(c + d*x)**2 +
2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
1)*acosh(c + d*x)/9 + b**2*d**2*e**2*x**3*acosh(c + d*x)**2/3 + 2*b**2*d**2
*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*aco
sh(c + d*x)/9 + 4*b**2*e**2*x/9 - 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acosh(c))**2,
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^2, x)
```

3.107 $\int (ce + dex)\left(a + b\cosh^{-1}(c + dx)\right)^2 dx$

Optimal. Leaf size=110

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c +
d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (e*(a + b*ArcCosh[c + d*x])^2)/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.250706, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c +
d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (e*(a + b*ArcCosh[c + d*x])^2)/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{d}$$
$$=-\frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{e(c+dx)^2\left(a+\right.}{}$$
$$=\frac{b^2e(c+dx)^2}{4d}-\frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e}{}$$

Mathematica [A] time = 0.217252, size = 167, normalized size = 1.52

$$\frac{e\left((c+dx)\left(2a^2(c+dx)-2ab\sqrt{c+dx-1}\sqrt{c+dx+1}+b^2(c+dx)\right)-2ab\log\left(\sqrt{c+dx-1}\sqrt{c+dx+1}+c+dx\right)-2b(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e*((c + d*x)*(2*a^2*(c + d*x) + b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]) - 2*b*(c + d*x)*(-2*a*(c + d*x) + b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-1 + 2*c^2 + 4*c*d*x + 2*d^2*x^2)*
ArcCosh[c + d*x]^2 - 2*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]]))/(4*d)
```

Maple [B] time = 0.036, size = 334, normalized size = 3.

$$\frac{a^2ex^2d}{2}+xa^2ce+\frac{a^2c^2e}{2\,d}+\frac{deb^2\left(\text{arccosh}\left(dx+c\right)\right)^2x^2}{2}+eb^2\left(\text{arccosh}\left(dx+c\right)\right)^2xc+\frac{eb^2\left(\text{arccosh}\left(dx+c\right)\right)^2c^2}{2\,d}-\frac{eb^2\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*e*x^2*d+x*a^2*c*e+1/2/d*a^2*c^2*e+1/2*d*e*b^2*arccosh(d*x+c)^2*x^2+
e*b^2*arccosh(d*x+c)^2*x*c+1/2/d*e*b^2*arccosh(d*x+c)^2*c^2-1/2*e*b^2*arcco
sh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-1/2/d*e*b^2*arccosh(d*x+c)*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-1/4/d*e*b^2*arccosh(d*x+c)^2+1/4*b^2*d*e*x^2+
1/2*e*b^2*x*c+1/4/d*e*b^2*c^2+d*arccosh(d*x+c)*x^2*a*b*e+2*arccosh(d*x+c)*x
*a*b*c*e+1/d*arccosh(d*x+c)*a*b*c^2*e-1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x
*a*b*e-1/2/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a*b*c*e-1/2/d*e*a*b*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38381, size = 533, normalized size = 4.85

$$\frac{\left(2a^2+b^2\right)d^2ex^2+2\left(2a^2+b^2\right)cdex+\left(2b^2d^2ex^2+4b^2cdex+\left(2b^2c^2-b^2\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right)^2+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2 +
 4*b^2*c*d*e*x + (2*b^2*c^2 - b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 - a*b)*e -
(b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1))/d
```

Sympy [A] time = 1.18923, size = 335, normalized size = 3.05

$$\begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\operatorname{acosh}(c+dx)}{d} + 2abcex\operatorname{acosh}(c+dx) - \frac{abce\sqrt{c^2+2cdx+d^2x^2-1}}{2d} + abdex^2\operatorname{acosh}(c+dx) - \frac{abex\sqrt{c^2+2cdx+d^2x^2 \ldots}}{2} \\ cex\,(a+b\operatorname{acosh}(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*acosh(c + d*x)/d + 2*a
*b*c*e*x*acosh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(2*d
) + a*b*d*e*x**2*acosh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/2 - a*b*e*acosh(c + d*x)/(2*d) + b**2*c**2*e*acosh(c + d*x)**2/(2*d) +
b**2*c*e*x*acosh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/(2*d) + b**2*d*e*x**2*acosh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)/2 - b**2*e*acosh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))
**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b\operatorname{arcosh}(dx + c) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

3.108 $\int \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$-\frac{2b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]
))/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d
```

Rubi [A] time = 0.124695, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5864, 5654, 5718, 8}

$$-\frac{2b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]
))/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
```

```
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}$$
$$= 2b^2x - \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\cosh^{-1}(c+$$

Mathematica [A] time = 0.0835136, size = 105, normalized size = 1.64

$$\frac{a^2(c+dx) - 2ab\sqrt{c+dx-1}\sqrt{c+dx+1} - 2b\cosh^{-1}(c+dx)\left(b\sqrt{c+dx-1}\sqrt{c+dx+1} - a(c+dx)\right) + 2b^2(c+dx) + b^2(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x] - 2*b*(-(a*(c + d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[
c + d*x] + b^2*(c + d*x)*ArcCosh[c + d*x]^2)/d
```

Maple [A] time = 0.003, size = 100, normalized size = 1.6

$$\frac{1}{d}\left((dx+c)\,a^2 + b^2\left((\text{arccosh}\,(dx+c))^2\,(dx+c) - 2\,\text{arccosh}\,(dx+c)\,\sqrt{dx+c-1}\sqrt{dx+c+1} + 2\,dx + 2\,c\right) + 2\,ab\left((dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*((d*x+c)*a^2+b^2*(arccosh(d*x+c)^2*(d*x+c)-2*arccosh(d*x+c)*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+2*a*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.37372, size = 333, normalized size = 5.2

$$\frac{\left(a^2+2\,b^2\right)dx+\left(b^2dx+b^2c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2-2\,\sqrt{d^2x^2+2\,cdx+c^2-1}ab+2\left(abdx+abc-\sqrt{d}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)))/d
```

Sympy [A] time = 0.479235, size = 143, normalized size = 2.23

$$\begin{cases} a^2x+\frac{2abc\operatorname{acosh}(c+dx)}{d}+2abx\operatorname{acosh}(c+dx)-\frac{2ab\sqrt{c^2+2cdx+d^2x^2-1}}{d}+\frac{b^2c\operatorname{acosh}^2(c+dx)}{d}+b^2x\operatorname{acosh}^2(c+dx)+2b^2x-\frac{2b^2\sqrt{c^2}}{} \\ x\left(a+b\operatorname{acosh}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*acosh(c + d*x)/d + 2*a*b*x*acosh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + b**2*c*acosh(c + d*x)**2/d + b*
*2*x*acosh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)*acosh(c + d*x)/d, Ne(d, 0)), (x*(a + b*acosh(c))**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2, x)
```

3.109 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=118

$$-\frac{b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde}+$$

```
[Out] (a + b*ArcCosh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcCosh[c + d*x])^2*Log[1 +
E^(-2*ArcCosh[c + d*x])])/(d*e) - (b*(a + b*ArcCosh[c + d*x])*PolyLog[2, -E
^(-2*ArcCosh[c + d*x])])/(d*e) - (b^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])])
/(2*d*e)
```

Rubi [A] time = 0.190075, antiderivative size = 117, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5660, 3718, 2190, 2531, 2282, 6589}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde}+\frac{\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcCosh[c + d*x])^2*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e) + (b*(a + b*ArcCosh[c + d*x])*PolyLog[2, -E
^(2*ArcCosh[c + d*x])])/(d*e) - (b^2*PolyLog[3, -E^(2*ArcCosh[c + d*x])])/(
2*d*e)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} - \frac{(2b)\,\text{Sub}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b\left(a+b\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b\left(a+b\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b\left(a+b\right.}{}$$

Mathematica [A] time = 0.392605, size = 140, normalized size = 1.19

$$\frac{-b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right) - \frac{1}{2}b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right) + a^2\log(c+dx) + ab\cosh^{-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a*b*ArcCosh[c + d*x]^2 + (b^2*ArcCosh[c + d*x]^3)/3 + 2*a*b*ArcCosh[c + d*
x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*
ArcCosh[c + d*x])] + a^2*Log[c + d*x] - b*(a + b*ArcCosh[c + d*x])*PolyLog[
2, -E^(-2*ArcCosh[c + d*x])] - (b^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])])/2
)/(d*e)
```

Maple [A] time = 0.031, size = 263, normalized size = 2.2

$$\frac{a^2\ln(dx+c)}{de}-\frac{b^2(\mathrm{arccosh}(dx+c))^3}{3de}+\frac{b^2(\mathrm{arccosh}(dx+c))^2}{de}\ln\left(\left(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1}\right)^2+1\right)+\frac{b^2\mathrm{arcco}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^2/e*ln(d*x+c)-1/3/d*b^2/e*arccosh(d*x+c)^3+1/d*b^2/e*arccosh(d*x+c)^2
*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+1/d*b^2/e*arccosh(d*x+c)*p
olylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/2/d*b^2/e*polylog(3,
-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a*b/e*arccosh(d*x+c)^2+2/d*
a*b/e*arccosh(d*x+c)*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+1/d*a*
b/e*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{arcosh}(dx+c)^2+2\,ab\,\mathrm{arcosh}(dx+c)+a^2}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d*e*x + c
*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*acosh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^2}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

3.110 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{2ib^2\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}+\frac{2ib^2\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{4b\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de\ldots}$$

```
[Out] -((a + b*ArcCosh[c + d*x])^2/(d*e^2*(c + d*x))) + (4*b*(a + b*ArcCosh[c + d
*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((2*I)*b^2*PolyLog[2, (-I)*E^Arc
Cosh[c + d*x]])/(d*e^2) + ((2*I)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e
^2)
```

Rubi [A] time = 0.242946, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5662, 5761, 4180, 2279, 2391}

$$-\frac{2ib^2\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}+\frac{2ib^2\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{4b\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^2/(d*e^2*(c + d*x))) + (4*b*(a + b*ArcCosh[c + d
*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((2*I)*b^2*PolyLog[2, (-I)*E^Arc
Cosh[c + d*x]])/(d*e^2) + ((2*I)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e
^2)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}x\sqrt{1+x}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int (a+bx)\mathbf{sech}(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{\left(2ib^2\right)\text{Sub}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{\left(2ib^2\right)\text{Sub}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{2ib^2\mathbf{Li}_2\left(-\right.}{}$$

Mathematica [A] time = 0.721384, size = 161, normalized size = 1.46

$$\frac{-ib^2\left(2\text{PolyLog}\left(2, -ie^{-\cosh^{-1}(c+dx)}\right) - 2\text{PolyLog}\left(2, ie^{-\cosh^{-1}(c+dx)}\right) + \cosh^{-1}(c+dx)\left(-\frac{i\cosh^{-1}(c+dx)}{c+dx} + 2\log\left(1-ie^{-\cosh}\right.\right.\right.}{de^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) + 2*a*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[Ar
cCosh[c + d*x]/2]]) - I*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c + d*x])/(c +
 d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c + d*x]])
+ 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCosh[c + d*x]
]))/(d*e^2)
```

Maple [A] time = 0.049, size = 290, normalized size = 2.6

$$-\frac{a^2}{de^2(dx+c)}-\frac{b^2(\operatorname{arccosh}(dx+c))^2}{de^2(dx+c)}-\frac{2\,ib^2\operatorname{arccosh}(dx+c)}{de^2}\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1}\right)\right)+\frac{2\,ib^2\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2*arccosh(d*x+c)^2/(d*x+c)-2*I/d*b^2/e^2*arc
cosh(d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I/d*b^2/e^2*a
rccosh(d*x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2*I/d*b^2/e^2
*dilog(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I/d*b^2/e^2*dilog(1-I
*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2/d*a*b/e^2/(d*x+c)*arccosh(d*x+c
)-2/d*a*b/e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/
((d*x+c)^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\operatorname{arcosh}(dx+c)^2+2\,ab\operatorname{arcosh}(dx+c)+a^2}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d^2*e^2*x
^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*acosh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*acosh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.111 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=92

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{b^2\log(c+dx)}{de^3}$$

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(d*e^3*(c
 + d*x)) - (a + b*ArcCosh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) - (b^2*Log[c +
d*x])/(d*e^3)
```

Rubi [A] time = 0.213952, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5866, 12, 5662, 5724, 29}

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{b^2\log(c+dx)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(d*e^3*(c
 + d*x)) - (a + b*ArcCosh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) - (b^2*Log[c +
d*x])/(d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx,x,c+dx\right)}{de^3}\\
&= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{b^2\,\text{Subs}}{}\\
&= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{b^2\log(c}{de^3}
\end{aligned}$$

Mathematica [A] time = 0.204559, size = 81, normalized size = 0.88

$$\frac{b\left(\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{c+dx}-b\log(c+dx)\right)-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2(c+dx)^2}}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a + b*ArcCosh[c + d*x])^2/(2*(c + d*x)^2) + b*((Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(c + d*x) - b*Log[c + d*x]))/(d*e^3)
```

Maple [B] time = 0.063, size = 194, normalized size = 2.1

$$-\frac{a^2}{2\,de^3\,(dx+c)^2}+\frac{b^2\mathrm{arccosh}\,(dx+c)}{de^3}+\frac{b^2\mathrm{arccosh}\,(dx+c)}{de^3\,(dx+c)}\sqrt{dx+c-1}\sqrt{dx+c+1}-\frac{b^2\,(\mathrm{arccosh}\,(dx+c))^2}{2\,de^3\,(dx+c)^2}-\frac{b^2}{de^3}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^2/e^3/(d*x+c)^2+1/d*b^2/e^3*arccosh(d*x+c)+1/d*b^2/e^3*arccosh(d*x
+c)/(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)-1/2/d*b^2/e^3*arccosh(d*x+c)^2/
(d*x+c)^2-1/d*b^2/e^3*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)-1/d*a
*b/e^3/(d*x+c)^2*arccosh(d*x+c)+1/d*a*b/e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
/(d*x+c)
```

Maxima [B] time = 1.7738, size = 309, normalized size = 3.36

$$\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}d\,\mathrm{arcosh}\,(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log\,(dx+c)}{de^3}\right)b^2+ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}d}{d^3e^3x+cd^2e^3}-\frac{\mathrm{arcosh}\,(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*e^3*x + c*d^2*e^
3) - log(d*x + c)/(d*e^3))*b^2 + a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(
```

```
d^3*e^3*x + c*d^2*e^3) - arccosh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^
2*d*e^3)) - 1/2*b^2*arccosh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d
*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [B] time = 2.76066, size = 713, normalized size = 7.75

$$\frac{2\,abc^2d^2x^2 + 4\,abc^3dx + 2\,abc^4 - b^2c^2\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)^2 - a^2c^2 + 2\left(abd^2x^2 + 2\,abcdx + \left(b^2c^2dx + b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 - b^2*c^2*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - a^2*c^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d
*x + (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
+ b^2*c^4)*log(d*x + c) + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
- c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ab\operatorname{acosh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^2}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

3.112 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=186

$$-\frac{ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{(}{}$$

```
[Out] b^2/(3*d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x]))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcCosh[c + d*x])^2/(3*d*e^
4*(c + d*x)^3) + (2*b*(a + b*ArcCosh[c + d*x])*ArcTan[E^ArcCosh[c + d*x]])/
(3*d*e^4) - ((I/3)*b^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^4) + ((I/3
)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rubi [A] time = 0.386817, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5662, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```


```
[Out] b^2/(3*d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x]))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcCosh[c + d*x])^2/(3*d*e^
4*(c + d*x)^3) + (2*b*(a + b*ArcCosh[c + d*x])*ArcTan[E^ArcCosh[c + d*x]])/
(3*d*e^4) - ((I/3)*b^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^4) + ((I/3
)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^4)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{3de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}}{}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^3}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^3}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^3}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^3}$$

Mathematica [A] time = 0.956459, size = 251, normalized size = 1.35

$$\frac{b^2\left(-i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right)+i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)+\frac{1}{c+dx}-\frac{\cosh^{-1}(c+dx)^2}{(c+dx)^3}+\frac{\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)\cosh^{-1}(c+dx)}{(c+dx)^2}-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] (-(a^2/(c + d*x)^3) + a*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x
))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[ArcCosh[c
 + d*x]/2]]) + b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c + d*x)^3
- I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c + d*x]*Log
[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] + I*Poly
Log[2, I/E^ArcCosh[c + d*x]]))/(3*d*e^4)
```

Maple [A] time = 0.086, size = 381, normalized size = 2.1

$$-\frac{a^2}{3\,de^4\,(dx+c)^3}+\frac{b^2\text{arccosh}\,(dx+c)}{3\,de^4\,(dx+c)^2}\sqrt{dx+c-1}\sqrt{dx+c+1}-\frac{b^2\,(\text{arccosh}\,(dx+c))^2}{3\,de^4\,(dx+c)^3}+\frac{b^2}{3\,de^4\,(dx+c)}-\frac{\frac{i}{3}b^2\text{arccosh}}{de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^2/e^4/(d*x+c)^3+1/3/d*b^2/e^4/(d*x+c)^2*arccosh(d*x+c)*(d*x+c+1)^(
1/2)*(d*x+c-1)^(1/2)-1/3/d*b^2/e^4/(d*x+c)^3*arccosh(d*x+c)^2+1/3*b^2/d/e^4
/(d*x+c)-1/3*I/d*b^2/e^4*arccosh(d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)))+1/3*I/d*b^2/e^4*arccosh(d*x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)))-1/3*I/d*b^2/e^4*dilog(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)))+1/3*I/d*b^2/e^4*dilog(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
-2/3/d*a*b/e^4/(d*x+c)^3*arccosh(d*x+c)-1/3/d*a*b/e^4*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))+1/3/d*a*b/e^4*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{a^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}+\int\frac{}{3\left(d^7e^4x^7+7\,cd^6e^4x^6+c^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^4*e^4*x^3
+ 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 +
 b^2*d^3)*x^3 + 3*(c^3 - c)*a*b + (c^3 - c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^
2)*x^2 + (b^2*c^2 + 3*(c^2 - 1)*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*
c*d + b^2*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*d - d)*a*
b + (3*c^2*d - d)*b^2)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d
^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*d^
5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^3*d
^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*
d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^4
- 5*c^4*d*e^4)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arcosh}\left(dx+c\right)^2+2\,ab\,\text{arcosh}\left(dx+c\right)+a^2}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d^4*e^4*x
^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{b^2\,\text{acosh}^2\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{2ab\,\text{acosh}\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*acosh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

3.113 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=382

$$\frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d}+\frac{8b^2e^4(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c\right.}{5d}$$

```
[Out] (16*a*b^2*e^4*x)/25 - (4144*b^3*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(
5625*d) - (272*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5
625*d) - (6*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(625*
d) + (16*b^3*e^4*(c + d*x)*ArcCosh[c + d*x])/(25*d) + (8*b^2*e^4*(c + d*x)^
3*(a + b*ArcCosh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*ArcCosh[
c + d*x]))/(125*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/(25*d) - (4*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) - (3*b*e^4*Sqrt[-1 + c + d*x
]*(c + d*x)^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) + (e^4*(
c + d*x)^5*(a + b*ArcCosh[c + d*x])^3)/(5*d)
```

Rubi [A] time = 0.723173, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5718, 5654, 74, 100}

$$\frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d}+\frac{8b^2e^4(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```


```
[Out] (16*a*b^2*e^4*x)/25 - (4144*b^3*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(
5625*d) - (272*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5
625*d) - (6*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(625*
d) + (16*b^3*e^4*(c + d*x)*ArcCosh[c + d*x])/(25*d) + (8*b^2*e^4*(c + d*x)^
3*(a + b*ArcCosh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*ArcCosh[
c + d*x]))/(125*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/(25*d) - (4*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) - (3*b*e^4*Sqrt[-1 + c + d*x
]*(c + d*x)^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) + (e^4*(
c + d*x)^5*(a + b*ArcCosh[c + d*x])^3)/(5*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^4x^4 \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4 \text{Subst}\left(\int x^4 \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5 \left(a+b\cosh^{-1}(c+dx)\right)^3}{5d} - \frac{\left(3be^4\right) \text{Subst}\left(\int \frac{x^5\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}} dx, x\right.}{5d}$$

$$= -\frac{3be^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{25d} + \frac{e^4(c+d}{}$$

$$= \frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d} - \frac{4be^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c}}{25d}$$

$$= -\frac{6b^3e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{625d} + \frac{8b^2e^4(c+dx)^3\left(a+b\cosh^{-1}(c\right.}{75d}$$

$$= \frac{16}{25}ab^2e^4x - \frac{8b^3e^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{225d} - \frac{6b^3e^4\sqrt{-1+c+dx}(}{62}$$

$$= \frac{16}{25}ab^2e^4x - \frac{272b^3e^4\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5625d} - \frac{6b^3e^4\sqrt{-1+c+d}}{}$$

$$= \frac{16}{25}ab^2e^4x - \frac{32b^3e^4\sqrt{-1+c+dx}\sqrt{1+c+dx}}{45d} - \frac{272b^3e^4\sqrt{-1+c+dx}(c+d}{5625d}$$

$$= \frac{16}{25}ab^2e^4x - \frac{4144b^3e^4\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5625d} - \frac{272b^3e^4\sqrt{-1+c+dx}(c+}{5625d}$$

Mathematica [A] time = 0.586373, size = 404, normalized size = 1.06

$$\frac{e^4\left(3a\left(25a^2+6b^2\right)(c+dx)^5+\frac{1}{15}b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(-27\left(25a^2+2b^2\right)(c+dx)^4-4\left(225a^2+68b^2\right)(c+dx)^2-8\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) + 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8*(225*a^2 + 518*b^2)
```

```
- 4*(225*a^2 + 68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 -
 b*(-240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*
(c + d*x)^5 + 240*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 120*a*b*Sqrt[-
1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] + 90*a*b*Sqrt[-1 + c + d*x]*(c +
 d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 15*b^2*(-15*a*(c + d*x)^5 + 8
*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^
2*Sqrt[1 + c + d*x] + 3*b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])
*ArcCosh[c + d*x]^2 + 75*b^3*(c + d*x)^5*ArcCosh[c + d*x]^3))/(375*d)
```

Maple [A] time = 0.05, size = 602, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^3+e^4*b^3*(1/5*(d*x+c)^3*arccosh(d*x+c)^3*(d*x+c-1
)*(d*x+c+1)+1/5*arccosh(d*x+c)^3*(d*x+c-1)*(d*x+c+1)*(d*x+c)+1/5*arccosh(d*
x+c)^3*(d*x+c)-3/25*(d*x+c)^4*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)-8/25*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4/25*arccosh(d*x+
c)^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+6/125*arccosh(d*x+c)*(d*x+c-
1)*(d*x+c+1)*(d*x+c)^3+58/375*arccosh(d*x+c)*(d*x+c-1)*(d*x+c+1)*(d*x+c)+29
8/375*(d*x+c)*arccosh(d*x+c)-6/625*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)-4144/5625*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-272/5625*(d*x+c)^2*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2))+3*e^4*a*b^2*(1/5*(d*x+c)^3*arccosh(d*x+c)^2*(d*x+c-1)*
(d*x+c+1)+1/5*arccosh(d*x+c)^2*(d*x+c-1)*(d*x+c+1)*(d*x+c)+1/5*arccosh(d*x+
c)^2*(d*x+c)-2/25*(d*x+c)^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-
16/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-8/75*arccosh(d*x+c)*(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+2/125*(d*x+c-1)*(d*x+c+1)*(d*x+c)^3
+58/1125*(d*x+c-1)*(d*x+c+1)*(d*x+c)+298/1125*d*x+298/1125*c)+3*e^4*a^2*b*(
1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c
)^4+4*(d*x+c)^2+8)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.77614, size = 2325, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e^
4*x^4 + 150*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 + 450*(4*a*b^2
*c + (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 + 225*(8*a*b^2*c^2 + (25*a^3 + 6*a
*b^2)*c^4 + 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4*x^4
 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x + b^
3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 225*(15*a*b
^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3 + 150*a
*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3*b^3*d^4
*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 + 2*b^3)*d^2*e^4*x^2 + 4*(3*
b^3*c^3 + 2*b^3*c)*d*e^4*x + (3*b^3*c^4 + 4*b^3*c^2 + 8*b^3)*e^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^
2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*e^4*
x^4 + 10*(4*b^3 + 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 + 30*(4*b^3*c + 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 + 15*(8*b^3*c^2 + 3*(25*a^2*b + 2*b^3)*c
^4 + 16*b^3)*d*e^4*x + (40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 + 240*b^3*c)*
e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*(9*a*b^2*c^2 + 2
*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 + 2*a*b^2*c)*d*e^4*x + (3*a*b^2*c^4 +
4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (27*(25*a^2*b + 2*b^3)*d^4*e^4*x^4 +
 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 + 2*(450*a^2*b + 136*b^3 + 81*(25*a^2
*b + 2*b^3)*c^2)*d^2*e^4*x^2 + 4*(27*(25*a^2*b + 2*b^3)*c^3 + 2*(225*a^2*b
+ 68*b^3)*c)*d*e^4*x + (27*(25*a^2*b + 2*b^3)*c^4 + 1800*a^2*b + 4144*b^3 +
 4*(225*a^2*b + 68*b^3)*c^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 23.0477, size = 2518, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
*4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*acosh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*acosh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*acosh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*acosh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*acosh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25
- 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 3*a**2*b*d**4
*e**4*x**5*acosh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)/25 - 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 3*a*b*
*2*c**5*e**4*acosh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*acosh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*acosh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*acosh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*aco
sh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*c**2*
e**4*x/25 - 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*acosh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*c*d*e**4*x**2/25 - 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*acosh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*d*
*2*e**4*x**3/75 - 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
*acosh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*acosh(c + d*x)/(25*d) + b**3*c**5*e**4*acosh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*acosh(c + d*x)/(125*d) + b**3*c**4*e**4*x*acosh(
c + d*x)**3 + 6*b**3*c**4*e**4*x*acosh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*aco
sh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*acosh(c + d*x)/25 - 12*b**3*c**3*
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/625 + 8*b**3*c**3*e**4*aco
sh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*acosh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*acosh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
```

```
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/625 + 8*b**3*c**2*e**4*x*acosh(c + d*
x)/25 - 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x
)**2/(25*d) - 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*acosh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*aco
sh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)*acosh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/625 + 8*b**3*c*d*e**4*x**2*acosh(c + d*x)/25 - 8*b**3*c*e**4*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/5625 + 16*b**3*c*e**4*acosh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*acosh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*acosh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1)*acosh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/625 + 8*b**3*d**2*e**4*x**3*acosh(c + d*x)/75 - 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/5625 + 16*b**3*e**4*x*acos
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(c))**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^3, x)
```

3.114 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=307

$$\frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d}-3$$

```
[Out] (-45*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(256*d) - (3*b
^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(128*d) - (45*b^3*
e^3*ArcCosh[c + d*x])/(256*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d
*x]))/(32*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(32*d) - (9
*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*
x])^2)/(32*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^3)/(32*
d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^3)/(4*d)
```

Rubi [A] time = 0.6246, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5676, 90, 52, 100}

$$\frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d}-3$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```


```
[Out] (-45*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(256*d) - (3*b
^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(128*d) - (45*b^3*
e^3*ArcCosh[c + d*x])/(256*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d
*x]))/(32*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(32*d) - (9
*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*
x])^2)/(32*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^3)/(32*
d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^3)/(4*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}} dx, x\right.}{4d}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d} + \frac{e^3(c+d}{}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d} - \frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+}}{32d}$$

$$= -\frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{128d} + \frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c\right.}{32d}$$

$$= -\frac{9b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{64d} - \frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1}}{128d}$$

$$= -\frac{45b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{256d} - \frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1}}{128d}$$

$$= -\frac{45b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{256d} - \frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1}}{128d}$$

Mathematica [A] time = 0.525829, size = 359, normalized size = 1.17

$$\frac{e^3\left(8a\left(8a^2+3b^2\right)(c+dx)^4+3b\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\left(-2\left(8a^2+b^2\right)(c+dx)^2-3\left(8a^2+5b^2\right)\right)-9b\left(8a^2+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^3*(72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*Sqrt[-1
+ c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2
)*(c + d*x)^2) - 24*b*(c + d*x)*(-3*b^2*(c + d*x) - 8*a^2*(c + d*x)^3 - b^2
*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*a*b*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 24*b^2*(-3*a +
```

```
8*a*(c + d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*
Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 8*b^
3*(-3 + 8*(c + d*x)^4)*ArcCosh[c + d*x]^3 - 9*b*(8*a^2 + 5*b^2)*Log[c + d*x
 + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(256*d)
```

Maple [B] time = 0.044, size = 1554, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] -9/16*d*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-3/8*
d^2*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-9/32/d*e^3
*a^2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c
)^2-1)^(1/2))-3/8/d*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)*c^3-9/16/d*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-9/1
6*d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*a^2*b*c*e^3-9/8*e^3*a*b^2*arccosh(d
*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-9/8*d*e^3*a*b^2*arccosh(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-45/256*b^3*e^3*arccosh(d*x+c)/d+9/32*
d*e^3*b^3*arccosh(d*x+c)*x^2+3/8*e^3*b^3*arccosh(d*x+c)*x*c^3+9/16*e^3*b^3*
arccosh(d*x+c)*x*c-45/256*e^3*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+9/32*d*
e^3*a*b^2*x^2+3/32*d^3*e^3*a*b^2*x^4+3/2*d*x^2*a^3*c^2*e^3+d^2*x^3*a^3*c*e^
3+e^3*b^3*arccosh(d*x+c)^3*x*c^3+1/4/d*e^3*b^3*arccosh(d*x+c)^3*c^4+3/32/d*
e^3*b^3*arccosh(d*x+c)*c^4+9/32/d*e^3*b^3*arccosh(d*x+c)*c^2+9/16*e^3*a*b^2
*x*c+3/8*e^3*a*b^2*x*c^3+1/4*d^3*e^3*b^3*arccosh(d*x+c)^3*x^4+3/32*d^3*e^3*
b^3*arccosh(d*x+c)*x^4+9/32/d*e^3*a*b^2*c^2+3/32/d*e^3*a*b^2*c^4+d^2*e^3*b^
3*arccosh(d*x+c)^3*x^3*c+3/4/d*e^3*a*b^2*arccosh(d*x+c)^2*c^4-3/128/d*e^3*b
^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-45/256/d*e^3*b^3*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)*c-3/128*d^2*e^3*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3+3/4*d^
3*e^3*a*b^2*arccosh(d*x+c)^2*x^4+3/8*d^2*e^3*b^3*arccosh(d*x+c)*x^3*c+9/16*
d*e^3*a*b^2*x^2*c^2-9/32*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*x-9/32*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a^2*b*e^3+3*arccosh(d*x+c)*
x*a^2*b*c^3*e^3-9/128*e^3*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2+3/4/d*a
rccosh(d*x+c)*a^2*b*c^4*e^3+9/16*d*e^3*b^3*arccosh(d*x+c)*x^2*c^2+3/8*d^2*e
^3*a*b^2*x^3*c+3/2*d*e^3*b^3*arccosh(d*x+c)^3*x^2*c^2+1/4/d*a^3*c^4*e^3+x*a
^3*c^3*e^3+1/4*d^3*x^4*a^3*e^3-3/32/d*e^3*b^3*arccosh(d*x+c)^3-9/32/d*e^3*a
*b^2*arccosh(d*x+c)^2+3/4*d^3*arccosh(d*x+c)*x^4*a^2*b*e^3+3*e^3*a*b^2*arcc
osh(d*x+c)^2*x*c^3-9/16*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*x*c^2-3/16*d^2*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x^3-3/16*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*a^2*b*e^3-3/16/d*e^3*b^
```

```
3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-9/32/d*e^3*b^3*arcco
sh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/16/d*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)*a^2*b*c^3*e^3-9/32/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^2*b*c*e^3+
3*d^2*arccosh(d*x+c)*x^3*a^2*b*c*e^3+9/2*d*arccosh(d*x+c)*x^2*a^2*b*c^2*e^3
+3*d^2*e^3*a*b^2*arccosh(d*x+c)^2*x^3*c+9/2*d*e^3*a*b^2*arccosh(d*x+c)^2*x^
2*c^2-9/16*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-9/128
*d*e^3*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-9/16*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)*x*a^2*b*c^2*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66008, size = 1773, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^3
 + 24*(3*a*b^2 + 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 + 16*(9*a*b^2*c + 2*(
8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*x^3
 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^3)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 24*(8*a*b^2*d^4*e^3*x^4
 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d*e^3*x
 + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^3*x^2 +
 3*(2*b^3*c^2 + b^3)*d*e^3*x + (2*b^3*c^3 + 3*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*(8
*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 + 24*(b^3 +
 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 + 16*(3*b^3*c + 2*(8*a^2*b + b^3)*c^3)*
d*e^3*x + (24*b^3*c^2 + 8*(8*a^2*b + b^3)*c^4 - 24*a^2*b - 15*b^3)*e^3 - 16
*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2 + a*b^2)*d*e
```

```
^3*x + (2*a*b^2*c^3 + 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*(2*(8*a^2*b + b^3)*d^3*e
^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 + 3*(8*a^2*b + 5*b^3 + 2*(8*a^2*b
+ b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 + 3*(8*a^2*b + 5*b^3)*c)*e^3)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 13.486, size = 1828, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*acosh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*acosh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*acosh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*acosh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)/16 - 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*acosh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)/32 - 9*a**2*b*e**3*acosh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*acosh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*acosh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
*acosh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*acosh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)/8 + 9*a*b**2*c*e**3*x/16 - 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*acosh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/8 + 9*a*b**2*d
*e**3*x**2/32 - 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(
c + d*x)/16 - 9*a*b**2*e**3*acosh(c + d*x)**2/(32*d) + b**3*c**4*e**3*acosh
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*acosh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*acosh(c + d*x)**3 + 3*b**3*c**3*e**3*x*acosh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*acosh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*acosh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 -
```

```
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*c**2*
e**3*acosh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*acosh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*acosh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*c*e**3*x*acosh(c + d*x)/16 - 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(32*d) - 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(256*d) + b**3*d**3*e**3*
x**4*acosh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*acosh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*d*
e**3*x**2*acosh(c + d*x)/32 - 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)**2/32 - 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/256 - 3*b**3*e**3*acosh(c + d*x)**3/(32*d) - 45*b**3*e**3*acosh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*acosh(c))**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^3, x)
```

3.115 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=262

$$\frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d}+\frac{4}{3}ab^2e^2x-\frac{be^2\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{}}{}$$

```
[Out] (4*a*b^2*e^2*x)/3 - (40*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(27*d
) - (2*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(27*d) + (
4*b^3*e^2*(c + d*x)*ArcCosh[c + d*x])/(3*d) + (2*b^2*e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.472206, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5718, 5654, 74, 100}

$$\frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d}+\frac{4}{3}ab^2e^2x-\frac{be^2\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```


```
[Out] (4*a*b^2*e^2*x)/3 - (40*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(27*d
) - (2*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(27*d) + (
4*b^3*e^2*(c + d*x)*ArcCosh[c + d*x])/(3*d) + (2*b^2*e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x])^3)/(3*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^3}{3d}-\frac{\left(be^2\right)\text{Subst}\left(\int\frac{x^3\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c$$

$$=-\frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}+\frac{e^2(c+dx)^3}{}$$

$$=\frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{2be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\right.}{3d}$$

$$=\frac{4}{3}ab^2e^2x-\frac{2b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{27d}+\frac{2b^2e^2(c+dx)^3\left(a+b\,c\right.}{9d}$$

$$=\frac{4}{3}ab^2e^2x-\frac{2b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{27d}+\frac{4b^3e^2(c+dx)\cosh^{-1}(c}{3d}$$

$$=\frac{4}{3}ab^2e^2x-\frac{40b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{27d}-\frac{2b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1}}{27d}$$

Mathematica [A] time = 0.392803, size = 296, normalized size = 1.13

$$\frac{e^2\left(a\left(3a^2+2b^2\right)(c+dx)^3+\frac{1}{3}b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(-\left(9a^2+2b^2\right)(c+dx)^2-2\left(9a^2+20b^2\right)\right)-b\cosh^{-1}(c+dx)\left(-9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^2*(12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x]*(-2*(9*a^2 + 20*b^2) - (9*a^2 + 2*b^2)*(c + d*x)^2)
)/3 - b*(-12*b^2*(c + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 + 12*a*b
*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*a*b*Sqrt[-1 + c + d*x]*(c + d*x)^
2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-3*a*(c + d*x)^3 + 2*b*Sqrt[
-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 +
 c + d*x])*ArcCosh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcCosh[c + d*x]^3))/(9*d
)
```

Maple [A] time = 0.043, size = 396, normalized size = 1.5

$$\frac{1}{d}\left(\frac{(dx+c)^3 e^2a^3}{3}+e^2b^3\left(\frac{(\operatorname{arccosh}(dx+c))^3(dx+c-1)(dx+c+1)(dx+c)}{3}+\frac{(\operatorname{arccosh}(dx+c))^3(dx+c)}{3}-\frac{(\operatorname{arcco}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^3+e^2*b^3*(1/3*arccosh(d*x+c)^3*(d*x+c-1)*(d*x+c+1
)*(d*x+c)+1/3*arccosh(d*x+c)^3*(d*x+c)-1/3*arccosh(d*x+c)^2*(d*x+c)^2*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)-2/3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)+2/9*arccosh(d*x+c)*(d*x+c-1)*(d*x+c+1)*(d*x+c)+14/9*(d*x+c)*arccosh(d*
x+c)-2/27*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-40/27*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2))+3*e^2*a*b^2*(1/3*arccosh(d*x+c)^2*(d*x+c-1)*(d*x+c+1)*(d*x+
c)+1/3*arccosh(d*x+c)^2*(d*x+c)-2/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)*(d*x+c)^2-4/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2/27*(
d*x+c-1)*(d*x+c+1)*(d*x+c)+14/27*d*x+14/27*c)+3*e^2*a^2*b*(1/3*arccosh(d*x+
c)*(d*x+c)^3-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.54766, size = 1285, normalized size = 4.9

$$\frac{3\left(3\,a^3+2\,ab^2\right)d^3e^2x^3+9\left(3\,a^3+2\,ab^2\right)cd^2e^2x^2+9\left(4\,ab^2+\left(3\,a^3+2\,ab^2\right)c^2\right)de^2x+9\left(b^3d^3e^2x^3+3\,b^3cd^2e^2x^2+3\,b^3c^2de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2 +
 9*(4*a*b^2 + (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3*c
*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^2 +
9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x
+ (b^3*c^2 + 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x^3 + 3*
(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 + 3*(4*b^3 + (9*a^2*b + 2*b^3)*c^2)*d*e^2*x
 + (12*b^3*c + (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 + 2*a*b^2*
c*d*e^2*x + (a*b^2*c^2 + 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - ((9*a^2*b + 2*b^3)*d^2*e^
2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x + (18*a^2*b + 40*b^3 + (9*a^2*b + 2*b
^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 6.59907, size = 1173, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*acosh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*acosh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(3*d) + 3*a**2*b*c*d*e
```

```
**2*x**2*acosh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/3 + a**2*b*d**2*e**2*x**3*acosh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)/3 - 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)/(3*d) + a*b**2*c**3*e**2*acosh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*acosh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
acosh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*acosh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/3 + 4*a*b**2*e**2*x/3 - 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + b**3*c**3*e**
2*acosh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*acosh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*acosh(c + d*x)**3 + 2*b**3*c**2*e**2*x*acosh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + b**3*c*d*e**2*x*
*2*acosh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*acosh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 4*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27 + 4*b**3*c*e**2*acosh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*acosh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*aco
sh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27
 + 4*b**3*e**2*x*acosh(c + d*x)/3 - 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)*acosh(c + d*x)**2/(3*d) - 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acosh(c))**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^3, x)
```

3.116 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=175

$$\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{3be\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\,\mathrm{co}\right.}{2d}$$

```
[Out] (-3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(8*d) - (3*b^3*e*
ArcCosh[c + d*x])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(4
*d) - (3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x])^2)/(4*d) - (e*(a + b*ArcCosh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*
(a + b*ArcCosh[c + d*x])^3)/(2*d)
```

Rubi [A] time = 0.358665, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5866, 12, 5662, 5759, 5676, 90, 52}

$$\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{3be\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\,\mathrm{co}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(8*d) - (3*b^3*e*
ArcCosh[c + d*x])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(4
*d) - (3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x])^2)/(4*d) - (e*(a + b*ArcCosh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*
(a + b*ArcCosh[c + d*x])^3)/(2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
 - d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c+\right.}{2d}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d} + \frac{e(c+dx)^2\left(a\right.}{}$$

$$= \frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d} - \frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(\right.}{4d}$$

$$= -\frac{3b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{8d} + \frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right.}{4d}$$

$$= -\frac{3b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{8d} - \frac{3b^3e\cosh^{-1}(c+dx)}{8d} + \frac{3b^2e(c+dx)}{}$$

Mathematica [A] time = 0.301525, size = 244, normalized size = 1.39

$$\frac{e\left(2a\left(2a^2+3b^2\right)(c+dx)^2-3b\left(2a^2+b^2\right)\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}-3b\left(2a^2+b^2\right)\log\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*Sqrt[-1 + c + d*x]*
(c + d*x)*Sqrt[1 + c + d*x] - 6*b*(c + d*x)*(-2*a^2*(c + d*x) - b^2*(c + d*
x) + 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 6*b^2*(
-a + 2*a*(c + d*x)^2 - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*Ar
cCosh[c + d*x]^2 + 2*b^3*(-1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^3 - 3*b*(2*a
^2 + b^2)*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(8*d)
```

Maple [B] time = 0.036, size = 605, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] -3/2/d*e*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/4/d*e*a^2
*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-
1)^(1/2))-3/8*b^3*e*arccosh(d*x+c)/d+3/4*d*e*a*b^2*x^2+1/2/d*a^3*c^2*e+x*a^
3*c*e+1/2*d*x^2*a^3*e-1/4/d*e*b^3*arccosh(d*x+c)^3+3/4/d*e*a*b^2*c^2+3/2*e*
a*b^2*x*c+1/2*d*e*b^3*arccosh(d*x+c)^3*x^2+3/4*d*e*b^3*arccosh(d*x+c)*x^2+1
/2/d*e*b^3*arccosh(d*x+c)^3*c^2-3/8*e*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x
+3/4/d*e*b^3*arccosh(d*x+c)*c^2-3/4/d*e*a*b^2*arccosh(d*x+c)^2+3/2*e*b^3*ar
ccosh(d*x+c)*x*c+e*b^3*arccosh(d*x+c)^3*x*c+3/2*d*arccosh(d*x+c)*x^2*a^2*b*
e+3/2*d*e*a*b^2*arccosh(d*x+c)^2*x^2-3/8/d*e*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*c-3/4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a^2*b*e+3/2/d*e*a*b^2*arccosh
(d*x+c)^2*c^2+3/2/d*arccosh(d*x+c)*a^2*b*c^2*e+3*e*a*b^2*arccosh(d*x+c)^2*x
*c-3/4*e*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+3*arccosh(d
*x+c)*x*a^2*b*c*e-3/4/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^2*b*c*e-3/2*e*a*b
^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/4/d*e*b^3*arccosh(d*x
+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.51384, size = 891, normalized size = 5.09

$$\frac{2\left(2\,a^{3}+3\,ab^{2}\right)d^{2}ex^{2}+4\left(2\,a^{3}+3\,ab^{2}\right)cdex+2\left(2\,b^{3}d^{2}ex^{2}+4\,b^{3}cdex+\left(2\,b^{3}c^{2}-b^{3}\right)e\right)\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^3
*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 - b^3)*e)*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*a*b
^2*c^2 - a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*(2*(2*a^2*b + b^3)
*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x - (2*a^2*b + b^3 - 2*(2*a^2*b + b^3)
*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*((2*a^2*b + b^3)*d*e*x +
 (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 2.93132, size = 685, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*acosh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*acosh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)/(4*d) + 3*a**2*b*d*e*x**2*acosh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 - 3*a**2*b*e*acosh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*acosh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*acosh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*acosh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/2 - 3*a*
b**2*e*acosh(c + d*x)**2/(4*d) + b**3*c**2*e*acosh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*acosh(c + d*x)/(4*d) + b**3*c*e*x*acosh(c + d*x)**3 + 3*b**3*c*e*
x*acosh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(8*d) +
 b**3*d*e*x**2*acosh(c + d*x)**3/2 + 3*b**3*d*e*x**2*acosh(c + d*x)/4 - 3*b
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - b**3*e*acosh(c + d*x)**3/(4*d)
- 3*b**3*e*acosh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))**3, Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b\operatorname{arcosh}(dx + c) + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

3.117 $\int \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=114

$$6ab^2x-\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{6b^3\sqrt{c+dx-1}\sqrt{c+dx}}{d}$$

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (6*b^3*(c + d*
x)*ArcCosh[c + d*x])/d - (3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^3)/d
```

Rubi [A] time = 0.184149, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5864, 5654, 5718, 74}

$$6ab^2x-\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{6b^3\sqrt{c+dx-1}\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (6*b^3*(c + d*
x)*ArcCosh[c + d*x])/d - (3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^3)/d
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx\right)}{d}$$

$$=6ab^2x-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}\right.}{d}$$

$$=6ab^2x+\frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+d\right.}{d}$$

$$=6ab^2x-\frac{6b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d}+\frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{3b\sqrt{-1+c+dx}}{}$$

Mathematica [A] time = 0.173148, size = 168, normalized size = 1.47

$$\frac{a\left(a^2+6b^2\right)(c+dx)-3b\left(a^2+2b^2\right)\sqrt{c+dx-1}\sqrt{c+dx+1}-3b\cosh^{-1}(c+dx)\left(a^2(-(c+dx))+2ab\sqrt{c+dx-1}\sqrt{c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x] - 3*b*(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + b^3*(c + d*x)*ArcCosh[c
+ d*x]^3)/d
```

Maple [A] time = 0.056, size = 180, normalized size = 1.6

$$\frac{1}{d}\left(a^3(dx+c)+b^3\left((\operatorname{arccosh}(dx+c))^3(dx+c)-3(\operatorname{arccosh}(dx+c))^2\sqrt{dx+c-1}\sqrt{dx+c+1}+6(dx+c)\operatorname{arccosh}(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+b^3*(arccosh(d*x+c)^3*(d*x+c)-3*arccosh(d*x+c)^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))+3*a*b^2*(arccosh(d*x+c)^2*(d*x+c)-2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)+2*d*x+2*c)+3*a^2*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42965, size = 548, normalized size = 4.81

$$\frac{\left(b^3dx+b^3c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^3+\left(a^3+6\,ab^2\right)dx+3\left(ab^2dx+ab^2c-\sqrt{d^2x^2+2\,cdx+c^2-1}b^3\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - 3*(2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*(a^2*b + 2*b^3))/d
```

Sympy [A] time = 1.16201, size = 282, normalized size = 2.47

$$\begin{cases} a^3x + \frac{3a^2bc\operatorname{acosh}(c+dx)}{d} + 3a^2bx\operatorname{acosh}(c+dx) - \frac{3a^2b\sqrt{c^2+2cdx+d^2x^2-1}}{d} + \frac{3ab^2c\operatorname{acosh}^2(c+dx)}{d} + 3ab^2x\operatorname{acosh}^2(c+dx) + 6ab^2x \\ x(a+b\operatorname{acosh}(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*acosh(c + d*x)/d + 3*a**2*b*x*acosh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + 3*a*b**2*c*acosh(c + d
*x)**2/d + 3*a*b**2*x*acosh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + b**3*c*acosh(c + d*x)**3/d + 6
*b**3*c*acosh(c + d*x)/d + b**3*x*acosh(c + d*x)**3 + 6*b**3*x*acosh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d, Ne(d, 0)), (x*(a + b*acosh(c))**
3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3, x)
```

3.118 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{ce+dex}\,dx$

Optimal. Leaf size=159

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{2de}-\frac{3b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de}-\cdots$$

```
[Out] (a + b*ArcCosh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcCosh[c + d*x])^3*Log[1 +
E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b*(a + b*ArcCosh[c + d*x])^2*PolyLog[2
, -E^(-2*ArcCosh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])*Poly
Log[3, -E^(-2*ArcCosh[c + d*x])])/(2*d*e) - (3*b^3*PolyLog[4, -E^(-2*ArcCos
h[c + d*x])])/(4*d*e)
```

Rubi [A] time = 0.216049, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{2de}+\frac{3b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de}+\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcCosh[c + d*x])^3*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e) + (3*b*(a + b*ArcCosh[c + d*x])^2*PolyLog[2
, -E^(2*ArcCosh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])*PolyL
og[3, -E^(2*ArcCosh[c + d*x])])/(2*d*e) + (3*b^3*PolyLog[4, -E^(2*ArcCosh[c
 + d*x])])/(4*d*e)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{ex}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x}\,dx, x, c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^3\tanh(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} - \frac{(3b)\,\text{Subs}}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+b\right.}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+b\right.}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+b\right.}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+b\right.}{}$$

Mathematica [A] time = 0.531425, size = 217, normalized size = 1.36

$$\frac{-6b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right) - 6b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2 - 3b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (6*a^2*b*ArcCosh[c + d*x]^2 + 4*a*b^2*ArcCosh[c + d*x]^3 + b^3*ArcCosh[c +
d*x]^4 + 12*a^2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + 12*a*
b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*b^3*ArcCosh[c +
 d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*a^3*Log[c + d*x] - 6*b*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] - 6*b^2*(a + b*Arc
Cosh[c + d*x])*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*b^3*PolyLog[4, -E^(
-2*ArcCosh[c + d*x])])/(4*d*e)
```

Maple [B] time = 0.035, size = 471, normalized size = 3.

$$\frac{a^3\ln(dx+c)}{de} - \frac{b^3(\operatorname{arccosh}(dx+c))^4}{4de} + \frac{b^3(\operatorname{arccosh}(dx+c))^3}{de}\ln\left(\left(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1}\right)^2+1\right) + \frac{3b^3(\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^3/e*ln(d*x+c)-1/4/d*b^3/e*arccosh(d*x+c)^4+1/d*b^3/e*arccosh(d*x+c)^3
*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+3/2/d*b^3/e*arccosh(d*x+c)
^2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2/d*b^3/e*arccos
h(d*x+c)*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/4/d*b^3/e*
polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a*b^2/e*arccosh(d
*x+c)^3+3/d*a*b^2/e*arccosh(d*x+c)^2*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2))^2+1)+3/d*a*b^2/e*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))^2)-3/2/d*a*b^2/e*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2))^2)-3/2/d*a^2*b/e*arccosh(d*x+c)^2+3/d*a^2*b/e*arccosh(d*x+c)*ln((d*x+
c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+3/2/d*a^2*b/e*polylog(2,-(d*x+c+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{arcosh}\,(dx+c)^3+3\,ab^2\,\text{arcosh}\,(dx+c)^2+3\,a^2b\,\text{arcosh}\,(dx+c)+a^3}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c+dx}\,dx+\int \frac{b^3\,\text{acosh}^3\,(c+dx)}{c+dx}\,dx+\int \frac{3ab^2\,\text{acosh}^2\,(c+dx)}{c+dx}\,dx+\int \frac{3a^2b\,\text{acosh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*acosh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*acosh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
cosh(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

3.119 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{6ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+\frac{6ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+\frac{6ib^3}{}$$

```
[Out] -((a + b*ArcCosh[c + d*x])^3/(d*e^2*(c + d*x))) + (6*b*(a + b*ArcCosh[c + d
*x])^2*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^2*(a + b*ArcCosh[c +
d*x])*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^2*(a + b*ArcC
osh[c + d*x])*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^3*PolyLo
g[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^3*PolyLog[3, I*E^ArcCosh[
c + d*x]])/(d*e^2)
```

Rubi [A] time = 0.36201, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5761, 4180, 2531, 2282, 6589}

$$-\frac{6ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+\frac{6ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+\frac{6ib^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```


```
[Out] -((a + b*ArcCosh[c + d*x])^3/(d*e^2*(c + d*x))) + (6*b*(a + b*ArcCosh[c + d
*x])^2*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^2*(a + b*ArcCosh[c +
d*x])*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^2*(a + b*ArcC
osh[c + d*x])*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^3*PolyLo
g[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^3*PolyLog[3, I*E^ArcCosh[
c + d*x]])/(d*e^2)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^2x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^2}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}x\sqrt{1+x}}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\,\text{sech}(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{\left(6ib^2\right)\text{Su}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{6ib^2\left(a+\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{6ib^2\left(a+\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{6ib^2\left(a+\right.}{}$$

Mathematica [A] time = 1.09634, size = 327, normalized size = 1.76

$$-\frac{3iab^2\left(2\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right)-2\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)+\cosh^{-1}(c+dx)\left(-\frac{i\cosh^{-1}(c+dx)}{c+dx}+2\log\left(1-ie^{-\text{co}}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a^3/(c + d*x) + (3*a^2*b*ArcCosh[c + d*x])/(c + d*x) + 3*a^2*b*ArcTan[1/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])] + (3*I)*a*b^2*(ArcCosh[c + d*x]*(((
-I)*ArcCosh[c + d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1
 + I/E^ArcCosh[c + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyL
og[2, I/E^ArcCosh[c + d*x]]) + b^3*(ArcCosh[c + d*x]^3/(c + d*x) - (3*I)*(-
(ArcCosh[c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c
+ d*x]])) - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyL
og[2, I/E^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*Po
lyLog[3, I/E^ArcCosh[c + d*x]])))/(d*e^2))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{arccosh}(dx+c))^3}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{arcosh}\,(dx+c)^3+3\,ab^2\,\mathrm{arcosh}\,(dx+c)^2+3\,a^2b\,\mathrm{arcosh}\,(dx+c)+a^3}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*acosh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*acosh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.120 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=164

$$\frac{3b^3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)}{2de^3}-\frac{3b^2\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\,(a}{2de^3(c+d}$$

```
[Out] (-3*b*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3) + (3*b*Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcCo
sh[c + d*x])^3/(2*d*e^3*(c + d*x)^2) - (3*b^2*(a + b*ArcCosh[c + d*x])*Log[
1 + E^(-2*ArcCosh[c + d*x])])/(d*e^3) + (3*b^3*PolyLog[2, -E^(-2*ArcCosh[c
+ d*x])])/(2*d*e^3)
```

Rubi [A] time = 0.369211, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5662, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{3b^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)}{2de^3}-\frac{3b^2\log\left(e^{2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\,(a}{2de^3(c+dx}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (3*b*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3) + (3*b*Sqrt[-1 + c + d*x]*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcCos
h[c + d*x])^3/(2*d*e^3*(c + d*x)^2) - (3*b^2*(a + b*ArcCosh[c + d*x])*Log[1
 + E^(2*ArcCosh[c + d*x])])/(d*e^3) - (3*b^3*PolyLog[2, -E^(2*ArcCosh[c + d
*x])])/(2*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx, x, c+dx\right)}{2de^3}$$

$$= \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} - \frac{(3b^2 \cdots}{\cdots}$$

$$= \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} - \frac{(3b^2 \cdots}{\cdots}$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \cdots$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \cdots$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \cdots$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \cdots$$

Mathematica [A] time = 1.04348, size = 266, normalized size = 1.62

$$3b^3\left(\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)+\cosh^{-1}(c+dx)\left(\frac{\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)\cosh^{-1}(c+dx)}{c+dx}-\cosh^{-1}(c+dx)-2\log\left(e^{-2\cosh^{-1}(c+dx)}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^3/(c + d*x)^2) + (3*a^2*b*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^2
 + 2*c*d*x + d*x*(1 + d*x)) - ArcCosh[c + d*x]))/(c + d*x)^2 - (b^3*ArcCosh
[c + d*x]^3)/(c + d*x)^2 + 6*a*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*ArcCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2)
 - Log[c + d*x]) + 3*b^3*(ArcCosh[c + d*x]*(-ArcCosh[c + d*x] + (Sqrt[(-1 +
 c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) - 2*Log[
1 + E^(-2*ArcCosh[c + d*x])]) + PolyLog[2, -E^(-2*ArcCosh[c + d*x])]))/(2*d
*e^3)
```

Maple [B] time = 0.067, size = 375, normalized size = 2.3

$$-\frac{a^3}{2\,de^3\,(dx+c)^2}+\frac{3\,b^3\,(\operatorname{arccosh}(dx+c))^2}{2\,de^3\,(dx+c)}\sqrt{dx+c-1}\sqrt{dx+c+1}+\frac{3\,b^3\,(\operatorname{arccosh}(dx+c))^2}{2\,de^3}-\frac{b^3\,(\operatorname{arccosh}(dx+c))^3}{2\,de^3\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^3/e^3/(d*x+c)^2+3/2/d*b^3/e^3*arccosh(d*x+c)^2/(d*x+c)*(d*x+c+1)^(
1/2)*(d*x+c-1)^(1/2)+3/2/d*b^3/e^3*arccosh(d*x+c)^2-1/2/d*b^3/e^3*arccosh(d
*x+c)^3/(d*x+c)^2-3/d*b^3/e^3*arccosh(d*x+c)*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))^2+1)-3/2/d*b^3/e^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))^2)+3/d*a*b^2/e^3*arccosh(d*x+c)+3/d*a*b^2/e^3*arccosh(d*x+c)/(d*x+c
)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)-3/2/d*a*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)^
2-3/d*a*b^2/e^3*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)-3/2/d*a^2*b
/e^3/(d*x+c)^2*arccosh(d*x+c)+3/2/d*a^2*b/e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arcosh}(dx+c)^3 + 3\,ab^2 \operatorname{arcosh}(dx+c)^2 + 3\,a^2 b \operatorname{arcosh}(dx+c) + a^3}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3 \operatorname{acosh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2 \operatorname{acosh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b \operatorname{acosh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

3.121 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=297

$$-\frac{ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^3\text{Po}}{}$$

```
[Out] (b^2*(a + b*ArcCosh[c + d*x]))/(d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*
ArcCosh[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcCosh[c + d*x])^2*A
rcTan[E^ArcCosh[c + d*x]])/(d*e^4) - (b^3*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(d*e^4) - (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, (-I)*E^Ar
cCosh[c + d*x]])/(d*e^4) + (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, I*E^A
rcCosh[c + d*x]])/(d*e^4) + (I*b^3*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*
e^4) - (I*b^3*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rubi [A] time = 0.593879, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5662, 5748, 5761, 4180, 2531, 2282, 6589, 92, 203}

$$-\frac{ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^3\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```


```
[Out] (b^2*(a + b*ArcCosh[c + d*x]))/(d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*
ArcCosh[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcCosh[c + d*x])^2*A
rcTan[E^ArcCosh[c + d*x]])/(d*e^4) - (b^3*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(d*e^4) - (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, (-I)*E^Ar
cCosh[c + d*x]])/(d*e^4) + (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, I*E^A
rcCosh[c + d*x]])/(d*e^4) + (I*b^3*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*
e^4) - (I*b^3*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^4)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{de^4}$$

$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subs}}{}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{(a+}{}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{(a+}{}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{(a+}{}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{(a+}{}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{(a+}{}$$

Mathematica [A] time = 2.16661, size = 504, normalized size = 1.7

$$\frac{6ab^2\left(-i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right)+i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)+\frac{1}{c+dx}-\frac{\cosh^{-1}(c+dx)^2}{(c+dx)^3}+\frac{\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)\cosh^{-1}(c+dx)}{(c+dx)^2}-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*a^3)/(c + d*x)^3 + (3*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c +
 d*x)^2 - (6*a^2*b*ArcCosh[c + d*x])/(c + d*x)^3 - 3*a^2*b*ArcTan[1/(Sqrt[-
```

```
1 + c + d*x]*Sqrt[1 + c + d*x])] + 6*a*b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c
+ d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh
[c + d*x]^2/(c + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]]
+ I*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^Ar
cCosh[c + d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + b^3*((6*ArcCosh[c +
 d*x])/(c + d*x) + (3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcC
osh[c + d*x]^2)/(c + d*x)^2 - (2*ArcCosh[c + d*x]^3)/(c + d*x)^3 - (3*I)*((
-4*I)*ArcTan[Tanh[ArcCosh[c + d*x]/2]] + ArcCosh[c + d*x]^2*Log[1 - I/E^Arc
Cosh[c + d*x]] - ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + 2*ArcCo
sh[c + d*x]*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*ArcCosh[c + d*x]*PolyLo
g[2, I/E^ArcCosh[c + d*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - 2*Poly
Log[3, I/E^ArcCosh[c + d*x]])))/(6*d*e^4)
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{3}}{\left(dex+ce\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^4*e^4*x^3
+ 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 - c)*a*b
^2 + (c^3 - c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c
*d^2)*x^2 + (b^3*c^2 + 3*(c^2 - 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*
(3*a*b^2*c*d + b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*
d - d)*a*b^2 + (3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
```

```
 c - 1) + c)^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d - d)*a^2*b
*x + (c^3 - c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 - 1)*a^2*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^
5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4
 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^
6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 +
4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 +
 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*
c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arcosh}(dx+c)^3 + 3\,ab^2 \operatorname{arcosh}(dx+c)^2 + 3\,a^2b \operatorname{arcosh}(dx+c) + a^3}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4
*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3 \operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2 \operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3a^2b \operatorname{acosh}(c+\cdots}{c^4+4c^3dx+6c^2d^2x^2+4c\cdots}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*acosh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
```

```
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

3.122 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=377

$$-\frac{3b^3e^3\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{32d}-\frac{45b^3e^3\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c\right.}{64d}$$

```
[Out] (45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) - (45*b^
3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x
]))/(64*d) - (3*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a
 + b*ArcCosh[c + d*x]))/(32*d) - (45*b^2*e^3*(a + b*ArcCosh[c + d*x])^2)/(1
28*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(16*d) + (3*b^2*
e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*b*e^3*Sqrt[-1 + c +
 d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(8*d) - (b*e^
3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])
^3)/(4*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^4)/(32*d) + (e^3*(c + d*x)^4*(a
 + b*ArcCosh[c + d*x])^4)/(4*d)
```

Rubi [A] time = 1.17449, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$-\frac{3b^3e^3\sqrt{c+dx-1}(c+dx)^3\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{32d}-\frac{45b^3e^3\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```


```
[Out] (45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) - (45*b^
3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x
]))/(64*d) - (3*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a
 + b*ArcCosh[c + d*x]))/(32*d) - (45*b^2*e^3*(a + b*ArcCosh[c + d*x])^2)/(1
28*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(16*d) + (3*b^2*
e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*b*e^3*Sqrt[-1 + c +
 d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(8*d) - (b*e^
3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])
^3)/(4*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^4)/(32*d) + (e^3*(c + d*x)^4*(a
 + b*ArcCosh[c + d*x])^4)/(4*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^4}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c\right.}{d}$$

$$= -\frac{be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d} + \frac{e^3(c+dx)^4}{}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d} - \frac{3be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+}}{8d}$$

$$= -\frac{3b^3e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{32d} + \frac{9b^2e^3(c+}{}$$

$$= \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{64d}$$

$$= \frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^3e^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{64d}$$

Mathematica [A] time = 0.806236, size = 562, normalized size = 1.49

$$\frac{e^3\left(\left(24a^2b^2+32a^4+3b^4\right)(c+dx)^4+9b^2\left(8a^2+5b^2\right)(c+dx)^2+2ab\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\left(-2\left(8a^2+3b^2\right)(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^3*(9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
+ d*x)^4 + 2*a*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2
+ 15*b^2) - 2*(8*a^2 + 3*b^2)*(c + d*x)^2) + 2*b*(c + d*x)*(72*a*b^2*(c + d
*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 - 72*a^2*b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x] - 45*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 48*a^2
*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 6*b^3*Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-24*a^2 - 15*b
```

```
^2 + 24*b^2*(c + d*x)^2 + 64*a^2*(c + d*x)^4 + 8*b^2*(c + d*x)^4 - 48*a*b*S
qrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 32*a*b*Sqrt[-1 + c + d*x]*(
c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 16*b^3*(-3*a + 8*a*(c +
d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 +
 c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 4*b^4*(-3 + 8
*(c + d*x)^4)*ArcCosh[c + d*x]^4 - 6*a*b*(8*a^2 + 15*b^2)*Log[c + d*x + Sqr
t[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(128*d)
```

Maple [B] time = 0.05, size = 2465, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] -9/32*d*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-3/4*d*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*x^2*a^3*b*c*e^3-3/8/d*e^3*a^3*b*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))-3/4/d*e^3*a*b^3*a
rccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-9/32*d*e^3*b^4*arccosh(
d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-3/4*d*e^3*b^4*arccosh(d*x+c)^3
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-3/4/d*e^3*a^2*b^2*arccosh(d*x+c)*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-9/8/d*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*c-9/4*d*e^3*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*x^2*c-9/4*d*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)*x^2*c-9/32*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-3/32/
d*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-45/64/d*e^3*a*b^3*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*c-3/8/d*e^3*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)*c-3/32/d*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)*c^3-45/64/d*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-1/4/
d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^3*b*c^3*e^3-3/8/d*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)*a^3*b*c*e^3-1/4*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*a^3*b*e^
3+4*d^2*arccosh(d*x+c)*x^3*a^3*b*c*e^3+6*d*arccosh(d*x+c)*x^2*a^3*b*c^2*e^3
-3/32*d^2*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3+4*d^2*e^3*a*b^3*arc
cosh(d*x+c)^3*x^3*c+6*d*e^3*a*b^3*arccosh(d*x+c)^3*x^2*c^2-1/4*d^2*e^3*b^4*
arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3+3/2*d^2*e^3*a*b^3*arcc
osh(d*x+c)*x^3*c+9/4*d*e^3*a*b^3*arccosh(d*x+c)*x^2*c^2-3/4*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*x*a^3*b*c^2*e^3+45/128/d*e^3*b^4*c^2+3/128/d*e^3*b^4*c^4+1
/4/d*a^4*c^4*e^3+3/128*d^3*e^3*b^4*x^4+45/128*d*e^3*b^4*x^2+1/4*d^3*x^4*a^4
*e^3-45/128/d*e^3*b^4*arccosh(d*x+c)^2-3/32/d*e^3*b^4*arccosh(d*x+c)^4+45/6
4*e^3*b^4*x*c+3/32*e^3*b^4*x*c^3+x*a^4*c^3*e^3+3/16*d^3*e^3*a^2*b^2*x^4+9/1
6*d*e^3*a^2*b^2*x^2+d^2*x^3*a^4*c*e^3+3/32*d^2*e^3*b^4*x^3*c+9/64*d*e^3*b^4
```

```
*x^2*c^2+3/2*d*x^2*a^4*c^2*e^3+3/16*d^3*e^3*b^4*arccosh(d*x+c)^2*x^4+9/16*d
*e^3*b^4*arccosh(d*x+c)^2*x^2+1/4*d^3*e^3*b^4*arccosh(d*x+c)^4*x^4+e^3*b^4*
arccosh(d*x+c)^4*x*c^3+3/4*e^3*b^4*arccosh(d*x+c)^2*x*c^3+9/8*e^3*b^4*arcco
sh(d*x+c)^2*x*c-9/16/d*e^3*a^2*b^2*arccosh(d*x+c)^2-45/64/d*e^3*a*b^3*arcco
sh(d*x+c)-3/8/d*e^3*a*b^3*arccosh(d*x+c)^3+9/16/d*e^3*b^4*arccosh(d*x+c)^2*
c^2+1/4/d*e^3*b^4*arccosh(d*x+c)^4*c^4+3/4*e^3*a^2*b^2*x*c^3+9/8*e^3*a^2*b^
2*x*c+d^3*e^3*a*b^3*arccosh(d*x+c)^3*x^4+3/8*d^3*e^3*a*b^3*arccosh(d*x+c)*x
^4+9/8*d*e^3*a*b^3*arccosh(d*x+c)*x^2+3/2*d^3*e^3*a^2*b^2*arccosh(d*x+c)^2*
x^4-45/64*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+4*e^3*a*b^3*arccosh(d
*x+c)^3*x*c^3-3/8*e^3*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*
x+3/2*d*e^3*b^4*arccosh(d*x+c)^4*x^2*c^2+3/4*d^2*e^3*b^4*arccosh(d*x+c)^2*x
^3*c+9/8*d*e^3*b^4*arccosh(d*x+c)^2*x^2*c^2+3/2/d*e^3*a^2*b^2*arccosh(d*x+c
)^2*c^4+1/d*arccosh(d*x+c)*a^3*b*c^4*e^3+1/d*e^3*a*b^3*arccosh(d*x+c)^3*c^4
+d^2*e^3*b^4*arccosh(d*x+c)^4*x^3*c-3/8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a
^3*b*e^3+4*arccosh(d*x+c)*x*a^3*b*c^3*e^3+3/4*d^2*e^3*a^2*b^2*x^3*c+9/8*d*e
^3*a^2*b^2*x^2*c^2+3/2*e^3*a*b^3*arccosh(d*x+c)*x*c^3+9/4*e^3*a*b^3*arccosh
(d*x+c)*x*c-45/64*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+
6*e^3*a^2*b^2*arccosh(d*x+c)^2*x*c^3+3/8/d*e^3*a*b^3*arccosh(d*x+c)*c^4+9/8
/d*e^3*a*b^3*arccosh(d*x+c)*c^2+d^3*arccosh(d*x+c)*x^4*a^3*b*e^3+9/16/d*e^3
*a^2*b^2*c^2+3/16/d*e^3*a^2*b^2*c^4+3/16/d*e^3*b^4*arccosh(d*x+c)^2*c^4-9/3
2*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-3/4*e^3*b^4*
arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-3/32*d^2*e^3*b^4*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3+6*d^2*e^3*a^2*b^2*arccosh(d
*x+c)^2*x^3*c+9*d*e^3*a^2*b^2*arccosh(d*x+c)^2*x^2*c^2-1/4/d*e^3*b^4*arccos
h(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-9/8*e^3*a*b^3*arccosh(d*x+c)
^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-9/8*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*x-3/4*d^2*e^3*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*x^3-9/4*e^3*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)*x*c^2-9/4*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)*x*c^2-9/8/d*e^3*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
*c-3/4*d^2*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87925, size = 2631, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c*d^3*e^3*x^3 + 3*(24*a^2*b^2 + 15*b^4 + 2*(32*a^4 + 24*a^2*b^2 + 3
*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 + 9*(8*a^2*
b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 + 48*
b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4 + 32*
a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*x + (8*
a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2 + 3*(2*
b^4*c^2 + b^4)*d*e^3*x + (2*b^4*c^3 + 3*b^4*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 3*(8*(8*a^
2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 + 24*(b^4 + 2
*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 + 16*(3*b^4*c + 2*(8*a^2*b^2 + b^4)*c^3
)*d*e^3*x + (24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*a^2*b^2 - 15*b^4)*e^
3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*b^3*c^2 + a*b^
3)*d*e^3*x + (2*a*b^3*c^3 + 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 2*(8*(8*a^3*b + 3*
a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3 + 24*(3*a*b^3 + 2
*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 + 16*(9*a*b^3*c + 2*(8*a^3*b + 3*a*b^
3)*c^3)*d*e^3*x + (72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 - 24*a^3*b - 45
*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6*(8*a^2*b^2 + b^4)*c*d^
2*e^3*x^2 + 3*(8*a^2*b^2 + 5*b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d*e^3*x + (2*(8
*a^2*b^2 + b^4)*c^3 + 3*(8*a^2*b^2 + 5*b^4)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(2*(8*a^3*
b + 3*a*b^3)*d^3*e^3*x^3 + 6*(8*a^3*b + 3*a*b^3)*c*d^2*e^3*x^2 + 3*(8*a^3*b
 + 15*a*b^3 + 2*(8*a^3*b + 3*a*b^3)*c^2)*d*e^3*x + (2*(8*a^3*b + 3*a*b^3)*c
^3 + 3*(8*a^3*b + 15*a*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 27.3642, size = 2876, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**4,x)

[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*acosh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*acosh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*acosh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*ac
osh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/
4 - 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(8*d) + a**3*b*d**
3*e**3*x**4*acosh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)/4 - 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3
*a**3*b*e**3*acosh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*acosh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*acosh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2 + 9*a**2*b**2*c
**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*acosh(c + d*x)/4 + 9*a**2*b**2*c*e**3*x/8 - 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(8*d) + 3*a**2*
b**2*d**3*e**3*x**4*acosh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*
x)/4 + 9*a**2*b**2*d*e**3*x**2/16 - 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/8 - 9*a**2*b**2*e**3*acosh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*acosh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*acosh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*acosh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*ac
osh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*ac
osh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*acosh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*acosh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)/32 + 9*a*b**3*c**2*e**3*acosh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*acosh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*acosh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/32
 + 9*a*b**3*c*e**3*x*acosh(c + d*x)/4 - 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)**2/(8*d) - 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/(64*d) + a*b**3*d**3*e**3*x**4*acosh(c + d*x)**3 + 3
*a*b**3*d**3*e**3*x**4*acosh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/32 + 9*a*b**3*d*e**3*x**2*acosh(c + d*x
)/8 - 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**
2/8 - 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/64 - 3*a*b**3*e
**3*acosh(c + d*x)**3/(8*d) - 45*a*b**3*e**3*acosh(c + d*x)/(64*d) + b**4*c
```

```
**4*e**3*acosh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*acosh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*acosh(c + d*x)**4 + 3*b**4*c**3*e**3*x*acosh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*acosh(c + d*x)
**4/2 + 9*b**4*c**2*d*e**3*x**2*acosh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/32 + 9*b**4*c**2*e**3*acosh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*acosh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*acosh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*acosh(c + d*x)/32 + 9*b**4*c*e**3*x*acosh(c + d*x)**2/8 + 45*b**
4*c*e**3*x/64 - 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)**3/(8*d) - 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*acosh(c + d*x)**4/4 + 3*b**4*d**3*e*
*3*x**4*acosh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/32 + 9*b**4*d
*e**3*x**2*acosh(c + d*x)**2/16 + 45*b**4*d*e**3*x**2/128 - 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/8 - 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/64 - 3*b**4*e**3*acosh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*acosh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*acosh(c))**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^3(b\operatorname{arcosh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^4, x)
```

3.123 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=309

$$-\frac{8b^3e^2\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{27d}-\frac{160b^3e^2\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{27d}$$

```
[Out] (160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) - (160*b^3*e^2*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(27*d) - (8*b^3*e^2*
Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/
(27*d) + (8*b^2*e^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (4*b^2*e^
2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(9*d) - (8*b*e^2*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) - (4*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) +
 (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^4)/(3*d)
```

Rubi [A] time = 0.835134, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5718, 5654, 8, 30}

$$-\frac{8b^3e^2\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{27d}-\frac{160b^3e^2\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```


```
[Out] (160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) - (160*b^3*e^2*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(27*d) - (8*b^3*e^2*
Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/
(27*d) + (8*b^2*e^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (4*b^2*e^
2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(9*d) - (8*b*e^2*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) - (4*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) +
 (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^4)/(3*d)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^4}{3d} - \frac{\left(4be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c\right.}{3d}$$
$$= -\frac{4be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{9d} + \frac{e^2(c+dx}{}$$
$$= \frac{4b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{9d} - \frac{8be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+\right.}{9d}$$
$$= -\frac{8b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{27d} + \frac{8b^2e^2(c+}{}$$
$$= \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{27d} -$$
$$= \frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}\right.}{27d}$$

Mathematica [A] time = 0.587725, size = 475, normalized size = 1.54

$$\frac{e^2\left(\left(36a^2b^2+27a^4+8b^4\right)(c+dx)^3+24b^2\left(9a^2+20b^2\right)(c+dx)+12ab\sqrt{c+dx-1}\sqrt{c+dx+1}\left(-\left(3a^2+2b^2\right)(c+dx)^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^2*(24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
+ d*x)^3 + 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-6*a^2 - 40*b^2 - (
3*a^2 + 2*b^2)*(c + d*x)^2) + 12*b*(36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3
+ 6*a*b^2*(c + d*x)^3 - 18*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 40*
b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 9*a^2*b*Sqrt[-1 + c + d*x]*(c +
d*x)^2*Sqrt[1 + c + d*x] - 2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c
+ d*x])*ArcCosh[c + d*x] + 18*b^2*(12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 + 2
*b^2*(c + d*x)^3 - 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 6*a*b*Sqrt
[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 36*b^3*(
-3*a*(c + d*x)^3 + 2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c
 + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 27*b^4*(c + d*x
)^3*ArcCosh[c + d*x]^4))/(81*d)
```

Maple [B] time = 0.045, size = 632, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^4+e^2*b^4*(1/3*arccosh(d*x+c)^4*(d*x+c)*(d*x+c-1)*
(d*x+c+1)+1/3*arccosh(d*x+c)^4*(d*x+c)-4/9*arccosh(d*x+c)^3*(d*x+c)^2*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)-8/9*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)+4/9*arccosh(d*x+c)^2*(d*x+c-1)*(d*x+c+1)*(d*x+c)+28/9*arccosh(d*x+c)^2
*(d*x+c)-8/27*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2-160/
27*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+8/81*(d*x+c-1)*(d*x+c+1)*
(d*x+c)+488/81*d*x+488/81*c)+4*e^2*a*b^3*(1/3*arccosh(d*x+c)^3*(d*x+c-1)*(d
*x+c+1)*(d*x+c)+1/3*arccosh(d*x+c)^3*(d*x+c)-1/3*arccosh(d*x+c)^2*(d*x+c)^2
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)+2/9*arccosh(d*x+c)*(d*x+c-1)*(d*x+c+1)*(d*x+c)+14/9*(d*x+c)*arcc
osh(d*x+c)-2/27*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-40/27*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2))+6*e^2*a^2*b^2*(1/3*arccosh(d*x+c)^2*(d*x+c-1)*(d*x+c+
1)*(d*x+c)+1/3*arccosh(d*x+c)^2*(d*x+c)-2/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*(d*x+c)^2-4/9*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+2/27*(d*x+c-1)*(d*x+c+1)*(d*x+c)+14/27*d*x+14/27*c)+4*e^2*a^3*b*(1/3*arcc
osh(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63868, size = 1894, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
8*b^4)*c*d^2*e^2*x^2 + 3*(72*a^2*b^2 + 160*b^4 + (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2*d*
e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 3
6*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x + 3*a*
b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 + 2*b^4)*e^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d
^2*e^2*x^2 + 3*(4*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x + (12*b^4*c + (9*a
^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2*x + (a*b^
3*c^2 + 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^3*e^2*x^3 +
9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 + 9*(4*a*b^3 + (3*a^3*b + 2*a*b^3)*c^2)
*d*e^2*x + 3*(12*a*b^3*c + (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9*a^2*b^2 + 2*b
^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x + (18*a^2*b^2 + 40*b^4 +
(9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 12*((3*a^3*b + 2*a*b^3)*d^2*e^2*x^
2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x + (6*a^3*b + 40*a*b^3 + (3*a^3*b + 2*a*
b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 12.7758, size = 1889, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
4*a**3*b*c**3*e**2*acosh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*acosh(c + d*
x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + 4*a**3
*b*c*d*e**2*x**2*acosh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/9 + 4*a**3*b*d**2*e**2*x**3*acosh(c + d*x)/3 - 4*a**3*b*d*e**
2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 8*a**3*b*e**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 - 1)/(9*d) + 2*a**2*b**2*c**3*e**2*acosh(c + d*x)**2/d +
 6*a**2*b**2*c**2*e**2*x*acosh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*
a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*
d) + 6*a**2*b**2*c*d*e**2*x**2*acosh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**
2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d
*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*acosh(c + d*x)**2 + 4*a**2*b**2*d**2*e**
2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*aco
sh(c + d*x)/3 + 8*a**2*b**2*e**2*x/3 - 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*acosh(c + d*x)*
*3/(3*d) + 8*a*b**3*c**3*e**2*acosh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*a
cosh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*acosh(c + d*x)/3 - 4*a*b**3*c**2*e*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 8*a*b**3*
c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + 4*a*b**3*c*d*e**2*x
**2*acosh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*acosh(c + d*x)/3 - 8*a*b**3*
c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 16*a*b*
*3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27 + 16*a*b**3*c*e**2*acos
h(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*acosh(c + d*x)**3/3 + 8*a*b**3*d
**2*e**2*x**3*acosh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)/27 + 16*a*b**3*e**2*x*acosh(c + d*x)/3 - 8*a*b**3*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 160*a*b**3*
e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + b**4*c**3*e**2*acosh(c +
 d*x)**4/(3*d) + 4*b**4*c**3*e**2*acosh(c + d*x)**2/(9*d) + b**4*c**2*e**2*
x*acosh(c + d*x)**4 + 4*b**4*c**2*e**2*x*acosh(c + d*x)**2/3 + 8*b**4*c**2*
e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acos
h(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*acosh(c + d*x)**4 + 4*b**4*c*d*e**2*
x**2*acosh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/27 + 8*b**4*c*e**2*acosh(c +
 d*x)**2/(3*d) + b**4*d**2*e**2*x**3*acosh(c + d*x)**4/3 + 4*b**4*d**2*e**2
*x**3*acosh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/9 - 8*b**4*d*e**2*x**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/27 + 8*b**4*e**2*x*ac
```

```
osh(c + d*x)**2/3 + 160*b**4*e**2*x/27 - 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +
d**2*x**2 - 1)*acosh(c + d*x)**3/(9*d) - 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acos
h(c))**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^4, x)
```

3.124 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=209

$$-\frac{3b^3e\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{3b^2e\left(a+b\cos\right.}{4}$$

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
+ c + d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (3*b^2*e*(a + b*ArcCosh[c + d*
x])^2)/(4*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d) - (b*
e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3
)/d - (e*(a + b*ArcCosh[c + d*x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[
c + d*x])^4)/(2*d)
```

Rubi [A] time = 0.562042, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$-\frac{3b^3e\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{3b^2e\left(a+b\cos\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
+ c + d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (3*b^2*e*(a + b*ArcCosh[c + d*
x])^2)/(4*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d) - (b*
e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3
)/d - (e*(a + b*ArcCosh[c + d*x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[
c + d*x])^4)/(2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^4}{2d} - \frac{(2be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c \right.}{d}$$
$$= -\frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{d} + \frac{e(c+dx)^2\left(\right.}{}$$
$$= \frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d} - \frac{be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{d}$$
$$= -\frac{3b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{2d} + \frac{3b^2e(c+d}{}$$
$$= \frac{3b^4e(c+dx)^2}{4d} - \frac{3b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{2d}$$

Mathematica [A] time = 0.468614, size = 360, normalized size = 1.72

$$\frac{e\left(\left(6a^2b^2+2a^4+3b^4\right)(c+dx)^2-2ab\left(2a^2+3b^2\right)\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}-2ab\left(2a^2+3b^2\right)\log\left(\sqrt{c+dx-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*Sqrt[-1
 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*(c + d*x)*(-4*a^3*(c + d*x) -
 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 3*b^3*S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-2*a^2 - b^2
 + 4*a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*Sqrt[-1 + c + d*x]*(c + d*
x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 4*b^3*(-a + 2*a*(c + d*x)^2 - b*
Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + b^4*(-
1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^4 - 2*a*b*(2*a^2 + 3*b^2)*Log[c + d*x +
 Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(4*d)
```

Maple [B] time = 0.04, size = 933, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] -1/d*e*a^3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+(
(d*x+c)^2-1)^(1/2))-3/d*e*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*c-3/d*e*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/2/
d*e*a^2*b^2*arccosh(d*x+c)^2+3*e*a^2*b^2*x*c-1/4/d*e*b^4*arccosh(d*x+c)^4-3
/4/d*e*b^4*arccosh(d*x+c)^2+3/2/d*e*a^2*b^2*c^2+3/2*d*e*a^2*b^2*x^2+e*b^4*a
rccosh(d*x+c)^4*x*c+3*e*b^4*arccosh(d*x+c)^2*x*c+1/2/d*e*b^4*arccosh(d*x+c)
^4*c^2+3/2/d*e*b^4*arccosh(d*x+c)^2*c^2-1/d*e*a*b^3*arccosh(d*x+c)^3-3/2/d*
e*a*b^3*arccosh(d*x+c)+1/2*d*e*b^4*arccosh(d*x+c)^4*x^2+3/2*d*e*b^4*arccosh
(d*x+c)^2*x^2+3/4/d*e*b^4*c^2+1/2/d*a^4*c^2*e+x*a^4*c*e+3/2*e*b^4*x*c+1/2*d
*x^2*a^4*e+3/4*d*e*b^4*x^2+2/d*e*a*b^3*arccosh(d*x+c)^3*c^2+3/d*e*a*b^3*arc
cosh(d*x+c)*c^2+3/d*e*a^2*b^2*arccosh(d*x+c)^2*c^2+2/d*arccosh(d*x+c)*a^3*b
*c^2*e+2*d*e*a*b^3*arccosh(d*x+c)^3*x^2+3*d*e*a*b^3*arccosh(d*x+c)*x^2+3*d*
e*a^2*b^2*arccosh(d*x+c)^2*x^2-3/2*e*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)*x+4*e*a*b^3*arccosh(d*x+c)^3*x*c+6*e*a*b^3*arccosh(d*x+c)*x*c-
3/2*e*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x*a^3*b*e+4*arccosh(d*x+c)*x*a^3*b*c*e+6*e*a^2*b^2*arccosh(d*x+c)^2*x*c-
e*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3*e*a^2*b^2*arccos
h(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/2/d*e*a*b^3*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)*c-3/2/d*e*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*
c-1/d*e*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-1/d*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)*a^3*b*c*e-3*e*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*x+2*d*arccosh(d*x+c)*x^2*a^3*b*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61011, size = 1287, normalized size = 6.16

$$\frac{\left(2\,a^4+6\,a^2b^2+3\,b^4\right)d^2ex^2+2\left(2\,a^4+6\,a^2b^2+3\,b^4\right)cdex+\left(2\,b^4d^2ex^2+4\,b^4cdex+\left(2\,b^4c^2-b^4\right)e\right)\log\left(dx+c+\sqrt{d^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)*
c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 - b^4)*e)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b^3*
c*d*e*x + (2*a*b^3*c^2 - a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 3*(2
*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x - (2*a^2*b^2 + b
^4 - 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 +
2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x - (2*a^3
*b + 3*a*b^3 - 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4)*d*e*x +
(2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x + (2*a^3*b +
 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 6.27775, size = 1027, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*acosh(c + d*x)/d
+ 4*a**3*b*c*e*x*acosh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1)/d + 2*a**3*b*d*e*x**2*acosh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1) - a**3*b*e*acosh(c + d*x)/d + 3*a**2*b**2*c**2*e*acosh(c
 + d*x)**2/d + 6*a**2*b**2*c*e*x*acosh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*acosh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x) - 3*a**2*b**2*e*acosh
```

```
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*acosh(c + d*x)**3/d + 3*a*b**3*c**2*e*
acosh(c + d*x)/d + 4*a*b**3*c*e*x*acosh(c + d*x)**3 + 6*a*b**3*c*e*x*acosh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(2*d) + 2*a*b**3*
d*e*x**2*acosh(c + d*x)**3 + 3*a*b**3*d*e*x**2*acosh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)/2 - a*b**3*e*acosh(c + d*x)**3/d - 3*a*b*
*3*e*acosh(c + d*x)/(2*d) + b**4*c**2*e*acosh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*acosh(c + d*x)**2/(2*d) + b**4*c*e*x*acosh(c + d*x)**4 + 3*b**4*c*e*x*
acosh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
1)*acosh(c + d*x)/(2*d) + b**4*d*e*x**2*acosh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*acosh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)*acosh(c + d*x)/2 - b**4*e*acosh(c + d*x)**4/(4*d) - 3*b**4*e*acosh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)
```

3.125 $\int \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=129

$$-\frac{24b^3\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{c+dx-1}\sqrt{c+dx}}{}$$

```
[Out] 24*b^4*x - (24*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
d*x]))/d + (12*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d - (4*b*Sqrt[-1 +
 c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/d + ((c + d*x)*(a +
 b*ArcCosh[c + d*x])^4)/d
```

Rubi [A] time = 0.28408, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5864, 5654, 5718, 8}

$$-\frac{24b^3\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{c+dx-1}\sqrt{c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
d*x]))/d + (12*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d - (4*b*Sqrt[-1 +
 c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/d + ((c + d*x)*(a +
 b*ArcCosh[c + d*x])^4)/d
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4}{d} - \frac{(4b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c+dx\right)}{d}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} - \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+d\right.}{d}$$

$$= -\frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+\right.}{d}$$

$$= 24b^4x - \frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\cos\right.}{d}$$

Mathematica [B] time = 0.251944, size = 261, normalized size = 2.02

$$\frac{\left(12a^2b^2+a^4+24b^4\right)(c+dx)-4ab\left(a^2+6b^2\right)\sqrt{c+dx-1}\sqrt{c+dx+1}+6b^2\cosh^{-1}(c+dx)^2\left(a^2(c+dx)-2ab\sqrt{c+dx-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*
b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x])*ArcCosh[c + d*x] + 6*b^2*(a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b
*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 4*b^3*(-(a*(c +
 d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + b^4*(
c + d*x)*ArcCosh[c + d*x]^4)/d
```

Maple [B] time = 0.033, size = 275, normalized size = 2.1

$$\frac{1}{d}\Big((dx+c)\,a^4+b^4\Big((\operatorname{arccosh}(dx+c))^4\,(dx+c)-4\,(\operatorname{arccosh}(dx+c))^3\,\sqrt{dx+c-1}\sqrt{dx+c+1}+12\,(\operatorname{arccosh}(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*(arccosh(d*x+c)^4*(d*x+c)-4*arccosh(d*x+c)^3*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)+12*arccosh(d*x+c)^2*(d*x+c)-24*arccosh(d*x+c)*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+24*d*x+24*c)+4*a*b^3*(arccosh(d*x+c)^3*(d*x+c)-3*
arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))+6*a^2*b^2*(arccosh(d*x+c)^2*(d*x+c)-2*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+4*a^3*b*((d*x+c)*arc
cosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38357, size = 784, normalized size = 6.08

$$\left(b^4dx + b^4c\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)^4 + 4\left(ab^3dx + ab^3c - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}b^4\right)\log\left(dx + c + \sqrt{d^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [A] time = 2.42589, size = 444, normalized size = 3.44

$$\begin{cases} a^4x + \frac{4a^3bc\operatorname{acosh}(c+dx)}{d} + 4a^3bx\operatorname{acosh}(c+dx) - \frac{4a^3b\sqrt{c^2+2cdx+d^2x^2-1}}{d} + \frac{6a^2b^2c\operatorname{acosh}^2(c+dx)}{d} + 6a^2b^2x\operatorname{acosh}^2(c+dx) + 12a^2b \\ x\left(a + b\operatorname{acosh}(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*acosh(c + d*x)/d + 4*a**3*b*x*acosh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + 6*a**2*b**2*c*acosh(c
+ d*x)**2/d + 6*a**2*b**2*x*acosh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + 4*a*b**3*c*acosh
(c + d*x)**3/d + 24*a*b**3*c*acosh(c + d*x)/d + 4*a*b**3*x*acosh(c + d*x)**
3 + 24*a*b**3*x*acosh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)*acosh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d
 + b**4*c*acosh(c + d*x)**4/d + 12*b**4*c*acosh(c + d*x)**2/d + b**4*x*acos
h(c + d*x)**4 + 12*b**4*x*acosh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1)*acosh(c + d*x)/d, Ne(d, 0)), (x*(a + b*acosh(c))**4, Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4, x)
```

3.126 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{ce+dex}\,dx$

Optimal. Leaf size=192

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}-\frac{3b^3\text{PolyLog}\left(4,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-$$

```
[Out] (a + b*ArcCosh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcCosh[c + d*x])^4*Log[1 +
E^(-2*ArcCosh[c + d*x])])/(d*e) - (2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2
, -E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])^2*Poly
Log[3, -E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b^3*(a + b*ArcCosh[c + d*x])*P
olyLog[4, -E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, -E^(-2*ArcCo
sh[c + d*x])])/(2*d*e)
```

Rubi [A] time = 0.265944, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}+\frac{3b^3\text{PolyLog}\left(4,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}+\frac{2b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcCosh[c + d*x])^4*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e) + (2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2
, -E^(2*ArcCosh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])^2*PolyL
og[3, -E^(2*ArcCosh[c + d*x])])/(d*e) + (3*b^3*(a + b*ArcCosh[c + d*x])*Pol
yLog[4, -E^(2*ArcCosh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, -E^(2*ArcCosh[c
 + d*x])])/(2*d*e)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{ce+dex}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{ex}\,dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x}\,dx,x,c+dx\right)}{de}$$

$$=\frac{\text{Subst}\left(\int (a+bx)^4\tanh(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^4}{1+e^{2x}}\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}-\frac{(4b)\,\text{Subs}}{}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}+\frac{2b\left(a+b\right.}{}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}+\frac{2b\left(a+b\right.}{}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}+\frac{2b\left(a+b\right.}{}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}+\frac{2b\left(a+b\right.}{}$$

$$=-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}+\frac{2b\left(a+b\right.}{}$$

Mathematica [A] time = 0.762255, size = 308, normalized size = 1.6

$$\frac{-3b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2-3ab^3\text{PolyLog}\left(4,-e^{-2\cosh^{-1}(c+dx)}\right)-2b\text{PolyLog}\left(2,-e^{-2c\cdots}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (2*a^3*b*ArcCosh[c + d*x]^2 + 2*a^2*b^2*ArcCosh[c + d*x]^3 + a*b^3*ArcCosh[
c + d*x]^4 + (b^4*ArcCosh[c + d*x]^5)/5 + 4*a^3*b*ArcCosh[c + d*x]*Log[1 +
E^(-2*ArcCosh[c + d*x])] + 6*a^2*b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCo
sh[c + d*x])] + 4*a*b^3*ArcCosh[c + d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])]
 + b^4*ArcCosh[c + d*x]^4*Log[1 + E^(-2*ArcCosh[c + d*x])] + a^4*Log[c + d*
x] - 2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] -
3*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*a
*b^3*PolyLog[4, -E^(-2*ArcCosh[c + d*x])] - 3*b^4*ArcCosh[c + d*x]*PolyLog[
4, -E^(-2*ArcCosh[c + d*x])] - (3*b^4*PolyLog[5, -E^(-2*ArcCosh[c + d*x])])
/2)/(d*e)
```

Maple [B] time = 0.033, size = 727, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^4/e*ln(d*x+c)-1/5/d*b^4/e*arccosh(d*x+c)^5+1/d*b^4/e*arccosh(d*x+c)^4
*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+2/d*b^4/e*arccosh(d*x+c)^3
*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/d*b^4/e*arccosh(d*
x+c)^2*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/d*b^4/e*arcc
osh(d*x+c)*polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2/d*b^4/
e*polylog(5,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a*b^3/e*arccosh
(d*x+c)^4+4/d*a*b^3/e*arccosh(d*x+c)^3*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2))^2+1)+6/d*a*b^3/e*arccosh(d*x+c)^2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2)-6/d*a*b^3/e*arccosh(d*x+c)*polylog(3,-(d*x+c+(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2))^2)+3/d*a*b^3/e*polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2))^2)-2/d*a^2*b^2/e*arccosh(d*x+c)^3+6/d*a^2*b^2/e*arccosh(d*x+c
)^2*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)+6/d*a^2*b^2/e*arccosh(d
*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/d*a^2*b^2/e*p
```

```
olylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-2/d*a^3*b/e*arccosh(d*
x+c)^2+4/d*a^3*b/e*arccosh(d*x+c)*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
)^2+1)+2/d*a^3*b/e*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \operatorname{arcosh}(dx+c)^4 + 4\,ab^3 \operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4\,a^3b \operatorname{arcosh}(dx+c) + a^4}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3b \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*acosh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*acosh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
```

```
2*acosh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*acosh(c + d*x)/(c + d
*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx + c) + a)^4}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

3.127 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx$

Optimal. Leaf size=264

$$\frac{24ib^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{24ib^3\text{PolyLog}\left(3,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{12i\ldots}{\ldots}$$

```
[Out] -((a + b*ArcCosh[c + d*x])^4/(d*e^2*(c + d*x))) + (8*b*(a + b*ArcCosh[c + d
*x])^3*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((12*I)*b^2*(a + b*ArcCosh[c +
 d*x])^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((12*I)*b^2*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^3
*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((
24*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^2
) - ((24*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^4*
PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^2)
```

Rubi [A] time = 0.412514, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5662, 5761, 4180, 2531, 6609, 2282, 6589}

$$\frac{24ib^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{24ib^3\text{PolyLog}\left(3,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{12i\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```



```
[Out] -((a + b*ArcCosh[c + d*x])^4/(d*e^2*(c + d*x))) + (8*b*(a + b*ArcCosh[c + d
*x])^3*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((12*I)*b^2*(a + b*ArcCosh[c +
 d*x])^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((12*I)*b^2*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^3
*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((
24*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^2
) - ((24*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^4*
PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^2)
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}x\sqrt{1+x}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int (a+bx)^3\mathbf{sech}(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{\left(12ib^2\right)}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{12ib^2\left(a\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{12ib^2\left(a\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{12ib^2\left(a\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{12ib^2\left(a\right.}{}$$

Mathematica [B] time = 2.34485, size = 872, normalized size = 3.3

$$\frac{-\frac{a^4}{c+dx} + 4b\left(2\tan^{-1}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\right) - \frac{\cosh^{-1}(c+dx)}{c+dx}\right)a^3 - 6ib^2\left(\cosh^{-1}(c+dx)\left(-\frac{i\cosh^{-1}(c+dx)}{c+dx} + 2\log\left(1 - ie^{-\cdots}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^4/(c + d*x)) + 4*a^3*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[
ArcCosh[c + d*x]/2]]) - (6*I)*a^2*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c +
d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c
```

```
 + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCos
h[c + d*x]]) + 4*a*b^3*(-(ArcCosh[c + d*x]^3/(c + d*x)) + (3*I)*(-(ArcCosh[
c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c + d*x]]))
 - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyLog[2, I/E
^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*PolyLog[3,
I/E^ArcCosh[c + d*x]])) + b^4*(((-7*I)/16)*Pi^4 + (Pi^3*ArcCosh[c + d*x])/2
 - ((3*I)/2)*Pi^2*ArcCosh[c + d*x]^2 - 2*Pi*ArcCosh[c + d*x]^3 + I*ArcCosh[
c + d*x]^4 - ArcCosh[c + d*x]^4/(c + d*x) + (Pi^3*Log[1 + I/E^ArcCosh[c + d
*x]])/2 - (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - 6*Pi*
ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + (4*I)*ArcCosh[c + d*x]^3
*Log[1 + I/E^ArcCosh[c + d*x]] + (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 - I*E^Ar
cCosh[c + d*x]] + 6*Pi*ArcCosh[c + d*x]^2*Log[1 - I*E^ArcCosh[c + d*x]] - (
Pi^3*Log[1 + I*E^ArcCosh[c + d*x]])/2 - (4*I)*ArcCosh[c + d*x]^3*Log[1 + I*
E^ArcCosh[c + d*x]] + (Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[c + d*x])/4]])/2 +
(3*I)*(Pi - (2*I)*ArcCosh[c + d*x])^2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] -
 (12*I)*ArcCosh[c + d*x]^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + (3*I)*Pi^2
*PolyLog[2, I*E^ArcCosh[c + d*x]] + 12*Pi*ArcCosh[c + d*x]*PolyLog[2, I*E^A
rcCosh[c + d*x]] + 12*Pi*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - (24*I)*ArcCo
sh[c + d*x]*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + (24*I)*ArcCosh[c + d*x]*P
olyLog[3, (-I)*E^ArcCosh[c + d*x]] - 12*Pi*PolyLog[3, I*E^ArcCosh[c + d*x]]
 - (24*I)*PolyLog[4, (-I)/E^ArcCosh[c + d*x]] - (24*I)*PolyLog[4, (-I)*E^Ar
cCosh[c + d*x]]))/(d*e^2)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(dx + c))^4}{(dex + ce)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \operatorname{arcosh}(dx+c)^4 + 4\,ab^3 \operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4\,a^3b \operatorname{arcosh}(dx+c) + a^4}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2 \operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b \operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*acosh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*acosh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*acosh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*acosh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.128 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx$

Optimal. Leaf size=195

$$\frac{6b^3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b^4\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)}{de^3}-\frac{6b^2\log\left(e^{-2\cosh^{-1}(c+dx)}\right.}{}$$

```
[Out] (-2*b*(a + b*ArcCosh[c + d*x])^3)/(d*e^3) + (2*b*Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcCosh[c
 + d*x])^4/(2*d*e^3*(c + d*x)^2) - (6*b^2*(a + b*ArcCosh[c + d*x])^2*Log[1
+ E^(-2*ArcCosh[c + d*x])])/(d*e^3) + (6*b^3*(a + b*ArcCosh[c + d*x])*PolyL
og[2, -E^(-2*ArcCosh[c + d*x])])/(d*e^3) + (3*b^4*PolyLog[3, -E^(-2*ArcCosh
[c + d*x])])/(d*e^3)
```

Rubi [A] time = 0.426362, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5662, 5724, 5660, 3718, 2190, 2531, 2282, 6589}

$$-\frac{6b^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b^4\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)}{de^3}-\frac{6b^2\log\left(e^{2\cosh^{-1}(c+dx)}+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (2*b*(a + b*ArcCosh[c + d*x])^3)/(d*e^3) + (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 +
 c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcCosh[c
+ d*x])^4/(2*d*e^3*(c + d*x)^2) - (6*b^2*(a + b*ArcCosh[c + d*x])^2*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e^3) - (6*b^3*(a + b*ArcCosh[c + d*x])*PolyLog
[2, -E^(2*ArcCosh[c + d*x])])/(d*e^3) + (3*b^4*PolyLog[3, -E^(2*ArcCosh[c +
 d*x])])/(d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx, x, c+dx\right)}{de^3}$$

$$= \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} - \frac{(6b^2)}{}$$

$$= \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} - \frac{(6b^2)}{}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

Mathematica [B] time = 2.21836, size = 398, normalized size = 2.04

$$4ab^3\left(3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right) - \cosh^{-1}(c+dx)\left(\frac{\cosh^{-1}(c+dx)^2}{(c+dx)^2} - \frac{3\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)\cosh^{-1}(c+dx)}{c+dx} + 3\cosh^{-1}(c+dx) + 6\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^4/(c + d*x)^2) + (4*a^3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c + d
*x) - (4*a^3*b*ArcCosh[c + d*x])/(c + d*x)^2 - (b^4*ArcCosh[c + d*x]^4)/(c
+ d*x)^2 + 12*a^2*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Ar
cCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2) - Log[c + d*
x]) + 4*a*b^3*(-(ArcCosh[c + d*x]*(3*ArcCosh[c + d*x] - (3*Sqrt[(-1 + c + d
*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) + ArcCosh[c +
d*x]^2/(c + d*x)^2 + 6*Log[1 + E^(-2*ArcCosh[c + d*x])])) + 3*PolyLog[2, -E
^(-2*ArcCosh[c + d*x])]) + 2*b^4*(2*ArcCosh[c + d*x]^2*(-ArcCosh[c + d*x] +
 (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d
*x) - 3*Log[1 + E^(-2*ArcCosh[c + d*x])]) + 6*ArcCosh[c + d*x]*PolyLog[2, -
E^(-2*ArcCosh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcCosh[c + d*x])]))/(2*d*e^
3)
```

Maple [B] time = 0.066, size = 605, normalized size = 3.1

$$-\frac{a^4}{2\,de^3\,(dx+c)^2}+2\,\frac{b^4\,(\mathrm{arccosh}\,(dx+c))^3\,\sqrt{dx+c+1}\sqrt{dx+c-1}}{de^3\,(dx+c)}+2\,\frac{b^4\,(\mathrm{arccosh}\,(dx+c))^3}{de^3}-\frac{b^4\,(\mathrm{arccosh}\,(dx+c))^4}{2\,de^3\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^4/e^3/(d*x+c)^2+2/d*b^4/e^3*arccosh(d*x+c)^3/(d*x+c)*(d*x+c+1)^(1/
2)*(d*x+c-1)^(1/2)+2/d*b^4/e^3*arccosh(d*x+c)^3-1/2/d*b^4/e^3*arccosh(d*x+c
)^4/(d*x+c)^2-6/d*b^4/e^3*arccosh(d*x+c)^2*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2))^2+1)-6/d*b^4/e^3*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2))^2)+3/d*b^4/e^3*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))^2)+6/d*a*b^3/e^3*arccosh(d*x+c)^2/(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-
1)^(1/2)+6/d*a*b^3/e^3*arccosh(d*x+c)^2-2/d*a*b^3/e^3*arccosh(d*x+c)^3/(d*x
+c)^2-12/d*a*b^3/e^3*arccosh(d*x+c)*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2))^2+1)-6/d*a*b^3/e^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2
)+6/d*a^2*b^2/e^3*arccosh(d*x+c)+6/d*a^2*b^2/e^3*arccosh(d*x+c)/(d*x+c)*(d*
x+c+1)^(1/2)*(d*x+c-1)^(1/2)-3/d*a^2*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)^2-6/d
*a^2*b^2/e^3*ln((d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2+1)-2/d*a^3*b/e^3/
(d*x+c)^2*arccosh(d*x+c)+2/d*a^3*b/e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x
+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4\,\text{arcosh}\,(dx+c)^4+4\,ab^3\,\text{arcosh}\,(dx+c)^3+6\,a^2b^2\,\text{arcosh}\,(dx+c)^2+4\,a^3b\,\text{arcosh}\,(dx+c)+a^4}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{b^4\,\text{acosh}^4\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{4ab^3\,\text{acosh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{6a^2b^2\,\text{acosh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\frac{4a^3b\,\text{acosh}\,(c}{c^3+3c^2dx+3cd^2x}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*acosh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*acosh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx + c) + a)^4}{(dex + ce)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

3.129 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx$

Optimal. Leaf size=432

$$\frac{4ib^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{4ib^3\text{PolyLog}\left(3,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{2ib^2\text{P}}{}$$

```
[Out] (2*b^2*(a + b*ArcCosh[c + d*x])^2)/(d*e^4*(c + d*x)) + (2*b*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (
a + b*ArcCosh[c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcCosh[c +
 d*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcCosh[c + d*x])^
3*ArcTan[E^ArcCosh[c + d*x]])/(3*d*e^4) + ((4*I)*b^4*PolyLog[2, (-I)*E^ArcC
osh[c + d*x]])/(d*e^4) - ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, (
-I)*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[2, I*E^ArcCosh[c + d*
x]])/(d*e^4) + ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh
[c + d*x]])/(d*e^4) + ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E
^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3
, I*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c +
 d*x]])/(d*e^4) + ((4*I)*b^4*PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rubi [A] time = 0.841757, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5866, 12, 5662, 5748, 5761, 4180, 2531, 6609, 2282, 6589, 2279, 2391}

$$\frac{4ib^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{4ib^3\text{PolyLog}\left(3,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{2ib^2\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```


```
[Out] (2*b^2*(a + b*ArcCosh[c + d*x])^2)/(d*e^4*(c + d*x)) + (2*b*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (
a + b*ArcCosh[c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcCosh[c +
 d*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcCosh[c + d*x])^
3*ArcTan[E^ArcCosh[c + d*x]])/(3*d*e^4) + ((4*I)*b^4*PolyLog[2, (-I)*E^ArcC
osh[c + d*x]])/(d*e^4) - ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, (
-I)*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[2, I*E^ArcCosh[c + d*
x]])/(d*e^4) + ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh
```

```
[c + d*x]])/(d*e^4) + ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E
^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3
, I*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c +
 d*x]])/(d*e^4) + ((4*I)*b^4*PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{3de^4}$$

$$= \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(2b)}{}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} -$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} -$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} -$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} -$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} -$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} -$$

Mathematica [B] time = 8.80474, size = 1213, normalized size = 2.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] (-(a^4/(c + d*x)^3) + 2*a^3*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c +
 d*x))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[ArcCo
sh[c + d*x]/2]]) + 6*a^2*b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c
+ d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c
 + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c
+ d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]
] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + 2*a*b^3*((6*ArcCosh[c + d*x])/(c
+ d*x) + (3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*
x]^2)/(c + d*x)^2 - (2*ArcCosh[c + d*x]^3)/(c + d*x)^3 - (3*I)*((-4*I)*ArcT
an[Tanh[ArcCosh[c + d*x]/2]] + ArcCosh[c + d*x]^2*Log[1 - I/E^ArcCosh[c + d
*x]] - ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + 2*ArcCosh[c + d*x
]*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*ArcCosh[c + d*x]*PolyLog[2, I/E^A
rcCosh[c + d*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[3, I/E
^ArcCosh[c + d*x]])) + 3*b^4*(((-7*I)/96)*Pi^4 + (Pi^3*ArcCosh[c + d*x])/12
 - (I/4)*Pi^2*ArcCosh[c + d*x]^2 + (2*ArcCosh[c + d*x]^2)/(c + d*x) - (Pi*A
rcCosh[c + d*x]^3)/3 + (2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*
ArcCosh[c + d*x]^3)/(3*(c + d*x)^2) + (I/6)*ArcCosh[c + d*x]^4 - ArcCosh[c
+ d*x]^4/(3*(c + d*x)^3) + (4*I)*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d
*x]] + (Pi^3*Log[1 + I/E^ArcCosh[c + d*x]])/12 - (4*I)*ArcCosh[c + d*x]*Log
[1 + I/E^ArcCosh[c + d*x]] - (I/2)*Pi^2*ArcCosh[c + d*x]*Log[1 + I/E^ArcCos
h[c + d*x]] - Pi*ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + ((2*I)/
3)*ArcCosh[c + d*x]^3*Log[1 + I/E^ArcCosh[c + d*x]] + (I/2)*Pi^2*ArcCosh[c
+ d*x]*Log[1 - I*E^ArcCosh[c + d*x]] + Pi*ArcCosh[c + d*x]^2*Log[1 - I*E^Ar
cCosh[c + d*x]] - (Pi^3*Log[1 + I*E^ArcCosh[c + d*x]])/12 - ((2*I)/3)*ArcCo
sh[c + d*x]^3*Log[1 + I*E^ArcCosh[c + d*x]] + (Pi^3*Log[Tan[(Pi + (2*I)*Arc
Cosh[c + d*x])/4]])/12 + (I/2)*(8 + Pi^2 - (4*I)*Pi*ArcCosh[c + d*x] - 4*Ar
cCosh[c + d*x]^2)*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - (4*I)*PolyLog[2, I/
E^ArcCosh[c + d*x]] - (2*I)*ArcCosh[c + d*x]^2*PolyLog[2, (-I)*E^ArcCosh[c
+ d*x]] + (I/2)*Pi^2*PolyLog[2, I*E^ArcCosh[c + d*x]] + 2*Pi*ArcCosh[c + d*
x]*PolyLog[2, I*E^ArcCosh[c + d*x]] + 2*Pi*PolyLog[3, (-I)/E^ArcCosh[c + d*
x]] - (4*I)*ArcCosh[c + d*x]*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + (4*I)*Ar
cCosh[c + d*x]*PolyLog[3, (-I)*E^ArcCosh[c + d*x]] - 2*Pi*PolyLog[3, I*E^Ar
cCosh[c + d*x]] - (4*I)*PolyLog[4, (-I)/E^ArcCosh[c + d*x]] - (4*I)*PolyLog
[4, (-I)*E^ArcCosh[c + d*x]]))/(3*d*e^4)
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{arccosh}(dx + c))^4}{(dex + ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^4*e^4*x^3
+ 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 -
c)*a*b^3 + (c^3 - c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 +
 b^4*c*d^2)*x^2 + (b^4*c^2 + 3*(c^2 - 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^
2 + 2*(3*a*b^3*c*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(
3*c^2*d - d)*a*b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d
- d)*a^2*b^2*x + (c^3 - c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (
c^2 - 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (
3*c^2*d - d)*a^3*b*x + (c^3 - c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (
c^2 - 1)*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^
5*e^4 + (21*c^2*d^5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4
+ 5*(7*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)
*x^2 + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4
 - d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*
c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + (7*c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \operatorname{arcosh}(dx+c)^4 + 4\,ab^3 \operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4\,a^3b \operatorname{arcosh}(dx+c) + a^4}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4\operatorname{acosh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3\operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{6a^2b^2\operatorname{acosh}^2(c+\cdots}{c^4+4c^3dx+6c^2d^2x^2+4c\cdots}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*acosh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*acosh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*acosh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

3.130 $\int \frac{(ce+dex)^4}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=213

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16bd}-\frac{e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16bd}+\frac{e^4 c}{}$$

```
[Out] -(e^4*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(8*b*d) - (3*e^4*
CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(16*b*d) - (e^4
*CoshIntegral[(5*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(5*a)/b])/(16*b*d) + (e^
4*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(8*b*d) + (3*e^4*Cosh
[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(16*b*d) + (e^4*Cos
h[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c + d*x]))/b])/(16*b*d)
```

Rubi [A] time = 0.428566, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8bd}-\frac{3e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{16bd}-\frac{e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\cosh^{-1}(\right.}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e^4*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b])/(8*b*d) - (3*e^4*Cosh
Integral[(3*a)/b + 3*ArcCosh[c + d*x]]*Sinh[(3*a)/b])/(16*b*d) - (e^4*CoshI
ntegral[(5*a)/b + 5*ArcCosh[c + d*x]]*Sinh[(5*a)/b])/(16*b*d) + (e^4*Cosh[a
/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(8*b*d) + (3*e^4*Cosh[(3*a)/b]*Si
nhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(16*b*d) + (e^4*Cosh[(5*a)/b]*Sin
hIntegral[(5*a)/b + 5*ArcCosh[c + d*x]])/(16*b*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}+\frac{3\sinh(3x)}{16(a+bx)}+\frac{\sinh(5x)}{16(a+bx)}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{16d} + \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d} +$$

$$= \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d} + \frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh(}{a}\right.}{16d}$$

$$= -\frac{e^4\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{8bd} - \frac{3e^4\textbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)\sinh\left(\frac{3a}{b}\right)}{16bd} - \frac{e^4\text{C}}{}$$

Mathematica [A] time = 0.250472, size = 151, normalized size = 0.71

$$\frac{e^4\left(-2\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-3\sinh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\textbf{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^4*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*
(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c +
 d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*Si
nhIntegral[5*(a/b + ArcCosh[c + d*x])]))/(16*b*d)
```

Maple [A] time = 0.155, size = 194, normalized size = 0.9

$$\frac{1}{d}\left(\frac{e^4}{32\,b}e^{5\,\frac{a}{b}}\text{Ei}\left(1,5\,\text{arccosh}\,(dx+c)+5\,\frac{a}{b}\right)+\frac{3\,e^4}{32\,b}e^{3\,\frac{a}{b}}\text{Ei}\left(1,3\,\text{arccosh}\,(dx+c)+3\,\frac{a}{b}\right)+\frac{e^4}{16\,b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{arccosh}\,(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/32*e^4/b*exp(5*a/b)*Ei(1,5*arccosh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a/
b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*e^4/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b
)-1/16*e^4/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-3/32*e^4/b*exp(-3*a/b)*Ei(
1,-3*arccosh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*
a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4}{b\operatorname{arcosh}(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b*arccosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int\frac{c^4}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{d^4x^4}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{4cd^3x^3}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{6c^2d^2x^2}{a+b\operatorname{acosh}(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c)),x)
```

```
[Out] e**4*(Integral(c**4/(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/(a + b*
acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*acosh(c + d*x)), x) + I
ntegral(6*c**2*d**2*x**2/(a + b*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(
a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{b \operatorname{arcosh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

3.131 $\int \frac{(ce+dex)^3}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8bd}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^3}{}$$

```
[Out] -(e^3*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b])/(4*b*d) -
 (e^3*CoshIntegral[(4*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(4*a)/b])/(8*b*d) +
 (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(4*b*d) +
 (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/(8*b*d)
```

Rubi [A] time = 0.334497, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{8bd}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\cosh^{-1}\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e^3*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b])/(4*b*d) - (
e^3*CoshIntegral[(4*a)/b + 4*ArcCosh[c + d*x]]*Sinh[(4*a)/b])/(8*b*d) + (e^
3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(4*b*d) + (e^3*
Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c + d*x]])/(8*b*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d}+\frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d}$$

$$= \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh(}{a\cdot}\right.}{8c}$$

$$= -\frac{e^3\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{4bd}-\frac{e^3\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)\sinh\left(\frac{4a}{b}\right)}{8bd}+\frac{e^3}{}$$

Mathematica [A] time = 0.17883, size = 109, normalized size = 0.75

$$\frac{e^3\left(-2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-}\right.\right.\right.}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^3*(-2*CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - CoshInteg
ral[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegra
l[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh
[c + d*x])]))/(8*b*d)
```

Maple [A] time = 0.079, size = 134, normalized size = 0.9

$$\frac{1}{d}\left(\frac{e^3}{16\,b}e^{4\,\frac{a}{b}}\text{Ei}\left(1,4\,\text{arccosh}\,(dx+c)+4\,\frac{a}{b}\right)+\frac{e^3}{8\,b}e^{2\,\frac{a}{b}}\text{Ei}\left(1,2\,\text{arccosh}\,(dx+c)+2\,\frac{a}{b}\right)-\frac{e^3}{8\,b}e^{-2\,\frac{a}{b}}\text{Ei}\left(1,-2\,\text{arccosh}\,(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{b \operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}{b \operatorname{arcosh}(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcco
sh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^3x^3}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{3cd^2x^2}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{3c^2dx}{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c)),x)
```

```
[Out] e**3*(Integral(c**3/(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/(a + b*
acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*acosh(c + d*x)), x) + I
ntegral(3*c**2*d*x/(a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{b\,\mathrm{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

3.132 $\int \frac{(ce+dex)^2}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\cdots}{\cdots}$$

```
[Out] -(e^2*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(4*b*d) - (e^2*Co
shIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(4*b*d) + (e^2*Co
sh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(4*b*d) + (e^2*Cosh[(3*a)
/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(4*b*d)
```

Rubi [A] time = 0.298317, antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+\cdots\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e^2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b])/(4*b*d) - (e^2*CoshIn
tegral[(3*a)/b + 3*ArcCosh[c + d*x]]*Sinh[(3*a)/b])/(4*b*d) + (e^2*Cosh[a/b
]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(4*b*d) + (e^2*Cosh[(3*a)/b]*SinhIn
tegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(4*b*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d}$$
$$= \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}\right.}{a+b}\right.}{4d}$$
$$= -\frac{e^2\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{4bd} - \frac{e^2\mathbf{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)\sinh\left(\frac{3a}{b}\right)}{4bd} + \frac{e^2\,\text{co}}{}$$

Mathematica [A] time = 0.154902, size = 102, normalized size = 0.72

$$\frac{e^2\left(\sinh\left(\frac{a}{b}\right)\left(-\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx\right.\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^2*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) - CoshIntegral[3*(a
/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcCos
h[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]))/(4*b
*d)
```

Maple [A] time = 0.066, size = 130, normalized size = 0.9

$$\frac{1}{d}\left(\frac{e^2}{8b}e^{3\frac{a}{b}}\text{Ei}\left(1, 3\,\text{arccosh}(dx+c)+3\frac{a}{b}\right)+\frac{e^2}{8b}e^{\frac{a}{b}}\text{Ei}\left(1, \text{arccosh}(dx+c)+\frac{a}{b}\right)-\frac{e^2}{8b}e^{-\frac{a}{b}}\text{Ei}\left(1, -\text{arccosh}(dx+c)-\frac{a}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/8*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*E
i(1,arccosh(d*x+c)+a/b)-1/8*e^2/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8*e
^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{b\,\mathrm{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b\,\mathrm{arcosh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arccosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^2x^2}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{2cdx}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c)),x)
```

```
[Out] e**2*(Integral(c**2/(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
acosh(c + d*x)), x) + Integral(2*c*d*x/(a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

3.133 $\int \frac{ce+dex}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}$$

```
[Out] -(e*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b*d) + (
e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(2*b*d)
```

Rubi [A] time = 0.159692, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b])/(2*b*d) + (e*
Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(2*b*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ce+dex}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{x}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d} \\
&= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d} - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}\right.}{a+bx}\right.}{2d} \\
&= -\frac{e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2bd}
\end{aligned}
$$

Mathematica [A] time = 0.0789858, size = 61, normalized size = 0.88

$$
-\frac{e\left(\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\right)}{2bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e*(CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/
b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]))/(2*b*d)
```

Maple [A] time = 0.032, size = 66, normalized size = 1.

$$
\frac{1}{d}\left(\frac{e}{4\,b}\mathrm{e}^{2\,\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{arccosh}\,(dx+c)+2\,\frac{a}{b}\right)-\frac{e}{4\,b}\mathrm{e}^{-2\,\frac{a}{b}}\mathrm{Ei}\left(1,-2\,\mathrm{arccosh}\,(dx+c)-2\,\frac{a}{b}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arccosh(d*x+c)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{b \operatorname{arcosh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dex + ce}{b \operatorname{arcosh}(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a + b \operatorname{acosh}(c + dx)}\, dx + \int \frac{dx}{a + b \operatorname{acosh}(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

```
[Out] e*(Integral(c/(a + b*acosh(c + d*x)), x) + Integral(d*x/(a + b*acosh(c + d*
x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{b \operatorname{arcosh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

3.134 $\int \frac{1}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(b*d)) + (Cosh[a/b]*
SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(b*d)
```

Rubi [A] time = 0.0901135, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5864, 5658, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-1),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(b*d)) + (Cosh[a/b]*
SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(b*d)
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a\right.}{bd}$$

$$= -\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bd} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

Mathematica [A] time = 0.112652, size = 49, normalized size = 0.84

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-1),x]
```

```
[Out] (-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) + Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c + d*x]])/(b*d)
```

Maple [A] time = 0.027, size = 60, normalized size = 1.

$$\frac{1}{d}\left(\frac{1}{2b}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)-\frac{1}{2b}\mathrm{e}^{-\frac{a}{b}}\mathrm{Ei}\left(1,-\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/2/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh(
d*x+c)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b\operatorname{arcosh}(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{acosh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arcosh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccosh(d*x + c) + a), x)
```

3.135 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])), x]/e
```

Rubi [A] time = 0.0641847, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.861364, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])), x]
```

Maple [A] time = 0.18, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\,(a+b\operatorname{arccosh}(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{adex+ace+(bdex+bce)\operatorname{arcosh}(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arccosh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ac+adx+bc\operatorname{acosh}(c+dx)+bdx\operatorname{acosh}(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral(1/(a*c + a*d*x + b*c*acosh(c + d*x) + b*d*x*acosh(c + d*x)), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

3.136 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=263

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}-\frac{e^4\,\text{si}}{}$$

```
[Out] -((e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(8
*b^2*d) + (9*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]
)/(16*b^2*d) + (5*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcCosh[c + d*x]
))/b])/(16*b^2*d) - (e^4*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b]
)/(8*b^2*d) - (9*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x])
)/b])/(16*b^2*d) - (5*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c +
d*x]))/b])/(16*b^2*d)
```

Rubi [A] time = 0.393699, antiderivative size = 259, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8b^2d}+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{16b^2d}+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\cosh^{-1}(c\right.}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^4*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(8*b^2
*d) + (9*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(16*
b^2*d) + (5*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c + d*x]])/(
16*b^2*d) - (e^4*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(8*b^2*d)
- (9*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(16*b^2*
d) - (5*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c + d*x]])/(16*b
^2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\mathrm{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^4\,\mathrm{Subst}\left(\int\left(-\frac{\cosh(x)}{8(a+bx)}-\frac{9\cosh(3x)}{16(a+bx)}-\frac{5\cosh(5x)}{16(a+bx)}\right)\,d}{bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^4\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{8bd} + \frac{(5}{}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}\right.}{8bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8b^2d} + \frac{9e^4\,\mathrm{co}}{}$$

Mathematica [A] time = 2.0537, size = 293, normalized size = 1.11

$$\frac{e^4\left(-16\left(3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/
(a + b*ArcCosh[c + d*x]) - 16*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d
*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]
*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcCosh[c + d*x])]) + 5*(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
 + 5*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]
*CoshIntegral[5*(a/b + ArcCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b +
 ArcCosh[c + d*x]] - 5*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x]
```

```
)] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])]])))/(16*b^2*d)
```

Maple [B] time = 0.204, size = 665, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/32*(-16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)^2*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20*
(d*x+c)^3+5*d*x+5*c)*e^4/b/(a+b*arccosh(d*x+c))-5/32*e^4/b^2*exp(5*a/b)*Ei(
1,5*arccosh(d*x+c)+5*a/b)+3/32*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4/b/(a+b*arccosh
(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*(-(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^4/b/(a+b*arccosh(d*x+c))-1/16*e^4/b^2*ex
p(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16*e^4/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))/(a+b*arccosh(d*x+c))-1/16*e^4/b^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/
b)-3/32*e^4/b*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/32*e^4/b^2*ex
p(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/32*e^4/b*(16*(d*x+c)^5-20*(d*x+c)
^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d
*x+c))-5/32*e^4/b^2*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^5
*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d^
3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*d^5
*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^3*d^
3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d
```

```
*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^4 -
 5*c^4*d*e^4)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*
x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d
^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((5
*d^8*e^4*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 - 10*c^6*e^4 + 5*c^4*e^4 + 10*(
14*c^2*d^6*e^4 - d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 - 3*c*d^5*e^4)*x^5 + 5*(
70*c^4*d^4*e^4 - 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^3*e^4 - 10*c^
3*d^3*e^4 + c*d^3*e^4)*x^3 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^5 + 5*c^6*e^4
- 3*c^4*e^4 + 3*(25*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(25*c^3*d^3*e^4 - 3*c*d^
3*e^4)*x^3 + 3*(25*c^4*d^2*e^4 - 6*c^2*d^2*e^4)*x^2 + 6*(5*c^5*d*e^4 - 2*c^
3*d*e^4)*x)*(d*x + c + 1)*(d*x + c - 1) + 10*(14*c^6*d^2*e^4 - 15*c^4*d^2*e
^4 + 3*c^2*d^2*e^4)*x^2 + (10*d^7*e^4*x^7 + 70*c*d^6*e^4*x^6 + 10*c^7*e^4 -
 13*c^5*e^4 + 4*c^3*e^4 + (210*c^2*d^5*e^4 - 13*d^5*e^4)*x^5 + 5*(70*c^3*d^
4*e^4 - 13*c*d^4*e^4)*x^4 + 2*(175*c^4*d^3*e^4 - 65*c^2*d^3*e^4 + 2*d^3*e^4
)*x^3 + 2*(105*c^5*d^2*e^4 - 65*c^3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (70*c^6*d*
e^4 - 65*c^4*d*e^4 + 12*c^2*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 20*(2*c^7*d*e^4 - 3*c^5*d*e^4 + c^3*d*e^4)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x
^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2
*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b +
 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c
^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int \frac{c^4}{a^2 + 2ab\operatorname{acosh}(c+dx) + b^2\operatorname{acosh}^2(c+dx)}\,dx + \int \frac{d^4x^4}{a^2 + 2ab\operatorname{acosh}(c+dx) + b^2\operatorname{acosh}^2(c+dx)}\,dx + \int \frac{}{a^2 +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**4*(Integral(c**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**4*x**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh
(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x)
+ b**2*acosh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*acosh(c +
 d*x) + b**2*acosh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^2, x)
```

3.137 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\,\mathrm{si}}{}$$

```
[Out] -((e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x])
)/b])/(2*b^2*d) + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c + d*x
]))/b])/(2*b^2*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d
*x]))/b])/(2*b^2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c +
 d*x]))/b])/(2*b^2*d)
```

Rubi [A] time = 0.2984, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(\right.}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x
]])/(2*b^2*d) + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcCosh[c + d*x
]])/(2*b^2*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x
]])/(2*b^2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c + d*x
]])/(2*b^2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^3\,\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2(a+bx)}-\frac{\cosh(4x)}{2(a+bx)}\right)dx, x, \cosh^{-1}\right.}{bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2bd} + \cdots$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh\right.}{2bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2b^2d} + \cdots$$

Mathematica [A] time = 2.16606, size = 230, normalized size = 1.18

$$e^3\left(-3\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\log\left(a+b\cosh^{-1}(c+dx)\right)\right)+\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^3*((-2*b*(c + d*x)^3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(
a + b*ArcCosh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c +
 d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a
+ b*ArcCosh[c + d*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d
*x])] - 3*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a +
 b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x]
)]) - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c + d*x])]))/(2*b^2*d)
```

Maple [B] time = 0.115, size = 418, normalized size = 2.1

$$\frac{1}{d}\left(\frac{e^{3}}{(16\,a+16\,b\text{arccosh}\,(dx+c))\,b}\left(-8\,(dx+c)^{3}\sqrt{dx+c-1}\sqrt{dx+c+1}+4\sqrt{dx+c+1}\sqrt{dx+c-1}\,(dx+c)+8\,(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/16*(-8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c+1)^(1/2)*(
d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3/(a+b*arccosh(d*x+c))/
b-1/4*e^3/b^2*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*(-2*(d*x+c+1)^(1/
2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3/(a+b*arccosh(d*x+c))/b-1/4*e^
3/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8*e^3/b*(2*(d*x+c)^2-1+2*(d
*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4*e^3/b^2*exp
(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/16*e^3/b*(8*(d*x+c)^4-8*(d*x+c)^2+
8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/
2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-1/4*e^3/b^2*exp(-4*a/b)*Ei(1,-4*arccosh(
d*x+c)-4*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 - c^4*e^3 + (15*c^2*d^4*e^3 - d^4
*e^3)*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 - 2*c^2*d^
2*e^3)*x^2 + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^2*d
^3*e^3 - d^3*e^3)*x^3 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 -
 3*c^2*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(3*c^5*d*e^3 - 2*c
^3*d*e^3)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x +
a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c -
 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((4*d^7
*e^3*x^7 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 - 8*c^5*e^3 + 4*c^3*e^3 + 4*(21*c^2
*d^5*e^3 - 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 - 2*c*d^4*e^3)*x^4 + 4*(35*c^
4*d^3*e^3 - 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 2*(2*d^5*e^3*x^5 + 10*c*d^4*e^3
*x^4 + 2*c^5*e^3 - c^3*e^3 + (20*c^2*d^3*e^3 - d^3*e^3)*x^3 + (20*c^3*d^2*e
^3 - 3*c*d^2*e^3)*x^2 + (10*c^4*d*e^3 - 3*c^2*d*e^3)*x)*(d*x + c + 1)*(d*x
```

```
+ c - 1) + 4*(21*c^5*d^2*e^3 - 20*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (8*d^6*e
^3*x^6 + 48*c*d^5*e^3*x^5 + 8*c^6*e^3 - 10*c^4*e^3 + 3*c^2*e^3 + 10*(12*c^2
*d^4*e^3 - d^4*e^3)*x^4 + 40*(4*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(40*c^4*d^
2*e^3 - 20*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(24*c^5*d*e^3 - 20*c^3*d*e^3 + 3*
c*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 4*(7*c^6*d*e^3 - 10*c^4*d
*e^3 + 3*c^2*d*e^3)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)
*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d
*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*
c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 +
4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)
*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*
log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*arc
cosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{a^2 + 2ab\,\text{acosh}\,(c+dx) + b^2\,\text{acosh}^2\,(c+dx)}\,dx + \int \frac{d^3x^3}{a^2 + 2ab\,\text{acosh}\,(c+dx) + b^2\,\text{acosh}^2\,(c+dx)}\,dx + \int \frac{}{a^2 + 2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh
```

```
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2
*acosh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^3}{(b \operatorname{arcosh}(dx + c) + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^2, x)
```

3.138 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=191

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4b^2d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}-\frac{3e^2\sinh\left(\right.}{}$$

```
[Out] -((e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(4
*b^2*d) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]
)/(4*b^2*d) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(4*b
^2*d) - (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/
(4*b^2*d)
```

Rubi [A] time = 0.275911, antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{4b^2d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(4*b^2
*d) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(4*b
^2*d) - (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(4*b^2*d) - (3
*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(4*b^2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} - \frac{3\cosh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-}\right.}{bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{4bd} + \frac{(3}{}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}\right.}{4bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{4b^2d} + \frac{3e^2\,\text{cc}}{}$$

Mathematica [A] time = 1.70244, size = 150, normalized size = 0.79

$$\frac{e^2\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\right.}{4b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(
a + b*ArcCosh[c + d*x]) + Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]] - 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcC
osh[c + d*x])]))/(4*b^2*d)
```

Maple [B] time = 0.099, size = 374, normalized size = 2.

$$\frac{1}{d}\left(\frac{e^2}{8\,b\,(a+b\text{arccosh}\,(dx+c))}\left(-4\,(dx+c)^2\sqrt{dx+c-1}\sqrt{dx+c+1}+\sqrt{dx+c-1}\sqrt{dx+c+1}+4\,(dx+c)^3-3\,dx-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/8*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2/b/(a+b*arccosh(d*x+c))-3/8*e^2/b^2*e
xp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+d*x+c)*e^2/b/(a+b*arccosh(d*x+c))-1/8*e^2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c
)+a/b)-1/8*e^2/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))-1/8*e^2/b^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8*e^2/b*(4*(d*x+c)^3-3
*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2))/(a+b*arccosh(d*x+c))-3/8*e^2/b^2*exp(-3*a/b)*Ei(1,-3*arccosh(d*x
+c)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 - c^3*e^2 + (10*c^2*d^3*e^2 - d^3
*e^2)*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)*x^2 + (d^4*e^2*x^4 + 4*c*d^3*e^2
*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 -
 c*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^2 - 3*c^2*d*e
^2)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*
d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^
2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*l
og(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((3*d^6*e^2*x
^6 + 18*c*d^5*e^2*x^5 + 3*c^6*e^2 - 6*c^4*e^2 + 3*(15*c^2*d^4*e^2 - 2*d^4*e
^2)*x^4 + 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 - 2*c*d^3*e^2)*x^3 + (3*d^4*e^2*x^4
 + 12*c*d^3*e^2*x^3 + 3*c^4*e^2 - c^2*e^2 + (18*c^2*d^2*e^2 - d^2*e^2)*x^2
+ 2*(6*c^3*d*e^2 - c*d*e^2)*x)*(d*x + c + 1)*(d*x + c - 1) + 3*(15*c^4*d^2*
e^2 - 12*c^2*d^2*e^2 + d^2*e^2)*x^2 + (6*d^5*e^2*x^5 + 30*c*d^4*e^2*x^4 + 6
*c^5*e^2 - 7*c^3*e^2 + (60*c^2*d^3*e^2 - 7*d^3*e^2)*x^3 + 2*c*e^2 + 3*(20*c
^3*d^2*e^2 - 7*c*d^2*e^2)*x^2 + (30*c^4*d*e^2 - 21*c^2*d*e^2 + 2*d*e^2)*x)*
```

```
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 6*(3*c^5*d*e^2 - 4*c^3*d*e^2 + c*d*e^
2)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3
*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x
+ c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^
2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b
^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) +
(c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^
2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*
x + c + 1)*sqrt(d*x + c - 1) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^2\,\text{arcosh}\left(dx + c\right)^2 + 2\,ab\,\text{arcosh}\left(dx + c\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arccosh(d*x + c)^2 + 2*
a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a^2 + 2ab\,\text{acosh}\left(c + dx\right) + b^2\,\text{acosh}^2\left(c + dx\right)}\,dx + \int \frac{d^2x^2}{a^2 + 2ab\,\text{acosh}\left(c + dx\right) + b^2\,\text{acosh}^2\left(c + dx\right)}\,dx + \int \frac{}{a^2 + 2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d
*x)**2), x))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\,\mathrm{arcosh}\,(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^2, x)
```

3.139 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

```
[Out] -((e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c
+ d*x]))) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/
(b^2*d) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(b
^2*d)
```

Rubi [A] time = 0.148076, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c
+ d*x]))) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(b
^2*d) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(b^2*d
)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\cosh^{-1}\right.}{bd}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^2d} - \frac{e\sinh}{}$$

Mathematica [A] time = 0.798728, size = 110, normalized size = 1.

$$\frac{e\left(-\frac{b\left(c^2+c(2dx-1)+dx(dx-1)\right)}{\sqrt{\frac{c+dx-1}{c+dx+1}}\left(a+b\cosh^{-1}(c+dx)\right)}+\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\textbf{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*(c^2 + d*x*(-1 + d*x) + c*(-1 + 2*d*x)))/(Sqrt[(-1 + c + d*x)/(1 +
 c + d*x)]*(a + b*ArcCosh[c + d*x]))) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b +
 ArcCosh[c + d*x])] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])
]))/(b^2*d)
```

Maple [A] time = 0.048, size = 170, normalized size = 1.6

$$\frac{1}{d}\left(\frac{e}{(4a+4b\,\text{arccosh}\,(dx+c))\,b}\left(-2\sqrt{dx+c+1}\sqrt{dx+c-1}\,(dx+c)+2\,(dx+c)^2-1\right)-\frac{e}{2b^2}e^{2\frac{a}{b}}\text{Ei}\left(1,2\,\text{arccosh}\,(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e/(a+b*
arccosh(d*x+c))/b-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4*e/b
*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x
+c))-1/2*e/b^2*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e - c^2*e + (6*c^2*d^2*e - d^2*e)*x^2 + (
d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e - c*e + (3*c^2*d*e - d*e)*x)*sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + 2*(2*c^3*d*e - c*d*e)*x)/(a*b*d^3*x^2 + 2*a*b*c*d
^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2
*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^5*e - 4
*c^3*e + 4*(5*c^2*d^3*e - d^3*e)*x^3 + 2*(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2
*d*e*x + c^3*e)*(d*x + c + 1)*(d*x + c - 1) + 4*(5*c^3*d^2*e - 3*c*d^2*e)*x
^2 + (4*d^4*e*x^4 + 16*c*d^3*e*x^3 + 4*c^4*e - 4*c^2*e + 4*(6*c^2*d^2*e - d
^2*e)*x^2 + 8*(2*c^3*d*e - c*d*e)*x + e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + 2*c*e + 2*(5*c^4*d*e - 6*c^2*d*e + d*e)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x
^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2
*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b +
 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c
^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dex + ce}{b^2 \operatorname{arcosh}(dx + c)^2 + 2\,ab \operatorname{arcosh}(dx + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a^2 + 2ab\operatorname{acosh}(c+dx) + b^2\operatorname{acosh}^2(c+dx)}\,dx + \int \frac{dx}{a^2 + 2ab\operatorname{acosh}(c+dx) + b^2\operatorname{acosh}^2(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e*(Integral(c/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b\operatorname{arcosh}(dx+c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^2, x)
```

3.140 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

```
[Out] -((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c + d*x]))) +
(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(b^2*d) - (Sinh[a/b]*S
inhIntegral[(a + b*ArcCosh[c + d*x])/b])/(b^2*d)
```

Rubi [A] time = 0.252003, antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5864, 5656, 5781, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-2),x]
```

```
[Out] -((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c + d*x]))) +
(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(b^2*d) - (Sinh[a/b]*SinhI
ntegral[a/b + ArcCosh[c + d*x]])/(b^2*d)
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd} - \text{si}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d} - \frac{\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\right.}{b^2}$$

Mathematica [A] time = 0.990587, size = 143, normalized size = 1.46

$$\frac{\sqrt{\frac{c+dx-1}{c+dx+1}}\coth\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\left(\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\log\left(a+b\right.\right.}{b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])) + Log
[1 + (b*ArcCosh[c + d*x])/a] + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Coth[ArcC
osh[c + d*x]/2]*(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] - Log[a + b
*ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]))/(b^2*
d)
```

Maple [A] time = 0.041, size = 139, normalized size = 1.4

$$\frac{1}{d}\left(\frac{1}{2b\left(a+b\text{arccosh}\left(dx+c\right)\right)}\left(-\sqrt{dx+c-1}\sqrt{dx+c+1}+dx+c\right)-\frac{1}{2b^2}e^{\frac{a}{b}}\text{Ei}\left(1,\text{arccosh}\left(dx+c\right)+\frac{a}{b}\right)-\frac{}{2b\left(a+ba\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)/b/(a+b*arccosh(d*x+c))-1/
2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2))/(a+b*arccosh(d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x
+ (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)
*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)) + integrate((d^4*x^4 + 4*c*d^3*x^3 + c^4 + (d^2*x^2 + 2*c*d*
x + c^2 + 1)*(d*x + c + 1)*(d*x + c - 1) + 2*(3*c^2*d^2 - d^2)*x^2 + (2*d^3
*x^3 + 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d - d)*x - c)*sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) - 2*c^2 + 4*(c^3*d - c*d)*x + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3
+ 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*
(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*
d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1
)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^2 \operatorname{arcosh}(dx+c)^2 + 2\,ab \operatorname{arcosh}(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcosh}(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-2), x)
```

3.141 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2} dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2},x\right)}{e}$$

[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])^2), x]/e

---

Rubi [A] time = 0.0617535, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^2), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^2} dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^2} dx, x, c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 7.33278, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2), x]
```

Maple [A] time = 0.167, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\,(a+b\mathrm{arccosh}\,(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*
e*x^2 + (3*c^2*d^2*e - d^2*e)*a*b*x + (c^3*d*e - c*d*e)*a*b + (a*b*d^3*e*x^
2 + 2*a*b*c*d^2*e*x + a*b*c^2*d*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b
^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*c^2*d^2*e - d^2*e)*b^2*x + (c^3*d*e -
 c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*c*d^2*e*x + b^2*c^2*d*e)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
 + integrate((2*(d*x + c + 1)*(d*x + c)*(d*x + c - 1) + (2*d^2*x^2 + 4*c*d*
x + 2*c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))/(a*b*d^6*e*x^6 + 6*a*b*
c*d^5*e*x^5 + (15*c^2*d^4*e - 2*d^4*e)*a*b*x^4 + 4*(5*c^3*d^3*e - 2*c*d^3*e
)*a*b*x^3 + (15*c^4*d^2*e - 12*c^2*d^2*e + d^2*e)*a*b*x^2 + 2*(3*c^5*d*e -
4*c^3*d*e + c*d*e)*a*b*x + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 + 6*a*b*c^2*d
^2*e*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d*x + c + 1)*(d*x + c - 1) + (c^6*
e - 2*c^4*e + c^2*e)*a*b + 2*(a*b*d^5*e*x^5 + 5*a*b*c*d^4*e*x^4 + (10*c^2*d
```

```
^3*e - d^3*e)*a*b*x^3 + (10*c^3*d^2*e - 3*c*d^2*e)*a*b*x^2 + (5*c^4*d*e - 3
*c^2*d*e)*a*b*x + (c^5*e - c^3*e)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ (b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e - 2*d^4*e)*b^2*x^4 + 4
*(5*c^3*d^3*e - 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e - 12*c^2*d^2*e + d^2*e)*
b^2*x^2 + 2*(3*c^5*d*e - 4*c^3*d*e + c*d*e)*b^2*x + (b^2*d^4*e*x^4 + 4*b^2*
c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 + 4*b^2*c^3*d*e*x + b^2*c^4*e)*(d*x + c +
 1)*(d*x + c - 1) + (c^6*e - 2*c^4*e + c^2*e)*b^2 + 2*(b^2*d^5*e*x^5 + 5*b^
2*c*d^4*e*x^4 + (10*c^2*d^3*e - d^3*e)*b^2*x^3 + (10*c^3*d^2*e - 3*c*d^2*e)
*b^2*x^2 + (5*c^4*d*e - 3*c^2*d*e)*b^2*x + (c^5*e - c^3*e)*b^2)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{a^2dex + a^2ce + \left(b^2dex + b^2ce\right)\operatorname{arcosh}\left(dx + c\right)^2 + 2\left(abdex + abce\right)\operatorname{arcosh}\left(dx + c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arccosh(d*x + c)^2
+ 2*(a*b*d*e*x + a*b*c*e)*arccosh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{acosh}\left(c+dx\right)+2abdx\operatorname{acosh}\left(c+dx\right)+b^2c\operatorname{acosh}^2\left(c+dx\right)+b^2dx\operatorname{acosh}^2\left(c+dx\right)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*acosh(c + d*x) + 2*a*b*d*x*acosh(c
+ d*x) + b**2*c*acosh(c + d*x)**2 + b**2*d*x*acosh(c + d*x)**2), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)(b\operatorname{arcosh}(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2), x)
```

3.142 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=327

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{16b^3d}-\frac{27e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}+\ldots$$

```
[Out] -(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (5
*e^4*(c + d*x)^5)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e^4*CoshIntegral[(a
 + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(16*b^3*d) - (27*e^4*CoshIntegral[(3*(
a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(32*b^3*d) - (25*e^4*CoshIntegra
l[(5*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(5*a)/b])/(32*b^3*d) + (e^4*Cosh[a/b
]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(16*b^3*d) + (27*e^4*Cosh[(3*a)
/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(32*b^3*d) + (25*e^4*Cosh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c + d*x]))/b])/(32*b^3*d)
```

Rubi [A] time = 1.11063, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 26, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{16b^3d}-\frac{27e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{32b^3d}-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\cosh\ldots\right)}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (5
*e^4*(c + d*x)^5)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e^4*CoshIntegral[a/
b + ArcCosh[c + d*x]]*Sinh[a/b])/(16*b^3*d) - (27*e^4*CoshIntegral[(3*a)/b
+ 3*ArcCosh[c + d*x]]*Sinh[(3*a)/b])/(32*b^3*d) - (25*e^4*CoshIntegral[(5*a
)/b + 5*ArcCosh[c + d*x]]*Sinh[(5*a)/b])/(32*b^3*d) + (e^4*Cosh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]])/(16*b^3*d) + (27*e^4*Cosh[(3*a)/b]*SinhInte
gral[(3*a)/b + 3*ArcCosh[c + d*x]])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*Sinh
Integral[(5*a)/b + 5*ArcCosh[c + d*x]])/(32*b^3*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
```

```
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\left(2e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x,\right.}{bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\cos\right.}$$

Mathematica [A] time = 1.27099, size = 323, normalized size = 0.99

$$\frac{e^4\left(-\frac{16b^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 48\left(\sinh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \sinh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^2 + (16*b*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[c
+ d*x]) + 48*(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] + CoshIntegral
[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c + d*x]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])
+ 25*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*
(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c +
 d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*Si
nhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(32*b^3*d)
```

Maple [B] time = 0.237, size = 993, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/64*(-16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20
*(d*x+c)^3+5*d*x+5*c)*e^4*(5*b*arccosh(d*x+c)+5*a-b)/b^2/(b^2*arccosh(d*x+c
)^2+2*a*b*arccosh(d*x+c)+a^2)+25/64*e^4/b^3*exp(5*a/b)*Ei(1,5*arccosh(d*x+c
)+5*a/b)-3/64*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(3*b*arccosh(d*x+c)+3*a-b)/b^2/
(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+27/64*e^4/b^3*exp(3*a/b)*Ei
(1,3*arccosh(d*x+c)+3*a/b)-1/32*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^
4*(b*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2
)+1/32*e^4/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/32*e^4/b*(d*x+c+(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/32*e^4/b^2*(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/32*e^4/b^3*exp(-a/b)*Ei(1,
-arccosh(d*x+c)-a/b)-3/64*e^4/b*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^2-9/64*e^4/b^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-27/64*e^4/b
^3*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/64*e^4/b*(16*(d*x+c)^5-20*(d
*x+c)^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arcc
osh(d*x+c))^2-5/64*e^4/b^2*(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c)^4*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
```

```
/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-25/64*e^4/b^3*exp
(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^3\,\text{arcosh}\,(dx + c)^3 + 3\,ab^2\,\text{arcosh}\,(dx + c)^2 + 3\,a^2b\,\text{arcosh}\,(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*
arccosh(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{(b \operatorname{arcosh}(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^3, x)
```

3.143 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}+\cdots$$

```
[Out] -(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) -
(2*e^3*(c + d*x)^4)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (e^3*CoshIntegral[(2
*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[
(4*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(4*a)/b])/(b^3*d) + (e^3*Cosh[(2*a)/b]
*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(2*b^3*d) + (e^3*Cosh[(4*a)/
b]*SinhIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/(b^3*d)
```

Rubi [A] time = 0.898627, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{b^3d}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh\cdots\right)}{2b^3d}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) -
(2*e^3*(c + d*x)^4)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (e^3*CoshIntegral[(2
*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4
*a)/b + 4*ArcCosh[c + d*x]]*Sinh[(4*a)/b])/(b^3*d) + (e^3*Cosh[(2*a)/b]*Sin
hIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*Sin
hIntegral[(4*a)/b + 4*ArcCosh[c + d*x]])/(b^3*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\left(3e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c\right.}{2bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\cosh\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\cosh\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\cosh\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\cosh\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\cosh\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\cosh\right.}$$

Mathematica [A] time = 0.562718, size = 186, normalized size = 0.73

$$\frac{e^3\left(-\frac{b^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - \sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - 2\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) + \cosh\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^3*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcCo
sh[c + d*x])^2) + (b*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c + d*
x]) - CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegr
al[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcCosh[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[
c + d*x])]))/(2*b^3*d)
```

Maple [B] time = 0.138, size = 624, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/32*(-8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c+1)^(1/2)*
(d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(4*b*arccosh(d*x+c)+
4*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/2*e^3/b^3*exp(
4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)-1/16*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2
)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*
x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4*e^3/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+
c)+2*a/b)-1/16*e^3/b*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+
c))/(a+b*arccosh(d*x+c))^2-1/8*e^3/b^2*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*
x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4*e^3/b^3*exp(-2*a/b)*Ei(1,-2*
arccosh(d*x+c)-2*a/b)-1/32*e^3/b*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+
b*arccosh(d*x+c))^2-1/8*e^3/b^2*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b
*arccosh(d*x+c))-1/2*e^3/b^3*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^3\,\text{arcosh}\,(dx+c)^3 + 3\,ab^2\,\text{arcosh}\,(dx+c)^2 + 3\,a^2b\,\text{arcosh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*arc
cosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int\frac{c^3}{a^3 + 3a^2b\,\text{acosh}\,(c+dx) + 3ab^2\,\text{acosh}^2\,(c+dx) + b^3\,\text{acosh}^3\,(c+dx)}\,dx + \int\frac{d^3x^3}{a^3 + 3a^2b\,\text{acosh}\,(c+dx) + 3ab^2\,\text{aco}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c +
 d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dex+ce)^3}{(b\,\text{arcosh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^3, x)
```

3.144 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=252

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^3d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^3d}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^3d}+\frac{9e^2\cosh}{}$$

```
[Out] -(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (e^2*(c + d*x))/(b^2*d*(a + b*ArcCosh[c + d*x])) - (3*e^2
*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e^2*CoshIntegral[(a + b
*ArcCosh[c + d*x])/b]*Sinh[a/b])/(8*b^3*d) - (9*e^2*CoshIntegral[(3*(a + b*
ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(8*b^3*d) + (e^2*Cosh[a/b]*SinhIntegra
l[(a + b*ArcCosh[c + d*x])/b])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegra
l[(3*(a + b*ArcCosh[c + d*x]))/b])/(8*b^3*d)
```

Rubi [A] time = 0.799218, antiderivative size = 311, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301, 5658}

$$-\frac{9e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8b^3d}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^3d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (e^2*(c + d*x))/(b^2*d*(a + b*ArcCosh[c + d*x])) - (3*e^2
*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (9*e^2*CoshIntegral[a/b
+ ArcCosh[c + d*x]]*Sinh[a/b])/(8*b^3*d) + (e^2*CoshIntegral[(a + b*ArcCosh
[c + d*x])/b]*Sinh[a/b])/(b^3*d) - (9*e^2*CoshIntegral[(3*a)/b + 3*ArcCosh[
c + d*x]]*Sinh[(3*a)/b])/(8*b^3*d) + (9*e^2*Cosh[a/b]*SinhIntegral[a/b + Ar
cCosh[c + d*x]])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcCosh[c + d*x]])/(8*b^3*d) - (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c
 + d*x])/b])/(b^3*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+\right.}{bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\cos\right.}$$

Mathematica [A] time = 0.664219, size = 223, normalized size = 0.88

$$e^2\left(-\frac{4b^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 9\left(\sinh\left(\frac{a}{b}\right)\left(-\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) + \text{co}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^2*((-4*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^2 + (4*b*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c + d*
x]) + 8*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 8*Cosh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]] + 9*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*
Sinh[a/b]) - CoshIntegral[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[
a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a
/b + ArcCosh[c + d*x])])))/(8*b^3*d)
```

Maple [B] time = 0.116, size = 557, normalized size = 2.2

$$\frac{1}{d}\left(-\frac{e^2\,(3\,b\operatorname{arccosh}(dx+c)+3\,a-b)}{16\,b^2\left(b^2\,(\operatorname{arccosh}(dx+c))^2+2\,ab\operatorname{arccosh}(dx+c)+a^2\right)}\left(-4\,(dx+c)^2\,\sqrt{dx+c-1}\sqrt{dx+c+1}+\sqrt{dx+c-1}\sqrt{dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/16*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(3*b*arccosh(d*x+c)+3*a-b)/b^2/(b^
2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+9/16*e^2/b^3*exp(3*a/b)*Ei(1,3
*arccosh(d*x+c)+3*a/b)-1/16*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^2*(b
*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/
16*e^2/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16*e^2/b*(d*x+c+(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/16*e^2/b^2*(d*x+c+(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/16*e^2/b^3*exp(-a/b)*Ei(1,-arc
cosh(d*x+c)-a/b)-1/16*e^2/b*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2
-3/16*e^2/b^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/16*e^2/b^3*ex
p(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^3\operatorname{arcosh}\left(dx + c\right)^3 + 3\,ab^2\operatorname{arcosh}\left(dx + c\right)^2 + 3\,a^2b\operatorname{arcosh}\left(dx + c\right) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arccosh(d*x + c)^3 + 3*
a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int\frac{c^2}{a^3 + 3a^2b\operatorname{acosh}\left(c + dx\right) + 3ab^2\operatorname{acosh}^2\left(c + dx\right) + b^3\operatorname{acosh}^3\left(c + dx\right)}\,dx + \int\frac{d^2}{a^3 + 3a^2b\operatorname{acosh}\left(c + dx\right) + 3ab^2\operatorname{ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)*
*2 + b**3*acosh(c + d*x)**3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dex + ce\right)^2}{\left(b\operatorname{arcosh}\left(dx + c\right) + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^3, x)
```

3.145 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{}{2b^2d\left(a+b\,\text{c}\right.}$$

```
[Out] -(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCosh[c
 + d*x])^2) + e/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d
*(a + b*ArcCosh[c + d*x])) - (e*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b
]*Sinh[(2*a)/b])/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[
c + d*x]))/b])/(b^3*d)
```

Rubi [A] time = 0.50852, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301, 5676}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCosh[c
 + d*x])^2) + e/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d
*(a + b*ArcCosh[c + d*x])) - (e*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*
Sinh[(2*a)/b])/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[
c + d*x]])/(b^3*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}\right.}$$

Mathematica [A] time = 0.325081, size = 127, normalized size = 0.78

$$\frac{e\left(-\frac{b^2\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - 2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) + 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) + \frac{b(1-}{a+b\cosh}\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c
 + d*x])^2) + (b*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x]) - 2*CoshInte
gral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcCosh[c + d*x])]))/(2*b^3*d)
```

Maple [A] time = 0.056, size = 254, normalized size = 1.6

$$\frac{1}{d}\left(-\frac{e\left(2\,b\mathrm{arccosh}\left(dx+c\right)+2\,a-b\right)}{8\,b^{2}\left(b^{2}\left(\mathrm{arccosh}\left(dx+c\right)\right)^{2}+2\,ab\mathrm{arccosh}\left(dx+c\right)+a^{2}\right)}\left(-2\,\sqrt{dx+c+1}\sqrt{dx+c-1}\left(dx+c\right)+2\,\left(dx+c\right)^{2}-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/8*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*b
*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+
1/2*e/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8*e/b*(2*(d*x+c)^2-1+2*
(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/4*e/b^2*(
2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c
))-1/2*e/b^3*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dex+ce}{b^{3}\,\mathrm{arcosh}\left(dx+c\right)^{3}+3\,ab^{2}\,\mathrm{arcosh}\left(dx+c\right)^{2}+3\,a^{2}b\,\mathrm{arcosh}\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2
 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3ab^2\operatorname{acosh}^2(c+dx) + b^3\operatorname{acosh}^3(c+dx)}\,dx + \int \frac{dx}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3ab^2\operatorname{acos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2
+ b**3*acosh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*acosh(c + d*x
) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^3, x)
```

3.146 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=132

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{\sqrt{c+dx-1}\sqrt{c+d}}{2bd\left(a+b\cosh^{-1}(c-\right.}$$

```
[Out] -(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCosh[c + d*x])^2)
- (c + d*x)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (CoshIntegral[(a + b*ArcCo
sh[c + d*x])/b]*Sinh[a/b])/(2*b^3*d) + (Cosh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c + d*x])/b])/(2*b^3*d)
```

Rubi [A] time = 0.259714, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5864, 5656, 5775, 5658, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{\sqrt{c+dx-1}\sqrt{c+d}}{2bd\left(a+b\cosh^{-1}(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-3),x]
```

```
[Out] -(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCosh[c + d*x])^2)
- (c + d*x)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (CoshIntegral[(a + b*ArcCo
sh[c + d*x])/b]*Sinh[a/b])/(2*b^3*d) + (Cosh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c + d*x])/b])/(2*b^3*d)
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
```

```
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b\cosh^{-1}(x)}\,d\right.}{2b^2d}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,\right.}{2b}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh}{x}\right.}{}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{\textbf{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\text{si}}{2b^3d}$$

Mathematica [A] time = 0.40856, size = 109, normalized size = 0.83

$$-\frac{\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) - \cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \frac{b\left(ac+adx+b\sqrt{c+dx-1}\sqrt{c+dx+1}+b(c+dx)\cosh^{-1}(c+dx)\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-3),x]
```

```
[Out] -((b*(a*c + a*d*x + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*(c + d*x)*Ar
cCosh[c + d*x]))/(a + b*ArcCosh[c + d*x])^2 + CoshIntegral[a/b + ArcCosh[c
+ d*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(2*b^3*
d)
```

Maple [A] time = 0.059, size = 207, normalized size = 1.6

$$\frac{1}{d}\left(-\frac{b\operatorname{arccosh}(dx+c)+a-b}{4\,b^2\left(b^2\left(\operatorname{arccosh}(dx+c)\right)^2+2\,ab\operatorname{arccosh}(dx+c)+a^2\right)}\left(-\sqrt{dx+c-1}\sqrt{dx+c+1}+dx+c\right)+\frac{1}{4\,b^3}e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcco}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/4*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b*arccosh(d*x+c)+a-b)/b
^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4/b^3*exp(a/b)*Ei(1,ar
ccosh(d*x+c)+a/b)-1/4/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccos
h(d*x+c))^2-1/4/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*
x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b^3\operatorname{arcosh}(dx+c)^3+3\,ab^2\operatorname{arcosh}(dx+c)^2+3\,a^2b\operatorname{arcosh}(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*a
rccosh(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\text{acosh}\,(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{arcosh}\,(dx + c) + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3), x)
```

3.147 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])^3), x]/e
```

Rubi [A] time = 0.0635637, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^3), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 1.68012, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3), x]
```

Maple [A] time = 0.172, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\,(a+b\,\mathrm{arccosh}\,(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{a^3dex+a^3ce+\left(b^3dex+b^3ce\right)\mathrm{arcosh}\,(dx+c)^3+3\left(ab^2dex+ab^2ce\right)\mathrm{arcosh}\,(dx+c)^2+3\left(a^2bdex+a^2bce\right)\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arccosh(d*x + c)^3
+ 3*(a*b^2*d*e*x + a*b^2*c*e)*arccosh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c
*e)*arccosh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^3c+a^3dx+3a^2bc\operatorname{acosh}(c+dx)+3a^2bdx\operatorname{acosh}(c+dx)+3ab^2c\operatorname{acosh}^2(c+dx)+3ab^2dx\operatorname{acosh}^2(c+dx)+b^3c\operatorname{acosh}^3(c+dx)+b^3dx\operatorname{acosh}^3(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*acosh(c + d*x) + 3*a**2*b*d*x*ac
osh(c + d*x) + 3*a*b**2*c*acosh(c + d*x)**2 + 3*a*b**2*d*x*acosh(c + d*x)**
2 + b**3*c*acosh(c + d*x)**3 + b**3*d*x*acosh(c + d*x)**3), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3), x)
```

3.148 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=431

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{48b^4d}+\frac{27e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{96b^4d}-$$

```
[Out] -(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2)
- (5*e^4*(c + d*x)^5)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (2*e^4*Sqrt[-1
 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x]))
 - (25*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(6*b^3*d*(a +
b*ArcCosh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])
/b])/(48*b^4*d) + (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c +
d*x]))/b])/(32*b^4*d) + (125*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcCo
sh[c + d*x]))/b])/(96*b^4*d) - (e^4*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c
 + d*x])/b])/(48*b^4*d) - (27*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcC
osh[c + d*x]))/b])/(32*b^4*d) - (125*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*(a +
 b*ArcCosh[c + d*x]))/b])/(96*b^4*d)
```

Rubi [A] time = 1.10399, antiderivative size = 427, normalized size of antiderivative = 0.99, number of steps used = 24, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{48b^4d}+\frac{27e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{32b^4d}+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\cos\right.}{96b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2)
- (5*e^4*(c + d*x)^5)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (2*e^4*Sqrt[-1
 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x]))
 - (25*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(6*b^3*d*(a +
b*ArcCosh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])
```

```
/(48*b^4*d) + (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*
x]])/(32*b^4*d) + (125*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c
 + d*x]])/(96*b^4*d) - (e^4*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])
/(48*b^4*d) - (27*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*
x]])/(32*b^4*d) - (125*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c
 + d*x]])/(96*b^4*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx=\frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}-\frac{\left(4e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c\right.}{3bd}$$

$$=-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{5e^4(c}{6b^2d\left(a+b\cos\right.}$$

$$=-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{5e^4(c}{6b^2d\left(a+b\cos\right.}$$

$$=-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{5e^4(c}{6b^2d\left(a+b\cos\right.}$$

$$=-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{5e^4(c}{6b^2d\left(a+b\cos\right.}$$

$$=-\frac{e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{5e^4(c}{6b^2d\left(a+b\cos\right.}$$

Mathematica [A] time = 1.99598, size = 424, normalized size = 0.98

$$\frac{e^4\left(-\frac{32b^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{16b^2\left(4(c+dx)^3-5(c+dx)^5\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}+384\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^4*((-32*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^3 + (16*b^2*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[
c + d*x])^2 - (16*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-12*(c + d*x)^2 +
 25*(c + d*x)^4))/(a + b*ArcCosh[c + d*x]) + 384*(Cosh[a/b]*CoshIntegral[a/
b + ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]) - 5
44*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh
[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]) + 125*
(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + 5*Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
cCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - 5*Si
nh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[(5*a)/b]*SinhIn
tegral[5*(a/b + ArcCosh[c + d*x])])))/(96*b^4*d)
```

Maple [B] time = 0.272, size = 1375, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/192*(-16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20
*(d*x+c)^3+5*d*x+5*c)*e^4*(25*b^2*arccosh(d*x+c)^2+50*a*b*arccosh(d*x+c)-5*
arccosh(d*x+c)*b^2+25*a^2-5*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*ar
ccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-125/192*e^4/b^4*exp(5*a/b)*Ei(1,
5*arccosh(d*x+c)+5*a/b)+1/64*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(9*b^2*arccosh(d
*x+c)^2+18*a*b*arccosh(d*x+c)-3*arccosh(d*x+c)*b^2+9*a^2-3*a*b+2*b^2)/b^3/(
b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2
7/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/96*(-(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+d*x+c)*e^4*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-arcc
osh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x
+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/96*e^4/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)
+a/b)-1/48*e^4/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^3-1/96*e^4/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+
c))^2-1/96*e^4/b^3*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x
+c))-1/96*e^4/b^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/32*e^4/b*(4*(d*x+c)
^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-3/64*e^4/b^2*(4*(d*x+c)^3-3*d*x-3*c+4
*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
```

```
/(a+b*arccosh(d*x+c))^2-9/64*e^4/b^3*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(
d*x+c))-27/64*e^4/b^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/96*e^4/b*
(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d
*x+5*c-12*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))/(a+b*arccosh(d*x+c))^3-5/192*e^4/b^2*(16*(d*x+c)^5-20*(d*x+c)^3+1
6*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^2-25/192*e^4/b^3*(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-125/192*e^4/b^4*exp(-5*a
/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^4\,\text{arcosh}\,(dx+c)^4 + 4\,ab^3\,\text{arcosh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arcosh}\,(dx+c)^2 + 4\,a^3b\,\text{arcosh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^
2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{(b \operatorname{arcosh}(dx + c) + a)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^4, x)
```

3.149 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^4} dx$

Optimal. Leaf size=360

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{4e^3}{}$$

```
[Out] -(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) -
(2*e^3*(c + d*x)^4)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^3*Sqrt[-1 + c
 + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x])) - (8*
e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCo
sh[c + d*x])) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x])
)/b])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c + d
*x]))/b])/(3*b^4*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c +
 d*x]))/b])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh
[c + d*x]))/b])/(3*b^4*d)
```

Rubi [A] time = 0.887219, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301}

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{3b^4d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}\right.}{3b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) -
(2*e^3*(c + d*x)^4)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^3*Sqrt[-1 + c
 + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x])) - (8*
e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCo
sh[c + d*x])) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x
]])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcCosh[c + d
*x]])/(3*b^4*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d
```

```
*x]])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c +
 d*x]])/(3*b^4*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+\right.}{bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{2e^3(c}{3b^2d\left(a+b\,\text{c}\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{2e^3(c}{3b^2d\left(a+b\,\text{c}\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{2e^3(c}{3b^2d\left(a+b\,\text{c}\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{2e^3(c}{3b^2d\left(a+b\,\text{c}\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{2e^3(c}{3b^2d\left(a+b\,\text{c}\right.}$$

Mathematica [A] time = 1.31323, size = 330, normalized size = 0.92

$$\frac{e^3\left(-\frac{2b^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{b^2\left(3(c+dx)^2-4(c+dx)^4\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - 30\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^3*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^3 + (b^2*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c +
d*x])^2 - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-3*(c + d*x) + 8*(c +
d*x)^3))/(a + b*ArcCosh[c + d*x]) + 6*Log[a + b*ArcCosh[c + d*x]] - 30*(Cos
h[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a + b*ArcCosh[c +
 d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]) + 8*(4*Cos
h[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*CoshInt
egral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a + b*ArcCosh[c + d*x]] - 4*Sinh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])] - Sinh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

Maple [B] time = 0.164, size = 860, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(-8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c+1)^(1/2)*(
d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(8*b^2*arccosh(d*x+c)
^2+16*a*b*arccosh(d*x+c)-2*arccosh(d*x+c)*b^2+8*a^2-2*a*b+b^2)/b^3/(b^3*arc
cosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2/3*e^3/
b^4*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/24*(-2*(d*x+c+1)^(1/2)*(d*x+c
-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b^2*arccosh(d*x+c)^2+4*a*b*arccosh(
d*x+c)-arccosh(d*x+c)*b^2+2*a^2-a*b+b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*
arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*
arccosh(d*x+c)+2*a/b)-1/24*e^3/b*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)
^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/24*e^3/b^2*(2*(d*x+c)^2-1+2*(d*x+c
+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/12*e^3/b^3*(2*(
d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-
1/6*e^3/b^4*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/48*e^3/b*(8*(d*x+c)
^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^3-1/24*e^3/b^2*(8*(d*x+c)
^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^2-1/6*e^3/b^3*(8*(d*x+c)^
4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-2/3*e^3/b^4*exp(-4*a/b)*Ei
(1,-4*arccosh(d*x+c)-4*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^4\operatorname{arcosh}(dx+c)^4 + 4\,ab^3\operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2\operatorname{arcosh}(dx+c)^2 + 4\,a^3b\operatorname{arcosh}(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*arc
cosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2
 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{(b\operatorname{arcosh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^4, x)
```

3.150 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=352

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{24b^4d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^4d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{24b^4d}-\frac{9e^2\sinh}{}$$

```
[Out] -(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e
^2*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) - (3*e^2*Sqrt[-1
+ c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b^3*d*(a + b*ArcCosh[c + d*x])
) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(24*b^4*d) + (
9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(8*b^4*d)
 - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(24*b^4*d) - (9
*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(8*b^4*d)
```

Rubi [A] time = 0.964816, antiderivative size = 348, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301, 5656, 5781}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{24b^4d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{8b^4d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+a\right.}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e
^2*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) - (3*e^2*Sqrt[-1
+ c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b^3*d*(a + b*ArcCosh[c + d*x])
) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(24*b^4*d) + (9*e^
2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(8*b^4*d) - (e^
2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(24*b^4*d) - (9*e^2*Sinh[
(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(8*b^4*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{(2e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c\right.}{3bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2(c+}{2b^2d\left(a+b\cos\right.}$$

$$= -\frac{e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2(c+}{2b^2d\left(a+b\cos\right.}$$

Mathematica [A] time = 1.29055, size = 272, normalized size = 0.77

$$\frac{e^2\left(-\frac{8b^3\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{4b^2\left(2(c+dx)-3(c+dx)^3\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 27\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{co}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^3 + (4*b^2*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c +
d*x])^2 - (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-2 + 9*(c + d*x)^2))/(
a + b*ArcCosh[c + d*x]) - 80*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
 + 80*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + 27*(3*Cosh[a/b]*Cosh
Integral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcC
osh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3
*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])))/(24*b^4*d)
```

Maple [B] time = 0.141, size = 777, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(9*b^2*arccosh(d*x+c)^2+18*a*b*arcc
osh(d*x+c)-3*arccosh(d*x+c)*b^2+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^
3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-9/16*e^2/b^4*exp(3*a
/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/48*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x
+c)*e^2*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-arccosh(d*x+c)*b^2+a^2-a
*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccos
h(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/24*e^2/b*(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-1/48*e^2/b^2*(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/48*e^2/b^3*(
d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/48*e^2/b^4*ex
p(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/24*e^2/b*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*
arccosh(d*x+c))^3-1/16*e^2/b^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c)
)^2-3/16*e^2/b^3*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/16*e^2/b^4
*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^4\,\text{arcosh}\,(dx+c)^4 + 4\,ab^3\,\text{arcosh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arcosh}\,(dx+c)^2 + 4\,a^3b\,\text{arcosh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arccosh(d*x + c)^4 + 4*
a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d
*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{(b\,\text{arcosh}\,(dx+c) + a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^4, x)
```

3.151 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=218

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{e(c+dx)^2}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{2e\sqrt{c+d}}{3b^3d\left(\right.}$$

```
[Out] -(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c
 + d*x])^3) + e/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b
^2*d*(a + b*ArcCosh[c + d*x])^2) - (2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIn
tegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*Sinh
Integral[(2*(a + b*ArcCosh[c + d*x]))/b])/(3*b^4*d)
```

Rubi [A] time = 0.510687, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301, 5676}

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}-\frac{e(c+dx)^2}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c
 + d*x])^3) + e/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b
^2*d*(a + b*ArcCosh[c + d*x])^2) - (2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIn
tegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIn
tegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(3*b^4*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right.}{3bd}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e(c+}{3b^2d\left(a+b\cos\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e(c+}{3b^2d\left(a+b\cos\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e(c+}{3b^2d\left(a+b\cos\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e(c+}{3b^2d\left(a+b\cos\right.}$$

Mathematica [A] time = 0.993619, size = 195, normalized size = 0.89

$$\frac{e\left(-\frac{2b^3\sqrt{c+dx-1}(c+dx)\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{b^2\left(1-2(c+dx)^2\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}+4\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh\right.\right.\right.\right.}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[
c + d*x])^3 + (b^2*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x])^2 - (4*b*S
qrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x]) - 4
*Log[a + b*ArcCosh[c + d*x]] + 4*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCo
sh[c + d*x])] + Log[a + b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*
(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

Maple [A] time = 0.066, size = 353, normalized size = 1.6

$$\frac{1}{d}\left(\frac{e\left(2\,b^2\left(\operatorname{arccosh}(dx+c)\right)^2+4\,ab\operatorname{arccosh}(dx+c)-\operatorname{arccosh}(dx+c)\,b^2+2\,a^2-ab+b^2\right)}{12\,b^3\left(b^3\left(\operatorname{arccosh}(dx+c)\right)^3+3\,ab^2\left(\operatorname{arccosh}(dx+c)\right)^2+3\,a^2b\operatorname{arccosh}(dx+c)+a^3\right)}\left(-2\sqrt{dx+c+1}\sqrt{dx+c-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*b
^2*arccosh(d*x+c)^2+4*a*b*arccosh(d*x+c)-arccosh(d*x+c)*b^2+2*a^2-a*b+b^2)/
b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a
^3)-1/3*e/b^4*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/12*e/b*(2*(d*x+c)^2
-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/12*e
/b^2*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh
(d*x+c))^2-1/6*e/b^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+
c))/(a+b*arccosh(d*x+c))-1/3*e/b^4*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b
))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dex + ce}{b^4 \operatorname{arcosh}(dx + c)^4 + 4\,ab^3 \operatorname{arcosh}(dx + c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx + c)^2 + 4\,a^3b \operatorname{arcosh}(dx + c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3
 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^4, x)
```

3.152 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^4} dx$

Optimal. Leaf size=174

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{\sqrt{c+dx-1}\sqrt{c+dx-}}{6b^3d\left(a+b\cosh^{-1}(c+\right.}$$

```
[Out] -(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^3)
- (c + d*x)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (Sqrt[-1 + c + d*x]*Sqrt
[1 + c + d*x])/(6*b^3*d*(a + b*ArcCosh[c + d*x])) + (Cosh[a/b]*CoshIntegral
[(a + b*ArcCosh[c + d*x])/b])/(6*b^4*d) - (Sinh[a/b]*SinhIntegral[(a + b*Ar
cCosh[c + d*x])/b])/(6*b^4*d)
```

Rubi [A] time = 0.445229, antiderivative size = 170, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5864, 5656, 5775, 5781, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{6b^4d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{6b^4d}-\frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{\sqrt{c+dx-1}}{6b^3d\left(a+b\,\text{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-4),x]
```

```
[Out] -(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^3)
- (c + d*x)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (Sqrt[-1 + c + d*x]*Sqrt
[1 + c + d*x])/(6*b^3*d*(a + b*ArcCosh[c + d*x])) + (Cosh[a/b]*CoshIntegral
[a/b + ArcCosh[c + d*x]])/(6*b^4*d) - (Sinh[a/b]*SinhIntegral[a/b + ArcCosh
[c + d*x]])/(6*b^4*d)
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^2}\right.}{6b^2d}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+dx}\sqrt{1+c+}}{6b^3d\left(a+b\cosh^{-1}(c+\right.}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+dx}\sqrt{1+c+}}{6b^3d\left(a+b\cosh^{-1}(c+\right.}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+dx}\sqrt{1+c+}}{6b^3d\left(a+b\cosh^{-1}(c+\right.}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+dx}\sqrt{1+c+}}{6b^3d\left(a+b\cosh^{-1}(c+\right.}$$

Mathematica [A] time = 0.635213, size = 144, normalized size = 0.83

$$-\frac{\frac{2b^3\sqrt{c+dx-1}\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{b^2(c+dx)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - \cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \frac{b\sqrt{c+}}{a+b\,c}}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-4),x]
```

```
[Out] -((2*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])^3 +
 (b^2*(c + d*x))/(a + b*ArcCosh[c + d*x])^2 + (b*Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x])/(a + b*ArcCosh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcCosh
```

```
[c + d*x]] + Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(6*b^4*d)
```

Maple [A] time = 0.073, size = 295, normalized size = 1.7

$$\frac{1}{d}\left(\frac{b^2(\operatorname{arccosh}(dx+c))^2+2\,ab\operatorname{arccosh}(dx+c)-\operatorname{arccosh}(dx+c)\,b^2+a^2-ab+2\,b^2}{12\,b^3\left(b^3(\operatorname{arccosh}(dx+c))^3+3\,ab^2(\operatorname{arccosh}(dx+c))^2+3\,a^2b\operatorname{arccosh}(dx+c)+a^3\right)}\left(-\sqrt{dx+c-1}\sqrt{dx+c+1}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b^2*arccosh(d*x+c)^2+2*
a*b*arccosh(d*x+c)-arccosh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c
)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/12/b^4*exp(a/b)*
Ei(1,arccosh(d*x+c)+a/b)-1/6/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))^3-1/12/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*ar
ccosh(d*x+c))^2-1/12/b^3*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arcco
sh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b^4\operatorname{arcosh}(dx+c)^4+4\,ab^3\operatorname{arcosh}(dx+c)^3+6\,a^2b^2\operatorname{arcosh}(dx+c)^2+4\,a^3b\operatorname{arcosh}(dx+c)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2
*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arcosh}\,(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-4), x)
```

3.153 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])^4), x]/e
```

Rubi [A] time = 0.061085, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^4), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 13.9996, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4), x]
```

Maple [A] time = 0.184, size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)(a + b\operatorname{arccosh}(dx + c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^4dex + a^4ce + \left(b^4dex + b^4ce\right)\operatorname{arcosh}(dx + c)^4 + 4\left(ab^3dex + ab^3ce\right)\operatorname{arcosh}(dx + c)^3 + 6\left(a^2b^2dex + a^2b^2ce\right)\text{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arccosh(d*x + c)^4
+ 4*(a*b^3*d*e*x + a*b^3*c*e)*arccosh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b
^2*c*e)*arccosh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arccosh(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arcosh}\,(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4), x)
```

3.154 $\int (ce+dex)^4 \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=361

$$-\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}-\frac{\sqrt{\pi}}{}$$

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcCosh[c + d*x]])/(5*d) - (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(64*d) - (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Cosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) - (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rubi [A] time = 0.951133, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}-\frac{\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```


```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcCosh[c + d*x]])/(5*d) - (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(64*d) - (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Cosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) - (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^4\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^4x^4\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}}\,d\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\cosh^5(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int\left(\frac{5\cosh(x)}{8\sqrt{a+bx}}+\frac{5\cosh(3x)}{16\sqrt{a+bx}}+\frac{\text{co}}{16}\right.\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+d\right.}{320d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}}\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{\sqrt{b}e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d} - \frac{\sqrt{b}e}{}$$

Mathematica [A] time = 0.707211, size = 342, normalized size = 0.95

$$\frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\text{G}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcCosh[c + d*x]]*(150*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c
+ d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcCo
```

```
sh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcCosh[c + d*x]))/b] + 25*Sqrt[3]*E^((2
*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x])
)/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*Arc
Cosh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])
/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c + d*x]))/b] + 3*Sqrt[5]*E^((10*a)/b)*Sq
rt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcCosh[c + d*x]))/b
]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int (dex + ce)^4 \sqrt{a + b\operatorname{arccosh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 \sqrt{b\operatorname{arcosh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int c^4\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int d^4x^4\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int 4cd^3x^3\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int 6c^2d^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*acosh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.155 $\int (ce+dex)^3\sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{\pi}\sqrt{b}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-$$

```
[Out] (-3*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcCosh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d) - (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) - (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rubi [A] time = 0.793165, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (-3*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcCosh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d) - (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) - (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^3x^3\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}}\,dx,\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+d\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int\left(\frac{3}{8\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}+\frac{\cosh(4}{8\sqrt{a+b}}\right.\right.}{8d}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subs}}{}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subs}}{}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{e^3\,\text{Subst}\left(\right.}{}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\sqrt{b}e^3e^{\frac{4a}{b}}\sqrt{\pi}}{}$$

Mathematica [A] time = 0.506471, size = 223, normalized size = 0.82

$$\frac{e^3 e^{-\frac{4a}{b}} \sqrt{a + b\cosh^{-1}(c + dx)} \left(\sqrt{\frac{a}{b} + \cosh^{-1}(c + dx)}\text{Gamma}\left(\frac{3}{2}, -\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right) + 4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b} + \cosh^{-1}(c + dx)}\text{Ga}\right.}{128d\sqrt{-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCosh
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c + d
*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(128*d
*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int (dex + ce)^3 \sqrt{a + b\text{arccosh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 \sqrt{b\,\text{arcosh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int c^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int d^3x^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int 3cd^2x^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int 3c^2dx\sqrt{a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*acosh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.156 $\int (ce+dex)^2 \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}-\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\sqrt{\frac{\pi}{3}}\cdots$$

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(48*d) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rubi [A] time = 0.759198, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}-\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\sqrt{\frac{\pi}{3}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```


```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(48*d) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^2\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^2x^2\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}}\,d\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{\cosh^3(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx,\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+d\right.}{48d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}}\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}e}{}$$

Mathematica [A] time = 0.482069, size = 237, normalized size = 0.97

$$\frac{e^2 e^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\text{Gamm}\right.}{7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqrt[a
/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcCos
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]
)
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int (dex+ce)^2\sqrt{a+b\text{arccosh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^2\sqrt{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int c^2\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int d^2x^2\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int 2cdx\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*acosh(c + d*x)),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.157 $\int (ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}-\frac{e\sqrt{a}}{}$$

```
[Out] -(e*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcCosh
[c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rubi [A] time = 0.575408, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}-\frac{e\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```


```
[Out] -(e*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcCosh
[c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int ex\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c\right.}{4d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx,x,\cosh\right.}{4d}$$

$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{e\,\text{Subst}\left(\int e^{\frac{2a}{b}}\right.}{}$$

$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{\sqrt{b}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\right.}{}$$

Mathematica [B] time = 2.3693, size = 437, normalized size = 2.66

$$\frac{e\left(16ce^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)+8\sqrt{\pi}\sqrt{b}c\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e*(-32*c*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCosh[c +
 d*x]]*Cosh[2*ArcCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(S
qrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] + (16*c*Sqrt[a + b*ArcCosh[c
+ d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCos
h[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCo
sh[c + d*x])/b)]))/E^(a/b) - 8*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - Sqrt[b]*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[
(2*a)/b])))/(32*d)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (dex + ce)\sqrt{a + b\mathrm{arccosh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)\sqrt{b\,\mathrm{arcosh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int c\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int dx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c*sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x*sqrt(a + b*acos
h(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.158 $\int \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}$$

```
[Out] ((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/d - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rubi [A] time = 0.350021, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5864, 5654, 5781, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/d - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d} - \frac{b\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c+dx)}\right)}{2d} - \frac{\text{Subs}\cdots}{\cdots}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right.}{4d}$$

Mathematica [A] time = 0.181939, size = 110, normalized size = 0.96

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d*
x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b
)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \sqrt{a+b\text{arccosh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arcosh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.159 $\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[Sqrt[a + b*ArcCosh[c + d*x]]/(c + d*x), x]/e
```

Rubi [A] time = 0.0870347, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]

[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcCosh[x]]/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\cosh^{-1}(x)}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\cosh^{-1}(x)}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 2.04174, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x), x]
```

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int \frac{1}{dex + ce}\sqrt{a + b\operatorname{arccosh}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\operatorname{arcosh}(dx + c) + a}}{dex + ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+b\operatorname{acosh}(c+dx)}}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(1/2)/(d*e*x+c*e),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c + d*x))/(c + d*x), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.160 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=374

$$-\frac{3\sqrt{\pi}b^{3/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}b^{3/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right.}{2048d}$$

```
[Out] (-9*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh
[c + d*x]])/(64*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d
*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^
(3/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (3
*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(2048*d) - (3*b^(3/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2048*d*E^((4*a)/b)) + (3*b^(3/2)*e
^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d*
E^((2*a)/b))
```

Rubi [A] time = 1.41225, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5866, 12, 5664, 5759, 5676, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}b^{3/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}b^{3/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right.}{2048d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-9*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh
[c + d*x]])/(64*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d
*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^
(3/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (3
*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(2048*d) - (3*b^(3/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2048*d*E^((4*a)/b)) + (3*b^(3/2)*e
^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d*
```

```
E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+b\cosh^{-1}(x)}}{\sqrt{-1+x}\sqrt{1+x}} dx, x \ldots}{8d}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx \ldots}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} - \frac{3be^3\sqrt{-1 \ldots}}{\ldots}$$

Mathematica [A] time = 3.72672, size = 558, normalized size = 1.49

$$e^3\left(\frac{ae^{-\frac{4a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\text{Gamma}\left(\frac{3}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\text{Ga}\ldots}{128d\sqrt{\ldots}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e^3*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/
2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqr
t[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c
+ d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(12
8*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (Sqrt[b]*((8*a +
 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b
] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt[2*Pi]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2
*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c
+ d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh[c +
 d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[c + d*x]*Cosh[
4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(2048*d))
```

Maple [F] time = 0.249, size = 0, normalized size = 0.

$$\int (dex+ce)^3 (a+b\operatorname{arccosh}(dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^3 (b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.161 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=342

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+$$

```
[Out] -(b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/
(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*Sqrt[a + b*
ArcCosh[c + d*x]])/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(3/2))
/(3*d) - (3*b^(3/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/S
qrt[b]])/(32*d) - (b^(3/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d) + (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rubi [A] time = 1.10973, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {5866, 12, 5664, 5759, 5718, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```


```
[Out] -(b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/
(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*Sqrt[a + b*
ArcCosh[c + d*x]])/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(3/2))
/(3*d) - (3*b^(3/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/S
qrt[b]])/(32*d) - (b^(3/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d) + (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\sqrt{a+b\cosh^{-1}(x)}}{\sqrt{-1+x}\sqrt{1+x}} dx, x,\right.}{2d}$$

$$= -\frac{be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{e^2(c+dx)^3}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+c+dx}(c}{}$$

Mathematica [A] time = 2.41833, size = 592, normalized size = 1.73

$$e^2\left(\frac{ae^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\text{Gan}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e^2*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCos
h[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqr
t[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[
3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*Arc
Cosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^
2)]) + (Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c +
 d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c + d*x]*S
qrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[
3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] -
 Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*Arc
Cosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*ArcCos
h[c + d*x]])))/(288*d))
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int (dex + ce)^2 (a + b\operatorname{arccosh}(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2 \left(\int ac^2 \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int ad^2x^2 \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int bc^2 \sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)\, dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(a*c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*acosh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.162 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=212

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d}$$

```
[Out] (-3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c
 + d*x]])/(8*d) - (e*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2
*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2
]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*
e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^
((2*a)/b))
```

Rubi [A] time = 0.687308, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5664, 5759, 5676, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```


```
[Out] (-3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c
 + d*x]])/(8*d) - (e*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2
*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2
]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*
e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^
((2*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\sqrt{a+b\cosh^{-1}(x)}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c\right.}{4d}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{e(c+dx)^2\left(a\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} - \frac{e\left(a+b\cosh\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} - \frac{e\left(a+b\cosh\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} - \frac{e\left(a+b\cosh\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} - \frac{e\left(a+b\cosh\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} - \frac{e\left(a+b\cosh\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} - \frac{e\left(a+b\cosh\right.}{}$$

Mathematica [B] time = 7.85377, size = 1144, normalized size = 5.4

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e*((a*c*Sqrt[-1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma
[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -(
```

```
(a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(
a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (b*c*Sqrt[-1 +
 c + d*x]*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*
ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x
]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh
[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*d*Sqrt[(-1 + c + d
*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a*Sqrt[-1 + c + d*x]*(-32*c*(c + d
*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCosh[c + d*x]]*Cosh[2*Ar
cCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcCosh[c
+ d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]] - 8*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCos
h[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - Sqrt[b]*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Si
nh[(2*a)/b])))/(32*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x])
+ (Sqrt[-1 + c + d*x]*(-16*c*(-12*b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*b*(c + d*x)*ArcCosh[c + d*x]*Sqr
t[a + b*ArcCosh[c + d*x]] + Sqrt[b]*(2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[b]*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + S
qrt[b]*(4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + (4*a - 3*b)*Sqrt[b]*Sqrt[2*Pi]*Er
f[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*
a)/b]) + 8*b*Sqrt[a + b*ArcCosh[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCos
h[c + d*x]] - 3*Sinh[2*ArcCosh[c + d*x]])))/(128*d*Sqrt[(-1 + c + d*x)/(1 +
 c + d*x)]*Sqrt[1 + c + d*x]))
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int (dex + ce) (a + b\operatorname{arccosh}(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int ac\sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int adx\sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int bc\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx)\, dx + \int b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(a*c*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
acosh(c + d*x)), x) + Integral(b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.163 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\cosh^{-1}(c+}}{2d}$$

```
[Out] (-3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(2
*d) + ((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/d - (3*b^(3/2)*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rubi [A] time = 0.363021, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\cosh^{-1}(c+}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(2
*d) + ((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/d - (3*b^(3/2)*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\cosh^{-1}(x)}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{2d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{d}$$

Mathematica [A] time = 0.706777, size = 290, normalized size = 1.85

$$4ae^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)+\sqrt{\pi}\sqrt{b}(2a-3b)\left(\sinh\left(\frac{a}{b}\right)+\cosh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-12*b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[
c + d*x]] + 8*b*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (
4*a*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c +
 d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*(2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b])
```

```
+ (2*a - 3*b)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(C
osh[a/b] + Sinh[a/b]))/(8*d)
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{acosh}(c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.164 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[(a + b*ArcCosh[c + d*x])^(3/2)/(c + d*x), x]/e
```

Rubi [A] time = 0.101872, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{3/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{3/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.40762, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

Maple [A] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{1}{dex + ce} \left(a + b\operatorname{arccosh}\left(dx + c\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arcosh}\left(dx + c\right) + a\right)^{\frac{3}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+b\operatorname{acosh}(c+dx)}}{c+dx}\,dx + \int \frac{b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(3/2)/(d*e*x+c*e),x)
```

```
[Out] (Integral(a*sqrt(a + b*acosh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*acosh(c + d*x))*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.165 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=469

$$-\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+d}}{\sqrt{b}}\right.}{16384d}$$

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(2048*d) + (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcCosh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcCosh[c + d*x]])/(256*d) - (15*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*Sqrt[-1 +
c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(32*
d) - (3*e^3*(a + b*ArcCosh[c + d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a +
b*ArcCosh[c + d*x])^(5/2))/(4*d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf
[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16384*d) - (15*b^(5/2)*e^3*E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5
12*d) - (15*b^(5/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(16384*d*E^((4*a)/b)) - (15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(512*d*E^((2*a)/b))
```

Rubi [A] time = 2.23879, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5866, 12, 5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+d}}{\sqrt{b}}\right.}{16384d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(2048*d) + (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcCosh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcCosh[c + d*x]])/(256*d) - (15*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*Sqrt[-1 +
c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(32*
d) - (3*e^3*(a + b*ArcCosh[c + d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a +
b*ArcCosh[c + d*x])^(5/2))/(4*d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf
[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16384*d) - (15*b^(5/2)*e^3*E^(
```

```
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5
12*d) - (15*b^(5/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(16384*d*E^((4*a)/b)) - (15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(512*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3 \text{Subst}\left(\int x^3 \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{4d} - \frac{\left(5be^3\right) \text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}(x)\right)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}} d\right.}{8d}$$

$$= -\frac{5be^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d} + \frac{e^3(c}{}$$

$$= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} - \frac{15be^3\sqrt{-1+c+dx}(c+dx)\sqrt{1+}}{64}$$

$$= \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c}}{256d}$$

$$= \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c}}{256d}$$

$$= -\frac{45b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}$$

Mathematica [B] time = 10.8631, size = 968, normalized size = 2.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e^3*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[
3/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[
c + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (a*Sqrt[b]*((8
*a + 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*
a)/b] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt[2*Pi
]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sin
h[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcCos
h[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh
[c + d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[c + d*x]*C
osh[4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(1024*d) + (-(Sqrt[
b]*(64*a^2 + 48*a*b + 15*b^2)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]]*(Cosh[(4*a)/b] - Sinh[(4*a)/b])) - Sqrt[b]*(64*a^2 - 48*a*b + 15
*b^2)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b]
 + Sinh[(4*a)/b]) - 16*(Sqrt[b]*(16*a^2 + 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)
/b]) + Sqrt[b]*(16*a^2 - 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 8*b*Sqrt[a
+ b*ArcCosh[c + d*x]]*(b*(15 + 16*ArcCosh[c + d*x]^2)*Cosh[2*ArcCosh[c + d*
x]] + 4*(a - 5*b*ArcCosh[c + d*x])*Sinh[2*ArcCosh[c + d*x]])) + 8*b*Sqrt[a
+ b*ArcCosh[c + d*x]]*(b*(15 + 64*ArcCosh[c + d*x]^2)*Cosh[4*ArcCosh[c + d*
x]] + 8*(a - 5*b*ArcCosh[c + d*x])*Sinh[4*ArcCosh[c + d*x]]))/(16384*d))
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int (dex + ce)^3 \left(a + b\operatorname{arccosh}(dx + c)\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.166 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=408

$$-\frac{15\sqrt{\pi}b^{5/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}-\frac{15\sqrt{\pi}b^{5/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}$$

```
[Out] (5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(36*d) - (5*b*e^2*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(18
*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*
e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(64*d) - (5
*b^(5/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(576*d) - (15*b^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
+ d*x]]/Sqrt[b]])/(64*d*E^(a/b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rubi [A] time = 1.75142, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {5866, 12, 5664, 5759, 5718, 5654, 5781, 3307, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}b^{5/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}-\frac{15\sqrt{\pi}b^{5/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(36*d) - (5*b*e^2*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(18
*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*
e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(64*d) - (5
*b^(5/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(576*d) - (15*b^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
+ d*x]]/Sqrt[b]])/(64*d*E^(a/b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
```

```
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{3d} - \frac{\left(5be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}(x)\right)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right.}{6d}$$

$$= -\frac{5be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{18d} + \frac{e^2(c+}{}$$

$$= \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{36d} - \frac{5be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+\right.}{9d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx}}{36d}$$

Mathematica [B] time = 9.19207, size = 1008, normalized size = 2.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e^2*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh
[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/
b^2)]) + (a*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c + d*
x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcC
osh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*S
qrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/
b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b
*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*Ar
cCosh[c + d*x]])))/(144*d) + (-27*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqr
t[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) +
b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2
)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b
]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) - Sqrt[b]*(12*a^2 + 12*a*b + 5*b^2)
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a
)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Erf[(S
qrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b
]) + 12*b*Sqrt[a + b*ArcCosh[c + d*x]]*(b*(5 + 12*ArcCosh[c + d*x]^2)*Cosh[
3*ArcCosh[c + d*x]] + 2*(a - 5*b*ArcCosh[c + d*x])*Sinh[3*ArcCosh[c + d*x]]
))/(1728*d))
```

Maple [F] time = 0.242, size = 0, normalized size = 0.

$$\int (dex+ce)^2 (a+b\operatorname{arccosh}(dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^2(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.167 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=269

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+d}}{32d}$$

```
[Out] (-15*b^2*e*Sqrt[a + b*ArcCosh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqr
t[a + b*ArcCosh[c + d*x]])/(32*d) - (5*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(8*d) - (e*(a + b*ArcCosh[c
+ d*x])^(5/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d)
 - (15*b^(5/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rubi [A] time = 1.10796, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+d}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (-15*b^2*e*Sqrt[a + b*ArcCosh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqr
t[a + b*ArcCosh[c + d*x]])/(32*d) - (5*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(8*d) - (e*(a + b*ArcCosh[c
+ d*x])^(5/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d)
 - (15*b^(5/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d}-\frac{(5be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\cosh^{-1}(x)\right)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x\right.}{4d}$$

$$=-\frac{5be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{8d}+\frac{e(c+dx}{}$$

$$=\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+}}{8d}$$

$$=\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+}}{8d}$$

$$=\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+}}{8d}$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5}{}$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5}{}$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5}{}$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5}{}$$

Mathematica [B] time = 9.00476, size = 1846, normalized size = 6.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e*((a^2*c*Sqrt[-1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gam
ma[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2,
-((a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E
^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a*b*c*Sqrt[
-1 + c + d*x]*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a
+ b*ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c +
 d*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(
Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcC
osh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(4*d*Sqrt[(-1 + c
 + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) - (c*Sqrt[-1 + c + d*x]*(-4*b*Sqr
t[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*
x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) +
Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(1
6*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a^2*Sqrt[-1 +
c + d*x]*(-32*c*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCo
sh[c + d*x]]*Cosh[2*ArcCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - 8*Sqrt[b]*c*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]
*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b
]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a
)/b] - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])))/(32*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x
)]*Sqrt[1 + c + d*x]) + (a*Sqrt[-1 + c + d*x]*(-16*c*(-12*b*Sqrt[(-1 + c +
d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*b*(c + d
*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + Sqrt[b]*(2*a + 3*b)*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
(2*a - 3*b)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cos
h[a/b] + Sinh[a/b])) + Sqrt[b]*(4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + (4*a - 3*
b)*Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(
Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*b*Sqrt[a + b*ArcCosh[c + d*x]]*(4*ArcCos
h[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh[c + d*x]])))/(64*d*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) - (Sqrt[-1 + c + d*x]*
(-32*c*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d
*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh
[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*
b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] +
 Sinh[a/b])) + Sqrt[b]*(16*a^2 + 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sq
rt[b]*(16*a^2 - 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 8*b*Sqrt[a + b*ArcCo
```

```
sh[c + d*x]]*(b*(15 + 16*ArcCosh[c + d*x]^2)*Cosh[2*ArcCosh[c + d*x]] + 4*(
a - 5*b*ArcCosh[c + d*x])*Sinh[2*ArcCosh[c + d*x]])))/(512*d*Sqrt[(-1 + c +
 d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int (dex + ce) (a + b\text{arccosh} (dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arcosh} (dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.168 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=186

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b}{}$$

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) - (5*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(
a + b*ArcCosh[c + d*x])^(5/2))/d - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rubi [A] time = 0.609113, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5654, 5718, 5781, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) - (5*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(
a + b*ArcCosh[c + d*x])^(5/2))/d - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{d}-\frac{(5b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{2d}$$

$$=-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+\right.}{d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+\right.}{2d}$$

Mathematica [B] time = 3.30878, size = 494, normalized size = 2.66

$$8a^2e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\,\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)-\sqrt{\pi}\sqrt{b}\left(4a^2-12ab+15b^2\right)\left(\sin\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x
]^2)) + (8*a^2*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b +
ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCo
sh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) - Sqrt[b]*(4
*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]
*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 4*a*b*(-12*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]
] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*b
)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b
]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b
]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(16*d)
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.169 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[(a + b*ArcCosh[c + d*x])^(5/2)/(c + d*x), x]/e
```

Rubi [A] time = 0.100752, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(5/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{5/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{5/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.0607, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\operatorname{arccosh}\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\operatorname{arcosh}\left(dx+c\right)+a\right)^{\frac{5}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.170 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=509

$$-\frac{105\sqrt{\pi}b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}+\frac{105\sqrt{\pi}b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}$$

```
[Out] (-175*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d
*x]])/(54*d) - (35*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]
*Sqrt[a + b*ArcCosh[c + d*x]])/(216*d) + (35*b^2*e^2*(c + d*x)*(a + b*ArcCo
sh[c + d*x])^(3/2))/(18*d) + (35*b^2*e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x
])^(3/2))/(108*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt
[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(18*d) + (e^2*(c + d*x)^3*(a
+ b*ArcCosh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(128*d) - (35*b^(7/2)*e^2*E^((3*a)/b
)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3456*d)
+ (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12
8*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rubi [A] time = 2.14238, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {5866, 12, 5664, 5759, 5718, 5654, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{105\sqrt{\pi}b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}+\frac{105\sqrt{\pi}b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-175*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d
*x]])/(54*d) - (35*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]
*Sqrt[a + b*ArcCosh[c + d*x]])/(216*d) + (35*b^2*e^2*(c + d*x)*(a + b*ArcCo
sh[c + d*x])^(3/2))/(18*d) + (35*b^2*e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x
])^(3/2))/(108*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt
```

```
[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(18*d) + (e^2*(c + d*x)^3*(a
+ b*ArcCosh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(128*d) - (35*b^(7/2)*e^2*E^((3*a)/b
)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3456*d)
+ (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12
8*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{3d} - \frac{\left(7be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}(x)\right)^{5/2}}{\sqrt{-1+x}\sqrt{1+x}}\,d\right.}{6d}$$

$$= -\frac{7be^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{18d} + \frac{e^2(c\cdots}{}$$

$$= \frac{35b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{108d} - \frac{7be^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{9d}$$

$$= -\frac{35b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{216d} + \frac{35b^2e}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2\sqrt{-1+}}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2\sqrt{-1+}}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2\sqrt{-1+}}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2\sqrt{-1+}}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2\sqrt{-1+}}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2\sqrt{-1+}}{}$$

Mathematica [B] time = 13.5723, size = 1523, normalized size = 2.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] e^2*((a^3*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh
[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/
b^2)]) + (a^2*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c +
d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a
)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCos
h[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a +
 b*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*
ArcCosh[c + d*x]])))/(96*d) + (a*(-27*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2
*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]
) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15
*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh
[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) - Sqrt[b]*(12*a^2 + 12*a*b + 5*
b^2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[
(3*a)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*
a)/b]) + 12*b*Sqrt[a + b*ArcCosh[c + d*x]]*(b*(5 + 12*ArcCosh[c + d*x]^2)*C
osh[3*ArcCosh[c + d*x]] + 2*(a - 5*b*ArcCosh[c + d*x])*Sinh[3*ArcCosh[c + d
*x]])))/(576*d) + (-81*(4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[(-1 + c + d*
x)/(1 + c + d*x)]*(1 + c + d*x)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15
 + 4*ArcCosh[c + d*x]^2)) - 2*b*(c + d*x)*(-10*a + b*ArcCosh[c + d*x]*(35 +
 4*ArcCosh[c + d*x]^2))) + Sqrt[b]*(8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + Sqrt[b]*(-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(72*a^3 + 1
08*a^2*b + 90*a*b^2 + 35*b^3)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(-72*a^3 + 108
*a^2*b - 90*a*b^2 + 35*b^3)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c +
```

```
d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) - 12*b*Sqrt[a + b*ArcCosh[c
 + d*x]]*(-2*b*(-10*a + b*ArcCosh[c + d*x]*(35 + 36*ArcCosh[c + d*x]^2))*Co
sh[3*ArcCosh[c + d*x]] + (12*a^2 - 12*a*b*ArcCosh[c + d*x] + 7*b^2*(5 + 12*
ArcCosh[c + d*x]^2))*Sinh[3*ArcCosh[c + d*x]]))/(10368*d))
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int (dex + ce)^2 (a + b\operatorname{arccosh}(dx + c))^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b\operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.171 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=319

$$-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e\sqrt{c+dx-1}(c+dx)\sqrt{c-}}{12}$$

```
[Out] (-105*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCo
sh[c + d*x]])/(128*d) - (35*b^2*e*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) +
(35*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(3/2))/(32*d) - (7*b*e*Sqrt[
-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(
8*d) - (e*(a + b*ArcCosh[c + d*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*Arc
Cosh[c + d*x])^(7/2))/(2*d) - (105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2
*a)/b))
```

Rubi [A] time = 1.27648, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5664, 5759, 5676, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e\sqrt{c+dx-1}(c+dx)\sqrt{c-}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```


```
[Out] (-105*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCo
sh[c + d*x]])/(128*d) - (35*b^2*e*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) +
(35*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(3/2))/(32*d) - (7*b*e*Sqrt[
-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(
8*d) - (e*(a + b*ArcCosh[c + d*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*Arc
Cosh[c + d*x])^(7/2))/(2*d) - (105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2
*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{2d} - \frac{(7be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}(x)\right)^{5/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x,\right.}{4d}$$

$$= -\frac{7be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{8d} + \frac{e(c+dx)}{}$$

$$= \frac{35b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d} - \frac{7be\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+}}{8d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} + \frac{35b^2e(c}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} - \frac{35b^2e\left(a\right.}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} - \frac{35b^2e\left(a\right.}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} - \frac{35b^2e\left(a\right.}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} - \frac{35b^2e\left(a\right.}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} - \frac{35b^2e\left(a\right.}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{128d} - \frac{35b^2e\left(a\right.}{}$$

Mathematica [A] time = 5.43985, size = 288, normalized size = 0.9

$$\frac{e\left(8\sqrt{a+b\cosh^{-1}(c+dx)}\left(4a\left(16a^2+35b^2\right)\cosh\left(2\cosh^{-1}(c+dx)\right)+4b\cosh^{-1}(c+dx)\left(\left(48a^2+35b^2\right)\cosh\left(2\cosh^{-1}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e*(105*b^(7/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 105*b^(7/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*
Sqrt[a + b*ArcCosh[c + d*x]]*(4*a*(16*a^2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]
] + 64*b^3*ArcCosh[c + d*x]^3*Cosh[2*ArcCosh[c + d*x]] - 7*b*(16*a^2 + 15*b
^2)*Sinh[2*ArcCosh[c + d*x]] + 16*b^2*ArcCosh[c + d*x]^2*(12*a*Cosh[2*ArcCo
sh[c + d*x]] - 7*b*Sinh[2*ArcCosh[c + d*x]]) + 4*b*ArcCosh[c + d*x]*((48*a^
2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]] - 56*a*b*Sinh[2*ArcCosh[c + d*x]]))))/
(2048*d)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int (dex + ce) (a + b\mathrm{arccosh}\,(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b\,\mathrm{arcosh}\,(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.172 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=230

$$-\frac{105\sqrt{\pi}b^{7/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}b^{7/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{c}}}{8d}$$

```
[Out] (-105*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]
)/(8*d) + (35*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (7*b*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d) +
((c + d*x)*(a + b*ArcCosh[c + d*x])^(7/2))/d - (105*b^(7/2)*E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rubi [A] time = 0.642199, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{105\sqrt{\pi}b^{7/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}b^{7/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\text{c}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(7/2),x]
```


```
[Out] (-105*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]
)/(8*d) + (35*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (7*b*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d) +
((c + d*x)*(a + b*ArcCosh[c + d*x])^(7/2))/d - (105*b^(7/2)*E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b))
```


Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{d} - \frac{(7b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^{5/2}}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c+dx\right)}{2d}$$

$$= -\frac{7b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d} + \frac{(c+dx)\left(a+b\cosh^{-1}(c+\right.}{d}$$

$$= \frac{35b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{7b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c\right.}{2d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\cosh^{-}\right.}{4d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\cosh^{-}\right.}{4d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\cosh^{-}\right.}{4d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\cosh^{-}\right.}{4d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\cosh^{-}\right.}{4d}$$

Mathematica [B] time = 7.41564, size = 765, normalized size = 3.33

$$\frac{a^3e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}} + \frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)}{2d} + \frac{3a\left(-\sqrt{\pi}\sqrt{b}\left(4a^2-12ab+15b^2\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c
 + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x
])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b)) + (3*a^2*b*(-12*
Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x
]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*
b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/
b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[
b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*d) + (3*a*(4*b*Sqrt[a + b*ArcCosh
[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*Arc
Cosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) - Sqrt[b]*(4*a^2
+ 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cos
h[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a
 + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d) + (-4*b*Sq
rt[a + b*ArcCosh[c + d*x]]*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x
)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15 + 4*ArcCosh[c + d*x]^2)) - 2*
b*(c + d*x)*(-10*a + b*ArcCosh[c + d*x]*(35 + 4*ArcCosh[c + d*x]^2))) - Sqr
t[b]*(8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCo
sh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]) - Sqrt[b]*(-8*a^3 + 36*a^2*b
 - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(
Cosh[a/b] + Sinh[a/b]))/(32*d)
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.173 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[(a + b*ArcCosh[c + d*x])^(7/2)/(c + d*x), x]/e
```

Rubi [A] time = 0.100789, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(7/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{7/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{7/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.084, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\operatorname{arccosh}\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arcosh}\left(dx+c\right)+a\right)^{\frac{7}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.174 $\int \frac{(ce+dex)^4}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{E}}{}$$

```
[Out] -(e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[
b]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(32*Sqrt[b]*d) - (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) + (e^4*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((
3*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rubi [A] time = 0.622491, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[
b]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(32*Sqrt[b]*d) - (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) + (e^4*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((
3*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right) dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{16d} + \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d}$$

$$= -\frac{e^4\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{32d} + \frac{e^4\,\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{32d}$$

$$= -\frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{16bd} + \frac{e^4\,\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x, \sqrt{a+\right.}{16bd}$$

$$= -\frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\frac{\sqrt{}}{}\right.}{32}$$

Mathematica [A] time = 0.547762, size = 319, normalized size = 0.98

$$\frac{e^4e^{-\frac{5a}{b}}\left(10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(c-\right.}{b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*(10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[
c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a +
 b*ArcCosh[c + d*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c +
d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqrt
[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] +
 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*Ar
cCosh[c + d*x]))/b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gam
ma[1/2, (5*(a + b*ArcCosh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Arc
Cosh[c + d*x]])
```

Maple [F] time = 0.402, size = 0, normalized size = 0.

$$\int (dex + ce)^4 \frac{1}{\sqrt{a + b\text{arccosh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{\sqrt{b\,\text{arcosh}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int \frac{c^4}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{d^4x^4}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{4cd^3x^3}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{6c^2d^2x^2}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*acosh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.175 $\int \frac{(ce+dex)^3}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\operatorname{E}}{}$$

```
[Out] -(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(
32*Sqrt[b]*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcC
osh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) + (e^3*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b))
```

Rubi [A] time = 0.456234, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\operatorname{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```


```
[Out] -(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(
32*Sqrt[b]*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcC
osh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) + (e^3*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{a+bx}}+\frac{\sinh(4x)}{8\sqrt{a+bx}}\right)dx,x,\cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d} + \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}$$

$$= -\frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{16d} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{16d} -$$

$$= -\frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c+dx)}\right)}{8bd} + \frac{e^3\,\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx,x,\sqrt{a+b\right.}}{8bd}$$

$$= -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{\ }}{\ }\right.}{32\sqrt{\ }}$$

Mathematica [A] time = 0.390323, size = 205, normalized size = 0.94

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+2\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+\right.}{32d\sqrt{a+b\cosh^{-1}(c+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c +
 d*x]))/b] + 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcCosh[c
```

```
+ d*x]]*(2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)
*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[a + b
*ArcCosh[c + d*x]])
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int (dex + ce)^3 \frac{1}{\sqrt{a + b\operatorname{arccosh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^3}{\sqrt{b\operatorname{arcosh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{d^3x^3}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{3cd^2x^2}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{3c^2dx}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*acosh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.176 $\int \frac{(ce+dex)^2}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{3}}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\sqrt{\frac{\pi}{3}}e^2e^{-\frac{3a}{b}}\text{Er}$$

```
[Out] -(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b
]*d) - (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rubi [A] time = 0.459393, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{3}}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\sqrt{\frac{\pi}{3}}e^2e^{-\frac{3a}{b}}\text{Er}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b
]*d) - (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx,x,\cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d} + \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}$$

$$= -\frac{e^2\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d} - \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d}$$

$$= -\frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c+dx)}\right)}{4bd} - \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\,c}\right.}{4bd}$$

$$= -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\sqrt{\right.}{8\sqrt{}}$$

<u>Mathematica</u> [A] time = 0.342785, size = 216, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(c+\right.}{b}\right.\right.}{24d\sqrt{a+b\,c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcCosh[c + d*x]))/b] + 3*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]
```

```
*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int (dex + ce)^2 \frac{1}{\sqrt{a + b\mathrm{arccosh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{\sqrt{b\,\mathrm{arcosh}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int\frac{c^2}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int\frac{d^2x^2}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int\frac{2cdx}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*acosh(c + d*x)),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.177 $\int \frac{ce+dex}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

```
[Out] -(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]])/(4*Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rubi [A] time = 0.236956, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]])/(4*Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{ce+dex}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d}$$

$$= -\frac{e\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{2bd} + \frac{e\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\,\text{c}}\right.}{2bd}$$

$$= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

Mathematica [B] time = 1.3291, size = 306, normalized size = 2.71

$$\frac{e\left(\frac{4ce^{a/b}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{4ce^{-\frac{a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{4\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} \ldots\right.}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e*((4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/Sqrt[b
] - (E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/Sqrt[b] - (4*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(
```

```
Sqrt[b]*E^(a/b)) + (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/
Sqrt[b]])/(Sqrt[b]*E^((2*a)/b)) + (4*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]
*Gamma[1/2, a/b + ArcCosh[c + d*x]])/Sqrt[a + b*ArcCosh[c + d*x]] + (4*c*Sq
rt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]
)/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(8*d)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int (dex+ce)\frac{1}{\sqrt{a+b\mathrm{arccosh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex+ce}{\sqrt{b\,\mathrm{arcosh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx+\int \frac{dx}{\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x/sqrt(a + b*acos
h(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.178 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}} dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rubi [A] time = 0.128826, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5864, 5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}\right.}{2bd}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}}\right.}{bd}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Mathematica [A] time = 0.10334, size = 110, normalized size = 1.2

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]
] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x
])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\operatorname{arcosh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\,\operatorname{acosh}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.179 $\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}},x\right)}{e}$$

[Out] Unintegrable[1/((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]]), x]/e

---

Rubi [A] time = 0.0936042, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]

[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcCosh[x]]), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.0752057, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]), x]
```

Maple [A] time = 0.188, size = 0, normalized size = 0.

$$\int \frac{1}{dex + ce}\frac{1}{\sqrt{a + b\operatorname{arccosh}(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)\sqrt{b\operatorname{arcosh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\sqrt{a+b\operatorname{acosh}(c+dx)}+dx\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(c*sqrt(a + b*acosh(c + d*x)) + d*x*sqrt(a + b*acosh(c + d*x))),
 x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.180 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}}{}$$

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(8*b^(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*E^((5*a)/b)*Sqrt[5*
Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (
e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/
b)) + (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]
])/(16*b^(3/2)*d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rubi [A] time = 0.643751, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(8*b^(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*E^((5*a)/b)*Sqrt[5*
Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (
e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/
b)) + (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]
])/(16*b^(3/2)*d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*Ar
```

cCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{\left(2e^4\right)\text{Subst}\left(\int\left(-\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{9\cosh(3x)}{16\sqrt{a+bx}} - \frac{5\,\text{co}}{16\sqrt{}}\right.\right.}{bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{8bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c\right.}{4b^2d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \frac{3e^4e^{\frac{3a}{b}}\sqrt{}}{}$$

Mathematica [A] time = 1.45167, size = 396, normalized size = 1.06

$$\frac{e^4e^{-\frac{5a}{b}}\left(-2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(c\right.}{b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-4*E^((5*a)/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - 2*E
^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]]
+ Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh
[c + d*x]))/b] + 3*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*
Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*A
rcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - 3*Sqrt[3]*
E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d
*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (5*
(a + b*ArcCosh[c + d*x]))/b] - 6*E^((5*a)/b)*Sinh[3*ArcCosh[c + d*x]] - 2*E
^((5*a)/b)*Sinh[5*ArcCosh[c + d*x]]))/(16*b*d*E^((5*a)/b)*Sqrt[a + b*ArcCos
h[c + d*x]])
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int (dex+ce)^4 (a+b\operatorname{arccosh}(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arcosh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int \frac{c^4}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^4x^4}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acos}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt
(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.181 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\operatorname{Erf}}{}$$

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[
(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) + (e^3
*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2
)*d*E^((2*a)/b))
```

Rubi [A] time = 0.449448, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\operatorname{Erf}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[
(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) + (e^3
*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2
)*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{\left(2e^3\right)\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2\sqrt{a+bx}} - \frac{\cosh(4x)}{2\sqrt{a+bx}}\right)dx, x, c\right.}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{2bd} +$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c\right.}{b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A] time = 0.978821, size = 265, normalized size = 0.99

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c +
 d*x]))/b] + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[
```

```
1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] - E^((4*a)/b)*(8*(c + d*x)^3*Sqrt[(-1
 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((4*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
4*b*d*E^((4*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F] time = 0.251, size = 0, normalized size = 0.

$$\int (dex + ce)^{3} (a + b\operatorname{arccosh}(dx + c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)

[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^{3}}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^3x^3}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(3*c*d**2*x*
*2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b
*acosh(c + d*x))*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.182 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Er}}{}$$

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(4*b^(3/2)*d) + (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rubi [A] time = 0.449567, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Er}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```


```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(4*b^(3/2)*d) + (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}}-\frac{3\cosh(3x)}{4\sqrt{a+bx}}\right)dx,x\right.}{bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{2bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c\right.}{2b^2d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^2e^{\frac{3a}{b}}\sqrt{3}}{}$$

Mathematica [A] time = 1.54405, size = 265, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(-e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(c+\right.}{b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-(E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c
 + d*x]]) + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
```

```
 b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*
Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b] - 2*E^((3*a)/b
)*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sinh[3*ArcCosh[c + d*
x]])))/(4*b*d*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int (dex + ce)^2 (a + b\text{arccosh}(dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{(b\,\text{arcosh}(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^2x^2}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acos}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(2*c*d*x/(a*
sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.183 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCo
sh[c + d*x]]) + (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rubi [A] time = 0.225043, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCo
sh[c + d*x]]) + (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{bd} + \frac{e\,\text{S}}{}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c+}\right.}{b^2d}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{er}}{}$$

Mathematica [B] time = 6.38246, size = 314, normalized size = 2.03

$$e\left(-\frac{2\sqrt{b}e^{-\frac{a}{b}}\left(ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-c\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+e^{a/b}\sinh\left(2\cosh^{-1}(c+dx)\right)\right)}{\sqrt{a+b\cosh^{-1}(c+dx)}}-2\sqrt{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e*((-2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b) + (S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)
 - 2*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh
[a/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Co
sh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*(c*E^((2*a)/b)*Sqrt[a/b + ArcCosh
```

```
[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] - c*Sqrt[-((a + b*ArcCosh[c +
 d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^(a/b)*Sinh[2*ArcCo
sh[c + d*x]]))/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(2*b^(3/2)*d)
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int (dex+ce)\,(a+b\,\mathrm{arccosh}\,(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex+ce}{(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx+\int \frac{dx}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(c/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*
acosh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a
 + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.184 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCosh[c + d*x]]
) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d
) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b
))
```

Rubi [A] time = 0.354413, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5864, 5656, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{c+dx-1}\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```


```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCosh[c + d*x]]
) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d
) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b
))
```


Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{b^2d} + \frac{2\,\text{S}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}}}{\sqrt{b}}\right.}{b^{3/2}d}$$

Mathematica [A] time = 0.468769, size = 145, normalized size = 1.13

$$\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)-2\right.}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - E^((2*a)/b)*
Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] + Sqrt[-((a
 + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/(b*d*
E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.185 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}},x\right)}{e}$$

[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2)), x]/e

---

Rubi [A] time = 0.104409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(3/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 0.083416, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.224, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\mathrm{arccosh}\left(dx+c\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arcosh}\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ac\sqrt{a+b\operatorname{acosh}(c+dx)}+adx\sqrt{a+b\operatorname{acosh}(c+dx)}+bc\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+bdx\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/(a*c*sqrt(a + b*acosh(c + d*x)) + a*d*x*sqrt(a + b*acosh(c + d*x
)) + b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b*d*x*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x)), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.186 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}-\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{\sqrt{\pi}e^4e}{}$$

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcCosh[c
+ d*x]]) - (20*e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e
^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*
d) - (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*b^(5/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) + (3*e^4*Sqrt[3
*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^(
(3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(24*b^(5/2)*d*E^((5*a)/b))
```

Rubi [A] time = 1.76501, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}-\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{\sqrt{\pi}e^4e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcCosh[c
+ d*x]]) - (20*e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e
^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*
d) - (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*b^(5/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqr
```

```
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) + (3*e^4*Sqrt[3
*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^(
(3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(24*b^(5/2)*d*E^((5*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,\right.}{3bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{20e^4(c}{3b^2d\sqrt{a+b\,c}}$$

Mathematica [A] time = 3.2334, size = 615, normalized size = 1.39

$$e^4 e^{-5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(-10\sqrt{5}be^{5\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-18\sqrt{3}be^{\frac{2a}{b}+5\cosh^{-1}(c+d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-10*Sqrt[5]*b*E^(5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^
(3/2)*Gamma[1/2, (-5*(a + b*ArcCosh[c + d*x]))/b] - 18*Sqrt[3]*b*E^((2*a)/b
 + 5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3
*(a + b*ArcCosh[c + d*x]))/b] + 2*E^(4*(a/b + ArcCosh[c + d*x]))*(2*E^((2*a
)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x
])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c + d*x]*Sq
rt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCosh[c + d*x]
))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*ArcCosh[c + d
*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])) + 3*E^((5*a)/b +
 2*ArcCosh[c + d*x])*(b - 6*a*(1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c
+ d*x] - b*E^(6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3
*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c +
d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]) + 2*E^((5*a)/b)*(-(b*(-1
+ E^(10*ArcCosh[c + d*x])))/2 - 5*(1 + E^(10*ArcCosh[c + d*x]))*(a + b*ArcC
osh[c + d*x]) + 5*Sqrt[5]*E^(5*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh
[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*ArcCosh[c + d*x]))
/b])))/(48*b^2*d*E^(5*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3
/2))
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (dex+ce)^4 \left(a+b\text{arccosh}\left(dx+c\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.187 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}e^3e^{-}}{}$$

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (16*e^3*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e^
3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^
(5/2)*d) - (e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((4*a)/b)) + (e^3*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rubi [A] time = 1.44116, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}e^3e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (16*e^3*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e^
3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^
(5/2)*d) - (e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((4*a)/b)) + (e^3*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
```

```
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{\left(2e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,d\right.}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{16e^3(c}{3b^2d\sqrt{a+b\,c}}$$

Mathematica [A] time = 2.34505, size = 391, normalized size = 1.17

$$e^3e^{-4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(-16be^{4\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-8\sqrt{2}be^{\frac{2a}{b}+4\cosh^{-1}(c+dx)}\left(-\frac{a}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Ga
mma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] - 8*Sqrt[2]*b*E^((2*a)/b + 4*ArcC
osh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*A
rcCosh[c + d*x]))/b] + E^((4*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))^2*(b*(-1
 + E^(4*ArcCosh[c + d*x])) + 8*a*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh
[c + d*x])) + 8*b*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh[c + d*x]))*Arc
Cosh[c + d*x])) + 8*Sqrt[2]*E^((2*a)/b + 4*ArcCosh[c + d*x])*Sqrt[a/b + Arc
Cosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*ArcCosh[c + d*
x]))/b] + 16*E^(4*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a
 + b*ArcCosh[c + d*x])*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(24*b^
2*d*E^(4*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F] time = 0.251, size = 0, normalized size = 0.

$$\int (dex+ce)^3 \left(a+b\operatorname{arccosh}(dx+c)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{\left(b\operatorname{arcosh}(dx+c)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{a^2\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c+dx)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*
x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.188 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=328

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{I}}{}$$

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (4*e^2*(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e^2*E^(
a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) - (e
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]])/(2*b^(5/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]
])/(6*b^(5/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rubi [A] time = 1.23728, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 5658}

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (4*e^2*(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e^2*E^(
a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) - (e
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]])/(2*b^(5/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]
])/(6*b^(5/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
```

```
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\operatorname{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{\left(4e^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,\right.}{3bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4e^2(c}{b^2d\sqrt{a+b\,\mathrm{co}}}$$

Mathematica [A] time = 2.93635, size = 391, normalized size = 1.19

$$e^2 e^{-3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)} \left(-6\sqrt{3} b e^{3\cosh^{-1}(c+dx)} \left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2} \text{Gamma}\left(\frac{1}{2}, -\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right) - 2be^{\frac{2a}{b}+3\cosh^{-1}(c+dx)} \left(-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a +
b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
b*ArcCosh[c + d*x]))/b] - 2*b*E^((2*a)/b + 3*ArcCosh[c + d*x])*(-((a + b*Ar
cCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^((3
*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))*(a*(6 - 4*E^(2*ArcCosh[c + d*x]) + 6
*E^(4*ArcCosh[c + d*x])) + b*(-1 + 6*ArcCosh[c + d*x] - 4*E^(2*ArcCosh[c +
d*x])*ArcCosh[c + d*x] + E^(4*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]))))
 + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a
 + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b])))/(12*b^
2*d*E^(3*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int (dex+ce)^2 (a+b\operatorname{arccosh}(dx+c))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\,\text{arcosh}(dx+c)+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a^2\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c+dx)}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*acosh(c + d*x)) +
2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.189 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{2\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{}{3b^2d\sqrt{a+b}}$$

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh
[c + d*x])^(3/2)) + (4*e)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (8*e*(c
+ d*x)^2)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2
*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5
/2)*d*E^((2*a)/b))
```

Rubi [A] time = 0.779637, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 5676}

$$-\frac{2\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{}{3b^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh
[c + d*x])^(3/2)) + (4*e)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (8*e*(c
+ d*x)^2)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2
*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5
/2)*d*E^((2*a)/b))
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x\right.}{3bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{8e(c}{3b^2d\sqrt{a+b\,c}}$$

Mathematica [B] time = 5.47783, size = 687, normalized size = 3.18

$$e\left(-2b^{3/2}ce^{-\frac{a}{b}}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+2\sqrt{b}ce^{a/b}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e*(4*a*Sqrt[b]*c*(c + d*x) + 4*b^(3/2)*c*(c + d*x)*ArcCosh[c + d*x] - (2*S
qrt[b]*c*(1 + E^(2*ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x]))/E^ArcCosh[c
 + d*x] - 4*a*Sqrt[b]*Cosh[2*ArcCosh[c + d*x]] - 4*b^(3/2)*ArcCosh[c + d*x]
*Cosh[2*ArcCosh[c + d*x]] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cos
h[a/b]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - 2*Sqrt[2*Pi]*(a + b*ArcC
osh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]] - 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cosh[a/b]*Erfi[Sqr
t[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^
(3/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] +
2*Sqrt[b]*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*G
amma[1/2, a/b + ArcCosh[c + d*x]] - (2*b^(3/2)*c*(-((a + b*ArcCosh[c + d*x]
)/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/E^(a/b) + 2*c*Sqrt[P
i]*(a + b*ArcCosh[c + d*x])^(3/2)*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]
*Sinh[a/b] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^
(3/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 2
*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - b^(3/2)*Sinh[2*ArcCosh[c + d*x]]))/(3*
b^(5/2)*d*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (dex+ce)\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a^2\sqrt{a + b\operatorname{acosh}(c + dx)} + 2ab\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx) + b^2\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}^2(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acos
h(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)
**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.190 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2\sqrt{c+dx-1}\sqrt{c+}}{3bd\left(a+b\cosh^{-1}(c\right.}$$

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^(
3/2)) - (4*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rubi [A] time = 0.408877, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5656, 5775, 5658, 3308, 2180, 2205, 2204}

$$-\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2\sqrt{c+dx-1}\sqrt{c+}}{3bd\left(a+b\cosh^{-1}(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^(
3/2)) - (4*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{3bd}$$

$$=-\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh}}\right.}{3b^2}$$

$$=-\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\right.}{}$$

$$=-\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\right.}{}$$

$$=-\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx\right.}{}$$

$$=-\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cos}}{}\right.}{3b^{5/2}d}$$

Mathematica [A] time = 0.168889, size = 219, normalized size = 1.33

$$\frac{e^{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\left(2e^{\frac{2a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-2\left(be^{\text{co}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

```
[Out] (2*E^((2*a)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCo
sh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c
 + d*x]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCos
h[c + d*x]))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*Arc
Cosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(3*b^2
*d*E^((a + b*ArcCosh[c + d*x])/b)*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{acosh}(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.191 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])^(5/2)), x]/e
```

Rubi [A] time = 0.10754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(5/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 0.0880201, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce} \left(a+b\operatorname{arccosh}\left(dx+c\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\,\operatorname{arcosh}\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^{2}c\sqrt{a+b\operatorname{acosh}(c+dx)}+a^{2}dx\sqrt{a+b\operatorname{acosh}(c+dx)}+2abc\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+2abdx\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^{2}c\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acc}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(a**2*c*sqrt(a + b*acosh(c + d*x)) + a**2*d*x*sqrt(a + b*acosh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*c*sqrt(a + b*acosh(c + d*x))
*acosh(c + d*x)**2 + b**2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2)
, x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.192 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=552

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{\sqrt{\pi}e^4e^{-}}{}$$

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcCosh[c + d
*x])^(3/2)) - (4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (32*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a
 + b*ArcCosh[c + d*x]]) - (40*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c
 + d*x])/(3*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d) + (9*e^4*E^((3*a)/b)
*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2
)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) + (9*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2)*d*E^((3*a)/b)) + (5*e^4*Sq
rt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*
d*E^((5*a)/b))
```

Rubi [A] time = 1.77558, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{\sqrt{\pi}e^4e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```



```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcCosh[c + d
*x])^(3/2)) - (4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
```

```
+ (32*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a
 + b*ArcCosh[c + d*x]]) - (40*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c
 + d*x])/(3*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d) + (9*e^4*E^((3*a)/b)
*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2
)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) + (9*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2)*d*E^((3*a)/b)) + (5*e^4*Sq
rt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*
d*E^((5*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,\right.}{5bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a+\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a+\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a+\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a+\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a+\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a+\right.}$$

Mathematica [A] time = 4.65917, size = 654, normalized size = 1.18

$$e^4\left(-4\left(e^{-\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\text{Gamma}\left(\frac{1}{2},\frac{a}{b}\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^4*(-4*(3*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + ((a + b*
ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c +
 d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b
 + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] + ((a + b*ArcCosh[c + d*x])*(E^(a
/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*Arc
Cosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b
)) - 9*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c + d*x])/
b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b) + (2*(b
+ 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(6*ArcCosh
[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x
]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 5*(a + b*ArcCosh[c +
d*x])*((2*(b + 10*a*(-1 + E^(10*ArcCosh[c + d*x])) - 10*b*ArcCosh[c + d*x]
+ b*E^(10*ArcCosh[c + d*x])*(1 + 10*ArcCosh[c + d*x])))/E^(5*ArcCosh[c + d*
x]) + (20*Sqrt[5]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-5*(a
 + b*ArcCosh[c + d*x]))/b])/E^((5*a)/b) + 20*Sqrt[5]*E^((5*a)/b)*Sqrt[a/b +
 ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*ArcCosh[c
+ d*x]))/b]) - 18*b^2*Sinh[3*ArcCosh[c + d*x]] - 6*b^2*Sinh[5*ArcCosh[c + d
*x]]))/(240*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (dex + ce)^4 (a + b\operatorname{arccosh}(dx + c))^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.193 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=441

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\,\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+4\sqrt{2}$$

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcCosh[c + d*x
])^(3/2)) - (16*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (16*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a +
 b*ArcCosh[c + d*x]]) - (128*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c
+ d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (16*e^3*E^((4*a)/b)*Sqrt[
Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (4*e^3*
E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) + (4*e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rubi [A] time = 1.40616, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\,\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+4\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcCosh[c + d*x
])^(3/2)) - (16*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (16*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a +
 b*ArcCosh[c + d*x]]) - (128*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c
+ d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (16*e^3*E^((4*a)/b)*Sqrt[
```

```
Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (4*e^3*
E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) + (4*e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(6e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,\right.}{5bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{1}{15b^2d\left(a+\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{1}{15b^2d\left(a+\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{1}{15b^2d\left(a+\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{1}{15b^2d\left(a+\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{1}{15b^2d\left(a+\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{1}{15b^2d\left(a+\right.}$$

Mathematica [A] time = 3.01434, size = 445, normalized size = 1.01

$$e^3\left(-2\left(\left(a+b\cosh^{-1}(c+dx)\right)\left(8\sqrt{2}be^{-\frac{2a}{b}}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+8\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*((-4*(a + b*ArcCosh[c + d*x])*(16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*A
rcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + E^
((4*a)/b)*(b + 8*a*(-1 + E^(8*ArcCosh[c + d*x])) - 8*b*ArcCosh[c + d*x] + b
*E^(8*ArcCosh[c + d*x])*(1 + 8*ArcCosh[c + d*x]) + 16*E^(4*(a/b + ArcCosh[c
 + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2,
(4*(a + b*ArcCosh[c + d*x]))/b])))/E^(4*(a/b + ArcCosh[c + d*x])) - 2*((a +
 b*ArcCosh[c + d*x])*((2*(b + 4*a*(-1 + E^(4*ArcCosh[c + d*x])) - 4*b*ArcCo
sh[c + d*x] + b*E^(4*ArcCosh[c + d*x])*(1 + 4*ArcCosh[c + d*x])))/E^(2*ArcC
osh[c + d*x]) + (8*Sqrt[2]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-2*(a + b*ArcCosh[c + d*x]))/b])/E^((2*a)/b) + 8*Sqrt[2]*E^((2*a)/b)*Sq
rt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*Ar
cCosh[c + d*x]))/b]) + 3*b^2*Sinh[2*ArcCosh[c + d*x]]) - 3*b^2*Sinh[4*ArcCo
sh[c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (dex + ce)^{3} (a + b\operatorname{arccosh}(dx + c))^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^{3}}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(7/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")

[Out] sage0*x
```

3.194 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=431

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\cdots}{\cdots}$$

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x]
)^(3/2)) - (4*e^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) + (
16*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c
 + d*x]]) - (24*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^
3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (3*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d) + (e^2*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b)) +
(3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*
b^(7/2)*d*E^((3*a)/b))
```

Rubi [A] time = 1.47498, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205, 5656, 5781}

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```


```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x]
)^(3/2)) - (4*e^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) + (
16*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c
 + d*x]]) - (24*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^
3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
```

```
cCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (3*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d) + (e^2*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b)) +
(3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*
b^(7/2)*d*E^((3*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,\right.}{5bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a+\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a+\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a+\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a+\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a+\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a+\right.}$$

Mathematica [A] time = 2.78614, size = 452, normalized size = 1.05

$$\frac{e^2\left(-2e^{-\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(-6*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*A
rcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c +
d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b
+ ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*(a + b*ArcCosh[c + d*x])*(E^(
a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*Ar
cCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/
b) - 3*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c + d*x])/
b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b) + (2*(b
+ 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(6*ArcCosh
[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x
]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 6*b^2*Sinh[3*ArcCosh[
c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int (dex + ce)^2 (a + b\operatorname{arccosh}(dx + c))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{(b\operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.195 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=266

$$\frac{8\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{32e\sqrt{c+}}{15b^3}$$

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh
[c + d*x])^(5/2)) + (4*e)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*e*
(c + d*x)^2)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (32*e*Sqrt[-1 + c
+ d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]])
 + (8*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(15*b^(7/2)*d) + (8*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rubi [A] time = 0.784347, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205, 5676}

$$\frac{8\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{32e\sqrt{c+}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh
[c + d*x])^(5/2)) + (4*e)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*e*
(c + d*x)^2)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (32*e*Sqrt[-1 + c
+ d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]])
 + (8*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(15*b^(7/2)*d) + (8*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c\right.}{5bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

Mathematica [B] time = 4.52608, size = 916, normalized size = 3.44

$$\frac{e\left(8c\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)\cosh^{-1}(c+dx)^2b^{5/2} + 4c(c+dx)\cosh^{-1}(c+dx)b^{5/2} - 4\cosh^{-1}(c+dx)\cosh\left(2\cosh^{-1}(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e*(4*a*b^(3/2)*c*(c + d*x) + 8*a^2*Sqrt[b]*c*Sqrt[(-1 + c + d*x)/(1 + c +
d*x)]*(1 + c + d*x) + 4*b^(5/2)*c*(c + d*x)*ArcCosh[c + d*x] + 16*a*b^(3/2)
*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x] + 8*b^
(5/2)*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x]^2
 - 4*a*b^(3/2)*Cosh[2*ArcCosh[c + d*x]] - 4*b^(5/2)*ArcCosh[c + d*x]*Cosh[2
*ArcCosh[c + d*x]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*
Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c +
 d*x])^(5/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*Erfi[Sqrt[a + b
*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*C
osh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - (2*Sqrt
[b]*c*(a + b*ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b
+ ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*G
amma[1/2, a/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*Sqrt[b]*c*(a +
b*ArcCosh[c + d*x])*(E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c +
d*x]) + 2*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCo
sh[c + d*x])/b)]))/E^(a/b) - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Er
f[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 4*c*Sqrt[Pi]*(a + b*Arc
Cosh[c + d*x])^(5/2)*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] +
 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCos
h[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])
^(5/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] -
 16*a^2*Sqrt[b]*Sinh[2*ArcCosh[c + d*x]] - 3*b^(5/2)*Sinh[2*ArcCosh[c + d*x
]] - 32*a*b^(3/2)*ArcCosh[c + d*x]*Sinh[2*ArcCosh[c + d*x]] - 16*b^(5/2)*Ar
cCosh[c + d*x]^2*Sinh[2*ArcCosh[c + d*x]]))/(15*b^(7/2)*d*(a + b*ArcCosh[c
+ d*x])^(5/2))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^{-\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.196 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{c+dx-1}\sqrt{c+}}{15b^3d\sqrt{a+b\cosh^{-}}}$$

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh[c + d*x])^(
5/2)) - (4*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (
4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d
) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E
^(a/b))
```

Rubi [A] time = 0.627465, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5656, 5775, 5781, 3307, 2180, 2204, 2205}

$$\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{c+dx-1}\sqrt{c+}}{15b^3d\sqrt{a+b\cosh^{-}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```


```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh[c + d*x])^(
5/2)) - (4*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (
4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d
) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E
^(a/b))
```


Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{5bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4\,\text{Subst}\left(\int \frac{}{\left(a+b\cos\right.}\right.}{1}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1+c+dx}\sqrt{1}}{15b^3d\sqrt{a+b\cosh}}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1+c+dx}\sqrt{1}}{15b^3d\sqrt{a+b\cosh}}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1+c+dx}\sqrt{1}}{15b^3d\sqrt{a+b\cosh}}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1+c+dx}\sqrt{1}}{15b^3d\sqrt{a+b\cosh}}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1+c+dx}\sqrt{1}}{15b^3d\sqrt{a+b\cosh}}$$

Mathematica [A] time = 0.746831, size = 243, normalized size = 1.16

$$-\frac{\frac{2e^{-\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-2a-2b\cosh^{-1}(c+dx)+b\right)}{b^2} - \cdots}{15bd\left(a+b\cos\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*ArcCosh[c
+ d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c + d*x])*Sqr
t[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh
[c + d*x]]))/(b^2*E^ArcCosh[c + d*x]) - (2*(a + b*ArcCosh[c + d*x])*(E^(a/b
 + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*ArcCo
sh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(b^2*E^(
a/b)))/(15*b*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (a + b\text{arccosh}\,(dx + c))^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{arcosh}\,(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.197 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcCosh[c + d*x])^(7/2)), x]/e
```

Rubi [A] time = 0.103044, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(7/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.091956, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)), x]
```

Maple [A] time = 0.231, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\operatorname{arccosh}\left(dx+c\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}\left(dx+c\right)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.198 $\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=189

$$\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de}-\frac{28be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}}{405d}-\frac{28be^3\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\text{si}\right.}{135d\sqrt{-c-dx}\sqrt{c+d.}}$$

```
[Out] (-28*b*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(405*d
) - (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(7/2)*Sqrt[1 + c + d*x])/(81*d) +
 (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x]))/(9*d*e) - (28*b*e^3*Sqrt[
1 - c - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]],
 2])/(135*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rubi [A] time = 0.143705, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 114, 113}

$$\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de}-\frac{28be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}}{405d}-\frac{28be^3\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\text{si}\right.}{135d\sqrt{-c-dx}\sqrt{c+d.}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x]),x]
```


```
[Out] (-28*b*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(405*d
) - (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(7/2)*Sqrt[1 + c + d*x])/(81*d) +
 (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x]))/(9*d*e) - (28*b*e^3*Sqrt[
1 - c - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]],
 2])/(135*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```


Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{-1+x}\sqrt{1+x}}dx,x,c+d\right.}{9de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c\right.}{9de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c\right.}{9de}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{7}}{81d}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{7}}{81d}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{7}}{81d}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{7}}{81d}$$

Mathematica [C] time = 0.327371, size = 150, normalized size = 0.79

$$\frac{2(e(c+dx))^{7/2}\left(\frac{2b(c+dx)^{3/2}\left(-7\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},(c+dx)^2\right)+5\left(1-(c+dx)^2\right)(c+dx)^2+7\left(1-(c+dx)^2\right)\right)}{45\sqrt{c+dx-1}\sqrt{c+dx+1}}+(c+dx)^{9/2}\left(a+b\cosh\right.\right.}{9d(c+dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*((c + d*x)^(9/2)*(a + b*ArcCosh[c + d*x]) + (2*b*(c
+ d*x)^(3/2)*(7*(1 - (c + d*x)^2) + 5*(c + d*x)^2*(1 - (c + d*x)^2) - 7*Sqr
t[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2]))/(45*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(9*d*(c + d*x)^(7/2))
```

Maple [C] time = 0.064, size = 277, normalized size = 1.5

$$2\,{\frac {1}{de}\left(1/9\, \left(dex+ce \right) ^{9/2}a+b \left(1/9\, \left(dex+ce \right) ^{9/2}{\rm arccosh} \left({\frac {dex+ce}{e}}\right)-{\frac {2}{405\,e}\left(5\,\sqrt {-{e}^{-1}} \left(dex+ce \right) ^{11/2}+2\,\sqrt {-{e}^{-1}} \left(dex+ce \right) ^{7/2}\right.}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/9*(d*e*x+c*e)^(9/2)*a+b*(1/9*(d*e*x+c*e)^(9/2)*arccosh(1/e*(d*e*x+
c*e))-2/405/e*(5*(-1/e)^(1/2)*(d*e*x+c*e)^(11/2)+2*(-1/e)^(1/2)*(d*e*x+c*e)
^(7/2)*e^2-7*(-1/e)^(1/2)*(d*e*x+c*e)^(3/2)*e^4+21*e^5*((d*e*x+c*e+e)/e)^(1
/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-21
*e^5*((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticE((d*e*x+c*e)
^(1/2)*(-1/e)^(1/2),I))/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e)
/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\left(ad^{3}e^{3}x^{3} + 3\,acd^{2}e^{3}x^{2} + 3\,ac^{2}de^{3}x + ac^{3}e^{3} + \left(bd^{3}e^{3}x^{3} + 3\,bcd^{2}e^{3}x^{2} + 3\,bc^{2}de^{3}x + bc^{3}e^{3}\right){\rm arcosh}\left(dx + c\right)\right)\sqrt{dex\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^3*e^3*x^3 + 3*a*c*d^2*e^3*x^2 + 3*a*c^2*d*e^3*x + a*c^3*e^3 +
 (b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + b*c^3*e^3)*arccosh(
```

```
d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*acosh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{7}{2}}(b\,\mathrm{arcosh}\,(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^(7/2)*(b*arccosh(d*x + c) + a), x)
```

3.199 $\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=169

$$-\frac{20be^{5/2}\sqrt{-c-dx+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{147d\sqrt{c+dx-1}}+\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{20be^2\sqrt{c+dx-1}\sqrt{c-}}{147}$$

```
[Out] (-20*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(147*d)
- (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqrt[1 + c + d*x])/(49*d) + (
2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x]))/(7*d*e) - (20*b*e^(5/2)*Sqr
t[1 - c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(147*d*Sqr
t[-1 + c + d*x])
```

Rubi [A] time = 0.133221, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 117, 116}

$$\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{20be^2\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}}{147d}-\frac{20be^{5/2}\sqrt{-c-dx+1}F\left(\sin^{-1}\left(\frac{\sqrt{e(c+}}{\sqrt{e}}\right.\right.}{147d\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-20*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(147*d)
- (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqrt[1 + c + d*x])/(49*d) + (
2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x]))/(7*d*e) - (20*b*e^(5/2)*Sqr
t[1 - c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(147*d*Sqr
t[-1 + c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}}{\sqrt{-1+x}\sqrt{1+x}}dx,x,c+\right.}{7de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d} + \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(\right.}{7de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d} + \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(\right.}{7de}$$

$$= -\frac{20be^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{147d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/}}{49d}$$

$$= -\frac{20be^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{147d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/}}{49d}$$

$$= -\frac{20be^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{147d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/}}{49d}$$

$$= -\frac{20be^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{147d} - \frac{4b\sqrt{-1+c+dx}(e(c+dx))^{5/}}{49d}$$

Mathematica [C] time = 0.26376, size = 180, normalized size = 1.07

$$\frac{2(e(c+dx))^{5/2}\left(-10b\sqrt{1-(c+dx)^2}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right)+21a\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3-\right.}{147d\sqrt{\frac{c+dx-1}{c+dx}}(c+dx)^{5/2}\sqrt{c+dx+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(10*b - 4*b*(c + d*x)^2 - 6*b*(c + d*x)^4 + 21*a*Sqr
t[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x] + 21*b*Sqrt[-1 + c + d*x]*(c
+ d*x)^3*Sqrt[1 + c + d*x]*ArcCosh[c + d*x] - 10*b*Sqrt[1 - (c + d*x)^2]*Hy
pergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2]))/(147*d*Sqrt[(-1 + c + d*x)/(c
 + d*x)]*(c + d*x)^(5/2)*Sqrt[1 + c + d*x])
```

Maple [A] time = 0.023, size = 218, normalized size = 1.3

$$2\,{\frac{1}{de}\left(1/7\, \left(dex+ce \right) ^{7/2}a+b \left(1/7\, \left(dex+ce \right) ^{7/2}{\rm arccosh} \left({\frac{dex+ce}{e}}\right)-{\frac{2}{147\,e} \left(3\,\sqrt{-{e}^{-1}} \left(dex+ce \right) ^{9/2}+2\,\sqrt{-{e}^{-1}} \left(dex+ce \right) ^{5/2}e\right.}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arccosh(1/e*(d*e*x+
c*e))-2/147/e*(3*(-1/e)^(1/2)*(d*e*x+c*e)^(9/2)+2*(-1/e)^(1/2)*(d*e*x+c*e)^
(5/2)*e^2+5*e^4*((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticF(
(d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-5*(-1/e)^(1/2)*(d*e*x+c*e)^(1/2)*e^4)/(-1
/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(a d^{2} e^{2} x^{2} + 2 \, a c d e^{2} x + a c^{2} e^{2} + {\left(b d^{2} e^{2} x^{2} + 2 \, b c d e^{2} x + b c^{2} e^{2}\right)} \operatorname{arcosh}\left(d x + c\right)\right)} \sqrt{d e x + c e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^2*e^2*x^2 + 2*a*c*d*e^2*x + a*c^2*e^2 + (b*d^2*e^2*x^2 + 2*b*
c*d*e^2*x + b*c^2*e^2)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{5}{2}} (b \operatorname{arcosh}(dx + c) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a), x)
```

3.200 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=145

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}}{25d}-\frac{12be\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\sin^{-1}\right.}{25d\sqrt{-c-dx}\sqrt{c+dx-1}}$$

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(25*d) + (2
*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x]))/(5*d*e) - (12*b*e*Sqrt[1 - c
 - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/
(25*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rubi [A] time = 0.111114, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 114, 113}

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}}{25d}-\frac{12be\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\sin^{-1}\right.}{25d\sqrt{-c-dx}\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(25*d) + (2
*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x]))/(5*d*e) - (12*b*e*Sqrt[1 - c
 - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/
(25*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+d\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c\right.}{5de}$$

Mathematica [C] time = 0.451433, size = 109, normalized size = 0.75

$$\frac{2(e(c+dx))^{3/2}\left(5(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)-\frac{2b\left(\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},(c+dx)^2\right)+c^2+2cdx+d^2x^2-1\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right)}{25d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*(c + d*x)*(a + b*ArcCosh[c + d*x]) - (2*b*(-1 + c
^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4,
7/4, (c + d*x)^2]))/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(25*d)
```

Maple [C] time = 0.02, size = 254, normalized size = 1.8

$$2\,\frac{1}{de}\left(1/5\,(dex+ce)^{5/2}a+b\left(1/5\,(dex+ce)^{5/2}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2}{25}\,\frac{1}{e}\left(\sqrt{-e^{-1}}\,(dex+ce)^{7/2}+3\,\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex}{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arccosh(1/e*(d*e*x+
c*e))-2/25/e*((-1/e)^(1/2)*(d*e*x+c*e)^(7/2)+3*((d*e*x+c*e+e)/e)^(1/2)*(-(d
*e*x+c*e-e)/e)^(1/2)*e^3*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-3*e^3*
((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2
)*(-1/e)^(1/2),I)-(-1/e)^(1/2)*(d*e*x+c*e)^(3/2)*e^2)/(-1/e)^(1/2)/((d*e*x+
c*e+e)/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(adex + ace + (bdex + bce)\operatorname{arcosh}(dx + c)\right)\sqrt{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arccosh(d*x + c))*sqrt(d*e*x
+ c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c + dx))^{\frac{3}{2}} (a + b\operatorname{acosh}(c + dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}}(b\operatorname{arcosh}(dx + c) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a), x)
```

3.201 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=127

$$-\frac{4b\sqrt{e}\sqrt{-c-dx+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{9d\sqrt{c+dx-1}}+\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{c+dx-1}\sqrt{c+dx}}{9d}$$

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(9*d) + (2*(e
*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x]))/(3*d*e) - (4*b*Sqrt[e]*Sqrt[1 -
 c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(9*d*Sqrt[-1 +
c + d*x])
```

Rubi [A] time = 0.0971451, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 117, 116}

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}}{9d}-\frac{4b\sqrt{e}\sqrt{-c-dx+1}F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\right|}{9d\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(9*d) + (2*(e
*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x]))/(3*d*e) - (4*b*Sqrt[e]*Sqrt[1 -
 c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(9*d*Sqrt[-1 +
c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+d\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+\right.}{3de}$$

Mathematica [C] time = 0.43508, size = 131, normalized size = 1.03

$$\frac{\sqrt{e(c+dx)}\left(\frac{2}{3}(c+dx)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right) - \frac{4b\left(\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right)+c^2+2cdx+d^2x^2-1\right)}{9\sqrt{\frac{c+dx-1}{c+dx}}\sqrt{c+dx+1}}\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (Sqrt[e*(c + d*x)]*((2*(c + d*x)^(3/2)*(a + b*ArcCosh[c + d*x]))/3 - (4*b*(
-1 + c^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/4,
 1/2, 5/4, (c + d*x)^2]))/(9*Sqrt[(-1 + c + d*x)/(c + d*x)]*Sqrt[1 + c + d*
x])))/(d*Sqrt[c + d*x])
```

Maple [A] time = 0.016, size = 194, normalized size = 1.5

$$2\,\frac{1}{de}\left(1/3\,(dex+ce)^{3/2}\,a+b\left(1/3\,(dex+ce)^{3/2}\,\text{arccosh}\left(\frac{dex+ce}{e}\right)-2/9\,\frac{1}{e}\left(\sqrt{-e^{-1}}\,(dex+ce)^{5/2}+\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex}{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arccosh(1/e*(d*e*x+
c*e))-2/9/e*((-1/e)^(1/2)*(d*e*x+c*e)^(5/2)+((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*
x+c*e-e)/e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e^2-(-1/e)^(1
/2)*(d*e*x+c*e)^(1/2)*e^2)/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e
-e)/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dex+ce}(b\operatorname{arcosh}(dx+c)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e(c+dx)}\,(a+b\operatorname{acosh}(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex + ce}(b \operatorname{arcosh}(dx + c) + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a), x)
```

3.202 $\int \frac{a+b\cosh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\sqrt{c+dx-1}}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x]))/(d*e) - (4*b*Sqrt[1 - c - d*
x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(d*e*
Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rubi [A] time = 0.0878043, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5866, 5662, 114, 113}

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x]))/(d*e) - (4*b*Sqrt[1 - c - d*
x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(d*e*
Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\\
&= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{\left(\sqrt{2}b\sqrt{1-c-dx}\sqrt{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{-x}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{1+x}}\,d\right.}{de\sqrt{-c-dx}\sqrt{-1+c+dx}}\\
&= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{4b\sqrt{1-c-dx}\sqrt{e(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{1+c+dx}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\sqrt{-1+c+dx}}
\end{aligned}$$

Mathematica [C] time = 0.177272, size = 94, normalized size = 0.9

$$\frac{2\sqrt{e(c+dx)}\left(3\left(a+b\cosh^{-1}(c+dx)\right)-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},(c+dx)^2\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 -
(c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2])/(Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x])))/(3*d*e)
```

Maple [C] time = 0.019, size = 138, normalized size = 1.3

$$2\frac{1}{de}\left(a\sqrt{dex+ce}+b\left(\sqrt{dex+ce}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-2\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-e^{-1}},i\right)-\operatorname{EllipticE}\left(\sqrt{dex+ce}\sqrt{-e^{-1}},\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(a*(d*e*x+c*e)^(1/2)+b*((d*e*x+c*e)^(1/2)*arccosh(1/e*(d*e*x+c*e))-2*
(EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/2)*(-
1/e)^(1/2),I))*(-(d*e*x+c*e-e)/e)^(1/2)/(-1/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\operatorname{arcosh}(dx+c)+a}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acosh}(c + dx)}{\sqrt{e(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/sqrt(e*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcosh}(dx + c) + a}{\sqrt{dex + ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

3.203 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{4b\sqrt{-c-dx+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{de^{3/2}\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[1 - c - d
*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(d*e^(3/2)*Sqrt[-1 +
c + d*x])
```

Rubi [A] time = 0.0839204, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5866, 5662, 117, 116}

$$\frac{4b\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[1 - c - d
*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(d*e^(3/2)*Sqrt[-1 +
c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\
&= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\\
&= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}+\frac{\left(2b\sqrt{1-c-dx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de\sqrt{-1+c+dx}}\\
&= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}+\frac{4b\sqrt{1-c-dx}F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{-1+c+dx}}
\end{aligned}$$

Mathematica [C] time = 0.155461, size = 92, normalized size = 1.1

$$\frac{2\left(\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}-a-b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-a - b*ArcCosh[c + d*x] + (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeo
metric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x
```

```
])))/(d*e*Sqrt[e*(c + d*x)])
```

Maple [A] time = 0.016, size = 119, normalized size = 1.4

$$2\,\frac{1}{de}\left(-\frac{a}{\sqrt{dex+ce}}+b\left(-\frac{1}{\sqrt{dex+ce}}\mathrm{arccosh}\left(\frac{dex+ce}{e}\right)+2\,\frac{1}{e}\mathrm{EllipticF}\left(\sqrt{dex+ce}\sqrt{-e^{-1}},i\right)\sqrt{-\frac{dex+ce-e}{e}}\frac{1}{\sqrt{-e^{-1}}}\frac{1}{\sqrt{\frac{dex+ce-e}{e}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arccosh(1/e*(d*e*x+c*e)
)+2/e*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*(-(d*e*x+c*e-e)/e)^(1/2)/
(-1/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dex+ce}(b\,\mathrm{arcosh}\,(dx+c)+a)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{acosh}\,(c + dx)}{(e\,(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcosh}\,(dx + c) + a}{(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

3.204 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}}{3de^2\sqrt{e(c+dx)}}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-c-dx}\sqrt{c+dx-1}}$$

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*d*e^2*Sqrt[e*(c + d*x)]) - (2
*(a + b*ArcCosh[c + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*Sqrt[1 - c -
d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(3*
d*e^3*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rubi [A] time = 0.120569, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 5662, 104, 12, 16, 114, 113}

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}}{3de^2\sqrt{e(c+dx)}}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-c-dx}\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*d*e^2*Sqrt[e*(c + d*x)]) - (2
*(a + b*ArcCosh[c + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*Sqrt[1 - c -
d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(3*
d*e^3*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}(ex)^{3/2}\sqrt{1+x}}\,dx, x, c+dx\right)}{3de}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int -\frac{ex}{2\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,d\right.}{3de^3}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,\right.}{3de^2}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, c\right.}{3de^3}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{\left(\sqrt{2}b\sqrt{1-c-dx}\sqrt{e(c+dx)}\right)\text{Su}}{3de^3\sqrt{-c-dx}}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{4b\sqrt{1-c-dx}\sqrt{e(c+dx)}E\left(\sin\right.}{3de^3\sqrt{-c-dx}\sqrt{-1+c}}$$

Mathematica [C] time = 0.139273, size = 94, normalized size = 0.63

$$\frac{2\left(-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},(c+dx)^2\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}} - a - b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (2*(-a - b*ArcCosh[c + d*x] - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeo
metric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x])))/(3*d*e*(e*(c + d*x))^(3/2))
```

Maple [C] time = 0.026, size = 269, normalized size = 1.8

$$2\frac{1}{de}\left(-1/3\frac{a}{(dex+ce)^{3/2}}+b\left(-1/3\frac{1}{(dex+ce)^{3/2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)+2/3\frac{1}{e^3\sqrt{dex+ce}}\left(-\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex+ce-e}{e}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arccosh(1/e*(d*e*
x+c*e))+2/3/e^3*(-((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*(d*e*x+c
*e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e+((d*e*x+c*e+e)/e)^(
1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*(d*e*x+c*e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2)
*(-1/e)^(1/2),I)*e+(-1/e)^(1/2)*(d*e*x+c*e)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2
)/(d*e*x+c*e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dex+ce}(b\operatorname{arcosh}(dx+c)+a)}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{acosh}(c+dx)}{(e(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.205 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=130

$$\frac{4b\sqrt{-c-dx+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),-1\right)}{15de^{7/2}\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}}{15de^2(e(c+dx))^{3/2}}$$

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*d*e^2*(e*(c + d*x))^(3/2)) -
 (2*(a + b*ArcCosh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) + (4*b*Sqrt[1 - c
 - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(15*d*e^(7/2)*Sqr
t[-1 + c + d*x])
```

Rubi [A] time = 0.111154, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 5662, 104, 12, 16, 117, 116}

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{c+dx-1}\sqrt{c+dx+1}}{15de^2(e(c+dx))^{3/2}}+\frac{4b\sqrt{-c-dx+1}F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{15de^{7/2}\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*d*e^2*(e*(c + d*x))^(3/2)) -
 (2*(a + b*ArcCosh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) + (4*b*Sqrt[1 - c
 - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(15*d*e^(7/2)*Sqr
t[-1 + c + d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}(ex)^{5/2}\sqrt{1+x}}\,dx, x, c+dx\right)}{5de}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{ex}{2\sqrt{-1+x}(ex)^{3/2}\sqrt{1+x}}\right.}{15de^3}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}(ex)^{3/2}\sqrt{1+x}}\right.}{15de^2}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx\right.}{15de^3}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{\left(2b\sqrt{1-c-dx}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{15de^3\sqrt{-1+c}}$$

$$= \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{4b\sqrt{1-c-dx}F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx}}{\sqrt{e}}\right.\right.}{15de^{7/2}\sqrt{-1+c+dx}}$$

Mathematica [C] time = 0.151375, size = 94, normalized size = 0.72

$$\frac{2\left(-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},(c+dx)^2\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}} - 3\left(a+b\cosh^{-1}(c+dx)\right)\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hype
rgeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x])))/(15*d*e*(e*(c + d*x))^(5/2))
```

Maple [A] time = 0.027, size = 201, normalized size = 1.6

$$2\frac{1}{de}\left(-1/5\frac{a}{(dex+ce)^{5/2}} + b\left(-1/5\frac{1}{(dex+ce)^{5/2}}\text{arccosh}\left(\frac{dex+ce}{e}\right) + 2/15\frac{1}{e^3(dex+ce)^{3/2}}\left(\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{dex+ce}{e}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arccosh(1/e*(d*e*
x+c*e))+2/15/e^3*(((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*Elliptic
F((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*(d*e*x+c*e)^(3/2)+(-1/e)^(1/2)*(d*e*x+c
*e)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2)/(d*e*x+c*e)^(3/2)/((d*e*x+c*e+e)/e)^(1
/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dex+ce}(b\,\text{arcosh}\,(dx+c)+a)}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arcosh}(dx+c)+a}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

3.206 $\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{13/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{13}{4},\frac{13}{4}\right\},\left\{\frac{15}{4},\frac{17}{4}\right\},(c+dx)^2\right)}{1287de^3}-\frac{8b\sqrt{-c-dx+1}(e(c+dx))^{11/2}\text{Hypergeo}}{99}$$

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])^2)/(9*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(11/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/
2, 11/4, 15/4, (c + d*x)^2])/(99*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c
+ d*x))^(13/2)*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, (c + d*x)^2
])/(1287*d*e^3)
```

Rubi [A] time = 0.32138, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};(c+dx)^2\right)}{1287de^3}-\frac{8b\sqrt{1-(c+dx)^2}(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};(c+dx)^2\right)\left(a+b\cos\right.}{99de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F
1[1/2, 11/4, 15/4, (c + d*x)^2])/(99*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c +
d*x]) - (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1, 13/4, 13/4}, {15
/4, 17/4}, (c + d*x)^2])/(1287*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{9de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}} \ldots\right.}{9de}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{9de} - \frac{8b(e(c+dx))^{11/2}\sqrt{1-(c+dx)^2}\,\left(\ldots\right.}{99de^2\sqrt{-\ldots}}$$

Mathematica [A] time = 0.50461, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{9/2}\left(143\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{1,\frac{13}{4},\frac{13}{4}\right\},\left\{\frac{15}{4},\frac{17}{4}\right\},(c+\ldots\right.\right.\right.}{1287de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((13
*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 11/4
, 15/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x
)*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, (c + d*x)^2])))/(1287*d*
```

```
e)
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{7}{2}} (a+b\operatorname{arccosh}(dx+c))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\Big(a^2d^3e^3x^3 + 3\,a^2cd^2e^3x^2 + 3\,a^2c^2de^3x + a^2c^3e^3 + \big(b^2d^3e^3x^3 + 3\,b^2cd^2e^3x^2 + 3\,b^2c^2de^3x + b^2c^3e^3\big)\operatorname{arcosh}(dx + c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^3*e^3*x^3 + 3*a^2*c*d^2*e^3*x^2 + 3*a^2*c^2*d*e^3*x + a^2*c
^3*e^3 + (b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*b^2*c^2*d*e^3*x + b^2*c
^3*e^3)*arccosh(d*x + c)^2 + 2*(a*b*d^3*e^3*x^3 + 3*a*b*c*d^2*e^3*x^2 + 3*a
*b*c^2*d*e^3*x + a*b*c^3*e^3)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{7}{2}} (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arccosh(d*x + c) + a)^2, x)
```

3.207 $\int (ce+dex)^{5/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{11/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},(c+dx)^2\right)}{693de^3}-\frac{8b\sqrt{-c-dx+1}(e(c+dx))^{9/2}\text{Hypergeon}}{63}$$

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])^2)/(7*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 9/4, 13/4, (c + d*x)^2])/(63*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c +
d*x))^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])
/(693*d*e^3)
```

Rubi [A] time = 0.327672, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)}{693de^3}-\frac{8b\sqrt{1-(c+dx)^2}(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};(c+dx)^2\right)\left(a+b\cosh^{-}\right.}{63de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1
[1/2, 9/4, 13/4, (c + d*x)^2])/(63*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]) - (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4
, 15/4}, (c + d*x)^2])/(693*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\right.}{7de}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de} - \frac{8b(e(c+dx))^{9/2}\sqrt{1-(c+dx)^2}\left(a\right.}{63de^2\sqrt{-1}}$$

Mathematica [A] time = 0.463746, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{7/2}\left(99\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},(c+d\right.\right.\right.}{693de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((11*
Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 9/4,
13/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*
HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])))/(693*d*e)
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{5}{2}} (a + b\operatorname{arccosh}(dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a^2d^2e^2x^2 + 2\,a^2cde^2x + a^2c^2e^2 + \left(b^2d^2e^2x^2 + 2\,b^2cde^2x + b^2c^2e^2\right)\operatorname{arcosh}(dx + c)^2 + 2\left(abd^2e^2x^2 + 2\,abcde^2x +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arccosh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
+ 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{5}{2}} (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a)^2, x)
```

3.208 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{9/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{9}{4},\frac{9}{4}\right\},\left\{\frac{11}{4},\frac{13}{4}\right\},(c+dx)^2\right)}{315de^3}-\frac{8b\sqrt{-c-dx+1}(e(c+dx))^{7/2}\text{Hypergeomet}}{35de^2}$$

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^2)/(5*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 7/4, 11/4, (c + d*x)^2])/(35*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c +
d*x))^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])/(3
15*d*e^3)
```

Rubi [A] time = 0.326341, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)}{315de^3}-\frac{8b\sqrt{1-(c+dx)^2}(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c\right.}{35de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1
[1/2, 7/4, 11/4, (c + d*x)^2])/(35*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]) - (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 1
3/4}, (c + d*x)^2])/(315*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\right.}{5de}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de} - \frac{8b(e(c+dx))^{7/2}\sqrt{1-(c+dx)^2}\,\left(a\right.}{35de^2\sqrt{-1}}$$

Mathematica [A] time = 0.476456, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{5/2}\left(63\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{1,\frac{9}{4},\frac{9}{4}\right\},\left\{\frac{11}{4},\frac{13}{4}\right\},(c+dx\right.\right.\right.}{315de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((9*S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 1
1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*H
ypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])))/(315*d*e)
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}} (a + b\operatorname{arccosh}(dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2dex + a^2ce + \left(b^2dex + b^2ce\right)\operatorname{arcosh}(dx + c)^2 + 2\,(abdex + abce)\operatorname{arcosh}(dx + c)\right)\sqrt{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arccosh(d*x + c)^2 +
2*(a*b*d*e*x + a*b*c*e)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c + dx))^{\frac{3}{2}} (a + b\operatorname{acosh}(c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{3}{2}}(b\operatorname{arcosh}(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^2, x)
```

3.209 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{7/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},(c+dx)^2\right)}{105de^3}-\frac{8b\sqrt{-c-dx+1}(e(c+dx))^{5/2}\text{Hypergeometr}}{15de^2\sqrt{}}$$

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^2)/(3*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, (c + d*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c + d
*x))^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])/(105
*d*e^3)
```

Rubi [A] time = 0.315728, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)}{105de^3}-\frac{8b\sqrt{1-(c+dx)^2}(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+\right.}{15de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1
[1/2, 5/4, 9/4, (c + d*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x
]) - (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/
4}, (c + d*x)^2])/(105*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}} dx\right.}{3de}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de} - \frac{8b(e(c+dx))^{5/2}\sqrt{1-(c+dx)^2}\left(a+\right.}{15de^2\sqrt{-1+}}$$

Mathematica [A] time = 0.435795, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{3/2}\left(35\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},(c+dx)^2\right.\right.\right.}{105de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((7*S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9
/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*Hy
pergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])))/(105*d*e)
```

Maple [F] time = 0.37, size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{arccosh}\,(dx+c))^{2}\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2}\,\mathrm{arcosh}\,(dx+c)^{2}+2\,ab\,\mathrm{arcosh}\,(dx+c)+a^{2}\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\,(c+dx)}\,(a+b\,\mathrm{acosh}\,(c+dx))^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex+ce}(b\operatorname{arcosh}(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

3.210 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=151

$$-\frac{16b^2(e(c+dx))^{5/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},(c+dx)^2\right)}{15de^3}-\frac{8b\sqrt{-c-dx+1}(e(c+dx))^{3/2}\text{Hypergeometri}}{3de^2\sqrt{c}}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^2)/(d*e) - (8*b*Sqrt[1 - c -
d*x]*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 3/
4, 7/4, (c + d*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c + d*x))^
(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])/(15*d*e^3)
```

Rubi [A] time = 0.289153, antiderivative size = 163, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)}{15de^3}-\frac{8b\sqrt{1-(c+dx)^2}(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+\right.}{3de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 3/4, 7/4, (c + d*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) -
(16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c
 + d*x)^2])/(15*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de} - \frac{8b(e(c+dx))^{3/2}\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+\right.}{3de^2\sqrt{-1+c+dx}\sqrt{1+}}$$

Mathematica [A] time = 0.328131, size = 140, normalized size = 0.93

$$\frac{2\sqrt{e(c+dx)}\left(15\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},(c+dx)^2\right)+\right.\right.}{15de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((5*Sqr
t[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4
, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*Hype
```

```
rgeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])))/(15*d*e)
```

Maple [F] time = 0.412, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^2 \frac{1}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2 \operatorname{arcosh}(dx + c)^2 + 2\, ab \operatorname{arcosh}(dx + c) + a^2}{\sqrt{dex + ce}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/sqrt(d*e*x
 + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acosh}(c + dx))^2}{\sqrt{e(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx + c) + a)^2}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

3.211 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{16b^2(e(c+dx))^{3/2}\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{-c-dx+1}\sqrt{e(c+dx)}\text{Hypergeometric2I}}{de^2\sqrt{c+}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[1 - c -
 d*x]*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4, 1/2
, 5/4, (c + d*x)^2])/(d*e^2*Sqrt[-1 + c + d*x]) + (16*b^2*(e*(c + d*x))^(3/
2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2])/(3*d*e^3)
```

Rubi [A] time = 0.304253, antiderivative size = 161, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{1-(c+dx)^2}\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right.}{de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4,
 1/2, 5/4, (c + d*x)^2])/(d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + (16
*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, (c +
d*x)^2])/(3*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{e(c+dx)}\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\Big(}{de^2\sqrt{-1+c+dx}\sqrt{1+c+dx}}$$

Mathematica [A] time = 0.289743, size = 140, normalized size = 0.94

$$\frac{2\left(4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},(c+dx)^2\right)+\frac{3\sqrt{1-(c+dx)^2}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},(c+dx)^2\right.}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.}{3de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*((3*Sqrt[1 - (c + d*x)^2]
*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*HypergeometricPFQ[{3/4
```

```
, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2]))/(3*d*e*Sqrt[e*(c + d*x)])
```

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int (a + b\,\text{arccosh}\,(dx + c))^2 (dex + ce)^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2\right)\sqrt{dex+ce}}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}{(c + dx)})^2}{(e\,(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(dx + c) + a)^2}{(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

3.212 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{16b^2\sqrt{e(c+dx)}\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{4},1\right\},\left\{\frac{3}{4},\frac{5}{4}\right\},(c+dx)^2\right)}{3de^3}-\frac{8b\sqrt{-c-dx+1}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4}\right.}{3de^2\sqrt{c+dx-1}\sqrt{e(c}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*Sqrt[1 -
 c - d*x]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d
*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]) - (16*b^2*Sqrt[e*(c
+ d*x)]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])/(3*d*e^3
)
```

Rubi [A] time = 0.312231, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)}{3de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{3de^2\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}}-\frac{2\left(a+\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*Sqrt[1 -
 (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c
 + d*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x]
) - (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4},
(c + d*x)^2])/(3*d*e^3)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}(ex)^{3/2}\sqrt{1+x}}\,dx, x, c+dx\right)}{3de}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} - \frac{8b\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c\right.}{3de^2\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}$$

Mathematica [A] time = 0.357624, size = 140, normalized size = 0.92

$$\frac{2\left(4b(c+dx)\left(-2b(c+dx)\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{4},1\right\},\left\{\frac{3}{4},\frac{5}{4}\right\},(c+dx)^2\right)-\frac{\sqrt{1-(c+dx)^2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},(c+dx\right.}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (2*(-(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]*(
a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sq
```

```
rt[-1 + c + d*x]*Sqrt[1 + c + d*x])) - 2*b*(c + d*x)*HypergeometricPFQ[{1/4
, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2))
```

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(dx+c))^{2} (dex+ce)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^{2} \operatorname{arcosh}(dx+c)^{2}+2\,ab \operatorname{arcosh}(dx+c)+a^{2}\right)\sqrt{dex+ce}}{d^{3}e^{3}x^{3}+3\,cd^{2}e^{3}x^{2}+3\,c^{2}de^{3}x+c^{3}e^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^2}{(e\,(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.213 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=153

$$\frac{16b^2\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},1\right\},\left\{\frac{1}{4},\frac{3}{4}\right\},(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\sqrt{-c-dx+1}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},(c+dx)^2\right)}{15de^2\sqrt{c+dx-1}(e(c+dx))^{3/2}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*Sqrt[1 -
 c - d*x]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c + d
*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)) + (16*b^2*Hyperge
ometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])/(15*d*e^3*Sqrt[e*(c +
 d*x)])
```

Rubi [A] time = 0.317303, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5866, 5662, 5763}

$$\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{15de^2\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}}-\frac{2\left(a+b\cosh^{-1}\right.}{5de(e(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*Sqrt[1 -
 (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c
 + d*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d
*x]) + (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])
/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}(ex)^{5/2}\sqrt{1+x}}\,dx, x, c+dx\right)}{5de}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} - \frac{8b\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c\right.}{15de^2\sqrt{-1+c+dx}(e(c+dx))^{3/2}\sqrt{1+c+dx}}$$

Mathematica [A] time = 0.358658, size = 140, normalized size = 0.92

$$\frac{2\left(4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},1\right\},\left\{\frac{1}{4},\frac{3}{4}\right\},(c+dx)^2\right)-\frac{\sqrt{1-(c+dx)^2}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},(c+\right.}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.\right.}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]
*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(
```

```
Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])) + 2*b*(c + d*x)*HypergeometricPFQ[{-
1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])))/(15*d*e*(e*(c + d*x))^(5/2))
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(dx+c))^2 (dex+ce)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\operatorname{arcosh}(dx+c)^2+2\,ab\operatorname{arcosh}(dx+c)+a^2\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
+ c^4*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

3.214 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=88

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de}-\frac{6b\text{Unintegrable}\left(\frac{(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{5e}$$

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^3)/(5*d*e) - (6*b*Unintegra
ble[((e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^2)/(Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]), x])/(5*e)
```

Rubi [A] time = 0.320483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^{3/2} \left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de} - \frac{(6b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}} \, d\right.}{5de}$$

Mathematica [A] time = 100.013, size = 0, normalized size = 0.

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3, x]
```

Maple [A] time = 0.355, size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}} (a + b\operatorname{arccosh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3dex + a^3ce + \left(b^3dex + b^3ce\right)\operatorname{arcosh}(dx + c)^3 + 3\left(ab^2dex + ab^2ce\right)\operatorname{arcosh}(dx + c)^2 + 3\left(a^2bdex + a^2bce\right) \mathrm{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arccosh(d*x + c)^3 +
3*(a*b^2*d*e*x + a*b^2*c*e)*arccosh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e
```

```
)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}} (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^3, x)
```

3.215 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=86

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de}-\frac{2b\text{Unintegrable}\left(\frac{(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e}$$

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^3)/(3*d*e) - (2*b*Unintegra
ble[((e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^2)/(Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]), x])/e
```

Rubi [A] time = 0.310831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(d*e)
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de}-\frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,d\right.}{de}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.384, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(dx+c))^3\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3\operatorname{arcosh}(dx+c)^3+3\,ab^2\operatorname{arcosh}(dx+c)^2+3\,a^2b\operatorname{arcosh}(dx+c)+a^3\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex+ce}\left(b\operatorname{arcosh}\left(dx+c\right)+a\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

3.216 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de}-\frac{6b\text{Unintegrable}\left(\frac{\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^3)/(d*e) - (6*b*Unintegrable[
(Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^2)/(Sqrt[-1 + c + d*x]*Sqrt[1 +
 c + d*x]), x])/e
```

Rubi [A] time = 0.286593, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]), x],
 x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$

$$=\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de}-\frac{(6b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 22.32, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{arccosh}\,(dx+c))^3 \frac{1}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{arcosh}\,(dx+c)^3+3\,ab^2\,\mathrm{arcosh}\,(dx+c)^2+3\,a^2b\,\mathrm{arcosh}\,(dx+c)+a^3}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{acosh}\,(c + dx))^3}{\sqrt{e\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.217 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{6b\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Unintegrable
[(a + b*ArcCosh[c + d*x])^2/(Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 +
c + d*x]), x])/e
```

Rubi [A] time = 0.302954, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*Sqrt[e*x]*Sqrt[1 + x]), x],
x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 30.6674, size = 0, normalized size = 0.

$$\int \frac{\left(a + b\cosh^{-1}(c + dx)\right)^3}{(ce + dex)^{3/2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

Maple [A] time = 0.353, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccosh}(dx + c))^3 (dex + ce)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^3 \operatorname{arcosh}(dx + c)^3 + 3\,ab^2 \operatorname{arcosh}(dx + c)^2 + 3\,a^2 b \operatorname{arcosh}(dx + c) + a^3\right)\sqrt{dex + ce}}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2
), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^3}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(dx + c) + a)^3}{(dex + ce)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

3.218 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{2b\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}},x\right)}{e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Unintegr
able[(a + b*ArcCosh[c + d*x])^2/(Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqr
t[1 + c + d*x]), x])/e
```

Rubi [A] time = 0.321662, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*(e*x)^(3/2)*Sqrt[1 + x])
, x], x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}(ex)^{3/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 35.7995, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

Maple [A] time = 0.377, size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{3}\left(dex+ce\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{3}\,\mathrm{arcosh}\left(dx+c\right)^{3}+3\,ab^{2}\,\mathrm{arcosh}\left(dx+c\right)^{2}+3\,a^{2}b\,\mathrm{arcosh}\left(dx+c\right)+a^{3}\right)\sqrt{dex+ce}}{d^{3}e^{3}x^{3}+3\,cd^{2}e^{3}x^{2}+3\,c^{2}de^{3}x+c^{3}e^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c
^2*d*e^3*x + c^3*e^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^3}{(e\,(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.219 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{6b\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Unintegr
able[(a + b*ArcCosh[c + d*x])^2/(Sqrt[-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqr
t[1 + c + d*x]), x])/(5*e)
```

Rubi [A] time = 0.323943, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*(e*x)^(5/2)*Sqrt[1 + x])
, x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}(ex)^{5/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 115.254, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

Maple [A] time = 0.399, size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{arccosh}\,(dx+c))^3\,(dex+ce)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^3\,\mathrm{arcosh}\,(dx+c)^3+3\,ab^2\,\mathrm{arcosh}\,(dx+c)^2+3\,a^2b\,\mathrm{arcosh}\,(dx+c)+a^3\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c
^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

3.220 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=88

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de}-\frac{8b\text{Unintegrable}\left(\frac{(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{5e}$$

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^4)/(5*d*e) - (8*b*Unintegra
ble[((e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^3)/(Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]), x])/(5*e)
```

Rubi [A] time = 0.315273, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^4 \, dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} \, d\right.}{5de}$$

Mathematica [A] time = 70.6157, size = 0, normalized size = 0.

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4, x]
```

Maple [A] time = 0.37, size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}} (a + b\,\mathrm{arccosh}\,(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4dex + a^4ce + \left(b^4dex + b^4ce\right)\mathrm{arcosh}\,(dx + c)^4 + 4\left(ab^3dex + ab^3ce\right)\mathrm{arcosh}\,(dx + c)^3 + 6\left(a^2b^2dex + a^2b^2ce\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arccosh(d*x + c)^4 +
4*(a*b^3*d*e*x + a*b^3*c*e)*arccosh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b^2
```

```
*c*e)*arccosh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arccosh(d*x + c))*sq
rt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^4, x)
```

3.221 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=88

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de}-\frac{8b\text{Unintegrable}\left(\frac{(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{3e}$$

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^4)/(3*d*e) - (8*b*Unintegra
ble[((e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^3)/(Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]), x])/(3*e)
```

Rubi [A] time = 0.308667, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx &= \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}\\ &= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,d\right.}{3de}\end{aligned}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.424, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(dx+c))^4\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4\operatorname{arcosh}(dx+c)^4+4\,ab^3\operatorname{arcosh}(dx+c)^3+6\,a^2b^2\operatorname{arcosh}(dx+c)^2+4\,a^3b\operatorname{arcosh}(dx+c)+a^4\right)\sqrt{dex+ce}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex+ce}\,(b\operatorname{arcosh}\left(dx+c\right)+a)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)
```

3.222 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de}-\frac{8b\text{Unintegrable}\left(\frac{\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^4)/(d*e) - (8*b*Unintegrable[
(Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^3)/(Sqrt[-1 + c + d*x]*Sqrt[1 +
 c + d*x]), x])/e
```

Rubi [A] time = 0.280245, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]), x],
 x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\ &= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de}-\frac{(8b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 14.4313, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

Maple [A] time = 0.256, size = 0, normalized size = 0.

$$\int (a+b\mathrm{arccosh}\,(dx+c))^4 \frac{1}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^4\,\mathrm{arcosh}\,(dx+c)^4+4\,ab^3\,\mathrm{arcosh}\,(dx+c)^3+6\,a^2b^2\,\mathrm{arcosh}\,(dx+c)^2+4\,a^3b\,\mathrm{arcosh}\,(dx+c)+a^4}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^4}{\sqrt{e(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.223 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{8b\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Unintegrable
[(a + b*ArcCosh[c + d*x])^3/(Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 +
c + d*x]), x])/e
```

Rubi [A] time = 0.292829, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst]
[Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*Sqrt[e*x]*Sqrt[1 + x]), x],
x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 37.6135, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

Maple [A] time = 0.352, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(dx+c))^4 (dex+ce)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^4\operatorname{arcosh}(dx+c)^4+4\,ab^3\operatorname{arcosh}(dx+c)^3+6\,a^2b^2\operatorname{arcosh}(dx+c)^2+4\,a^3b\operatorname{arcosh}(dx+c)+a^4\right)\sqrt{dex+ce}}{d^2e^2x^2+2\,cde^2x+c^2e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^2*
e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^4}{(e(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcosh}(dx + c) + a)^4}{(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

3.224 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{8b\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}},x\right)}{3e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Unintegr
able[(a + b*ArcCosh[c + d*x])^3/(Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqr
t[1 + c + d*x]), x])/(3*e)
```

Rubi [A] time = 0.314641, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*(e*x)^(3/2)*Sqrt[1 + x])
, x], x, c + d*x])/(3*d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}(ex)^{3/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{3de}$$

Mathematica [A] time = 36.8726, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

Maple [A] time = 0.353, size = 0, normalized size = 0.

$$\int (a+b\mathrm{arccosh}\,(dx+c))^4\,(dex+ce)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^4\,\mathrm{arcosh}\,(dx+c)^4+4\,ab^3\,\mathrm{arcosh}\,(dx+c)^3+6\,a^2b^2\,\mathrm{arcosh}\,(dx+c)^2+4\,a^3b\,\mathrm{arcosh}\,(dx+c)+a^4\right)\sqrt{dex+}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^3*
e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^4}{(e(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.225 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{8b\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}$$

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Unintegr
able[(a + b*ArcCosh[c + d*x])^3/(Sqrt[-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqr
t[1 + c + d*x]), x])/(5*e)
```

Rubi [A] time = 0.313927, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*(e*x)^(5/2)*Sqrt[1 + x])
, x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}(ex)^{5/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 163.198, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

Maple [A] time = 0.358, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arccosh}(dx+c))^4 (dex+ce)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^4\operatorname{arcosh}(dx+c)^4+4ab^3\operatorname{arcosh}(dx+c)^3+6a^2b^2\operatorname{arcosh}(dx+c)^2+4a^3b\operatorname{arcosh}(dx+c)+a^4\right)\sqrt{dex+ce}}{d^4e^4x^4+4cd^3e^4x^3+6c^2d^2e^4x^2+4c^3de^4x+c^4e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^4*
e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

3.226 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=93

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de(m+1)}-\frac{4b\text{Unintegrable}\left(\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e(m+1)}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Uni
ntegrable[((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^3)/(Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x]), x])/(e*(1 + m))
```

Rubi [A] time = 0.290263, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqr
t[1 + x]), x], x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de(1+m)} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} \, d\right.}{de(1+m)}$$

Mathematica [A] time = 3.73641, size = 0, normalized size = 0.

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4, x]
```

Maple [A] time = 1.961, size = 0, normalized size = 0.

$$\int (dex + ce)^m (a + b\operatorname{arccosh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(b^4 \operatorname{arcosh}(dx + c)^4 + 4\,ab^3 \operatorname{arcosh}(dx + c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx + c)^2 + 4\,a^3b \operatorname{arcosh}(dx + c) + a^4\right)(dex + c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcosh}(dx + c) + a)^4 (dex + ce)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

3.227 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=93

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de(m+1)}-\frac{3b\text{Unintegrable}\left(\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e(m+1)}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Uni
ntegrable[((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^2)/(Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x]), x])/(e*(1 + m))
```

Rubi [A] time = 0.293982, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqr
t[1 + x]), x], x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx &= \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}\\ &= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de(1+m)}-\frac{(3b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\right.}{de(1+m)}\end{aligned}$$

Mathematica [A] time = 1.78223, size = 0, normalized size = 0.

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3, x]
```

Maple [A] time = 1.86, size = 0, normalized size = 0.

$$\int (dex + ce)^m (a + b\operatorname{arccosh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{arcosh}(dx + c)^3 + 3\,ab^2 \operatorname{arcosh}(dx + c)^2 + 3\,a^2b \operatorname{arcosh}(dx + c) + a^3\right)(dex + ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (e\left(c+dx\right))^{m}\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}\left(dx+c\right)+a)^{3}(dex+ce)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

3.228 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=206

$$-\frac{2b^2(e(c+dx))^{m+3}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b\sqrt{-c-dx+1}(e(c+dx))^{m}}{}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*Sqr
t[1 - c - d*x]*(e*(c + d*x))^(2 + m)*(a + b*ArcCosh[c + d*x])*Hypergeometri
c2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sqrt[-
1 + c + d*x]) - (2*b^2*(e*(c + d*x))^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/
2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, (c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(
3 + m))
```

Rubi [A] time = 0.318661, antiderivative size = 218, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5866, 5662, 5763}

$$-\frac{2b^2(e(c+dx))^{m+3}\,_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b\sqrt{1-(c+dx)^2}(e(c+dx))^{m+2}\,_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+\right.}{de^2(m+1)(m+2)\sqrt{c+dx-1}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]) - (2*b^2*(e*(c + d*x))^(3 + m)*Hypergeo
metricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, (c + d*x)^2])/(d
*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}} \cdots\right.}{de(1+m)}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{2b(e(c+dx))^{2+m}\sqrt{1-(c+dx)^2}\,(a \cdots}{de^2(1+m)(2 \cdots}$$

Mathematica [A] time = 0.429451, size = 178, normalized size = 0.86

$$\frac{(c+dx)(e(c+dx))^m\left(\left(a+b\cosh^{-1}(c+dx)\right)^2 - \frac{2b(c+dx)\left(\frac{b(c+dx)\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},(c+dx)^2\right)}{m+3} + \frac{\sqrt{1-(c+dx)^2}\text{Hypergeom}\cdots}{\cdots}\right.}{m+2}\cdots\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcCosh[c + d*x])^2 - (2*b*(c + d*x)*((S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, (2 + m
)/2, (4 + m)/2, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + (b*(
c + d*x)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 (c + d*x)^2])/(3 + m)))/(2 + m)))/(d*(1 + m))
```

Maple [F] time = 2.108, size = 0, normalized size = 0.

$$\int (dex+ce)^m (a+b\operatorname{arccosh}(dx+c))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{arcosh}(dx+c)^2 + 2\,ab \operatorname{arcosh}(dx+c) + a^2\right)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*(d*e*x + c
*e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c+dx))^m (a+b\operatorname{acosh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}(dx+c)+a)^2(dex+ce)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

3.229 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=118

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b\left(1-(c+dx)^2\right)(e(c+dx))^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+4}{2},(c+dx)^2\right)}{de^2(m+1)(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*(1 - (c + d*x)^2)*Hypergeometric2F1[1, (3 + m)/2, (4 + m)/2,
 (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])
```

Rubi [A] time = 0.0989388, antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 5662, 126, 365, 364}

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b\sqrt{1-(c+dx)^2}(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+dx)^2\right)}{de^2(m+1)(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
 m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c +
 d*x])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{-1+x}\sqrt{1+x}}\, dx, x, c+dx\right)}{de(1+m)}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{\left(b\sqrt{-1+(c+dx)^2}\right)\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{-1+x}}\right.}{de(1+m)\sqrt{-1+c+dx}\sqrt{1+}}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{\left(b\sqrt{1-(c+dx)^2}\right)\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{1-x^2}}\right.}{de(1+m)\sqrt{-1+c+dx}\sqrt{1+}}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b(e(c+dx))^{2+m}\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\right.}{de^2(1+m)(2+m)\sqrt{-1+c+}}$$

Mathematica [A] time = 0.192028, size = 106, normalized size = 0.9

$$\frac{(c+dx)(e(c+dx))^m\left(-\frac{b(c+dx)\sqrt{1-(c+dx)^2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},(c+dx)^2\right)}{(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}}+a+b\cosh^{-1}(c+dx)\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*(a + b*ArcCosh[c + d*x] - (b*(c + d*x)*Sqrt[1 -
(c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/((2
 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(d*(1 + m))
```

Maple [F] time = 1.967, size = 0, normalized size = 0.

$$\int (dex+ce)^m\,(a+b\mathrm{arccosh}\,(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\,\mathrm{arcosh}\,(dx+c)+a)(dex+ce)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c+dx))^m (a+b\operatorname{acosh}(c+dx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcosh}(dx+c)+a)(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

3.230 $\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(e(c+dx))^m}{a+b\cosh^{-1}(c+dx)},x\right)$$

[Out] Unintegrable[(e*(c + d*x))^m/(a + b*ArcCosh[c + d*x]), x]

Rubi [A] time = 0.0610087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]

[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcCosh[x]), x], x, c + d*x]/d

Rubi steps

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)} dx = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\cosh^{-1}(x)} dx, x, c+dx\right)}{d}$$

Mathematica [A] time = 1.13939, size = 0, normalized size = 0.

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]

```
[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]), x]
```

Maple [A] time = 0.881, size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^m}{a + b\operatorname{arccosh}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^m}{b\,\operatorname{arcosh}(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dex + ce)^m}{b\,\operatorname{arcosh}(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\,(c+dx))^m}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m/(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m/(a + b*acosh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^m}{b\,\mathrm{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

3.231 $\int \frac{\cosh^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=54

$$\frac{1}{10}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^5)}\right)-\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(e^{2\cosh^{-1}(ax^5)}+1\right)$$

```
[Out] -ArcCosh[a*x^5]^2/10 + (ArcCosh[a*x^5]*Log[1 + E^(2*ArcCosh[a*x^5])])/5 + P
olyLog[2, -E^(2*ArcCosh[a*x^5])]/10
```

Rubi [A] time = 0.0668968, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5891, 3718, 2190, 2279, 2391}

$$\frac{1}{10}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^5)}\right)-\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(e^{2\cosh^{-1}(ax^5)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x^5]/x,x]
```

```
[Out] -ArcCosh[a*x^5]^2/10 + (ArcCosh[a*x^5]*Log[1 + E^(2*ArcCosh[a*x^5])])/5 + P
olyLog[2, -E^(2*ArcCosh[a*x^5])]/10
```

Rule 5891

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(ax^5\right)\right)\\ &= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{2}{5}\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(ax^5\right)\right)\\ &= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}\left(ax^5\right)}\right)-\frac{1}{5}\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \cosh\right.\\ &= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}\left(ax^5\right)}\right)-\frac{1}{10}\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh}\right.\\ &= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}\left(ax^5\right)}\right)+\frac{1}{10}\text{Li}_2\left(-e^{2\cosh^{-1}\left(ax^5\right)}\right)\end{aligned}$$

Mathematica [A] time = 0.0418792, size = 50, normalized size = 0.93

$$\frac{1}{10}\left(\cosh^{-1}\left(ax^5\right)\left(\cosh^{-1}\left(ax^5\right)+2\log\left(e^{-2\cosh^{-1}\left(ax^5\right)}+1\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}\left(ax^5\right)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x^5]/x,x]
```

```
[Out] (ArcCosh[a*x^5]*(ArcCosh[a*x^5] + 2*Log[1 + E^(-2*ArcCosh[a*x^5])]) - PolyL
og[2, -E^(-2*ArcCosh[a*x^5])])/10
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccosh}\left(a x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x^5)/x,x)
```

```
[Out] int(arccosh(a*x^5)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(a x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x^5)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}\left(a x^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x^5)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}\left(a x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x**5)/x,x)
```

```
[Out] Integral(acosh(a*x**5)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x^5)/x, x)
```

3.232 $\int x^2 \cosh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=117

$$-\frac{1}{18}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{5}{72}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{5}{48}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/48 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/72 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/18 - (5*ArcCosh[Sqrt[x]])/48 + (x^3*ArcCosh[Sqrt[x]])/3
```

Rubi [A] time = 0.0704343, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5903, 12, 323, 330, 52}

$$-\frac{1}{18}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{5}{72}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{5}{48}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/48 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/72 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/18 - (5*ArcCosh[Sqrt[x]])/48 + (x^3*ArcCosh[Sqrt[x]])/3
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x^2\cosh^{-1}\left(\sqrt{x}\right)dx = \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$
$$= \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$
$$= -\frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{5}{36}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$
$$= -\frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{5}{48}\int \frac{}{\sqrt{-1+}}$$
$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{3}x^3 \text{co}$$
$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{3}x^3 \text{co}$$
$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{5}{48}\text{cos}$$

Mathematica [A] time = 0.0539353, size = 79, normalized size = 0.68

$$\frac{1}{144}\left(-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(8x^2+10x+15\right)\sqrt{x}+48x^3\cosh^{-1}\left(\sqrt{x}\right)-30\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2)) + 48*x
^3*ArcCosh[Sqrt[x]] - 30*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/144
```

Maple [A] time = 0.035, size = 75, normalized size = 0.6

$$\frac{x^3}{3}\text{arccosh}\left(\sqrt{x}\right) - \frac{1}{144}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(8\,x^{5/2}\sqrt{-1+x}+10\,x^{3/2}\sqrt{-1+x}+15\,\sqrt{x}\sqrt{-1+x}+15\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(x^(1/2)),x)
```

```
[Out] 1/3*x^3*arccosh(x^(1/2))-1/144*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*x^(5
/2)*(-1+x)^(1/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1
/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A] time = 1.02065, size = 76, normalized size = 0.65

$$\frac{1}{3}x^3 \operatorname{arcosh}\left(\sqrt{x}\right) - \frac{1}{18}\sqrt{x-1}x^{\frac{5}{2}} - \frac{5}{72}\sqrt{x-1}x^{\frac{3}{2}} - \frac{5}{48}\sqrt{x-1}\sqrt{x} - \frac{5}{48}\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(sqrt(x)) - 1/18*sqrt(x - 1)*x^(5/2) - 5/72*sqrt(x - 1)*x^(3
/2) - 5/48*sqrt(x - 1)*sqrt(x) - 5/48*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 2.17917, size = 128, normalized size = 1.09

$$-\frac{1}{144}\left(8x^2 + 10x + 15\right)\sqrt{x-1}\sqrt{x} + \frac{1}{48}\left(16x^3 - 5\right)\log\left(\sqrt{x-1} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 + 10*x + 15)*sqrt(x - 1)*sqrt(x) + 1/48*(16*x^3 - 5)*log(sqrt
(x - 1) + sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acosh}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(x**(1/2)),x)
```

```
[Out] Integral(x**2*acosh(sqrt(x)), x)
```

Giac [A] time = 1.16154, size = 82, normalized size = 0.7

$$\frac{1}{3}x^3\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{144}\left(2\left(4x+5\right)x+15\right)\sqrt{x-1}\sqrt{x}+\frac{5}{48}\log\left(\left|\sqrt{x-1}-\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/144*(2*(4*x
+ 5)*x + 15)*sqrt(x - 1)*sqrt(x) + 5/48*log(abs(sqrt(x - 1) - sqrt(x)))
```

3.233 $\int x \cosh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=86

$$-\frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (-3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/16 - (Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/8 - (3*ArcCosh[Sqrt[x]])/16 + (x^2*ArcCosh[Sqrt[
x]])/2
```

Rubi [A] time = 0.0503724, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5903, 12, 323, 330, 52}

$$-\frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/16 - (Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/8 - (3*ArcCosh[Sqrt[x]])/16 + (x^2*ArcCosh[Sqrt[
x]])/2
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int x\cosh^{-1}\left(\sqrt{x}\right)dx = \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{x^{3/2}}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$

$$= \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right) - \frac{3}{16}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$

$$= -\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right) - \frac{3}{32}\int \frac{1}{\sqrt{-1+\sqrt{x}}\ldots}$$

$$= -\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right) - \frac{3}{16}\text{Subst}\left(\int \frac{}{\sqrt{-\ldots}}\right.$$

$$= -\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right) + \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0383801, size = 74, normalized size = 0.86

$$\frac{1}{16}\left(8x^2\cosh^{-1}\left(\sqrt{x}\right)-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(2x+3)\sqrt{x}-6\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x)) + 8*x^2*ArcCosh[
Sqrt[x]] - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/16
```

Maple [A] time = 0.005, size = 65, normalized size = 0.8

$$\frac{x^2}{2}\mathrm{arccosh}\left(\sqrt{x}\right)-\frac{1}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(2\,x^{3/2}\sqrt{-1+x}+3\,\sqrt{x}\sqrt{-1+x}+3\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(x^(1/2)),x)
```

```
[Out] 1/2*x^2*arccosh(x^(1/2))-1/16*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x^(3/
2)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(
1/2)
```

Maxima [A] time = 1.02128, size = 62, normalized size = 0.72

$$\frac{1}{2}\,x^2\,\mathrm{arcosh}\left(\sqrt{x}\right)-\frac{1}{8}\,\sqrt{x-1}x^{\frac{3}{2}}-\frac{3}{16}\,\sqrt{x-1}\sqrt{x}-\frac{3}{16}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccosh(sqrt(x)) - 1/8*sqrt(x - 1)*x^(3/2) - 3/16*sqrt(x - 1)*sqrt(
x) - 3/16*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 1.95737, size = 112, normalized size = 1.3

$$-\frac{1}{16}\,(2\,x+3)\sqrt{x-1}\sqrt{x}+\frac{1}{16}\left(8\,x^2-3\right)\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x + 3)*sqrt(x - 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x - 1) + sq
rt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{acosh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(x**(1/2)),x)
```

```
[Out] Integral(x*acosh(sqrt(x)), x)
```

Giac [A] time = 1.16708, size = 76, normalized size = 0.88

$$\frac{1}{2}\,x^2\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{16}\,(2\,x+3)\sqrt{x-1}\sqrt{x}+\frac{3}{16}\,\log\left(\left|\sqrt{x-1}-\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/16*(2*x + 3)
*sqrt(x - 1)*sqrt(x) + 3/16*log(abs(sqrt(x - 1) - sqrt(x)))
```

3.234 $\int \cosh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=50

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/2 - ArcCosh[Sqrt[x]]/2 + x*
ArcCosh[Sqrt[x]]
```

Rubi [A] time = 0.0313341, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5901, 12, 323, 330, 52}

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]],x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/2 - ArcCosh[Sqrt[x]]/2 + x*
ArcCosh[Sqrt[x]]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
```

```
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}\left(\sqrt{x}\right)\,dx &= x\cosh^{-1}\left(\sqrt{x}\right)-\int \frac{\sqrt{x}}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\
&= x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\
&= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\
&= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)+x\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0233279, size = 64, normalized size = 1.28

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[Sqrt[x]],x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/2 + x*ArcCosh[Sqrt[x]] - Ar
cTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]]
```

Maple [A] time = 0.003, size = 49, normalized size = 1.

$$x\mathrm{arccosh}\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(\sqrt{x}\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2)),x)
```

```
[Out] x*arccosh(x^(1/2))-1/2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x^(1/2)*(-1+x)
^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A] time = 1.01828, size = 45, normalized size = 0.9

$$x\,\mathrm{arcosh}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x-1}\sqrt{x}-\frac{1}{2}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arccosh(sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x) - 1/2*log(2*sqrt(x - 1) + 2*sq
rt(x))
```

Fricas [A] time = 2.0325, size = 92, normalized size = 1.84

$$\frac{1}{2}\,(2\,x-1)\log\left(\sqrt{x-1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x-1}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - 1)*log(sqrt(x - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x)
```

Sympy [A] time = 0.343594, size = 29, normalized size = 0.58

$$-\frac{\sqrt{x}\sqrt{x-1}}{2}+x\operatorname{acosh}\left(\sqrt{x}\right)-\frac{\operatorname{acosh}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(x - 1)/2 + x*acosh(sqrt(x)) - acosh(sqrt(x))/2
```

Giac [A] time = 1.16816, size = 65, normalized size = 1.3

$$x\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x-1}\sqrt{x}+\frac{1}{2}\log\left(\left|\sqrt{x-1}-\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt
(x) + 1/2*log(abs(sqrt(x - 1) - sqrt(x)))
```

3.235 $\int \frac{\cosh^{-1}(\sqrt{x})}{x} dx$

Optimal. Leaf size=46

$$\text{PolyLog}\left(2,-e^{2\cosh^{-1}(\sqrt{x})}\right)-\cosh^{-1}\left(\sqrt{x}\right)^2+2\cosh^{-1}\left(\sqrt{x}\right)\log\left(e^{2\cosh^{-1}(\sqrt{x})}+1\right)$$

```
[Out] -ArcCosh[Sqrt[x]]^2 + 2*ArcCosh[Sqrt[x]]*Log[1 + E^(2*ArcCosh[Sqrt[x]])] +
PolyLog[2, -E^(2*ArcCosh[Sqrt[x]])]
```

Rubi [A] time = 0.0638731, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5891, 3718, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,-e^{2\cosh^{-1}(\sqrt{x})}\right)-\cosh^{-1}\left(\sqrt{x}\right)^2+2\cosh^{-1}\left(\sqrt{x}\right)\log\left(e^{2\cosh^{-1}(\sqrt{x})}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x,x]
```

```
[Out] -ArcCosh[Sqrt[x]]^2 + 2*ArcCosh[Sqrt[x]]*Log[1 + E^(2*ArcCosh[Sqrt[x]])] +
PolyLog[2, -E^(2*ArcCosh[Sqrt[x]])]
```

Rule 5891

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\sqrt{x}\right)\right)\\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 4\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\sqrt{x}\right)\right)\\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) - 2\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \cosh^{-1}\left(\right.\right.\\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) - \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)\\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) + \text{Li}_2\left(-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.0348489, size = 46, normalized size = 1.

$$\cosh^{-1}\left(\sqrt{x}\right)\left(\cosh^{-1}\left(\sqrt{x}\right) + 2\log\left(e^{-2\cosh^{-1}\left(\sqrt{x}\right)}+1\right)\right) - \text{PolyLog}\left(2, -e^{-2\cosh^{-1}\left(\sqrt{x}\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x,x]
```

```
[Out] ArcCosh[Sqrt[x]]*(ArcCosh[Sqrt[x]] + 2*Log[1 + E^(-2*ArcCosh[Sqrt[x]])]) -
PolyLog[2, -E^(-2*ArcCosh[Sqrt[x]])]
```

Maple [A] time = 0.031, size = 65, normalized size = 1.4

$$-\left(\operatorname{arccosh}\left(\sqrt{x}\right)\right)^{2}+2\operatorname{arccosh}\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)^{2}\right)+\operatorname{polylog}\left(2,-\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x,x)
```

```
[Out] -arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(sqrt(x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(sqrt(x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x,x)
```

```
[Out] Integral(acosh(sqrt(x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(sqrt(x))/x, x)
```

3.236 $\int \frac{\cosh^{-1}(\sqrt{x})}{x^2} dx$

Optimal. Leaf size=40

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}(\sqrt{x})}{x}$$

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Rubi [A] time = 0.0256731, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5903, 12, 265}

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
```

```
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}+\int \frac{1}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx\\ &= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx\\ &= \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0135517, size = 40, normalized size = 1.

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Maple [A] time = 0.003, size = 29, normalized size = 0.7

$$-\frac{1}{x}\operatorname{arccosh}\left(\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x^2,x)
```

```
[Out] -arccosh(x^(1/2))/x+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Maxima [A] time = 1.54244, size = 26, normalized size = 0.65

$$\frac{\sqrt{x-1}}{\sqrt{x}}-\frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(x - 1)/sqrt(x) - arccosh(sqrt(x))/x
```

Fricas [A] time = 2.17386, size = 73, normalized size = 1.82

$$\frac{\sqrt{x-1}\sqrt{x}-\log\left(\sqrt{x-1}+\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(x - 1)*sqrt(x) - log(sqrt(x - 1) + sqrt(x)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x**2,x)
```

```
[Out] Integral(acosh(sqrt(x))/x**2, x)
```

Giac [A] time = 1.12095, size = 61, normalized size = 1.52

$$-\frac{\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{x}+\frac{2}{\left(\sqrt{x-1}-\sqrt{x}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x + 2/((sqrt(x - 1) - s
qrt(x))^2 + 1)
```

3.237 $\int \frac{\cosh^{-1}(\sqrt{x})}{x^3} dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{6x^{3/2}} - \frac{\cosh^{-1}(\sqrt{x})}{2x^2} + \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(6*x^(3/2)) + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])/(3*Sqrt[x]) - ArcCosh[Sqrt[x]]/(2*x^2)
```

Rubi [A] time = 0.0406338, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5903, 12, 272, 265}

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{6x^{3/2}} - \frac{\cosh^{-1}(\sqrt{x})}{2x^2} + \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(6*x^(3/2)) + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])/(3*Sqrt[x]) - ArcCosh[Sqrt[x]]/(2*x^2)
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx \\
&= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx \\
&= \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{6x^{3/2}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{6}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx \\
&= \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{6x^{3/2}}+\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.0223006, size = 49, normalized size = 0.64

$$
\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}(2x+1)-3\cosh^{-1}\left(\sqrt{x}\right)}{6x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 + 2*x) - 3*ArcCosh[Sqrt[x]
])/(6*x^2)
```

Maple [A] time = 0.004, size = 35, normalized size = 0.5

$$-\frac{1}{2x^2}\text{arccosh}\left(\sqrt{x}\right)+\frac{1+2x}{6}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arccosh(x^(1/2))/x^2+1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(1+2*x)/
x^(3/2)
```

Maxima [A] time = 1.53534, size = 41, normalized size = 0.54

$$\frac{\sqrt{x-1}}{3\sqrt{x}}+\frac{\sqrt{x-1}}{6x^{\frac{3}{2}}}-\frac{\text{arcosh}\left(\sqrt{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x - 1)/sqrt(x) + 1/6*sqrt(x - 1)/x^(3/2) - 1/2*arccosh(sqrt(x))/x^
2
```

Fricas [A] time = 1.99472, size = 97, normalized size = 1.28

$$\frac{(2x+1)\sqrt{x-1}\sqrt{x}-3\log\left(\sqrt{x-1}+\sqrt{x}\right)}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*((2*x + 1)*sqrt(x - 1)*sqrt(x) - 3*log(sqrt(x - 1) + sqrt(x)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x**3,x)
```

```
[Out] Integral(acosh(sqrt(x))/x**3, x)
```

Giac [A] time = 1.13663, size = 84, normalized size = 1.11

$$-\frac{\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{2\,x^2}+\frac{2\left(3\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)}{3\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(
x - 1) - sqrt(x))^2 + 1)/((sqrt(x - 1) - sqrt(x))^2 + 1)^3
```

3.238 $\int \cosh^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x+1}}\sqrt{x+1}\sin^{-1}(x)+x\text{sech}^{-1}(x)$$

```
[Out] x*ArcSech[x] + Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcSin[x]
```

Rubi [A] time = 0.008589, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5893, 6277, 216}

$$\sqrt{\frac{1}{x+1}}\sqrt{x+1}\sin^{-1}(x)+x\text{sech}^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[x^(-1)],x]
```

```
[Out] x*ArcSech[x] + Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcSin[x]
```

Rule 5893

```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6277

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cosh^{-1}\left(\frac{1}{x}\right)dx = \int \operatorname{sech}^{-1}(x)\,dx$$
$$= x\operatorname{sech}^{-1}(x) + \left(\sqrt{\frac{1}{1+x}}\sqrt{1+x}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= x\operatorname{sech}^{-1}(x) + \sqrt{\frac{1}{1+x}}\sqrt{1+x}\sin^{-1}(x)$$

Mathematica [A] time = 0.0458795, size = 46, normalized size = 1.92

$$x\cosh^{-1}\left(\frac{1}{x}\right) - \frac{\sqrt{\frac{1}{x^2}-1}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)}{\sqrt{\frac{1}{x}-1}\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[x^(-1)],x]
```

```
[Out] x*ArcCosh[x^(-1)] - (Sqrt[-1 + x^(-2)]*ArcTan[Sqrt[-1 + x^(-2)]])/(Sqrt[-1
+ x^(-1)]*Sqrt[1 + x^(-1)])
```

Maple [A] time = 0.025, size = 38, normalized size = 1.6

$$\operatorname{arccosh}\left(x^{-1}\right)x + \sqrt{x^{-1}-1}\sqrt{x^{-1}+1}\arctan\left(\frac{1}{\sqrt{x^{-2}-1}}\right)\frac{1}{\sqrt{x^{-2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(1/x),x)
```

```
[Out] arccosh(1/x)*x+(1/x-1)^(1/2)*(1/x+1)^(1/2)/(1/x^2-1)^(1/2)*arctan(1/(1/x^2-
1)^(1/2))
```

Maxima [B] time = 1.53753, size = 23, normalized size = 0.96

$$x\operatorname{arcosh}\left(\frac{1}{x}\right) - \arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="maxima")
```

```
[Out] x*arccosh(1/x) - arctan(sqrt(1/x^2 - 1))
```

Fricas [B] time = 2.15316, size = 173, normalized size = 7.21

$$(x-2)\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}+1}{x}\right)-2\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)-2\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="fricas")
```

```
[Out] (x - 2)*log((x*sqrt(-(x^2 - 1)/x^2) + 1)/x) - 2*arctan((x*sqrt(-(x^2 - 1)/x
^2) - 1)/x) - 2*log((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acosh}\left(\frac{1}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(1/x),x)
```

```
[Out] Integral(acosh(1/x), x)
```

Giac [B] time = 1.10403, size = 30, normalized size = 1.25

$$x\log\left(\sqrt{\frac{1}{x^2}-1}+\frac{1}{x}\right)+\frac{\arcsin(x)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="giac")
```

```
[Out] x*log(sqrt(1/x^2 - 1) + 1/x) + arcsin(x)/sgn(x)
```

3.239 $\int \frac{\cosh^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=60

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^n)}\right)}{2n}-\frac{\cosh^{-1}(ax^n)^2}{2n}+\frac{\cosh^{-1}(ax^n)\log\left(e^{2\cosh^{-1}(ax^n)}+1\right)}{n}$$

```
[Out] -ArcCosh[a*x^n]^2/(2*n) + (ArcCosh[a*x^n]*Log[1 + E^(2*ArcCosh[a*x^n])])/n
+ PolyLog[2, -E^(2*ArcCosh[a*x^n])]/(2*n)
```

Rubi [A] time = 0.0654295, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5891, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^n)}\right)}{2n}-\frac{\cosh^{-1}(ax^n)^2}{2n}+\frac{\cosh^{-1}(ax^n)\log\left(e^{2\cosh^{-1}(ax^n)}+1\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x^n]/x,x]
```

```
[Out] -ArcCosh[a*x^n]^2/(2*n) + (ArcCosh[a*x^n]*Log[1 + E^(2*ArcCosh[a*x^n])])/n
+ PolyLog[2, -E^(2*ArcCosh[a*x^n])]/(2*n)
```

Rule 5891

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax^n)}{x}\,dx &= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax^n)\right)}{n}\\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax^n)\right)}{n}\\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \cosh^{-1}(ax^n)\right)}{n}\\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(ax^n)}\right)}{2n}\\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}(ax^n)}\right)}{2n}
\end{aligned}
$$

Mathematica [B] time = 0.468645, size = 179, normalized size = 2.98

$$
\frac{a\sqrt{1-a^2x^{2n}}\left(-\text{PolyLog}\left(2, e^{-2\sinh^{-1}\left(\sqrt{-a^2}x^n\right)}\right) - 2n\log(x)\log\left(\sqrt{-a^2}x^n+\sqrt{1-a^2x^{2n}}\right) + \sinh^{-1}\left(\sqrt{-a^2}x^n\right)^2 + 2\sinh^{-1}\left(\right.\right.}{2\sqrt{-a^2}n\sqrt{ax^n-1}\sqrt{ax^n+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x^n]/x,x]
```

```
[Out] ArcCosh[a*x^n]*Log[x] + (a*Sqrt[1 - a^2*x^(2*n)]*(ArcSinh[Sqrt[-a^2]*x^n]^2
 + 2*ArcSinh[Sqrt[-a^2]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] - 2*n*
Log[x]*Log[Sqrt[-a^2]*x^n + Sqrt[1 - a^2*x^(2*n)]] - PolyLog[2, E^(-2*ArcSi
nh[Sqrt[-a^2]*x^n])]))/(2*Sqrt[-a^2]*n*Sqrt[-1 + a*x^n]*Sqrt[1 + a*x^n])
```

Maple [A] time = 0.049, size = 91, normalized size = 1.5

$$-\frac{(\operatorname{arccosh}(ax^n))^2}{2n} + \frac{\operatorname{arccosh}(ax^n)}{n}\ln\left(1+\left(ax^n+\sqrt{ax^n-1}\sqrt{ax^n+1}\right)^2\right) + \frac{1}{2n}\operatorname{polylog}\left(2,-\left(ax^n+\sqrt{ax^n-1}\sqrt{ax^n+1}\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x^n)/x,x)
```

```
[Out] -1/2*arccosh(a*x^n)^2/n+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1
)^(1/2))^2)/n+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$an\int\frac{x^n\log(x)}{a^3xx^{3n}-axx^n+\left(a^2xx^{2n}-x\right)\sqrt{ax^n+1}\sqrt{ax^n-1}}\,dx-\frac{1}{2}n\log(x)^2+n\int\frac{\log(x)}{2(axx^n+x)}\,dx-n\int\frac{\log(x)}{2(axx^n-x)}\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) - a*x*x^n + (a^2*x*x^(2*n) - x)*sqr
t(a*x^n + 1)*sqrt(a*x^n - 1)), x) - 1/2*n*log(x)^2 + n*integrate(1/2*log(x)
/(a*x*x^n + x), x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(a*x^n +
 sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acosh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x**n)/x,x)
```

```
[Out] Integral(acosh(a*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x^n)/x, x)
```

3.240 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=145

$$-\frac{192b^3\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2-\frac{8b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

```
[Out] 384*b^4*x - (192*b^3*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*
x^2]*Sqrt[2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[1 + d*x^2])^2 - (8*b*(2*x^2
 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x
*(a + b*ArcCosh[1 + d*x^2])^4
```

Rubi [A] time = 0.0360027, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$-\frac{192b^3\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2-\frac{8b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*
x^2]*Sqrt[2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[1 + d*x^2])^2 - (8*b*(2*x^2
 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x
*(a + b*ArcCosh[1 + d*x^2])^4
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4\,dx = -\frac{8b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4 + \left(48b^2\right)\int$$
$$= -\frac{192b^3\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}} + 48b^2x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2 - \frac{8b}{}$$
$$= 384b^4x - \frac{192b^3\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}} + 48b^2x\left(a+b\cosh^{-1}\left(1+dx^2\right.\right.$$

Mathematica [A] time = 0.220184, size = 264, normalized size = 1.82

$$\frac{dx^2\left(48a^2b^2+a^4+384b^4\right)-8ab\left(a^2+24b^2\right)\sqrt{dx^2}\sqrt{dx^2+2}+6b^2\cosh^{-1}\left(dx^2+1\right)^2\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2+2}+8b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

```
[Out] ((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqrt
[2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[2
+ d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2] + 6*b^2*(
a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*
x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^
2]^3 + b^4*d*x^2*ArcCosh[1 + d*x^2]^4)/(d*x)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(dx^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^4,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08347, size = 626, normalized size = 4.32

$$\frac{b^4dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^4+\left(a^4+48\,a^2b^2+384\,b^4\right)dx^2+4\left(ab^3dx^2-2\,\sqrt{d^2x^4+2\,dx^2}b^4\right)\log\left(dx^2+\sqrt{d^2x^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^4 + (a^4 + 48*a^2*b^2 +
 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^4)*log(d*x^2
 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 - 6*(4*sqrt(d^2*x^4 + 2*d*x^2)*a*b^3 - (a
^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + 4*((a^3
*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 + 2*d*x^2)*(a^2*b^2 + 8*b^4))*log(d*x
^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 8*sqrt(d^2*x^4 + 2*d*x^2)*(a^3*b + 24*a
*b^3))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**4,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**4, x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.241 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3 dx$

Optimal. Leaf size=125

$$24ab^2x-\frac{6b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2+2}}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^3-\frac{48b^3\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+24b^3x\cosh^{-}$$

[Out] 24*a*b^2*x - (48*b^3*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + 24*b^3*
x*ArcCosh[1 + d*x^2] - (6*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^2)/(
x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^3

Rubi [A] time = 0.0625657, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5880, 5901, 12, 6719, 261}

$$24ab^2x-\frac{6b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2+2}}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^3-\frac{48b^3\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+24b^3x\cosh^{-}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[1 + d*x^2])^3,x]

[Out] 24*a*b^2*x - (48*b^3*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + 24*b^3*
x*ArcCosh[1 + d*x^2] - (6*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^2)/(
x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^3

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] && !FunctionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3\,dx = -\frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3 + \left(24b^2\right)\int$$

$$= 24ab^2x - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3 + (\,$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\,(a\,-$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\,(a\,-$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\,(a\,-$$

$$= 24ab^2x - \frac{48b^3\sqrt{\frac{dx^2}{2+dx^2}}\left(2+dx^2\right)}{dx} + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\,\mathrm{co}\right.}{x\sqrt{dx^2}\sqrt{2+}}$$

Mathematica [A] time = 0.118374, size = 171, normalized size = 1.37

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2}\sqrt{dx^2+2}+3b\cosh^{-1}\left(dx^2+1\right)\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2+2}+8b^2dx^2\right)+3b^2\cos}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[2 + d*x^2] + 3
*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1
+ d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*
x^2]^2 + b^3*d*x^2*ArcCosh[1 + d*x^2]^3)/(d*x)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{arccosh}\left(dx^2+1\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^3,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08461, size = 441, normalized size = 3.53

$$\frac{b^3dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^3+\left(a^3+24\,ab^2\right)dx^2+3\left(ab^2dx^2-2\sqrt{d^2x^4+2\,dx^2}b^3\right)\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 + (a^3 + 24*a*b^2)*d*
x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^2*
x^4 + 2*d*x^2) + 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 + 2*d*x^2
)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 6*sqrt(d^2*x^4 + 2*d*x^
2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{acosh}\left(dx^2 + 1\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.242 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2 dx$

Optimal. Leaf size=72

$$-\frac{4b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+8b^2x$$

```
[Out] 8*b^2*x - (4*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*x^2]*S
qrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^2
```

Rubi [A] time = 0.0143908, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$-\frac{4b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*x^2]*S
qrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^2
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2 dx = -\frac{4b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2 + \left(8b^2\right)\int 1$$
$$= 8b^2x - \frac{4b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2$$

Mathematica [A] time = 0.0606404, size = 104, normalized size = 1.44

$$x\left(a^2+8b^2\right) - \frac{4ab\sqrt{dx^2}\sqrt{dx^2+2}}{dx} + \frac{2b\cosh^{-1}\left(dx^2+1\right)\left(adx^2-2b\sqrt{dx^2}\sqrt{dx^2+2}\right)}{dx} + b^2x\cosh^{-1}\left(dx^2+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

```
[Out] (a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*x) + (2*b*(a*d*x^2
 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2])/(d*x) + b^2*x*ArcCo
sh[1 + d*x^2]^2
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^2,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^2,x)
```

Maxima [A] time = 1.29604, size = 173, normalized size = 2.4

$$b^2x\,\text{arcosh}\left(dx^2+1\right)^2 + 4\,b^2d\left(\frac{2\,x}{d} - \frac{\left(d^{\frac{3}{2}}x^2+2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2+2}\sqrt{dx^2}+1\right)}{\sqrt{dx^2+2}d^2}\right) + 2\left(x\,\text{arcosh}\left(dx^2+1\right) - \frac{2\left(d^{\frac{3}{2}}x^2\right.}{\sqrt{dx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(d*x^2 + 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*log
(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 2*(x*arc
cosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a*b + a^
2*x
```

Fricas [A] time = 2.02929, size = 277, normalized size = 3.85

$$\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^2+\left(a^2+8\,b^2\right)dx^2-4\,\sqrt{d^2x^4+2\,dx^2}ab+2\left(abdx^2-2\,\sqrt{d^2x^4+2\,dx^2}b^2\right)\log\left(dx^2+\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + (a^2 + 8*b^2)*d*x^2
 - 4*sqrt(d^2*x^4 + 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)
*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.243 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right) dx$

Optimal. Leaf size=49

$$ax-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+bx\cosh^{-1}\left(dx^2+1\right)$$

```
[Out] a*x - (2*b*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + b*x*ArcCosh[1 + d
*x^2]
```

Rubi [A] time = 0.0390155, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5901, 12, 6719, 261}

$$ax-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+bx\cosh^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + b*x*ArcCosh[1 + d
*x^2]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
```

```
[v, x] &&  !FreeQ[w, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)\,dx &= ax+b\int \cosh^{-1}\left(1+dx^2\right)\,dx\\
&= ax+bx\cosh^{-1}\left(1+dx^2\right)-b\int 2\sqrt{\frac{dx^2}{2+dx^2}}\,dx\\
&= ax+bx\cosh^{-1}\left(1+dx^2\right)-(2b)\int \sqrt{\frac{dx^2}{2+dx^2}}\,dx\\
&= ax+bx\cosh^{-1}\left(1+dx^2\right)-\frac{\left(2b\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}\right)\int \frac{x}{\sqrt{2+dx^2}}\,dx}{x}\\
&= ax-\frac{2b\sqrt{\frac{dx^2}{2+dx^2}}\left(2+dx^2\right)}{dx}+bx\cosh^{-1}\left(1+dx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0588238, size = 37, normalized size = 0.76

$$ax-\frac{2bx}{\sqrt{\frac{dx^2}{dx^2+2}}}+bx\cosh^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*x)/Sqrt[(d*x^2)/(2 + d*x^2)] + b*x*ArcCosh[1 + d*x^2]
```

Maple [A] time = 0.004, size = 37, normalized size = 0.8

$$ax+b\left(x\mathrm{arccosh}\left(dx^2+1\right)-2\,\frac{x\sqrt{dx^2+2}}{\sqrt{dx^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x^2+1),x)
```

```
[Out] a*x+b*(x*arccosh(d*x^2+1)-2/(d*x^2)^(1/2)*x*(d*x^2+2)^(1/2))
```

Maxima [A] time = 1.03273, size = 59, normalized size = 1.2

$$\left(x \operatorname{arcosh}\left(dx^2+1\right) - \frac{2\left(d^{\frac{3}{2}}x^2 + 2\sqrt{d}\right)}{\sqrt{dx^2+2}d}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="maxima")
```

```
[Out] (x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*b
+ a*x
```

Fricas [A] time = 1.98192, size = 132, normalized size = 2.69

$$\frac{bdx^2 \log\left(dx^2 + \sqrt{d^2x^4 + 2\,dx^2} + 1\right) + adx^2 - 2\sqrt{d^2x^4 + 2\,dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) + a*d*x^2 - 2*sqrt(d^2*x^
4 + 2*d*x^2)*b)/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{acosh}\left(dx^2+1\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x**2+1),x)
```

```
[Out] Integral(a + b*acosh(d*x**2 + 1), x)
```

Giac [A] time = 1.12393, size = 88, normalized size = 1.8

$$\left(2\,d\left(\frac{\sqrt{2}\mathrm{sgn}\left(x\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{d^{2}x^{2}+2\,d}}{d^{2}\mathrm{sgn}\left(x\right)}\right)+x\log\left(dx^{2}+\sqrt{\left(dx^{2}+1\right)^{2}-1}+1\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="giac")
```

```
[Out] (2*d*(sqrt(2)*sgn(x)/d^(3/2) - sqrt(d^2*x^2 + 2*d)/(d^2*sgn(x))) + x*log(d*
x^2 + sqrt((d*x^2 + 1)^2 - 1) + 1))*b + a*x
```

3.244 $\int \frac{1}{a+b\cosh^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=98

$$\frac{x\cosh\left(\frac{a}{2b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

```
[Out] (x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b
*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rubi [A] time = 0.0318139, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5881}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b
*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 5881

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Co
sh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt
[d*x^2]), x] - Simp[(x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}\left(1+dx^2\right)}\,dx = \frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(1+dx^2\right)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}} - \frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(1+dx^2\right)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

Mathematica [A] time = 0.127426, size = 118, normalized size = 1.2

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)\right)}{b\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1
+ d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)]))/(b*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1)),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arcosh}\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\,\text{arcosh}\left(dx^2+1\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\text{acosh}\left(dx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1)),x)
```

```
[Out] Integral(1/(a + b*acosh(d*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arcosh}\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

3.245 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2+2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}$$

```
[Out] -(Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2])) - (x*Co
shIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(2*Sqrt[2]*b^2*
Sqrt[d*x^2]) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*
b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rubi [A] time = 0.0216295, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5887}

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2+2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

```
[Out] -(Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2])) - (x*Co
shIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(2*Sqrt[2]*b^2*
Sqrt[d*x^2]) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*
b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rule 5887

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[(Sqr
t[d*x^2]*Sqrt[2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2])), x] + (-Simp
[(x*Sinh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(2*Sqrt[2
]*b^2*Sqrt[d*x^2]), x] + Simp[(x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[
1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2]), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}\,dx = -\frac{\sqrt{dx^2}\sqrt{2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)} - \frac{x\text{Chi}\left(\frac{a+b\cosh^{-1}\left(1+dx^2\right)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}} + \frac{x\cosh\left(\frac{a}{2b}\right)\text{Sh}}{2\sqrt{2}}$$

Mathematica [A] time = 0.880592, size = 130, normalized size = 0.87

$$-\frac{x^2\text{csch}\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)\right)+\frac{2b\sqrt{dx^2}\sqrt{dx^2+2}}{ad+bd\cosh^{-1}\left(dx^2+1\right)}}{4b^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

```
[Out] -((2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(a*d + b*d*ArcCosh[1 + d*x^2]) + x^2*Cs
ch[ArcCosh[1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]*Si
nh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])
)/(4*b^2*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int\left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^2x^4+3\,dx^2+\left(d^{\frac{3}{2}}x^3+2\,\sqrt{d}x\right)\sqrt{dx^2+2}+2}{2\left(abd^2x^3+2\,abdx+\left(b^2d^2x^3+2\,b^2dx+\left(b^2d^{\frac{3}{2}}x^2+b^2\sqrt{d}\right)\sqrt{dx^2+2}\right)\log\left(dx^2+\sqrt{dx^2+2}\sqrt{d}x+1\right)+\left(abd^{\frac{3}{2}}x^2+ab\sqrt{d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2)/
(a*b*d^2*x^3 + 2*a*b*d*x + (b^2*d^2*x^3 + 2*b^2*d*x + (b^2*d^(3/2)*x^2 + b^
2*sqrt(d))*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (a
*b*d^(3/2)*x^2 + a*b*sqrt(d))*sqrt(d*x^2 + 2)) + integrate(1/2*(d^3*x^6 + 3
*d^2*x^4 + (d^2*x^4 + d*x^2 + 2)*(d*x^2 + 2) + (2*d^(5/2)*x^5 + 4*d^(3/2)*x
^3 + sqrt(d)*x)*sqrt(d*x^2 + 2) - 4)/(a*b*d^3*x^6 + 4*a*b*d^2*x^4 + 4*a*b*d
*x^2 + (a*b*d^2*x^4 + 2*a*b*d*x^2 + a*b)*(d*x^2 + 2) + (b^2*d^3*x^6 + 4*b^2
*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 + 2*b^2*d*x^2 + b^2)*(d*x^2 + 2) + 2*
(b^2*d^(5/2)*x^5 + 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2))*lo
g(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 2*(a*b*d^(5/2)*x^5 + 3*a*b*d^(3/
2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 \operatorname{arcosh}\left(dx^2+1\right)^2+2\,ab \operatorname{arcosh}\left(dx^2+1\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x^2 + 1)^2 + 2*a*b*arccosh(d*x^2 + 1) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b \operatorname{acosh}\left(dx^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2+1\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-2), x)
```

3.246 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=180

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}-\frac{}{4bx\sqrt{dx^2}\sqrt{dx^2}}$$

```
[Out] -(2*x^2 + d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^2) - x/(8*b^2*(a + b*ArcCosh[1 + d*x^2])) + (x*Cosh[a/(2*b)]*CoshIntegr
al[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) - (x*Sinh
[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sq
rt[d*x^2])
```

Rubi [A] time = 0.0380817, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5889, 5881}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}-\frac{}{4bx\sqrt{dx^2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

```
[Out] -(2*x^2 + d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^2) - x/(8*b^2*(a + b*ArcCosh[1 + d*x^2])) + (x*Cosh[a/(2*b)]*CoshIntegr
al[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) - (x*Sinh
[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sq
rt[d*x^2])
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
```

```
EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 5881

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Co
sh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt
[d*x^2]), x] - Simp[(x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}\,dx = -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}+\frac{\int \frac{}{a+b}}{}$$

$$= -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}+\frac{x\cos}{}$$

Mathematica [A] time = 0.457619, size = 152, normalized size = 0.84

$$\frac{-\frac{2b^2\sqrt{dx^2}\sqrt{dx^2+2}}{d\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}+\frac{\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)\right)}{d}-\frac{bx^2}{a+b\cosh^{-1}\left(dx^2+1\right)}}{8b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

```
[Out] ((-2*b^2*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*(a + b*ArcCosh[1 + d*x^2])^2) - (b
*x^2)/(a + b*ArcCosh[1 + d*x^2]) + (Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b
)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhInteg
ral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \left(a+b\text{arccosh}\left(dx^2+1\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 + 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
(11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 + 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*(3
*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 + (3*a*d^3 + 8*b*d^3)*x^
5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 + 4*(a*d + b*d)*x)*(d*x^2 + 2)^(3/2) + (3*(a*
d^4 + 2*b*d^4)*sqrt(d)*x^8 + 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*a*d^2
 + 25*b*d^2)*sqrt(d)*x^4 + 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*sqrt(d)
)*(d*x^2 + 2) + (b*d^(11/2)*x^10 + 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^6 + 2*b
*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 + 3*b*d^3*x^5 + 4*b*d^2*x^3 +
4*b*d*x)*(d*x^2 + 2)^(3/2) + (3*b*d^(9/2)*x^8 + 12*b*d^(7/2)*x^6 + 16*b*d^(
5/2)*x^4 + 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2) + (3*b*d^5*x^9 + 15*
b*d^4*x^7 + 23*b*d^3*x^5 + 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2) - 8*b*sqr
t(d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (3*(a*d^5 + 2*b*d^5)*x^9
 + 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 + (7*a*d^2 + 64*b
*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2) - 8*a*sqrt(d))/(a^2*b^2*d^
(11/2)*x^9 + 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5 + 8*a^2*b^2*d^(
5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*b^4*d^(9/2)*x^7 + 12*b^4*d^(7/2)*x^5 + 8*b
^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*b^4*d^3*x^4 + 3*b^4*d^2*x^2 + b^4*d)*(d*x
^2 + 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 + 4*b^4*d^(7/2)*x^5 + 5*b^4*d^(5/2)*x^3
+ 2*b^4*d^(3/2)*x)*(d*x^2 + 2) + 3*(b^4*d^5*x^8 + 5*b^4*d^4*x^6 + 8*b^4*d^3
*x^4 + 4*b^4*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*
x + 1)^2 + (a^2*b^2*d^4*x^6 + 3*a^2*b^2*d^3*x^4 + 3*a^2*b^2*d^2*x^2 + a^2*b
^2*d)*(d*x^2 + 2)^(3/2) + 3*(a^2*b^2*d^(9/2)*x^7 + 4*a^2*b^2*d^(7/2)*x^5 +
5*a^2*b^2*d^(5/2)*x^3 + 2*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2) + 2*(a*b^3*d^(11/2
)*x^9 + 6*a*b^3*d^(9/2)*x^7 + 12*a*b^3*d^(7/2)*x^5 + 8*a*b^3*d^(5/2)*x^3 +
(a*b^3*d^4*x^6 + 3*a*b^3*d^3*x^4 + 3*a*b^3*d^2*x^2 + a*b^3*d)*(d*x^2 + 2)^(
3/2) + 3*(a*b^3*d^(9/2)*x^7 + 4*a*b^3*d^(7/2)*x^5 + 5*a*b^3*d^(5/2)*x^3 + 2
*a*b^3*d^(3/2)*x)*(d*x^2 + 2) + 3*(a*b^3*d^5*x^8 + 5*a*b^3*d^4*x^6 + 8*a*b^
3*d^3*x^4 + 4*a*b^3*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*s
```

```
qrt(d)*x + 1) + 3*(a^2*b^2*d^5*x^8 + 5*a^2*b^2*d^4*x^6 + 8*a^2*b^2*d^3*x^4
+ 4*a^2*b^2*d^2*x^2)*sqrt(d*x^2 + 2)) + integrate(1/8*(d^6*x^12 + 8*d^5*x^1
0 + 27*d^4*x^8 + 56*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*d^3*x^6 + 3*d^2*x^4
 - 8*d*x^2 + 4)*(d*x^2 + 2)^2 + 96*d*x^2 + 2*(2*d^(9/2)*x^9 + 10*d^(7/2)*x^
7 + 15*d^(5/2)*x^5 - d^(3/2)*x^3 - 11*sqrt(d)*x)*(d*x^2 + 2)^(3/2) + 3*(2*d
^5*x^10 + 12*d^4*x^8 + 26*d^3*x^6 + 24*d^2*x^4 + 3*d*x^2 - 10)*(d*x^2 + 2)
+ 2*(2*d^(11/2)*x^11 + 14*d^(9/2)*x^9 + 39*d^(7/2)*x^7 + 61*d^(5/2)*x^5 + 6
1*d^(3/2)*x^3 + 30*sqrt(d)*x)*sqrt(d*x^2 + 2) + 48)/(a*b^2*d^6*x^12 + 8*a*b
^2*d^5*x^10 + 24*a*b^2*d^4*x^8 + 32*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b
^2*d^4*x^8 + 4*a*b^2*d^3*x^6 + 6*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2 + a*b^2)*(d*
x^2 + 2)^2 + 4*(a*b^2*d^(9/2)*x^9 + 5*a*b^2*d^(7/2)*x^7 + 9*a*b^2*d^(5/2)*x
^5 + 7*a*b^2*d^(3/2)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 + 2)^(3/2) + 6*(a*b^2*
d^5*x^10 + 6*a*b^2*d^4*x^8 + 13*a*b^2*d^3*x^6 + 12*a*b^2*d^2*x^4 + 4*a*b^2*
d*x^2)*(d*x^2 + 2) + (b^3*d^6*x^12 + 8*b^3*d^5*x^10 + 24*b^3*d^4*x^8 + 32*b
^3*d^3*x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 + 4*b^3*d^3*x^6 + 6*b^3*d^2*x^4
+ 4*b^3*d*x^2 + b^3)*(d*x^2 + 2)^2 + 4*(b^3*d^(9/2)*x^9 + 5*b^3*d^(7/2)*x^7
 + 9*b^3*d^(5/2)*x^5 + 7*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 + 2)^(3/
2) + 6*(b^3*d^5*x^10 + 6*b^3*d^4*x^8 + 13*b^3*d^3*x^6 + 12*b^3*d^2*x^4 + 4*
b^3*d*x^2)*(d*x^2 + 2) + 4*(b^3*d^(11/2)*x^11 + 7*b^3*d^(9/2)*x^9 + 18*b^3*
d^(7/2)*x^7 + 20*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d*x^2 + 2))*log(
d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 4*(a*b^2*d^(11/2)*x^11 + 7*a*b^2*d
^(9/2)*x^9 + 18*a*b^2*d^(7/2)*x^7 + 20*a*b^2*d^(5/2)*x^5 + 8*a*b^2*d^(3/2)*
x^3)*sqrt(d*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3\operatorname{arcosh}\left(dx^2+1\right)^3+3\,ab^2\operatorname{arcosh}\left(dx^2+1\right)^2+3\,a^2b\operatorname{arcosh}\left(dx^2+1\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x^2 + 1)^3 + 3*a*b^2*arccosh(d*x^2 + 1)^2 + 3*a^2
*b*arccosh(d*x^2 + 1) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2+1\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3), x)
```

3.247 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=147

$$\frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+\frac{8b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

```
[Out] 384*b^4*x + (192*b^3*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d
*x^2]*Sqrt[-2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[-1 + d*x^2])^2 + (8*b*(2*
x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]
) + x*(a + b*ArcCosh[-1 + d*x^2])^4
```

Rubi [A] time = 0.0337011, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$\frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+\frac{8b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```


```
[Out] 384*b^4*x + (192*b^3*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d
*x^2]*Sqrt[-2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[-1 + d*x^2])^2 + (8*b*(2*
x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]
) + x*(a + b*ArcCosh[-1 + d*x^2])^4
```


Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4 dx = \frac{8b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4 + \left(48b^2\right)$$

$$= \frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + 48b^2x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2 +$$

$$= 384b^4x + \frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + 48b^2x\left(a+b\cosh^{-1}\left(-1+\right.\right.$$

Mathematica [A] time = 0.225346, size = 264, normalized size = 1.8

$$\frac{dx^2\left(48a^2b^2+a^4+384b^4\right)-8ab\left(a^2+24b^2\right)\sqrt{dx^2}\sqrt{dx^2-2}+6b^2\cosh^{-1}\left(dx^2-1\right)^2\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2-2}+8b^2d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

```
[Out] ((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2] + 6*b
^2*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-
1 + d*x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-
1 + d*x^2]^3 + b^4*d*x^2*ArcCosh[-1 + d*x^2]^4)/(d*x)
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(dx^2-1\right)\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^4,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18158, size = 626, normalized size = 4.26

$$\frac{b^4dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^4+\left(a^4+48\,a^2b^2+384\,b^4\right)dx^2+4\left(ab^3dx^2-2\,\sqrt{d^2x^4-2\,dx^2}b^4\right)\log\left(dx^2+\sqrt{d^2x^4-\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^4 + (a^4 + 48*a^2*b^2 +
 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^4)*log(d*x^2
 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 - 6*(4*sqrt(d^2*x^4 - 2*d*x^2)*a*b^3 - (a
^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + 4*((a^3
*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 - 2*d*x^2)*(a^2*b^2 + 8*b^4))*log(d*x
^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 8*sqrt(d^2*x^4 - 2*d*x^2)*(a^3*b + 24*a
*b^3))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**4,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**4, x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.248 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3\,dx$

Optimal. Leaf size=110

$$24ab^2x+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3-48b^3x\sqrt{1-\frac{2}{dx^2}}+24b^3x\cosh^{-1}\left(dx^2\right.$$

```
[Out] 24*a*b^2*x - 48*b^3*Sqrt[1 - 2/(d*x^2)]*x + 24*b^3*x*ArcCosh[-1 + d*x^2] +
(6*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^2)/(x*Sqrt[d*x^2]*Sqrt[-2
+ d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^3
```

Rubi [A] time = 0.0456404, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5880, 5901, 12, 191}

$$24ab^2x+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3-48b^3x\sqrt{1-\frac{2}{dx^2}}+24b^3x\cosh^{-1}\left(dx^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - 48*b^3*Sqrt[1 - 2/(d*x^2)]*x + 24*b^3*x*ArcCosh[-1 + d*x^2] +
(6*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^2)/(x*Sqrt[d*x^2]*Sqrt[-2
+ d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^3
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] && !FunctionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3\,dx = \frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3+\left(24b^2\right)$$

$$= 24ab^2x+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3$$

$$= 24ab^2x+24b^3x\cosh^{-1}\left(-1+dx^2\right)+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x$$

$$= 24ab^2x+24b^3x\cosh^{-1}\left(-1+dx^2\right)+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x$$

$$= 24ab^2x-48b^3\sqrt{1-\frac{2}{dx^2}}x+24b^3x\cosh^{-1}\left(-1+dx^2\right)+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh\right.}{x\sqrt{dx^2}\sqrt{-2+d}}$$

Mathematica [A] time = 0.119618, size = 171, normalized size = 1.55

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2}\sqrt{dx^2-2}+3b\cosh^{-1}\left(dx^2-1\right)\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2-2}+8b^2dx^2\right)+3b^2\cos}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[-2 + d*x^2] +
3*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[
-1 + d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1
 + d*x^2]^2 + b^3*d*x^2*ArcCosh[-1 + d*x^2]^3)/(d*x)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{arccosh}\left(dx^2 - 1\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^3,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10917, size = 441, normalized size = 4.01

$$\frac{b^3dx^2 \log\left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right)^3 + \left(a^3 + 24\,ab^2\right)dx^2 + 3\left(ab^2dx^2 - 2\sqrt{d^2x^4 - 2\,dx^2}b^3\right)\log\left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 + (a^3 + 24*a*b^2)*d*
x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^2*
x^4 - 2*d*x^2) - 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 - 2*d*x^2
)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 6*sqrt(d^2*x^4 - 2*d*x^
2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{acosh}\left(dx^2 - 1\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.249 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=73

$$\frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+8b^2x$$

```
[Out] 8*b^2*x + (4*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d*x^2]*
Sqrt[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^2
```

Rubi [A] time = 0.0138888, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$\frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

```
[Out] 8*b^2*x + (4*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d*x^2]*
Sqrt[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^2
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\,dx = \frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2 + \left(8b^2\right)\int$$
$$= 8b^2x + \frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2$$

Mathematica [A] time = 0.0608725, size = 104, normalized size = 1.42

$$x\left(a^2+8b^2\right) - \frac{4ab\sqrt{dx^2}\sqrt{dx^2-2}}{dx} + \frac{2b\cosh^{-1}\left(dx^2-1\right)\left(adx^2-2b\sqrt{dx^2}\sqrt{dx^2-2}\right)}{dx} + b^2x\cosh^{-1}\left(dx^2-1\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

```
[Out] (a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*x) + (2*b*(a*d*x^
2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2])/(d*x) + b^2*x*Ar
cCosh[-1 + d*x^2]^2
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(dx^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^2,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^2,x)
```

Maxima [A] time = 1.32562, size = 173, normalized size = 2.37

$$b^2x\operatorname{arcosh}\left(dx^2-1\right)^2 + 4b^2d\left(\frac{2x}{d} - \frac{\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2-2}\sqrt{dx^2}-1\right)}{\sqrt{dx^2-2}\,d^2}\right) + 2\left(x\operatorname{arcosh}\left(dx^2-1\right) - \frac{2\left(d^{\frac{3}{2}}x^2\right.}{\sqrt{dx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(d*x^2 - 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*log
(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 2*(x*arc
cosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a*b + a^
2*x
```

Fricas [A] time = 2.23492, size = 277, normalized size = 3.79

$$\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^2+\left(a^2+8\,b^2\right)dx^2-4\,\sqrt{d^2x^4-2\,dx^2}ab+2\left(abdx^2-2\,\sqrt{d^2x^4-2\,dx^2}b^2\right)\log\left(dx^2+\sqrt{\ldots}\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + (a^2 + 8*b^2)*d*x^2
 - 4*sqrt(d^2*x^4 - 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)
*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.250 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right) dx$

Optimal. Leaf size=33

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

```
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Rubi [A] time = 0.0168854, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5901, 12, 191}

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[-1 + d*x^2],x]
```

```
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\,dx &= ax+b\int \cosh^{-1}\left(-1+dx^2\right)\,dx\\ &= ax+bx\cosh^{-1}\left(-1+dx^2\right)-b\int \frac{2}{\sqrt{1-\frac{2}{dx^2}}}\,dx\\ &= ax+bx\cosh^{-1}\left(-1+dx^2\right)-(2b)\int \frac{1}{\sqrt{1-\frac{2}{dx^2}}}\,dx\\ &= ax-2b\sqrt{1-\frac{2}{dx^2}}x+bx\cosh^{-1}\left(-1+dx^2\right)\end{aligned}$$

Mathematica [A] time = 0.0234999, size = 33, normalized size = 1.

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[-1 + d*x^2],x]
```

```
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Maple [A] time = 0.006, size = 37, normalized size = 1.1

$$ax+b\left(x\mathrm{arccosh}\left(dx^2-1\right)-2\,\frac{x\sqrt{dx^2-2}}{\sqrt{dx^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x^2-1),x)
```

```
[Out] a*x+b*(x*arccosh(d*x^2-1)-2/(d*x^2)^(1/2)*x*(d*x^2-2)^(1/2))
```

Maxima [A] time = 0.99553, size = 59, normalized size = 1.79

$$\left(x\,\mathrm{arcosh}\left(dx^2-1\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)}{\sqrt{dx^2-2}d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="maxima")
```

```
[Out] (x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*b
+ a*x
```

Fricas [B] time = 2.1458, size = 132, normalized size = 4.

$$\frac{bdx^2 \log\left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right) + adx^2 - 2\,\sqrt{d^2x^4 - 2\,dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) + a*d*x^2 - 2*sqrt(d^2*x^
4 - 2*d*x^2)*b)/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{acosh}\left(dx^2 - 1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x**2-1),x)
```

```
[Out] Integral(a + b*acosh(d*x**2 - 1), x)
```

Giac [B] time = 1.11448, size = 95, normalized size = 2.88

$$\left(2\,d\left(\frac{\sqrt{2}\sqrt{-d}\text{sgn}\left(x\right)}{d^2} - \frac{\sqrt{d^2x^2 - 2\,d}}{d^2\text{sgn}\left(x\right)}\right) + x\log\left(dx^2 + \sqrt{\left(dx^2 - 1\right)^2 - 1} - 1\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="giac")
```

```
[Out] (2*d*(sqrt(2)*sqrt(-d)*sgn(x)/d^2 - sqrt(d^2*x^2 - 2*d)/(d^2*sgn(x))) + x*l
og(d*x^2 + sqrt((d*x^2 - 1)^2 - 1) - 1))*b + a*x
```

3.251 $\int \frac{1}{a+b\cosh^{-1}(-1+dx^2)}\,dx$

Optimal. Leaf size=98

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

```
[Out] -((x*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(Sqrt[2
]*b*Sqrt[d*x^2])) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2
])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rubi [A] time = 0.0153676, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5882}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

```
[Out] -((x*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(Sqrt[2
]*b*Sqrt[d*x^2])) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2
])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 5882

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x*
Sinh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]), x] + Simp[(x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*
x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx = -\frac{x\text{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}} + \frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

Mathematica [A] time = 0.129587, size = 86, normalized size = 0.88

$$-\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)\right)}{bdx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

```
[Out] -((Cosh[ArcCosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2
*b)]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])
/(2*b)]))/(b*d*x))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1)),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arcosh}\left(dx^2-1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\,\text{arcosh}\left(dx^2-1\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\text{acosh}\left(dx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1)),x)
```

```
[Out] Integral(1/(a + b*acosh(d*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arcosh}\left(dx^2-1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

3.252 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=150

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}$$

```
[Out] -(Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2])) + (x*
Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b
^2*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])
/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rubi [A] time = 0.0192627, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5888}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

```
[Out] -(Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2])) + (x*
Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b
^2*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])
/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rule 5888

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[(Sq
rt[d*x^2]*Sqrt[-2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2])), x] + (Si
mp[(x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqr
t[2]*b^2*Sqrt[d*x^2]), x] - Simp[(x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCo
sh[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2]), x]) /; FreeQ[{a, b, d}
, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}\,dx = -\frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right.}{}$$

Mathematica [A] time = 0.698854, size = 141, normalized size = 0.94

$$\frac{\frac{\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)\right)}{\sqrt{1-\frac{2}{dx^2}}}-\frac{b\sqrt{dx^2}\sqrt{dx^2-2}}{a+b\cosh^{-1}\left(dx^2-1\right)}}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

```
[Out] (-((b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(a + b*ArcCosh[-1 + d*x^2])) + (Sinh[Ar
cCosh[-1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2]
)/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]))/
Sqrt[1 - 2/(d*x^2)])/(2*b^2*d*x)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(a+b\text{arccosh}\left(dx^2-1\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^2x^4-3\,dx^2+\left(d^{\frac{3}{2}}x^3-2\,\sqrt{d}x\right)\sqrt{dx^2-2}+2}{2\left(abd^2x^3-2\,abdx+\left(b^2d^2x^3-2\,b^2dx+\left(b^2d^{\frac{3}{2}}x^2-b^2\sqrt{d}\right)\sqrt{dx^2-2}\right)\log\left(dx^2+\sqrt{dx^2-2}\sqrt{d}x-1\right)+\left(abd^{\frac{3}{2}}x^2-ab\sqrt{d}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2)/
(a*b*d^2*x^3 - 2*a*b*d*x + (b^2*d^2*x^3 - 2*b^2*d*x + (b^2*d^(3/2)*x^2 - b^
2*sqrt(d))*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (a
*b*d^(3/2)*x^2 - a*b*sqrt(d))*sqrt(d*x^2 - 2)) + integrate(1/2*(d^3*x^6 - 3
*d^2*x^4 + (d^2*x^4 - d*x^2 + 2)*(d*x^2 - 2) + (2*d^(5/2)*x^5 - 4*d^(3/2)*x
^3 + sqrt(d)*x)*sqrt(d*x^2 - 2) + 4)/(a*b*d^3*x^6 - 4*a*b*d^2*x^4 + 4*a*b*d
*x^2 + (a*b*d^2*x^4 - 2*a*b*d*x^2 + a*b)*(d*x^2 - 2) + (b^2*d^3*x^6 - 4*b^2
*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 - 2*b^2*d*x^2 + b^2)*(d*x^2 - 2) + 2*
(b^2*d^(5/2)*x^5 - 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2))*lo
g(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 2*(a*b*d^(5/2)*x^5 - 3*a*b*d^(3/
2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2\,\text{arcosh}\left(dx^2-1\right)^2+2\,ab\,\text{arcosh}\left(dx^2-1\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x^2 - 1)^2 + 2*a*b*arccosh(d*x^2 - 1) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\,\text{acosh}\left(dx^2-1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2-1\right)+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-2), x)
```

3.253 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}+\frac{\cdots}{4bx\sqrt{dx^2}\sqrt{dx\cdots}}$$

```
[Out] (2*x^2 - d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^2) - x/(8*b^2*(a + b*ArcCosh[-1 + d*x^2])) - (x*CoshIntegral[(a + b*Ar
cCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*C
osh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^
3*Sqrt[d*x^2])
```

Rubi [A] time = 0.0334418, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5889, 5882}

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}+\frac{\cdots}{4bx\sqrt{dx^2}\sqrt{dx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

```
[Out] (2*x^2 - d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^2) - x/(8*b^2*(a + b*ArcCosh[-1 + d*x^2])) - (x*CoshIntegral[(a + b*Ar
cCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*C
osh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^
3*Sqrt[d*x^2])
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
```

```
EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 5882

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x*
Sinh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]), x] + Simp[(x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*
x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}\,dx = \frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)} + \cdots$$

$$= \frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)} - \cdots$$

Mathematica [A] time = 0.643935, size = 168, normalized size = 0.93

$$-\frac{\frac{2b^2\sqrt{dx^2}\sqrt{dx^2-2}}{d\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2} + \frac{1}{2}x^2\sqrt{1-\frac{2}{dx^2}}\operatorname{csch}\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}\cdots}{2\cdots}\right)\cdots\right.}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

```
[Out] -((2*b^2*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*(a + b*ArcCosh[-1 + d*x^2])^2) +
(b*x^2)/(a + b*ArcCosh[-1 + d*x^2]) + (Sqrt[1 - 2/(d*x^2)]*x^2*Csch[ArcCosh
[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*
b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]))/2)/(8
*b^3*x)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(dx^2-1\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 - 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
(11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 - 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*(3
*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*a*d^3 + 8*b*d^3)*x^
5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(a*d + b*d)*x)*(d*x^2 - 2)^(3/2) + (3*(a*
d^4 + 2*b*d^4)*sqrt(d)*x^8 - 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*a*d^2
 + 25*b*d^2)*sqrt(d)*x^4 - 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*sqrt(d)
)*(d*x^2 - 2) + (b*d^(11/2)*x^10 - 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^6 - 2*b
*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 - 3*b*d^3*x^5 + 4*b*d^2*x^3 -
4*b*d*x)*(d*x^2 - 2)^(3/2) + (3*b*d^(9/2)*x^8 - 12*b*d^(7/2)*x^6 + 16*b*d^(
5/2)*x^4 - 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2) + (3*b*d^5*x^9 - 15*
b*d^4*x^7 + 23*b*d^3*x^5 - 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2) + 8*b*sqr
t(d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 - (7*a*d^2 + 64*b
*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2) + 8*a*sqrt(d))/(a^2*b^2*d^
(11/2)*x^9 - 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5 - 8*a^2*b^2*d^(
5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*b^4*d^(9/2)*x^7 + 12*b^4*d^(7/2)*x^5 - 8*b
^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*b^4*d^3*x^4 + 3*b^4*d^2*x^2 - b^4*d)*(d*x
^2 - 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 - 4*b^4*d^(7/2)*x^5 + 5*b^4*d^(5/2)*x^3
- 2*b^4*d^(3/2)*x)*(d*x^2 - 2) + 3*(b^4*d^5*x^8 - 5*b^4*d^4*x^6 + 8*b^4*d^3
*x^4 - 4*b^4*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*
x - 1)^2 + (a^2*b^2*d^4*x^6 - 3*a^2*b^2*d^3*x^4 + 3*a^2*b^2*d^2*x^2 - a^2*b
^2*d)*(d*x^2 - 2)^(3/2) + 3*(a^2*b^2*d^(9/2)*x^7 - 4*a^2*b^2*d^(7/2)*x^5 +
5*a^2*b^2*d^(5/2)*x^3 - 2*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2) + 2*(a*b^3*d^(11/2
)*x^9 - 6*a*b^3*d^(9/2)*x^7 + 12*a*b^3*d^(7/2)*x^5 - 8*a*b^3*d^(5/2)*x^3 +
(a*b^3*d^4*x^6 - 3*a*b^3*d^3*x^4 + 3*a*b^3*d^2*x^2 - a*b^3*d)*(d*x^2 - 2)^(
3/2) + 3*(a*b^3*d^(9/2)*x^7 - 4*a*b^3*d^(7/2)*x^5 + 5*a*b^3*d^(5/2)*x^3 - 2
*a*b^3*d^(3/2)*x)*(d*x^2 - 2) + 3*(a*b^3*d^5*x^8 - 5*a*b^3*d^4*x^6 + 8*a*b^
3*d^3*x^4 - 4*a*b^3*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*s
```

```
qrt(d)*x - 1) + 3*(a^2*b^2*d^5*x^8 - 5*a^2*b^2*d^4*x^6 + 8*a^2*b^2*d^3*x^4
- 4*a^2*b^2*d^2*x^2)*sqrt(d*x^2 - 2)) + integrate(1/8*(d^6*x^12 - 8*d^5*x^1
0 + 27*d^4*x^8 - 56*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*d^3*x^6 + 3*d^2*x^4
 + 8*d*x^2 + 4)*(d*x^2 - 2)^2 - 96*d*x^2 + 2*(2*d^(9/2)*x^9 - 10*d^(7/2)*x^
7 + 15*d^(5/2)*x^5 + d^(3/2)*x^3 - 11*sqrt(d)*x)*(d*x^2 - 2)^(3/2) + 3*(2*d
^5*x^10 - 12*d^4*x^8 + 26*d^3*x^6 - 24*d^2*x^4 + 3*d*x^2 + 10)*(d*x^2 - 2)
+ 2*(2*d^(11/2)*x^11 - 14*d^(9/2)*x^9 + 39*d^(7/2)*x^7 - 61*d^(5/2)*x^5 + 6
1*d^(3/2)*x^3 - 30*sqrt(d)*x)*sqrt(d*x^2 - 2) + 48)/(a*b^2*d^6*x^12 - 8*a*b
^2*d^5*x^10 + 24*a*b^2*d^4*x^8 - 32*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b
^2*d^4*x^8 - 4*a*b^2*d^3*x^6 + 6*a*b^2*d^2*x^4 - 4*a*b^2*d*x^2 + a*b^2)*(d*
x^2 - 2)^2 + 4*(a*b^2*d^(9/2)*x^9 - 5*a*b^2*d^(7/2)*x^7 + 9*a*b^2*d^(5/2)*x
^5 - 7*a*b^2*d^(3/2)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 - 2)^(3/2) + 6*(a*b^2*
d^5*x^10 - 6*a*b^2*d^4*x^8 + 13*a*b^2*d^3*x^6 - 12*a*b^2*d^2*x^4 + 4*a*b^2*
d*x^2)*(d*x^2 - 2) + (b^3*d^6*x^12 - 8*b^3*d^5*x^10 + 24*b^3*d^4*x^8 - 32*b
^3*d^3*x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 - 4*b^3*d^3*x^6 + 6*b^3*d^2*x^4
- 4*b^3*d*x^2 + b^3)*(d*x^2 - 2)^2 + 4*(b^3*d^(9/2)*x^9 - 5*b^3*d^(7/2)*x^7
 + 9*b^3*d^(5/2)*x^5 - 7*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 - 2)^(3/
2) + 6*(b^3*d^5*x^10 - 6*b^3*d^4*x^8 + 13*b^3*d^3*x^6 - 12*b^3*d^2*x^4 + 4*
b^3*d*x^2)*(d*x^2 - 2) + 4*(b^3*d^(11/2)*x^11 - 7*b^3*d^(9/2)*x^9 + 18*b^3*
d^(7/2)*x^7 - 20*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d*x^2 - 2))*log(
d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 4*(a*b^2*d^(11/2)*x^11 - 7*a*b^2*d
^(9/2)*x^9 + 18*a*b^2*d^(7/2)*x^7 - 20*a*b^2*d^(5/2)*x^5 + 8*a*b^2*d^(3/2)*
x^3)*sqrt(d*x^2 - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3\operatorname{arcosh}\left(dx^2-1\right)^3+3\,ab^2\operatorname{arcosh}\left(dx^2-1\right)^2+3\,a^2b\operatorname{arcosh}\left(dx^2-1\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x^2 - 1)^3 + 3*a*b^2*arccosh(d*x^2 - 1)^2 + 3*a^2
*b*arccosh(d*x^2 - 1) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2-1\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3), x)
```

3.254 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=280

$$\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{}$$

```
[Out] (-5*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(5/2) - (15*b^(5/2)*Sqrt[Pi/2]*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (15*b^(5/2)*Sqrt[Pi/2]*Erf[
Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(
2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (30*b^2*Sqrt[a + b*ArcCosh[1 + d
*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rubi [A] time = 0.108965, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5878}

$$\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```


```
[Out] (-5*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(5/2) - (15*b^(5/2)*Sqrt[Pi/2]*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (15*b^(5/2)*Sqrt[Pi/2]*Erf[
Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(
2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (30*b^2*Sqrt[a + b*ArcCosh[1 + d
*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```


Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
```

```
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5878

```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sqr
t[a + b*ArcCosh[1 + d*x^2]]*Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2)/(d*x), x] + (
Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x] -
 Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCos
h[1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x]
) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}dx = -\frac{5b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2} + \left(15b^2\right.$$

$$= -\frac{5b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2} - \frac{15b^{5/}}{}$$

Mathematica [A] time = 3.30582, size = 311, normalized size = 1.11

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(4\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}\left(\left(a^2+15b^2\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)-5ab\cosh\left(\frac{1}{2}\cosh^{-1}\left(d\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCo
sh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 15*b^(5
/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[
a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-5*a*b*Cosh[A
rcCosh[1 + d*x^2]/2] + (a^2 + 15*b^2)*Sinh[ArcCosh[1 + d*x^2]/2] + b^2*ArcC
osh[1 + d*x^2]^2*Sinh[ArcCosh[1 + d*x^2]/2] - b*ArcCosh[1 + d*x^2]*(5*b*Cos
h[ArcCosh[1 + d*x^2]/2] - 2*a*Sinh[ArcCosh[1 + d*x^2]/2]))))/(2*Sqrt[d*x^2]
```

```
*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{arccosh}\left(d x^{2}+1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{arcosh}\left(d x^{2}+1\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.255 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2} dx$

Optimal. Leaf size=238

$$\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{s}}{}$$

```
[Out] (-3*b*(2*x^2 + d*x^4)*Sqrt[a + b*ArcCosh[1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[2
 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (3*b^(3/2)*Sqrt[Pi/2]*Erf[Sqrt
[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)
```

Rubi [A] time = 0.0967674, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5883}

$$\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```



```
[Out] (-3*b*(2*x^2 + d*x^4)*Sqrt[a + b*ArcCosh[1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[2
 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (3*b^(3/2)*Sqrt[Pi/2]*Erf[Sqrt
[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)
```


Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
```

```
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5883

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] + Simp[(Sqrt[Pi/2
]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}\,dx=-\frac{3b\left(2x^2+dx^4\right)\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}+\left(3b^2\right)\int$$

$$=-\frac{3b\left(2x^2+dx^4\right)\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}+\frac{3b^{3/2}\sqrt{}}{}$$

Mathematica [A] time = 0.657109, size = 254, normalized size = 1.07

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(3\sqrt{2\pi}b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)+3\sqrt{2\pi}b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh
[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 3*b^(3/2)
*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-3*b*Cosh[ArcCos
h[1 + d*x^2]/2] + a*Sinh[ArcCosh[1 + d*x^2]/2] + b*ArcCosh[1 + d*x^2]*Sinh[
ArcCosh[1 + d*x^2]/2])))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 +
d*x^2])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{arccosh}\left(d x^{2}+1\right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{arcosh}\left(d x^{2}+1\right)+a\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{acosh}\left(d x^{2}+1\right)\right)^{\frac{3}{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.256 $\int \sqrt{a+b\cosh^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh}{}$$

```
[Out] -((Sqrt[b]*Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])
]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)) + (Sqr
t[b]*Sqrt[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (2*Sqrt[a +
b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rubi [A] time = 0.0264593, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5878}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] -((Sqrt[b]*Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])
]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)) + (Sqr
t[b]*Sqrt[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (2*Sqrt[a +
b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rule 5878

```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sqr
t[a + b*ArcCosh[1 + d*x^2]]*Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2)/(d*x), x] + (
Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x] -
 Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCos
h[1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x]
```

```
) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\,dx = -\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)}{dx} + \cdots$$

Mathematica [A] time = 0.291669, size = 210, normalized size = 1.02

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\right)\cdots}{2\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[1
 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[b]*Sq
rt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b
)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x
^2]/2]))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \sqrt{a+b\operatorname{arccosh}\left(dx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arcosh}\left(dx^2+1\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b \operatorname{acosh}\left(dx^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(d*x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.257 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(1+dx^2)}}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cos\ldots\right.}{\sqrt{b}\ldots}$$

```
[Out] (Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt[P
i/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
 Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x)
```

Rubi [A] time = 0.0245432, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5883}

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cos\ldots\right.}{\sqrt{b}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt[P
i/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
 Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x)
```

Rule 5883

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] + Simp[(Sqrt[Pi/2
]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)}{\sqrt{b}dx} + \frac{\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A] time = 0.304812, size = 166, normalized size = 1.01

$$\frac{\sqrt{\frac{\pi}{2}}x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)+\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\,\mathrm{co}}}{}\right.\right.}{\sqrt{b}\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*x*(Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] - Sinh[a/(2*b)]) + Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[
b]*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\mathrm{arccosh}\left(dx^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{arcosh}\left(dx^2+1\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}\left(dx^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(d*x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.258 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\right.}{}$$

```
[Out] -((Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])) + (
Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x) - (Sqrt[Pi
/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x)
```

Rubi [A] time = 0.0431451, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5885}

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

```
[Out] -((Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])) + (
Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x) - (Sqrt[Pi
/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x)
```

Rule 5885

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(S
qrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]), x] + (-
Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]
*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Simp[(S
```

```
qrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ[{a, b
, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{dx^2}\sqrt{2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{b^{3/2}dx}$$

Mathematica [A] time = 1.04592, size = 242, normalized size = 1.14

$$-\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\left(\sinh\right.\right.}{2b^{3/2}\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

```
[Out] -(x*(4*Sqrt[b]*Cosh[ArcCosh[1 + d*x^2]/2] + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[1
 + d*x^2]]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/
(2*b)] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Erf[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b
)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(2*b^(3/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d
*x^2)]*Sqrt[2 + d*x^2]*Sqrt[a + b*ArcCosh[1 + d*x^2]])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(dx^2+1\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2+1\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b \operatorname{acosh}\left(dx^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-3/2), x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.259 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\mathrm{c}\right.}{3b}$$

```
[Out] -(2*x^2 + d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erfi[Sq
rt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*
b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x) + (Sqrt[Pi/2]*Erf[Sqrt[a +
 b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*S
inh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x)
```

Rubi [A] time = 0.0715158, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5883}

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\mathrm{c}\right.}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```


```
[Out] -(2*x^2 + d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erfi[Sq
rt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*
b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x) + (Sqrt[Pi/2]*Erf[Sqrt[a +
 b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*S
inh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x)
```


Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
```

```
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 5883

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] + Simp[(Sqrt[Pi/2
]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx = -\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}} + \frac{\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 0.971358, size = 273, normalized size = 1.08

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{3/2}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\left(\cos\cdots\right.\right.}{6b^{5/2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*Erf[Sqrt[a + b*Ar
cCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sq
rt[b]*(-(b*Cosh[ArcCosh[1 + d*x^2]/2]) - (a + b*ArcCosh[1 + d*x^2])*Sinh[Ar
cCosh[1 + d*x^2]/2])))/(6*b^(5/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqr
```

```
t[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^2])^(3/2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{arccosh}\left(dx^2+1\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\mathrm{arcosh}\left(dx^2+1\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.260 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\right.}{1}$$

```
[Out] -(2*x^2 + d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*Sqr
t[2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x) - (Sqrt[Pi/2]*Erf[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b
)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x)
```

Rubi [A] time = 0.0774556, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5885}

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

```
[Out] -(2*x^2 + d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*Sqr
t[2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x) - (Sqrt[Pi/2]*Erf[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b
)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x)
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 5885

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(S
qrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]), x] + (-
Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]
*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Simp[(S
qrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ[{a, b
, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx = -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}+$$

$$= -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}-$$

Mathematica [A] time = 1.22105, size = 291, normalized size = 0.97

$$-\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(4\sqrt{b}\left(\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+3b^2\right)+b\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

```
[Out] -(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(5/2)
*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + S
```

```
inh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(5/2)*Erf[Sqrt[a + b*
ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4*
Sqrt[b]*((3*b^2 + (a + b*ArcCosh[1 + d*x^2])^2)*Cosh[ArcCosh[1 + d*x^2]/2]
+ b*(a + b*ArcCosh[1 + d*x^2])*Sinh[ArcCosh[1 + d*x^2]/2])))/(30*b^(7/2)*Sq
rt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(5/2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{arccosh}\left(dx^2+1\right)\right)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.261 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2} dx$

Optimal. Leaf size=281

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2\right.\right.}{}$$

```
[Out] (5*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(5/2) + (30*b^2*Sqrt[a + b*Ar
cCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (15*b^(5/2)*Sqrt[
Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (15*b^(5/2)*Sqrt[Pi
/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2
]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rubi [A] time = 0.0593825, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5879}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```


```
[Out] (5*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(5/2) + (30*b^2*Sqrt[a + b*Ar
cCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (15*b^(5/2)*Sqrt[
Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (15*b^(5/2)*Sqrt[Pi
/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2
]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```


Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
```

```
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5879

```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sq
rt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2)/(d*x), x]
+ (-Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*Arc
Cosh[-1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(d*x)
, x] - Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*
ArcCosh[-1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(
d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx=\frac{5b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}+\left(15b\right.$$

$$=\frac{5b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}+\frac{30b}{}$$

Mathematica [A] time = 1.56875, size = 277, normalized size = 0.99

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(4\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}\left(\left(a^2+15b^2\right)\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)+b\cosh^{-1}\left(dx^2-1\right)\left(2a\,\mathrm{co}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCos
h[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 15*b^(5
/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*((a^2 + 15*b
^2)*Cosh[ArcCosh[-1 + d*x^2]/2] + b^2*ArcCosh[-1 + d*x^2]^2*Cosh[ArcCosh[-1
 + d*x^2]/2] - 5*a*b*Sinh[ArcCosh[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*(2
*a*Cosh[ArcCosh[-1 + d*x^2]/2] - 5*b*Sinh[ArcCosh[-1 + d*x^2]/2]))))/(2*d*x
```

)

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{arccosh}\left(dx^2 - 1\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arcosh}\left(dx^2 - 1\right) + a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.262 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=239

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-\right.\right.}{}$$

```
[Out] (3*b*(2*x^2 - d*x^4)*Sqrt[a + b*ArcCosh[-1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (3*b^(3/2)*Sqrt[Pi/2]*Cosh
[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rubi [A] time = 0.0542718, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5884}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```


```
[Out] (3*b*(2*x^2 - d*x^4)*Sqrt[a + b*ArcCosh[-1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (3*b^(3/2)*Sqrt[Pi/2]*Cosh
[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```


Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
```

```
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5884

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sq
rt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] - Simp[(Sqrt[P
i/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}dx=\frac{3b\left(2x^2-dx^4\right)\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}+\left(3b^2\right)$$

$$=\frac{3b\left(2x^2-dx^4\right)\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}+\frac{3b^{3/2}}{}$$

Mathematica [A] time = 0.533798, size = 221, normalized size = 0.92

$$\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(-3\sqrt{2\pi}b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+3\sqrt{2\pi}b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[
-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 3*b^(3/2)
*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*(a*Cosh[ArcCosh
[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*Cosh[ArcCosh[-1 + d*x^2]/2] - 3*b*S
inh[ArcCosh[-1 + d*x^2]/2])))/(2*d*x)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(d x^{2}-1\right)\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arcosh}\left(dx^{2}-1\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b \operatorname{acosh}\left(dx^{2}-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.263 $\int \sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx$

Optimal. Leaf size=206

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)}{}$$

```
[Out] (2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (
Sqrt[b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 +
 d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (Sqrt[
b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rubi [A] time = 0.0263156, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5879}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```


```
[Out] (2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (
Sqrt[b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 +
 d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (Sqrt[
b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```


Rule 5879

```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sq
rt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2)/(d*x), x]
+ (-Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*Arc
Cosh[-1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(d*x)
, x] - Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*
ArcCosh[-1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(
```

```
d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx = \frac{2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\cosh^2\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)}{dx} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(-\right.\right.}{}$$

Mathematica [A] time = 0.250142, size = 178, normalized size = 0.86

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(-\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\right)\mathrm{E}\right.}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCos
h[-1 + d*x^2]/2] + Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/
(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) - Sqrt[b]*Sqrt[2*Pi]*Er
f[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[
a/(2*b)])))/(2*d*x)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \sqrt{a+b\operatorname{arccosh}\left(dx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arcosh}\left(dx^2 - 1\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{acosh}\left(dx^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(d*x**2 - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.264 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\right.}{\sqrt{}}$$

```
[Out] (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]
]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sqrt
[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x)
```

Rubi [A] time = 0.025722, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5884}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]
]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sqrt
[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x)
```

Rule 5884

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sq
rt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] - Simp[(Sqrt[P
i/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}\,dx=\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{b}dx}-$$

Mathematica [A] time = 0.254311, size = 134, normalized size = 0.81

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)+\left(\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cos}}{\sqrt{}}\right.\right.}{\sqrt{b}dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x
^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Erf[Sqrt[a + b*A
rcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])))/(S
qrt[b]*d*x))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\text{arccosh}\left(dx^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\text{arcosh}\left(dx^2-1\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\operatorname{acosh}\left(dx^2 - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(d*x**2 - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.265 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right.\right.}{b^{3/2}}$$

```
[Out] -((Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])) +
 (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(b^(3/2)*d*x) + (Sqr
t[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sq
rt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b^(3/2)*d*x)
```

Rubi [A] time = 0.0273975, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5886}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2\right.\right.}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```


```
[Out] -((Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])) +
 (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(b^(3/2)*d*x) + (Sqr
t[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sq
rt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b^(3/2)*d*x)
```


Rule 5886

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(
Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]), x] +
 (Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]
/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Sim
```

```
p[(Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*E
rfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ
[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}} + \frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}}}{\sqrt{2}}\right.}{b^{3/2}dx}$$

Mathematica [A] time = 0.973442, size = 209, normalized size = 0.99

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\left(\cosh\right.\right.}{2b^{3/2}dx\sqrt{a+b\cosh^{-1}\left(dx\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Er
fi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh
[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Erf[Sqrt[a + b*ArcC
osh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) - 4*Sqr
t[b]*Sinh[ArcCosh[-1 + d*x^2]/2]))/(2*b^(3/2)*d*x*Sqrt[a + b*ArcCosh[-1 + d
*x^2]])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{arccosh}\left(dx^2-1\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2 - 1\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^2 - 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-3/2), x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.266 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\right.}{3b}$$

```
[Out] (2*x^2 - d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]*Cosh[
ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(3*b^(5/2)*d*x) - (Sqrt[Pi/2]*Cosh[Arc
Cosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*
(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3*b^(5/2)*d*x)
```

Rubi [A] time = 0.055022, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5884}

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\right.}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```



```
[Out] (2*x^2 - d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]*Cosh[
ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(3*b^(5/2)*d*x) - (Sqrt[Pi/2]*Cosh[Arc
Cosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*
(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3*b^(5/2)*d*x)
```


Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
```

```
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 5884

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sq
rt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] - Simp[(Sqrt[P
i/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx = \frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}$$

$$= \frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}$$

Mathematica [A] time = 0.847156, size = 238, normalized size = 0.94

$$-\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{3/2}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\left(\mathrm{si}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

```
[Out] -(Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(3/2)
*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] +
Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(3/2)*Erf[Sqrt[a +
b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) +
 4*Sqrt[b]*((a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2] + b*Sin
```

```
h[ArcCosh[-1 + d*x^2]/2])))/(6*b^(5/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(3/2
))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{arccosh}\left(d x^{2}-1\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\mathrm{arcosh}\left(dx^{2}-1\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b \operatorname{acosh}\left(d x^2-1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.267 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=302

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\right.}{15b^{7}}$$

```
[Out] (2*x^2 - d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[-1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*S
qrt[-2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]
*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*
Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(15*b^(7/2)*d*x) + (Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqr
t[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b^(7/2)*d*x)
```

Rubi [A] time = 0.0601722, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5886}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\right.}{15b^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```


```
[Out] (2*x^2 - d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[-1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*S
qrt[-2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]
*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*
Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(15*b^(7/2)*d*x) + (Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqr
t[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b^(7/2)*d*x)
```


Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 5886

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(
Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]), x] +
 (Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]
/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Sim
p[(Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*E
rfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ
[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx=\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right.}$$

$$=\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right.}$$

Mathematica [A] time = 1.01791, size = 260, normalized size = 0.86

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(4\sqrt{b}\left(-\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+3b^2\right)-b\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*
Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Si
```

```
nh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*Erf[Sqrt[a + b*
ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4
*Sqrt[b]*(-(b*(a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2]) - (3
*b^2 + (a + b*ArcCosh[-1 + d*x^2])^2)*Sinh[ArcCosh[-1 + d*x^2]/2])))/(30*b^
(7/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(5/2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{arccosh}\left(d x^{2}-1\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\mathrm{arcosh}\left(d x^{2}-1\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.268 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),
x]
```

Rubi [A] time = 0.0513401, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.132985, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A] time = 0.588, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\operatorname{arccosh}\left(\sqrt{-cx+1}\,\frac{1}{\sqrt{cx+1}}\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.269 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=265

$$\frac{3b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}$$

```
[Out] -(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c) - ((a + b*ArcCosh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (3*b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2
, -E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*ArcC
osh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^(-2*ArcCosh[Sqrt[1 - c*x]/S
qrt[1 + c*x]])])/(2*c) + (3*b^3*PolyLog[4, -E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/(4*c)
```

Rubi [A] time = 0.220762, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6681, 5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c) - ((a + b*ArcCosh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (3*b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2,
-E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])])/(2*c) - (3*b^3*PolyLog[4, -E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/(4*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^3\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(3b)\,\text{Su}}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+\right.}{}$$

Mathematica [F] time = 0.366583, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] time = 0.942, size = 870, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)+1/4*b^3/c*arccosh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^4-b^3/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)^(1/2))^2+1)-3/2*b^3/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*b^3/c*arccosh((-c*x
+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)
^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-3/
4*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1
/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+a*b^2/c*arccosh((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)
*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2+1)-3*a*b^2/c*arccosh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)
/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*a*b^
2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*a^2*b/c*arccosh((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2+1)-3/2*a^2*b/c*polylog(2,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2
```

)/(c*x+1)^(1/2)+1)^(1/2))^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) + sqrt(-c*x + 1))^3/c + integrate(1/8*(((c*x + 1)*sqrt(-c*x
 + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*log(c*x + 1)^3 - 6*((c*x + 1)*sqrt(-c*x +
 1)*a*b^2 - (-c*x + 1)^(3/2)*a*b^2)*log(c*x + 1)^2 - 6*((4*a*b^2 + (b^3*c*x
 + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x + 1))*(c*x + 1)*sqrt(-c*x +
 1) - (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x +
1))*(-c*x + 1)^(3/2) + ((4*a*b^2 + (b^3*c*x - b^3)*log(c*x + 1) - (b^3*c*x
- b^3)*log(-c*x + 1))*(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) -
 (b^3*c*x + b^3)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^3 + (c*x - 1)*b^
3)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
 sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*
log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1)
+ sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2 + (((c*x + 1)*b^3 + (c*x - 1)*b^3)*lo
g(c*x + 1)^3 - 6*((c*x + 1)*a*b^2 + (c*x - 1)*a*b^2)*log(c*x + 1)^2 + 12*((
c*x + 1)*a^2*b + (c*x - 1)*a^2*b)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-
c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + 12*((c*x + 1)*sqrt(-c*x +
 1)*a^2*b - (-c*x + 1)^(3/2)*a^2*b)*log(c*x + 1) - 6*(4*(c*x + 1)*sqrt(-c*x
 + 1)*a^2*b - 4*(-c*x + 1)^(3/2)*a^2*b + ((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-
c*x + 1)^(3/2)*b^3)*log(c*x + 1)^2 + (4*(c*x + 1)*a^2*b + 4*(c*x - 1)*a^2*b
 + ((c*x + 1)*b^3 + (c*x - 1)*b^3)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b^2 + (c
*x - 1)*a*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqr
t(c*x + 1) + sqrt(-c*x + 1)) - 4*((c*x + 1)*sqrt(-c*x + 1)*a*b^2 - (-c*x +
1)^(3/2)*a*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt
(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*(c*x +
1)*sqrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x +
1) + (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab^2\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^3}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccosh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.270 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=196

$$\frac{b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

```
[Out] -(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c) - ((a + b*ArcCosh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -E
^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, -E^(-2*Arc
Cosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rubi [A] time = 0.195145, antiderivative size = 197, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 5660, 3718, 2190, 2531, 2282, 6589}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c) - ((a + b*ArcCosh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -E^(
2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, -E^(2*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(2b)\,\text{Su}}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\left(a+\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\left(a+\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\left(a+\right.}{}$$

Mathematica [F] time = 0.97689, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [A] time = 0.007, size = 491, normalized size = 2.5

$$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}+\frac{b^2}{3c}\left(\operatorname{arccosh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^3-\frac{b^2}{c}\left(\operatorname{arccosh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2\ln\left(\left(\sqrt{-cx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+1/3*b^2/c*arccosh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3-b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)^(1/2))^2+1)-b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*po
lylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/
2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+1/2*b^2/c*polylog(3,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)^(1/2))^2)+a*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-
2*a*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
^(1/2))^2+1)-a*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)
/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2/c + integrate(-1/4*(((c*x + 1)*sqrt(-c*
x + 1)*b^2 - (-c*x + 1)^(3/2)*b^2)*log(c*x + 1)^2 + (((c*x + 1)*b^2 + (c*x
- 1)*b^2)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b + (c*x - 1)*a*b)*log(c*x + 1))*
sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1))
- 4*((c*x + 1)*sqrt(-c*x + 1)*a*b - (-c*x + 1)^(3/2)*a*b)*log(c*x + 1) + 2*
((4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*
x + 1)*sqrt(-c*x + 1) - (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x +
```

```
b^2)*log(-c*x + 1))*(-c*x + 1)^(3/2) + ((4*a*b + (b^2*c*x - b^2)*log(c*x +
1) - (b^2*c*x - b^2)*log(-c*x + 1))*(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*lo
g(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^2 +
(c*x - 1)*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqr
t(c*x + 1) + sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^2 - (-c*x + 1)
^(3/2)*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*(c*x + 1)*s
qrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) +
 (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c
*x + 1) + sqrt(-c*x + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccosh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.271 $\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=133

$$\frac{b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

```
[Out] -(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c) - ((a + b*ArcCosh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c + (b*PolyLog[2, -E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2
*c)
```

Rubi [A] time = 0.119465, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {206, 6681, 5660, 3718, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c) - ((a + b*ArcCosh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x
]])])/c - (b*PolyLog[2, -E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
```

```
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\text{Li}_2\left(-e^{2\,\text{co}}\right.}{2}$$

Mathematica [F] time = 2.97766, size = 0, normalized size = 0.

$$\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

Maple [A] time = 0.008, size = 207, normalized size = 1.6

$$-\frac{a\ln(cx-1)}{2c} + \frac{a\ln(cx+1)}{2c} + \frac{b}{2c}\left(\text{arccosh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2 - \frac{b}{c}\text{arccosh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\ln\left(\left(\sqrt{-cx+1}\frac{}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+1/2*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2-b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+1)^(1/2))^2+1)-1/2*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+
1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}b\left(\frac{2(\log(cx+1)-\log(-cx+1))\log(cx+1)-\log(cx+1)^2+2\log(cx+1)\log(-cx+1)-\log(-cx+1)^2-4(\log(c}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-
c*x + 1)) + sqrt(-c*x + 1)))/c + 8*integrate(1/2*(c*x + 1)*sqrt(-c*x + 1)*(
log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1)*sqrt(-c*x + 1) - (c^
2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) + (c^2*x^2 - 1)*(c*x
 - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x
 + 1))), x) + 8*integrate(-1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))
/((c^2*x^2 - 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 8*integ
rate(1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c
*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(
c*x - 1)/c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b\,\mathrm{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.272 $\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
, x]
```

Rubi [A] time = 0.0463448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.141791, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A] time = 0.324, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\operatorname{arccosh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{1}{ac^2x^2+\left(bc^2x^2-b\right)\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.273 $\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^
2), x]
```

Rubi [A] time = 0.0436054, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 5.09053, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A] time = 0.319, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\mathrm{arccosh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 2*(2*c*x*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c
*x + 1)) + (c*x + 1)*sqrt(-c*x + 1) - (-c*x + 1)^(3/2))/(2*(c*x + 1)*sqrt(-
c*x + 1)*a*b*c - 2*(-c*x + 1)^(3/2)*a*b*c - ((c*x - 1)*b^2*c*log(c*x + 1) -
 2*(c*x - 1)*a*b*c)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1
) + sqrt(-c*x + 1)) - ((c*x + 1)*sqrt(-c*x + 1)*b^2*c - (-c*x + 1)^(3/2)*b^
2*c)*log(c*x + 1) + 2*((c*x - 1)*b^2*c*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))
*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + (c*x + 1)*sqrt(-c*x + 1)*b^2*c - (
-c*x + 1)^(3/2)*b^2*c)*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(
c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))) - integrate(-2*(2*(c*x + 1)*s
qrt(-c*x + 1)*(sqrt(c*x + 1) + sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x +
 1)) + ((c*x + 1)^2 + 2*(c*x + 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x
```

```
 + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/(2*(a*b*c^2*x^2 - a*b)*(c*x +
 1)^2*sqrt(-c*x + 1) - 4*(a*b*c^2*x^2 - a*b)*(c*x + 1)*(-c*x + 1)^(3/2) + 2
*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(5/2) + ((b^2*c^2*x^2 - b^2)*(-c*x + 1)^(3/
2)*log(c*x + 1) - 2*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(3/2))*(sqrt(c*x + 1) +
sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) + 2*(2*(a*b*c^2*x^2 - a*b)
*(c*x + 1)*(c*x - 1) + 2*(a*b*c^2*x^2 - a*b)*(c*x - 1)^2 - ((b^2*c^2*x^2 -
b^2)*(c*x + 1)*(c*x - 1) + (b^2*c^2*x^2 - b^2)*(c*x - 1)^2)*log(c*x + 1))*s
qrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) -
 ((b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(-c*x + 1) - 2*(b^2*c^2*x^2 - b^2)*(c
*x + 1)*(-c*x + 1)^(3/2) + (b^2*c^2*x^2 - b^2)*(-c*x + 1)^(5/2))*log(c*x +
1) - 2*((b^2*c^2*x^2 - b^2)*(-c*x + 1)^(3/2)*(sqrt(c*x + 1) + sqrt(-c*x + 1
))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) - (b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(
-c*x + 1) + 2*(b^2*c^2*x^2 - b^2)*(c*x + 1)*(-c*x + 1)^(3/2) - (b^2*c^2*x^2
 - b^2)*(-c*x + 1)^(5/2) - 2*((b^2*c^2*x^2 - b^2)*(c*x + 1)*(c*x - 1) + (b^
2*c^2*x^2 - b^2)*(c*x - 1)^2)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1)))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqr
t(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{a^2c^2x^2 + \left(b^2c^2x^2 - b^2\right)\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 - a^2 + 2\left(abc^2x^2 - ab\right)\operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] Timed out
```

3.274 $\int \cosh^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=76

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\cosh^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

```
[Out] -ArcCosh[c*E^(a + b*x)]^2/(2*b) + (ArcCosh[c*E^(a + b*x)]*Log[1 + E^(2*ArcC
osh[c*E^(a + b*x)])])/b + PolyLog[2, -E^(2*ArcCosh[c*E^(a + b*x)])]/(2*b)
```

Rubi [A] time = 0.073347, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2282, 5660, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\cosh^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*E^(a + b*x)],x]
```

```
[Out] -ArcCosh[c*E^(a + b*x)]^2/(2*b) + (ArcCosh[c*E^(a + b*x)]*Log[1 + E^(2*ArcC
osh[c*E^(a + b*x)])])/b + PolyLog[2, -E^(2*ArcCosh[c*E^(a + b*x)])]/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \cosh^{-1}\left(ce^{a+bx}\right)dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(ce^{a+bx}\right)\right)}{b}$$

$$= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(ce^{a+bx}\right)\right)}{b}$$

$$= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)dx, x, \cos\right.}{b}$$

$$= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}\left(\right.}\right.}{2b}$$

$$= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

Mathematica [F] time = 0.673708, size = 0, normalized size = 0.

$$\int \cosh^{-1}\left(ce^{a+bx}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[c*E^(a + b*x)],x]
```

```
[Out] Integrate[ArcCosh[c*E^(a + b*x)], x]
```

Maple [A] time = 0.02, size = 115, normalized size = 1.5

$$-\frac{\left(\operatorname{arccosh}\left(ce^{bx+a}\right)\right)^2}{2b}+\frac{\operatorname{arccosh}\left(ce^{bx+a}\right)}{b}\ln\left(1+\left(ce^{bx+a}+\sqrt{ce^{bx+a}-1}\sqrt{ce^{bx+a}+1}\right)^2\right)+\frac{1}{2b}\operatorname{polylog}\left(2,-\left(ce^{bx+a}+\sqrt{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*exp(b*x+a)),x)
```

```
[Out] -1/2*arccosh(c*exp(b*x+a))^2/b+arccosh(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+(c*
exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b+1/2*polylog(2,-(c*exp(b*x+
a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$bc\int\frac{xe^{(bx+a)}}{c^3e^{(3bx+3a)}-ce^{(bx+a)}+\left(c^2e^{(2bx+2a)}-1\right)e^{\left(\frac{1}{2}\log\left(ce^{(bx+a)}+1\right)+\frac{1}{2}\log\left(ce^{(bx+a)}-1\right)\right)}}\,dx+x\log\left(ce^{(bx+a)}+\sqrt{ce^{(bx+a)}+1}\sqrt{ce^{(bx+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) - c*e^(b*x + a) + (c^2*e^(
2*b*x + 2*a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(c*e^(b*x + a) - 1
))), x) + x*log(c*e^(b*x + a) + sqrt(c*e^(b*x + a) + 1)*sqrt(c*e^(b*x + a)
- 1)) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a)))/b - 1/2*(b
```

```
*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acosh}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*exp(b*x+a)),x)
```

```
[Out] Integral(acosh(c*exp(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcosh}\left(ce^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccosh(c*e^(b*x + a)), x)
```

3.275 $\int e^{\cosh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=165

$$-\frac{\left(4a^2+3\right)ae^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{\left(4a^2+3\right)a\cosh^{-1}(a+bx)}{8b^4}+\frac{\left(6a^2+1\right)e^{-\cosh^{-1}(a+bx)}}{8b^4}+\frac{\left(6a^2+1\right)e^{3\cosh^{-1}(a+bx)}}{24b^4}-\frac{3ae^{-2\cos}}{16}$$

```
[Out] 1/(48*b^4*E^(3*ArcCosh[a + b*x])) - (3*a)/(16*b^4*E^(2*ArcCosh[a + b*x])) +
 (1 + 6*a^2)/(8*b^4*E^ArcCosh[a + b*x]) - (a*(3 + 4*a^2)*E^(2*ArcCosh[a + b
*x]))/(16*b^4) + ((1 + 6*a^2)*E^(3*ArcCosh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcCosh[a + b*x]))/(32*b^4) + E^(5*ArcCosh[a + b*x])/(80*b^4) + (a*(3 + 4*a
^2)*ArcCosh[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.16092, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5899, 2282, 12, 1628}

$$-\frac{\left(4a^2+3\right)ae^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{\left(4a^2+3\right)a\cosh^{-1}(a+bx)}{8b^4}+\frac{\left(6a^2+1\right)e^{-\cosh^{-1}(a+bx)}}{8b^4}+\frac{\left(6a^2+1\right)e^{3\cosh^{-1}(a+bx)}}{24b^4}-\frac{3ae^{-2\cos}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x^3,x]
```



```
[Out] 1/(48*b^4*E^(3*ArcCosh[a + b*x])) - (3*a)/(16*b^4*E^(2*ArcCosh[a + b*x])) +
 (1 + 6*a^2)/(8*b^4*E^ArcCosh[a + b*x]) - (a*(3 + 4*a^2)*E^(2*ArcCosh[a + b
*x]))/(16*b^4) + ((1 + 6*a^2)*E^(3*ArcCosh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcCosh[a + b*x]))/(32*b^4) + E^(5*ArcCosh[a + b*x])/(80*b^4) + (a*(3 + 4*a
^2)*ArcCosh[a + b*x])/(8*b^4)
```


Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int e^{\cosh^{-1}(a+bx)}x^3\,dx &= \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^3\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1-2ax+x^2)^3}{16b^3x^4}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1-2ax+x^2)^3}{x^4}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{16b^4}\\
&= \frac{\text{Subst}\left(\int\left(-\frac{1}{x^4}+\frac{6a}{x^3}-\frac{2(1+6a^2)}{x^2}+\frac{2a(3+4a^2)}{x}-2a\left(3+4a^2\right)x+2\left(1+6a^2\right)x^2-6ax^3+x^4\right)dx,x,e^{\cos}\right.}{16b^4}\\
&= \frac{e^{-3\cosh^{-1}(a+bx)}}{48b^4}-\frac{3ae^{-2\cosh^{-1}(a+bx)}}{16b^4}+\frac{\left(1+6a^2\right)e^{-\cosh^{-1}(a+bx)}}{8b^4}-\frac{a\left(3+4a^2\right)e^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{(1+6a}{}
\end{aligned}$$

Mathematica [A] time = 0.0885515, size = 138, normalized size = 0.84

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-2\left(3a^2+4\right)b^2x^2+\left(6a^2+29\right)abx-6a^4-83a^2+6ab^3x^3+24b^4x^4-16\right)+15a\left(4a^2+3\right)\log\left(\right.}{120b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-16 - 83
*a^2 - 6*a^4 + a*(29 + 6*a^2)*b*x - 2*(4 + 3*a^2)*b^2*x^2 + 6*a*b^3*x^3 + 2
4*b^4*x^4) + 15*a*(3 + 4*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a +
 b*x]])/(120*b^4)
```

Maple [C] time = 0.047, size = 376, normalized size = 2.3

$$\frac{\operatorname{csgn}(b)}{120\,b^4}\sqrt{bx+a-1}\sqrt{bx+a+1}\left(24\operatorname{csgn}(b)\,x^4b^4\sqrt{b^2x^2+2\,xab+a^2-1}+6\operatorname{csgn}(b)\,x^3ab^3\sqrt{b^2x^2+2\,xab+a^2-1}-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x)
```

```
[Out] 1/120*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(24*csgn(b)*x^4*b^4*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)+6*csgn(b)*x^3*a*b^3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*csgn(b)*x^
2*a^2*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csg
n(b)*x*a^3*b-8*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x^2*b^2-6*(b^2*x^2+2*a
*b*x+a^2-1)^(1/2)*csgn(b)*a^4+29*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x*a*
b-83*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2+60*ln(((b^2*x^2+2*a*b*x+a^2-
1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a^3-16*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(
b)+45*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn(b)/
b^4/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/5*b*x^5+1/4*x^4*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89125, size = 327, normalized size = 1.98

$$\frac{24\,b^5x^5+30\,ab^4x^4+\left(24\,b^4x^4+6\,ab^3x^3-2\left(3\,a^2+4\right)b^2x^2-6\,a^4+\left(6\,a^3+29\,a\right)bx-83\,a^2-16\right)\sqrt{bx+a+1}\sqrt{bx+a-}}{120\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="fricas"
)
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 + 4
)*b^2*x^2 - 6*a^4 + (6*a^3 + 29*a)*b*x - 83*a^2 - 16)*sqrt(b*x + a + 1)*sqr
t(b*x + a - 1) - 15*(4*a^3 + 3*a)*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a
 - 1) - a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a+bx+\sqrt{a+bx-1}\sqrt{a+bx+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

Giac [A] time = 1.27554, size = 348, normalized size = 2.11

$$\frac{\left(\left(2\,(bx+a+1)\left(3\,(bx+a+1)\left(\frac{4\,(bx+a+1)}{b^3}-\frac{15\,ab^{12}+16\,b^{12}}{b^{15}}\right)+\frac{60\,a^2b^{12}+135\,ab^{12}+68\,b^{12}}{b^{15}}\right)-\frac{5\left(12\,a^3b^{12}+48\,a^2b^{12}+45\,ab^{12}+16\,b^{12}\right)}{b^{15}}\right)(bx+a\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/120*(((2*(b*x + a + 1)*(3*(b*x + a + 1)*(4*(b*x + a + 1)/b^3 - (15*a*b^12
 + 16*b^12)/b^15) + (60*a^2*b^12 + 135*a*b^12 + 68*b^12)/b^15) - 5*(12*a^3*
b^12 + 48*a^2*b^12 + 45*a*b^12 + 16*b^12)/b^15)*(b*x + a + 1) + 15*(4*a^3*b
^12 + 3*a*b^12)/b^15)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 30*(b*x^4 - (a^
4 + 4*a^3 + 6*a^2 + 4*a + 1)/b^3)*a + 24*(b*x^5 + (a^5 + 5*a^4 + 10*a^3 + 1
0*a^2 + 5*a + 1)/b^4)*b - 30*(4*a^3 + 3*a)*log(abs(-sqrt(b*x + a + 1) + sqr
t(b*x + a - 1)))/b^3)/b
```

3.276 $\int e^{\cosh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=115

$$\frac{\left(4a^2+1\right)e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{\left(4a^2+1\right)\cosh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}$$

```
[Out] 1/(16*b^3*E^(2*ArcCosh[a + b*x])) - a/(2*b^3*E^ArcCosh[a + b*x]) + ((1 + 4*
a^2)*E^(2*ArcCosh[a + b*x]))/(16*b^3) - (a*E^(3*ArcCosh[a + b*x]))/(6*b^3)
+ E^(4*ArcCosh[a + b*x])/(32*b^3) - ((1 + 4*a^2)*ArcCosh[a + b*x])/(8*b^3)
```

Rubi [A] time = 0.12071, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5899, 2282, 12, 1628}

$$\frac{\left(4a^2+1\right)e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{\left(4a^2+1\right)\cosh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x^2,x]
```

```
[Out] 1/(16*b^3*E^(2*ArcCosh[a + b*x])) - a/(2*b^3*E^ArcCosh[a + b*x]) + ((1 + 4*
a^2)*E^(2*ArcCosh[a + b*x]))/(16*b^3) - (a*E^(3*ArcCosh[a + b*x]))/(6*b^3)
+ E^(4*ArcCosh[a + b*x])/(32*b^3) - ((1 + 4*a^2)*ArcCosh[a + b*x])/(8*b^3)
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}x^2\,dx = \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$

$$=\frac{\text{Subst}\left(\int\frac{(-1+x^2)(1-2ax+x^2)^2}{8b^2x^3}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}$$

$$=\frac{\text{Subst}\left(\int\frac{(-1+x^2)(1-2ax+x^2)^2}{x^3}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{8b^3}$$

$$=\frac{\text{Subst}\left(\int\left(-\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1-4a^2}{x}+(1+4a^2)x-4ax^2+x^3\right)dx,x,e^{\cosh^{-1}(a+bx)}\right)}{8b^3}$$

$$=\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}+\frac{(1+4a^2)e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}-\frac{(1+$$

Mathematica [A] time = 0.123629, size = 119, normalized size = 1.03

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-2a^2bx+2a^3+a\left(2b^2x^2+13\right)+6b^3x^3-3bx\right)-3\left(4a^2+1\right)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+\right.}{24b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(2*a^3 - 3*
b*x - 2*a^2*b*x + 6*b^3*x^3 + a*(13 + 2*b^2*x^2)) - 3*(1 + 4*a^2)*Log[a + b
*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(24*b^3)
```

Maple [C] time = 0.01, size = 288, normalized size = 2.5

$$\frac{\operatorname{csgn}(b)}{24\,b^3}\sqrt{bx+a-1}\sqrt{bx+a+1}\left(6\operatorname{csgn}(b)\,x^3b^3\sqrt{b^2x^2+2\,xab+a^2-1}+2\operatorname{csgn}(b)\,x^2ab^2\sqrt{b^2x^2+2\,xab+a^2-1}-2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x)
```

```
[Out] 1/24*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(6*csgn(b)*x^3*b^3*(b^2*x^2+2*a*b*x+a^
2-1)^(1/2)+2*csgn(b)*x^2*a*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-2*(b^2*x^2+2*a
*b*x+a^2-1)^(1/2)*csgn(b)*x*a^2*b+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a
^3-3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x*b+13*(b^2*x^2+2*a*b*x+a^2-1)^(
1/2)*csgn(b)*a-12*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))
*a^2-3*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b)))*csgn(b)/b
^3/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/4*b*x^4+1/3*x^3*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9775, size = 271, normalized size = 2.36

$$\frac{6\,b^4x^4+8\,ab^3x^3+\left(6\,b^3x^3+2\,ab^2x^2+2\,a^3-\left(2\,a^2+3\right)bx+13\,a\right)\sqrt{bx+a+1}\sqrt{bx+a-1}+3\left(4\,a^2+1\right)\log\left(-bx+\sqrt{b\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="fricas"
)
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 +
 3)*b*x + 13*a)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 3*(4*a^2 + 1)*log(-b*
```

```
x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+bx+\sqrt{a+bx-1}\sqrt{a+bx+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

Giac [A] time = 1.2641, size = 271, normalized size = 2.36

$$\frac{\left((bx+a+1)\left(2\,(bx+a+1)\left(\frac{3\,(bx+a+1)}{b^2}-\frac{8\,ab^6+9\,b^6}{b^8}\right)+\frac{12\,a^2b^6+32\,ab^6+15\,b^6}{b^8}\right)-\frac{3\left(4\,a^2b^6+b^6\right)}{b^8}\right)\sqrt{bx+a+1}\sqrt{bx+a-1}+8\left(bx^3+\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/24*(((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*x + a + 1)/b^2 - (8*a*b^6 + 9*b
^6)/b^8) + (12*a^2*b^6 + 32*a*b^6 + 15*b^6)/b^8) - 3*(4*a^2*b^6 + b^6)/b^8)
*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 8*(b*x^3 + (a^3 + 3*a^2 + 3*a + 1)/b
^2)*a + 6*(b*x^4 - (a^4 + 4*a^3 + 6*a^2 + 4*a + 1)/b^3)*b + 6*(4*a^2 + 1)*l
og(abs(-sqrt(b*x + a + 1) + sqrt(b*x + a - 1)))/b^2)/b
```

3.277 $\int e^{\cosh^{-1}(a+bx)} x \, dx$

Optimal. Leaf size=67

$$-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}+\frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}$$

```
[Out] 1/(4*b^2*E^ArcCosh[a + b*x]) - (a*E^(2*ArcCosh[a + b*x]))/(4*b^2) + E^(3*Ar
cCosh[a + b*x])/(12*b^2) + (a*ArcCosh[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.0702901, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5899, 2282, 12, 1628}

$$-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}+\frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcCosh[a + b*x]) - (a*E^(2*ArcCosh[a + b*x]))/(4*b^2) + E^(3*Ar
cCosh[a + b*x])/(12*b^2) + (a*ArcCosh[a + b*x])/(2*b^2)
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int e^{\cosh^{-1}(a+bx)}x\,dx &= \frac{\operatorname{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(-1+2ax-x^2)}{4bx^2}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(-1+2ax-x^2)}{x^2}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{4b^2}\\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{2a}{x}-2ax+x^2\right)dx,x,e^{\cosh^{-1}(a+bx)}\right)}{4b^2}\\
&= \frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.152642, size = 93, normalized size = 1.39

$$\frac{1}{6}\left(\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-a^2+abx+2b^2x^2-2\right)}{b^2}+\frac{3a\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)}{b^2}+3ax^2+2bx^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-2 - a^2 + a*b*
x + 2*b^2*x^2))/b^2 + (3*a*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*
x]])/b^2)/6
```

Maple [C] time = 0.012, size = 194, normalized size = 2.9

$$\frac{\operatorname{csgn}(b)}{6\,b^2}\sqrt{bx+a-1}\sqrt{bx+a+1}\left(2\sqrt{b^2x^2+2\,xab+a^2-1}\operatorname{csgn}(b)\,x^2b^2+\sqrt{b^2x^2+2\,xab+a^2-1}\operatorname{csgn}(b)\,xab-\sqrt{b^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x)
```

```
[Out] 1/6*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b
)*x^2*b^2+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x*a*b-(b^2*x^2+2*a*b*x+a^2-
1)^(1/2)*csgn(b)*a^2-2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+3*ln(((b^2*x^2
+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn(b)/b^2/(b^2*x^2+2*a*b
*x+a^2-1)^(1/2)+1/3*b*x^3+1/2*a*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99024, size = 217, normalized size = 3.24

$$\frac{2\,b^3x^3+3\,ab^2x^2+\left(2\,b^2x^2+abx-a^2-2\right)\sqrt{bx+a+1}\sqrt{bx+a-1}-3\,a\log\left(-bx+\sqrt{bx+a+1}\sqrt{bx+a-1}-a\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + (2*b^2*x^2 + a*b*x - a^2 - 2)*sqrt(b*x + a +
 1)*sqrt(b*x + a - 1) - 3*a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1)
- a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+bx+\sqrt{a+bx-1}\sqrt{a+bx+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x,x)
```

```
[Out] Integral(x*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

Giac [A] time = 1.27061, size = 176, normalized size = 2.63

$$\frac{2\left(bx^3+\frac{a^3+3a^2+3a+1}{b^2}\right)b+\frac{3\left((bx+a+1)^2-2(bx+a+1)a-2bx-2a-2\right)a}{b}+\frac{((2bx-a-2)(bx+a+1)+3a)\sqrt{bx+a+1}\sqrt{bx+a-1}-6a\log\left(\left|-\sqrt{bx+a+1}+\sqrt{bx+a-1}\right|\right)}{b}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b*x^3 + (a^3 + 3*a^2 + 3*a + 1)/b^2)*b + 3*((b*x + a + 1)^2 - 2*(b*
x + a + 1)*a - 2*b*x - 2*a - 2)*a/b + (((2*b*x - a - 2)*(b*x + a + 1) + 3*a
)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 6*a*log(abs(-sqrt(b*x + a + 1) + sq
rt(b*x + a - 1))))/b)/b
```

3.278 $\int e^{\cosh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}$$

```
[Out] E^(2*ArcCosh[a + b*x])/(4*b) - ArcCosh[a + b*x]/(2*b)
```

Rubi [A] time = 0.0173356, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5897, 2282, 12, 14}

$$\frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x],x]
```

```
[Out] E^(2*ArcCosh[a + b*x])/(4*b) - ArcCosh[a + b*x]/(2*b)
```

Rule 5897

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Sinh[x], x], x, ArcCosh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int e^{\cosh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int e^x\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)dx, x, e^{\cosh^{-1}(a+bx)}\right)}{2b}\\ &= \frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.0274963, size = 69, normalized size = 2.23

$$\frac{(a+bx)\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)-\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x],x]
```

```
[Out] ((a + b*x)*(a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) - Log[a + b*x +
 Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(2*b)
```

Maple [B] time = 0.007, size = 147, normalized size = 4.7

$$\frac{bx^2}{2}+ax+\frac{1}{2\,b}\sqrt{bx+a-1}\,(bx+a+1)^{\frac{3}{2}}-\frac{1}{2\,b}\sqrt{bx+a-1}\sqrt{bx+a+1}-\frac{1}{2}\sqrt{(bx+a+1)\,(bx+a-1)}\ln\left(\left(\frac{b\,(1+a)}{2}+\frac{(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x)
```

```
[Out] 1/2*b*x^2+a*x+1/2/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(3/2)-1/2*(b*x+a-1)^(1/2)*(b*
x+a+1)^(1/2)/b-1/2*((b*x+a+1)*(b*x+a-1))^(1/2)/(b*x+a+1)^(1/2)/(b*x+a-1)^(1
/2)*ln((1/2*b*(1+a)+1/2*(a-1)*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(b*(1+a)+(a-1)*
b)*x+(1+a)*(a-1))^(1/2))/(b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00254, size = 174, normalized size = 5.61

$$\frac{b^2x^2 + 2\,abx + \sqrt{bx+a+1}(bx+a)\sqrt{bx+a-1} + \log\left(-bx + \sqrt{bx+a+1}\sqrt{bx+a-1} - a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b*x + a + 1)*(b*x + a)*sqrt(b*x + a - 1) + lo
g(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx + \sqrt{a+bx-1}\sqrt{a+bx+1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2),x)
```

```
[Out] Integral(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1), x)
```

Giac [A] time = 1.32156, size = 82, normalized size = 2.65

$$\frac{1}{2}\,bx^2 + ax + \frac{\sqrt{bx+a+1}(bx+a)\sqrt{bx+a-1} + 2\,\log\left(\left|-\sqrt{bx+a+1}+\sqrt{bx+a-1}\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*(sqrt(b*x + a + 1)*(b*x + a)*sqrt(b*x + a - 1) + 2*lo
g(abs(-sqrt(b*x + a + 1) + sqrt(b*x + a - 1))))/b
```

3.279 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=100

$$2\sqrt{1-a^2}\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}+2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)+a\log(x)+bx$$

```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*a*ArcSinh[Sqrt[-1 + a + b*x]
/Sqrt[2]] + 2*Sqrt[1 - a^2]*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
+ a]*Sqrt[-1 + a + b*x])] + a*Log[x]
```

Rubi [A] time = 0.091578, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5909, 14, 101, 157, 63, 215, 93, 205}

$$2\sqrt{1-a^2}\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}+2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)+a\log(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x,x]
```


```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*a*ArcSinh[Sqrt[-1 + a + b*x]
/Sqrt[2]] + 2*Sqrt[1 - a^2]*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
+ a]*Sqrt[-1 + a + b*x])] + a*Log[x]
```


Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
```

```
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\cosh^{-1}(a+bx)}}{x}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}\,dx\\ &= \int \left(b+\frac{a}{x}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}\right)dx\\ &= bx+a\log(x)+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}\,dx\\ &= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)-\int \frac{1-a^2-abx}{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,dx\\ &= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)-\left(1-a^2\right)\int \frac{1}{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,dx+(ab)\int \frac{}{\sqrt{}}\\ &= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)+(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx,x,\sqrt{-1+a+bx}\right)-\left(2\left(1-\right.\right.\\ &= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+2a\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)+2\sqrt{1-a^2}\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+b}}{\sqrt{1+a}\sqrt{-1+a+}}\right.\end{aligned}$$

Mathematica [C] time = 0.0918227, size = 141, normalized size = 1.41

$$i\sqrt{1-a^2}\log\left(\frac{2\sqrt{a+bx-1}\sqrt{a+bx+1}}{\left(a^2-1\right)x}+\frac{2i\left(a^2+abx-1\right)}{\sqrt{1-a^2}\left(a^2-1\right)x}\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}+a\log\left(\sqrt{a+bx-1}\sqrt{a+bx+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + a*Log[x] + a*Log[a + b*x + Sqr
t[-1 + a + b*x]*Sqrt[1 + a + b*x]] + I*Sqrt[1 - a^2]*Log[(2*Sqrt[-1 + a + b
*x]*Sqrt[1 + a + b*x])/((-1 + a^2)*x) + ((2*I)*(-1 + a^2 + a*b*x))/(Sqrt[1
- a^2]*(-1 + a^2)*x)]
```

Maple [C] time = 0.013, size = 156, normalized size = 1.6

$$\mathrm{csgn}(b)\sqrt{bx+a-1}\sqrt{bx+a+1}\left(-\mathrm{csgn}(b)\ln\left(2\,\frac{xab+\sqrt{a^2-1}\sqrt{b^2x^2+2\,xab+a^2-1}+a^2-1}{x}\right)\sqrt{a^2-1}+\sqrt{b^2x^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x)
```

```
[Out] (b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(-csgn(b)*ln(2*(x*a*b+(a^2-1)^(1/2)*(b^2*x^
2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*(a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2
)*csgn(b)+ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn
(b)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+b*x+a*ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07392, size = 647, normalized size = 6.47

$$\left[bx - a\log\left(-bx + \sqrt{bx + a + 1}\sqrt{bx + a - 1} - a\right) + a\log(x) + \sqrt{a^2 - 1}\log\left(\frac{a^2bx + a^3 + \left(a^2 - \sqrt{a^2 - 1}a - 1\right)\sqrt{bx + a + 1}\sqrt{}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] [b*x - a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + a*log(x) + s
qrt(a^2 - 1)*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a - 1)*sqrt(b*x + a
+ 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + sqrt(b*x
 + a + 1)*sqrt(b*x + a - 1), b*x - a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x
+ a - 1) - a) + a*log(x) + 2*sqrt(-a^2 + 1)*arctan(-(sqrt(-a^2 + 1)*b*x - s
qrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + sqrt(b*x +
a + 1)*sqrt(b*x + a - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+\sqrt{a+bx-1}\sqrt{a+bx+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1))/x, x)
```

Giac [A] time = 1.23964, size = 159, normalized size = 1.59

$$bx - a\log\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2\right) + a\log\left(|bx|\right) - a\log\left(|-a-1|\right) - \frac{2\left(a^2-1\right)\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2a}{2\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="giac")
```

```
[Out] b*x - a*log((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2) + a*log(abs(b*x)) -
a*log(abs(-a - 1)) - 2*(a^2 - 1)*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x
+ a - 1))^2 - 2*a)/sqrt(-a^2 + 1))/sqrt(-a^2 + 1) + sqrt(b*x + a + 1)*sqrt(
b*x + a - 1) + a + 1
```

3.280 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2ab\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)-\frac{a}{x}+b\log(x)$$

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + 2*b*ArcSinh[Sqrt[-1 + a
 + b*x]/Sqrt[2]] - (2*a*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 +
a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2] + b*Log[x]
```

Rubi [A] time = 0.0737779, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5909, 14, 97, 157, 63, 215, 93, 205}

$$-\frac{2ab\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + 2*b*ArcSinh[Sqrt[-1 + a
 + b*x]/Sqrt[2]] - (2*a*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 +
a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2] + b*Log[x]
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^2}\,dx$$
$$= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^2}\right)dx$$
$$= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^2}\,dx$$
$$= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}+b\log(x)+\int \frac{ab+b^2x}{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,dx$$
$$= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}+b\log(x)+(ab)\int \frac{1}{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,dx+b^2\int \frac{}{\sqrt{-1+a}}$$
$$= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}+b\log(x)+(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx,x,\sqrt{-1+a+bx}\right)+(2ab)\,\mathrm{S}$$
$$= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)-\frac{2ab\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}+b\log(x$$

Mathematica [C] time = 0.151557, size = 140, normalized size = 1.28

$$-\frac{iab\log\left(\frac{2\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+\frac{i(a^2+abx-1)}{\sqrt{1-a^2}}\right)}{abx}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{x}+b\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)-\frac{a}{x}+b\log$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + b*Log[x] + b*Log[a + b*
x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]] - (I*a*b*Log[(2*(Sqrt[-1 + a + b*
x]*Sqrt[1 + a + b*x] + (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(a*b*x)])/Sqr
t[1 - a^2]
```

Maple [C] time = 0.014, size = 237, normalized size = 2.2

$$\frac{\mathrm{csgn}(b)}{(a^2-1)x}\left(-\mathrm{csgn}(b)\sqrt{a^2-1}\ln\left(2\,\frac{xab+\sqrt{a^2-1}\sqrt{b^2x^2+2xab+a^2-1}+a^2-1}{x}\right)xab+\ln\left(\left(\sqrt{b^2x^2+2xab+a^2-1}\,\mathrm{csg}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x)
```

```
[Out] (-csgn(b)*(a^2-1)^(1/2)*ln(2*(x*a*b+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(
1/2)+a^2-1)/x)*x*a*b+ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(
b))*x*a^2*b-(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2-ln(((b^2*x^2+2*a*b*x+
a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*x*b+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn
(b))*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*csgn(b)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/
(a^2-1)/x-a/x+b*ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22424, size = 852, normalized size = 7.82

$$\left[\frac{\sqrt{a^2-1}abx\log\left(\frac{a^2bx+a^3+\left(a^2-\sqrt{a^2-1}a-1\right)\sqrt{bx+a+1}\sqrt{bx+a-1}-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-\left(a^2-1\right)bx\log\left(-bx+\sqrt{bx+a+1}\sqrt{bx+a-1}\right.}{\left(a^2-1\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="fricas"
)
```

```
[Out] [(sqrt(a^2 - 1)*a*b*x*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a - 1)*sqrt
(b*x + a + 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) -
 (a^2 - 1)*b*x*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + (a^2 -
 1)*b*x*log(x) - a^3 - (a^2 - 1)*b*x - (a^2 - 1)*sqrt(b*x + a + 1)*sqrt(b*x
```

```
 + a - 1) + a)/((a^2 - 1)*x), (2*sqrt(-a^2 + 1)*a*b*x*arctan(-(sqrt(-a^2 +
1)*b*x - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) - (
a^2 - 1)*b*x*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + (a^2 - 1
)*b*x*log(x) - a^3 - (a^2 - 1)*b*x - (a^2 - 1)*sqrt(b*x + a + 1)*sqrt(b*x +
 a - 1) + a)/((a^2 - 1)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + \sqrt{a + bx - 1}\sqrt{a + bx + 1}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**2,x)
```

```
[Out] Integral((a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1))/x**2, x)
```

Giac [B] time = 1.28769, size = 317, normalized size = 2.91

$$-\frac{\frac{2\,ab^2\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,a}{2\,\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}} + b^2\log\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2\right) - b^2\log\left(|bx|\right) - \frac{4\left(ab^2\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\cdots\right.}{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4\,a\left(\sqrt{bx+a+1}-\cdots\right.}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -(2*a*b^2*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/sqrt
(-a^2 + 1))/sqrt(-a^2 + 1) + b^2*log((sqrt(b*x + a + 1) - sqrt(b*x + a - 1)
)^2) - b^2*log(abs(b*x)) - 4*(a*b^2*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))
^2 - 2*b^2)/((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a
+ 1) - sqrt(b*x + a - 1))^2 + 4) + (a*b^2*log(abs(-a - 1)) + b^2*log(abs(-a
 - 1)) - b^2)/(a + 1) + ((b*x + a + 1)*b^2 - b^2)/(b*x))/b
```

3.281 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{b^2\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\sqrt{a+bx-1}(a+bx+1)^{3/2}}{2(a+1)x^2}-\frac{a}{2x^2}-\frac{b}{x}$$

```
[Out] -a/(2*x^2) - b/x + (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x)
 - (Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 + a)*x^2) - (b^2*ArcTan[(
Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b*x])])/(1 - a^2)
^(3/2)
```

Rubi [A] time = 0.108985, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5909, 14, 94, 93, 205}

$$-\frac{b^2\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\sqrt{a+bx-1}(a+bx+1)^{3/2}}{2(a+1)x^2}-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^3,x]
```

```
[Out] -a/(2*x^2) - b/x + (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x)
 - (Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 + a)*x^2) - (b^2*ArcTan[(
Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b*x])])/(1 - a^2)
^(3/2)
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\cosh^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^3}\,dx \\
&= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^3}\right)dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b\int \frac{\sqrt{1+a+bx}}{x^2\sqrt{-1+a+bx}}\,dx}{2(1+a)} \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b^2\int \frac{1}{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,d}{2\left(1-a^2\right)} \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{-1-a-(1-a)x^2}\right.}{1-a^2} \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1+a)x^2}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\left(1-a^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.240234, size = 142, normalized size = 1.03

$$
\frac{1}{2}\left(-\frac{ib^2\log\left(\frac{4i\sqrt{1-a^2}\left(-i\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+a^2+abx-1\right)}{b^2x}\right)}{\left(1-a^2\right)^{3/2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(a^2+abx-1\right)}{\left(a^2-1\right)x^2}-\frac{a}{x^2}-\frac{2b}{x}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-1 + a^2 + a*b
*x))/((-1 + a^2)*x^2) - (I*b^2*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x -
 I*Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]))/(b^2*x)])/(1 - a^2)
^(3/2))/2
```

Maple [B] time = 0.016, size = 236, normalized size = 1.7

$$\frac{1}{2\left(a^{2}-1\right)^{2}x^{2}}\sqrt{bx+a-1}\sqrt{bx+a+1}\left(\sqrt{a^{2}-1}\ln\left(2\frac{xab+\sqrt{a^{2}-1}\sqrt{b^{2}x^{2}+2xab+a^{2}-1}+a^{2}-1}{x}\right)x^{2}b^{2}-xa^{3}b\sqrt{b^{2}x^{2}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x)
```

```
[Out] 1/2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*((a^2-1)^(1/2)*ln(2*(x*a*b+(a^2-1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^2*b^2-x*a^3*b*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)-a^4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2
)*x*a*b+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a^2-(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/
(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/(a^2-1)^2/x^2-b/x-1/2*a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0167, size = 818, normalized size = 5.93

$$\left[\frac{\sqrt{a^{2}-1}b^{2}x^{2}\log\left(\frac{a^{2}bx+a^{3}+\left(a^{2}+\sqrt{a^{2}-1}a-1\right)\sqrt{bx+a+1}\sqrt{bx+a-1}+\left(abx+a^{2}-1\right)\sqrt{a^{2}-1}-a}{x}\right)-a^{5}-\left(a^{3}-a\right)b^{2}x^{2}+2a^{3}-2\left(a^{4}-2a^{2}+1\right)bx}{2\left(a^{4}-2a^{2}+1\right)x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/2*(sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + (a^2 + sqrt(a^2 - 1)*a - 1
)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a
)/x) - a^5 - (a^3 - a)*b^2*x^2 + 2*a^3 - 2*(a^4 - 2*a^2 + 1)*b*x - (a^4 + (
a^3 - a)*b*x - 2*a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a)/((a^4 -
2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^2 + 1)*b*x
 - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + a^5 + (
a^3 - a)*b^2*x^2 - 2*a^3 + 2*(a^4 - 2*a^2 + 1)*b*x + (a^4 + (a^3 - a)*b*x -
 2*a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + a)/((a^4 - 2*a^2 + 1)*x^2
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39922, size = 471, normalized size = 3.41

$$\frac{\frac{2\,b^3\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,a}{2\,\sqrt{-a^2+1}}\right)}{\left(a^2-1\right)\sqrt{-a^2+1}}-\frac{ab^3+2\,b^3}{a^2+2\,a+1}+\frac{4\left(2\,a^2b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-4\,a^3b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-2\,ab^3\left(\sqrt{b\cdots}\right.\right.}{\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4\,a\left(\sqrt{bx+a+1}\cdots\right.\right.}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/sqr
t(-a^2 + 1))/((a^2 - 1)*sqrt(-a^2 + 1)) - (a*b^3 + 2*b^3)/(a^2 + 2*a + 1) +
 4*(2*a^2*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 4*a^3*b^3*(sqrt(b
*x + a + 1) - sqrt(b*x + a - 1))^4 - b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))^6 - 2*a*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 + 8*a^2*b^3*(sq
rt(b*x + a + 1) - sqrt(b*x + a - 1))^2 + 4*b^3*(sqrt(b*x + a + 1) - sqrt(b*
x + a - 1))^2 - 8*a*b^3)/(((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*
(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 + 4)^2*(a^2 - 1)) - (2*(b*x + a +
 1)*b^3 - a*b^3 - 2*b^3)/(b^2*x^2))/b
```

3.282 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^4} dx$

Optimal. Leaf size=189

$$\frac{ab^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)^2 x}-\frac{ab^3\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{5/2}}+\frac{(a+bx-1)^{3/2}(a+bx+1)^{3/2}}{3\left(1-a^2\right)x^3}-\frac{ab\sqrt{a+bx-1}(a+bx+1)^{3/2}}{2(1-a)(a+1)^2x^2}-$$

```
[Out] -a/(3*x^3) - b/(2*x^2) + (a*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1
 - a^2)^2*x) - (a*b*Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a)*(1 +
 a)^2*x^2) + ((-1 + a + b*x)^(3/2)*(1 + a + b*x)^(3/2))/(3*(1 - a^2)*x^3) -
 (a*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b
*x])])/(1 - a^2)^(5/2)
```

Rubi [A] time = 0.156407, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5909, 14, 96, 94, 93, 205}

$$\frac{ab^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)^2 x}-\frac{ab^3\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{5/2}}+\frac{(a+bx-1)^{3/2}(a+bx+1)^{3/2}}{3\left(1-a^2\right)x^3}-\frac{ab\sqrt{a+bx-1}(a+bx+1)^{3/2}}{2(1-a)(a+1)^2x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(2*x^2) + (a*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1
 - a^2)^2*x) - (a*b*Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a)*(1 +
 a)^2*x^2) + ((-1 + a + b*x)^(3/2)*(1 + a + b*x)^(3/2))/(3*(1 - a^2)*x^3) -
 (a*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b
*x])])/(1 - a^2)^(5/2)
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^4}\,dx$$
$$= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^4}\right)dx$$
$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^4}\,dx$$
$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3\left(1-a^2\right)x^3}+\frac{(ab)\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^3}\,dx}{1-a^2}$$
$$= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3\left(1-a^2\right)x^3}+\frac{\left(ab^2\right)\int \frac{\sqrt{1+a+bx}}{x^2\sqrt{-1+a+b}}}{2(1-a)(1+a)}$$
$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1}{3\left(1-a^2\right)}$$
$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1}{3\left(1-a^2\right)}$$
$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1}{3\left(1-a^2\right)}$$

Mathematica [C] time = 0.156376, size = 179, normalized size = 0.95

$$\frac{1}{6}\left(\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(a^2\left(b^2x^2+4\right)-a^3bx-2a^4+abx+2b^2x^2-2\right)}{\left(a^2-1\right)^2x^3}-\frac{3iab^3\log\left(\frac{4\left(1-a^2\right)^{3/2}\left(\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+ia^2\right.}{ab^3x}\right.}{\left(1-a^2\right)^{5/2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-2 - 2*a^4
 + a*b*x - a^3*b*x + 2*b^2*x^2 + a^2*(4 + b^2*x^2)))/((-1 + a^2)^2*x^3) - (
(3*I)*a*b^3*Log[(4*(1 - a^2)^(3/2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sq
rt[-1 + a + b*x]*Sqrt[1 + a + b*x]))/(a*b^3*x)])/(1 - a^2)^(5/2))/6
```

Maple [B] time = 0.014, size = 374, normalized size = 2.

$$-\frac{1}{6\left(a^{2}-1\right)^{3}x^{3}}\sqrt{bx+a-1}\sqrt{bx+a+1}\left(3\sqrt{a^{2}-1}\ln\left(2\frac{xab+\sqrt{a^{2}-1}\sqrt{b^{2}x^{2}+2xab+a^{2}-1}+a^{2}-1}{x}\right)x^{3}ab^{3}-x^{2}a^{4}b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x)
```

```
[Out] -1/6*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(3*(a^2-1)^(1/2)*ln(2*(x*a*b+(a^2-1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^3*a*b^3-x^2*a^4*b^2*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)+x*a^5*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-(b^2*x^2+2*a*b*
x+a^2-1)^(1/2)*x^2*a^2*b^2+2*a^6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-2*x*a^3*b*(b
^2*x^2+2*a*b*x+a^2-1)^(1/2)+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^2*b^2-6*a^4*(
b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x*a*b+6*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)*a^2-2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/(b^2*x^2+2*a*b*x
+a^2-1)^(1/2)/(a^2-1)^3/x^3-1/2*b/x^2-1/3*a/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04357, size = 1035, normalized size = 5.48

$$\left[\frac{3\sqrt{a^{2}-1}ab^{3}x^{3}\log\left(\frac{a^{2}bx+a^{3}+\left(a^{2}-\sqrt{a^{2}-1}a-1\right)\sqrt{bx+a+1}\sqrt{bx+a-1}-\left(abx+a^{2}-1\right)\sqrt{a^{2}-1}-a}{x}\right)-2a^{7}+\left(a^{4}+a^{2}-2\right)b^{3}x^{3}+6a^{5}-6a^{3}-3}{6\left(a^{6}-3a^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/6*(3*sqrt(a^2 - 1)*a*b^3*x^3*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a
 - 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1)
 - a)/x) - 2*a^7 + (a^4 + a^2 - 2)*b^3*x^3 + 6*a^5 - 6*a^3 - 3*(a^6 - 3*a^4
 + 3*a^2 - 1)*b*x - (2*a^6 - (a^4 + a^2 - 2)*b^2*x^2 - 6*a^4 + (a^5 - 2*a^3
 + a)*b*x + 6*a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*a)/((a^6 - 3
*a^4 + 3*a^2 - 1)*x^3), 1/6*(6*sqrt(-a^2 + 1)*a*b^3*x^3*arctan(-(sqrt(-a^2
+ 1)*b*x - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) -
 2*a^7 + (a^4 + a^2 - 2)*b^3*x^3 + 6*a^5 - 6*a^3 - 3*(a^6 - 3*a^4 + 3*a^2 -
 1)*b*x - (2*a^6 - (a^4 + a^2 - 2)*b^2*x^2 - 6*a^4 + (a^5 - 2*a^3 + a)*b*x
+ 6*a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*a)/((a^6 - 3*a^4 + 3*a
^2 - 1)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64251, size = 695, normalized size = 3.68

$$-\frac{\frac{6\,ab^4\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,a}{2\,\sqrt{-a^2+1}}\right)}{\left(a^4-2\,a^2+1\right)\sqrt{-a^2+1}}-\frac{ab^4+3\,b^4}{a^3+3\,a^2+3\,a+1}-\frac{4\left(12\,a^4b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^8-16\,a^5b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-3\,ab^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*b^4*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/
sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - (a*b^4 + 3*b^4)/(a^3 +
 3*a^2 + 3*a + 1) - 4*(12*a^4*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8
```

```
 - 16*a^5*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 3*a*b^4*(sqrt(b*x
 + a + 1) - sqrt(b*x + a - 1))^10 + 6*a^2*b^4*(sqrt(b*x + a + 1) - sqrt(b*x
 + a - 1))^8 - 56*a^3*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 + 48*a^
4*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 + 12*b^4*(sqrt(b*x + a + 1)
 - sqrt(b*x + a - 1))^8 - 48*a*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^
6 + 192*a^2*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 96*a^3*b^4*(sqr
t(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 144*a*b^4*(sqrt(b*x + a + 1) - sqrt
(b*x + a - 1))^2 + 32*a^2*b^4 + 64*b^4)/((a^4 - 2*a^2 + 1)*((sqrt(b*x + a +
 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2
+ 4)^3) + (3*(b*x + a + 1)*b^4 - a*b^4 - 3*b^4)/(b^3*x^3))/b
```

3.283 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^5} \, dx$

Optimal. Leaf size=238

$$\frac{(2a^2+3)\, b^2 \sqrt{a+bx-1}\sqrt{a+bx+1}}{24(1-a^2)^2 x^2} + \frac{a(2a^2+13)\, b^3 \sqrt{a+bx-1}\sqrt{a+bx+1}}{24(1-a^2)^3 x} - \frac{(4a^2+1)\, b^4 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{4(1-a^2)^{7/2}} + ab$$

```
[Out] -a/(4*x^4) - b/(3*x^3) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*x^4) + (
a*b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(12*(1 - a^2)*x^3) + ((3 + 2*a^2)
*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^2*x^2) + (a*(13 +
2*a^2)*b^3*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^3*x) - ((1 +
 4*a^2)*b^4*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a
 + b*x])])/(4*(1 - a^2)^(7/2))
```

Rubi [A] time = 0.201372, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5909, 14, 97, 151, 12, 93, 205}

$$\frac{(2a^2+3)\, b^2 \sqrt{a+bx-1}\sqrt{a+bx+1}}{24(1-a^2)^2 x^2} + \frac{a(2a^2+13)\, b^3 \sqrt{a+bx-1}\sqrt{a+bx+1}}{24(1-a^2)^3 x} - \frac{(4a^2+1)\, b^4 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{4(1-a^2)^{7/2}} + ab$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^5,x]
```


```
[Out] -a/(4*x^4) - b/(3*x^3) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*x^4) + (
a*b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(12*(1 - a^2)*x^3) + ((3 + 2*a^2)
*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^2*x^2) + (a*(13 +
2*a^2)*b^3*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^3*x) - ((1 +
 4*a^2)*b^4*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a
 + b*x])])/(4*(1 - a^2)^(7/2))
```


Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x^5}\,dx = \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^5}\,dx$$
$$= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^5}\right)dx$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^5}\,dx$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{1}{4}\int \frac{ab+b^2x}{x^4\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,dx$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\int \frac{\left(3+2a^2\right)b^2+2ab^3x}{x^3\sqrt{-1+a+bx}\sqrt{1+a+bx}}\,dx}{12\left(1-a^2\right)}$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2\sqrt{-1+a+b}}{24\left(1-a^2\right)^2}$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2\sqrt{-1+a+b}}{24\left(1-a^2\right)^2}$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2\sqrt{-1+a+b}}{24\left(1-a^2\right)^2}$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2\sqrt{-1+a+b}}{24\left(1-a^2\right)^2}$$
$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2\sqrt{-1+a+b}}{24\left(1-a^2\right)^2}$$

Mathematica [C] time = 0.254085, size = 198, normalized size = 0.83

$$\frac{1}{24}\left(-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(\frac{a\left(2a^2+13\right)b^3x^3}{\left(a^2-1\right)^3}-\frac{\left(2a^2+3\right)b^2x^2}{\left(a^2-1\right)^2}+\frac{2abx}{a^2-1}+6\right)}{x^4}-\frac{3i\left(4a^2+1\right)b^4\log\left(\frac{16i\left(1-a^2\right)^{5/2}\left(-i\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx}\right.}{b^4\left(4a^2x+x\right)}\right.}{\left(1-a^2\right)^{7/2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(6 + (2*a*b
*x)/(-1 + a^2) - ((3 + 2*a^2)*b^2*x^2)/(-1 + a^2)^2 + (a*(13 + 2*a^2)*b^3*x
^3)/(-1 + a^2)^3))/x^4 - ((3*I)*(1 + 4*a^2)*b^4*Log[((16*I)*(1 - a^2)^(5/2)
*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]))
/(b^4*(x + 4*a^2*x))])/(1 - a^2)^(7/2))/24
```

Maple [B] time = 0.017, size = 603, normalized size = 2.5

$$\frac{1}{24\left(a^2-1\right)^4 x^4}\sqrt{bx+a-1}\sqrt{bx+a+1}\left(12\sqrt{a^2-1}\ln\left(2\,\frac{xab+\sqrt{a^2-1}\sqrt{b^2x^2+2\,xab+a^2-1}+a^2-1}{x}\right)x^4a^2b^4-2\,x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x)
```

```
[Out] 1/24*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(12*(a^2-1)^(1/2)*ln(2*(x*a*b+(a^2-1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^4*a^2*b^4-2*x^3*a^5*b^3*(b^
2*x^2+2*a*b*x+a^2-1)^(1/2)+2*x^2*a^6*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+3*(a
^2-1)^(1/2)*ln(2*(x*a*b+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/
x)*x^4*b^4-11*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^3*a^3*b^3-2*x*a^7*b*(b^2*x^2+
2*a*b*x+a^2-1)^(1/2)-x^2*a^4*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*a^8*(b^2*x
^2+2*a*b*x+a^2-1)^(1/2)+13*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^3*a*b^3+6*x*a^5*
b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^2*a^2*b^2
+24*a^6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*x*a^3*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/
2)+3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^2*b^2-36*a^4*(b^2*x^2+2*a*b*x+a^2-1)^(
1/2)+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x*a*b+24*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
*a^2-6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/(a^2-1)
^4/x^4-1/4*a/x^4-1/3*b/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07963, size = 1337, normalized size = 5.62

$$\left[\frac{3\left(4a^2+1\right)\sqrt{a^2-1}b^4x^4\log\left(\frac{a^2bx+a^3+\left(a^2+\sqrt{a^2-1}a-1\right)\sqrt{bx+a+1}\sqrt{bx+a-1}+\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-6a^9-\left(2a^5+11a^3-13a\right)b^4x^4+2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/24*(3*(4*a^2 + 1)*sqrt(a^2 - 1)*b^4*x^4*log((a^2*b*x + a^3 + (a^2 + sqrt
(a^2 - 1)*a - 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + (a*b*x + a^2 - 1)*sq
rt(a^2 - 1) - a)/x) - 6*a^9 - (2*a^5 + 11*a^3 - 13*a)*b^4*x^4 + 24*a^7 - 36
*a^5 + 24*a^3 - 8*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 +
 11*a^3 - 13*a)*b^3*x^3 - 24*a^6 - (2*a^6 - a^4 - 4*a^2 + 3)*b^2*x^2 + 36*a
^4 + 2*(a^7 - 3*a^5 + 3*a^3 - a)*b*x - 24*a^2 + 6)*sqrt(b*x + a + 1)*sqrt(b
*x + a - 1) - 6*a)/((a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*x^4), -1/24*(6*(4*a^2
 + 1)*sqrt(-a^2 + 1)*b^4*x^4*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(-a^2 + 1)*s
qrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + 6*a^9 + (2*a^5 + 11*a^3 -
13*a)*b^4*x^4 - 24*a^7 + 36*a^5 - 24*a^3 + 8*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 +
 1)*b*x + (6*a^8 + (2*a^5 + 11*a^3 - 13*a)*b^3*x^3 - 24*a^6 - (2*a^6 - a^4
- 4*a^2 + 3)*b^2*x^2 + 36*a^4 + 2*(a^7 - 3*a^5 + 3*a^3 - a)*b*x - 24*a^2 +
6)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 6*a)/((a^8 - 4*a^6 + 6*a^4 - 4*a^2
 + 1)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.06225, size = 1148, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(4*a^2*b^5 + b^5)*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^2 - 2*a)/sqrt(-a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*sqrt(-a^2 + 1)) - (a
*b^5 + 4*b^5)/(a^4 + 4*a^3 + 6*a^2 + 4*a + 1) + 2*(128*a^6*b^5*(sqrt(b*x +
a + 1) - sqrt(b*x + a - 1))^10 + 12*a^2*b^5*(sqrt(b*x + a + 1) - sqrt(b*x +
 a - 1))^14 - 128*a^7*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 - 168*a
^3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^12 + 448*a^4*b^5*(sqrt(b*x +
 a + 1) - sqrt(b*x + a - 1))^10 + 3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a -
 1))^14 - 1216*a^5*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 - 42*a*b^5
*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^12 + 512*a^6*b^5*(sqrt(b*x + a + 1
) - sqrt(b*x + a - 1))^6 + 768*a^2*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a -
1))^10 - 2544*a^3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 + 5632*a^4*
b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 84*b^5*(sqrt(b*x + a + 1) -
 sqrt(b*x + a - 1))^10 - 1536*a^5*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^4 - 312*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 + 1920*a^2*b^5*(
sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 7552*a^3*b^5*(sqrt(b*x + a + 1)
- sqrt(b*x + a - 1))^4 + 1024*a^4*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^2 + 336*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 992*a*b^5*(sqrt(
b*x + a + 1) - sqrt(b*x + a - 1))^4 + 5888*a^2*b^5*(sqrt(b*x + a + 1) - sqr
t(b*x + a - 1))^2 - 256*a^3*b^5 - 192*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a
 - 1))^2 - 1664*a*b^5)/((a^6 - 3*a^4 + 3*a^2 - 1)*((sqrt(b*x + a + 1) - sqr
t(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 + 4)^4) -
 (4*(b*x + a + 1)*b^5 - a*b^5 - 4*b^5)/(b^4*x^4))/b
```

3.284 $\int e^{\cosh^{-1}(a+bx)^2} x^3 \, dx$

Optimal. Leaf size=359

$$\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^4}-\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^4}+\frac{3\sqrt{\pi}a^2\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^4}+\frac{3\sqrt{\pi}}{}$$

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1 -
 ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcCosh[a + b*x]])/(32*b
^4*E^4) + (Sqrt[Pi]*Erfi[1 + ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi
]*Erfi[1 + ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcCosh[a + b*
x]])/(32*b^4*E^4) + (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/(16*b^
4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/
4)) + (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3
*a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt[
Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi[
(3 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rubi [A] time = 0.737565, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5899, 6741, 12, 6742, 5512, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^4}-\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^4}+\frac{3\sqrt{\pi}a^2\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^4}+\frac{3\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x^3,x]
```


```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1 -
 ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcCosh[a + b*x]])/(32*b
^4*E^4) + (Sqrt[Pi]*Erfi[1 + ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi
]*Erfi[1 + ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcCosh[a + b*
x]])/(32*b^4*E^4) + (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/(16*b^
4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/
4)) + (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3
*a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt[
Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi[
(3 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2}x^3\,dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^3\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(-a+\cosh(x))^3\sinh(x)}{b^3}\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}(-a+\cosh(x))^3\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^4}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3e^{x^2}\sinh(x)+3a^2e^{x^2}\cosh(x)\sinh(x)-3ae^{x^2}\cosh^2(x)\sinh(x)+e^{x^2}\cosh^3(x)\sinh(\right.\right.}{b^4}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh^3(x)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\,\text{Subst}\left(\int e^{x^2}\cosh^2(x)\sinh(x)\,dx,x,\right.}{b^4}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{16}e^{-4x+x^2}-\frac{1}{8}e^{-2x+x^2}+\frac{1}{8}e^{2x+x^2}+\frac{1}{16}e^{4x+x^2}\right)dx,x,\cosh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\,\text{Subst}\left(\int\left(\right.\right.}{}$$
$$= -\frac{\text{Subst}\left(\int e^{-4x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{16b^4} + \frac{\text{Subst}\left(\int e^{4x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{16b^4} - \frac{\text{Subst}\left(\int e^{-}\right.}{}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{16b^4e^4} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{16b^4e^4} + \frac{(3a)\,\text{S}}{}$$
$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{16b^4e} + \frac{3a^2\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^4e} + \frac{\sqrt{\pi}\,\text{erfi}\left(2-\cosh^{-1}(a+bx)\right)}{32b^4e^4}$$

Mathematica [A] time = 0.368927, size = 198, normalized size = 0.55

$$\frac{\sqrt{\pi}\left(-8e^{15/4}a^3\text{Erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)+12e^3a^2\text{Erfi}\left(\cosh^{-1}(a+bx)+1\right)-2e^{15/4}\left(4a^2+3\right)a\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*(3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcCosh[a + b*x]] + 2*(1 +
6*a^2)*E^3*Erfi[1 - ArcCosh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 - ArcCosh[a +
b*x]] + Erfi[2 - ArcCosh[a + b*x]] - 6*a*E^(15/4)*Erfi[1/2 + ArcCosh[a + b*
x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcCosh[a + b*x]] + 2*E^3*Erfi[1 + ArcCosh[
a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcCosh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
```

```
ArcCosh[a + b*x]] + Erfi[2 + ArcCosh[a + b*x]]))/(32*b^4*E^4)
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$\int e^{(\operatorname{arccosh}(bx+a))^{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3} e^{\left(\operatorname{arcosh}(bx+a)^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arccosh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^{3} e^{\left(\operatorname{arcosh}(bx+a)^{2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arccosh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{\mathrm{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(acosh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(arccosh(b*x + a)^2), x)
```

3.285 $\int e^{\cosh^{-1}(a+bx)^2}x^2\,dx$

Optimal. Leaf size=251

$$-\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^3}-\frac{\sqrt{\pi}a\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4eb^3}-\frac{\sqrt{\pi}a}{}$$

```
[Out] -(a*Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(4*b^3*E) - (a*Sqrt[Pi]*Erfi[1 + A
rcCosh[a + b*x]])/(4*b^3*E) - (Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/
(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(
1/4)) - (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) +
(Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[Pi
]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2
*ArcCosh[a + b*x])/2])/(16*b^3*E^(9/4))
```

Rubi [A] time = 0.49307, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5899, 6741, 12, 6742, 5512, 2234, 2204, 5514}

$$-\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^3}-\frac{\sqrt{\pi}a\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4eb^3}-\frac{\sqrt{\pi}a}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x^2,x]
```



```
[Out] -(a*Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(4*b^3*E) - (a*Sqrt[Pi]*Erfi[1 + A
rcCosh[a + b*x]])/(4*b^3*E) - (Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/
(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(
1/4)) - (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) +
(Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[Pi
]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2
*ArcCosh[a + b*x])/2])/(16*b^3*E^(9/4))
```


Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2}x^2\,dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(a-\cosh(x))^2\sinh(x)}{b^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}(a-\cosh(x))^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int\left(a^2e^{x^2}\sinh(x)-2ae^{x^2}\cosh(x)\sinh(x)+e^{x^2}\cosh^2(x)\sinh(x)\right)dx,x,\cosh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh^2(x)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx,x,\right.}{b^3}$$

$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{8}e^{-3x+x^2}-\frac{1}{8}e^{-x+x^2}+\frac{e^{x+x^2}}{8}+\frac{1}{8}e^{3x+x^2}\right)dx,x,\cosh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int\left(-\frac{1}{4}e\right.\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{-3x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{a\,\text{Su}}{}$$

$$= -\frac{a\sqrt{\pi}\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4b^3e} - \frac{a\sqrt{\pi}\text{erfi}\left(1+\cosh^{-1}(a+bx)\right)}{4b^3e} - \frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}\left(-3+2\cosh^{-1}(a\right.\right.}{16b^3e^{9/4}}$$

Mathematica [A] time = 0.21469, size = 136, normalized size = 0.54

$$\frac{\sqrt{\pi}\left(4e^2a^2\text{Erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)+e^2\left(4a^2+1\right)\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)-4e^{5/4}a\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)-4e^{5/4}a\right.}{16e^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x^2,x]
```

```
[Out] (Sqrt[Pi]*((1 + 4*a^2)*E^2*Erfi[1/2 - ArcCosh[a + b*x]] - 4*a*E^(5/4)*Erfi[
1 - ArcCosh[a + b*x]] + Erfi[3/2 - ArcCosh[a + b*x]] + E^2*Erfi[1/2 + ArcCo
sh[a + b*x]] + 4*a^2*E^2*Erfi[1/2 + ArcCosh[a + b*x]] - 4*a*E^(5/4)*Erfi[1
+ ArcCosh[a + b*x]] + Erfi[3/2 + ArcCosh[a + b*x]]))/(16*b^3*E^(9/4))
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \mathrm{e}^{(\operatorname{arccosh}(bx+a))^2} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x^2,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arccosh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(arccosh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x**2,x)
```

```
[Out] Integral(x**2*exp(acosh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(arccosh(b*x + a)^2), x)
```

3.286 $\int e^{\cosh^{-1}(a+bx)^2} x\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\pi}\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\text{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{8eb^2}+\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^2}-\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\cosh\right.\right.}{4\sqrt[4]{}}$$

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcCos
h[a + b*x]])/(8*b^2*E) + (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rubi [A] time = 0.27846, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5899, 6741, 12, 6742, 5512, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\text{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{8eb^2}+\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^2}-\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\cosh\right.\right.}{4\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x,x]
```


```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcCos
h[a + b*x]])/(8*b^2*E) + (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```


Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2} x\,dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(-a+\cosh(x))\sinh(x)}{b}\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}(-a+\cosh(x))\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-ae^{x^2}\sinh(x)+e^{x^2}\cosh(x)\sinh(x)\right)dx, x, \cosh^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int e^{x^2}\sinh(x)\,dx, x, \cosh^{-1}(a+bx\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)dx, x, \cosh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \left(-\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx, x, \cos\right.}{b^2}$$
$$= -\frac{\text{Subst}\left(\int e^{-2x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2} + \frac{\text{Subst}\left(\int e^{2x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2} + \frac{a\,\text{Subst}\left(\int e\right.}{}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2e} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2e} + \frac{a\,\text{Subs}}{}$$
$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^2e} + \frac{\sqrt{\pi}\,\text{erfi}\left(1+\cosh^{-1}(a+bx)\right)}{8b^2e} + \frac{a\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}(a+bx\right.\right.}{4b^2\sqrt[4]{e}}$$

Mathematica [A] time = 0.12821, size = 76, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(-2e^{3/4}a\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)-2e^{3/4}a\text{Erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)+\text{Erfi}\left(\cosh^{-1}(a+\right.\right.}{8eb^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*E^(3/4)*Erfi[1/2 - ArcCosh[a + b*x]] + Erfi[1 - ArcCosh[a +
 b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcCosh[a + b*x]] + Erfi[1 + ArcCosh[a + b*
x]]))/(8*b^2*E)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \mathrm{e}^{(\operatorname{arccosh}(bx+a))^2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(arccosh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(xe^{\left(\operatorname{arcosh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(arccosh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{\operatorname{acosh}^{2}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x,x)
```

```
[Out] Integral(x*exp(acosh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(arccosh(b*x + a)^2), x)
```

3.287 $\int e^{\cosh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b}-\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b}$$

```
[Out] -(Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erf
i[(1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4))
```

Rubi [A] time = 0.054025, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5897, 5512, 2234, 2204}

$$\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b}-\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2,x]
```

```
[Out] -(Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erf
i[(1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 5897

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Sinh[x], x], x, ArcCosh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{\cosh^{-1}(a+bx)^2}\,dx &= \frac{\mathrm{Subst}\left(\int e^{x^2}\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}\\ &= \frac{\mathrm{Subst}\left(\int \left(-\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx, x, \cosh^{-1}(a+bx)\right)}{b}\\ &= -\frac{\mathrm{Subst}\left(\int e^{-x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b}+\frac{\mathrm{Subst}\left(\int e^{x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b}\\ &= -\frac{\mathrm{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}+\frac{\mathrm{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}\\ &= -\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}\left(1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}\end{aligned}$$

Mathematica [A] time = 0.0349985, size = 42, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+\mathrm{Erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)\right)}{4\sqrt[4]{e}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*(Erfi[1/2 - ArcCosh[a + b*x]] + Erfi[1/2 + ArcCosh[a + b*x]]))/(4
*b*E^(1/4))
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int e^{(\mathrm{arccosh}(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2),x)
```

```
[Out] int(exp(arccosh(b*x+a)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(\operatorname{arcosh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] integral(e^(arccosh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2),x)
```

```
[Out] Integral(exp(acosh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="giac")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2), x)
```

3.288 $\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} \, dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\frac{e^{\cosh^{-1}(a+bx)^2}}{x},x\right)$$

```
[Out] CannotIntegrate[E^ArcCosh[a + b*x]^2/x, x]
```

Rubi [A] time = 0.0402105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcCosh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][E^ArcCosh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} \, dx = \int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} \, dx$$

Mathematica [A] time = 0.134319, size = 0, normalized size = 0.

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcCosh[a + b*x]^2/x,x]
```

```
[Out] Integrate[E^ArcCosh[a + b*x]^2/x, x]
```

Maple [A] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{(\operatorname{arccosh}(bx+a))^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)/x,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arccosh(b*x + a)^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{acosh}^2(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)/x,x)
```

```
[Out] Integral(exp(acosh(a + b*x)**2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x, x)
```

3.289 $\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} \, dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\frac{e^{\cosh^{-1}(a+bx)^2}}{x^2},x\right)$$

```
[Out] CannotIntegrate[E^ArcCosh[a + b*x]^2/x^2, x]
```

Rubi [A] time = 0.0388042, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcCosh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][E^ArcCosh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} \, dx = \int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} \, dx$$

Mathematica [A] time = 0.435765, size = 0, normalized size = 0.

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcCosh[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[E^ArcCosh[a + b*x]^2/x^2, x]
```

Maple [A] time = 0.006, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{(\operatorname{arccosh}(bx+a))^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)/x^2,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(arccosh(b*x + a)^2)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{acosh}^2(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)/x**2,x)
```

```
[Out] Integral(exp(acosh(a + b*x)**2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x^2, x)
```

3.290 $\int \frac{\cosh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=60

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(a+bx)}\right)}{2d}-\frac{\cosh^{-1}(a+bx)^2}{2d}+\frac{\cosh^{-1}(a+bx)\log\left(e^{2\cosh^{-1}(a+bx)}+1\right)}{d}$$

```
[Out] -ArcCosh[a + b*x]^2/(2*d) + (ArcCosh[a + b*x]*Log[1 + E^(2*ArcCosh[a + b*x]
)])/d + PolyLog[2, -E^(2*ArcCosh[a + b*x])]/(2*d)
```

Rubi [A] time = 0.0898679, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5866, 12, 5660, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(a+bx)}\right)}{2d}-\frac{\cosh^{-1}(a+bx)^2}{2d}+\frac{\cosh^{-1}(a+bx)\log\left(e^{2\cosh^{-1}(a+bx)}+1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -ArcCosh[a + b*x]^2/(2*d) + (ArcCosh[a + b*x]*Log[1 + E^(2*ArcCosh[a + b*x]
)])/d + PolyLog[2, -E^(2*ArcCosh[a + b*x])]/(2*d)
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\cosh^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(a+bx)\right)}{d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \cos\right.}{d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(\right.}{2d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}(a+bx)}\right)}{2d}$$

Mathematica [A] time = 0.0427777, size = 53, normalized size = 0.88

$$\frac{\cosh^{-1}(a+bx)\left(\cosh^{-1}(a+bx)+2\log\left(e^{-2\cosh^{-1}(a+bx)}+1\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(a+bx)}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (ArcCosh[a + b*x]*(ArcCosh[a + b*x] + 2*Log[1 + E^(-2*ArcCosh[a + b*x])]) -
 PolyLog[2, -E^(-2*ArcCosh[a + b*x])])/(2*d)
```

Maple [A] time = 0.039, size = 85, normalized size = 1.4

$$-\frac{(\text{arccosh}\,(bx+a))^2}{2\,d} + \frac{\text{arccosh}\,(bx+a)}{d}\ln\left(1+\left(bx+a+\sqrt{bx+a-1}\sqrt{bx+a+1}\right)^2\right) + \frac{1}{2\,d}\text{polylog}\left(2,-\left(bx+a+\sqrt{\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] -1/2*arccosh(b*x+a)^2/d+arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^2)/d+1/2*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b \operatorname{arcosh}(bx+a)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccosh(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{\operatorname{acosh}(a+bx)}{a+bx}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acosh(a + b*x)/(a + b*x), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcosh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

3.291 $\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\mathrm{Chi}\left(\cosh^{-1}(x)\right)$$

```
[Out] CoshIntegral[ArcCosh[x]]
```

Rubi [A] time = 0.120613, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5781, 3301}

$$\mathrm{Chi}\left(\cosh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]

[Out] CoshIntegral[ArcCosh[x]]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)}\,dx = \text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(x)\right)$$
$$= \textbf{Chi}\left(\cosh^{-1}(x)\right)$$

Mathematica [B] time = 0.0543266, size = 19, normalized size = 6.33

$$\frac{1}{2}(x-1)\textbf{Chi}\left(\cosh^{-1}(x)\right)\textbf{csch}^2\left(\frac{1}{2}\cosh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

```
[Out] ((-1 + x)*CoshIntegral[ArcCosh[x]]*Csch[ArcCosh[x]/2]^2)/2
```

Maple [B] time = 0.14, size = 78, normalized size = 26.

$$-\frac{\text{Ei}\,(1,\text{arccosh}\,(x))}{4\,x^2-4}\sqrt{2+2\,x}\sqrt{-2+2\,x}\sqrt{-1+x}\sqrt{1+x}-\frac{\text{Ei}\,(1,-\text{arccosh}\,(x))}{4\,x^2-4}\sqrt{2+2\,x}\sqrt{-2+2\,x}\sqrt{-1+x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] -1/4*(2+2*x)^(1/2)*(-2+2*x)^(1/2)*(-1+x)^(1/2)*(1+x)^(1/2)*Ei(1,arccosh(x))
/(x^2-1)-1/4*(2+2*x)^(1/2)*(-2+2*x)^(1/2)*(-1+x)^(1/2)*(1+x)^(1/2)*Ei(1,-ar
ccosh(x))/(x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x+1}\sqrt{x-1}\,\text{arcosh}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x+1}\sqrt{x-1}x}{\left(x^2-1\right)\text{arcosh}\left(x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x + 1)*sqrt(x - 1)*x/((x^2 - 1)*arccosh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x-1}\sqrt{x+1}\,\text{acosh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(x)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x - 1)*sqrt(x + 1)*acosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x+1}\sqrt{x-1}\,\text{arcosh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

3.292 $\int x^3 \cosh^{-1}\left(a+bx^4\right) dx$

Optimal. Leaf size=54

$$\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{a+bx^4-1}\sqrt{a+bx^4+1}}{4b}$$

```
[Out] -(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4])/(4*b) + ((a + b*x^4)*ArcCosh[a
+ b*x^4])/(4*b)
```

Rubi [A] time = 0.0521281, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 5864, 5654, 74}

$$\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{a+bx^4-1}\sqrt{a+bx^4+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a + b*x^4],x]
```

```
[Out] -(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4])/(4*b) + ((a + b*x^4)*ArcCosh[a
+ b*x^4])/(4*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\,\text{Subst}\left(\int\cosh^{-1}(a+bx)\,dx,x,x^4\right)\\ &= \frac{\text{Subst}\left(\int\cosh^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\ &= \frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int\frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx^4\right)}{4b}\\ &= -\frac{\sqrt{-1+a+bx^4}\sqrt{1+a+bx^4}}{4b}+\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0332246, size = 50, normalized size = 0.93

$$\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)-\sqrt{a+bx^4-1}\sqrt{a+bx^4+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a + b*x^4],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4]) + (a + b*x^4)*ArcCosh[a + b*x^
4])/(4*b)
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{1}{4\,b}\left(\left(bx^4+a\right)\text{arccosh}\left(bx^4+a\right)-\sqrt{bx^4+a-1}\sqrt{bx^4+a+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(b*x^4+a),x)
```

```
[Out] 1/4/b*((b*x^4+a)*arccosh(b*x^4+a)-(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2))
```

Maxima [A] time = 0.972995, size = 50, normalized size = 0.93

$$\frac{(bx^4+a)\,\mathrm{arcosh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2-1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arccosh(b*x^4 + a) - sqrt((b*x^4 + a)^2 - 1))/b
```

Fricas [A] time = 2.08293, size = 151, normalized size = 2.8

$$\frac{(bx^4+a)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2-1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))/b
```

Sympy [A] time = 1.21483, size = 61, normalized size = 1.13

$$\begin{cases} \frac{a\,\mathrm{acosh}\left(a+bx^4\right)}{4b}+\frac{x^4\,\mathrm{acosh}\left(a+bx^4\right)}{4}-\frac{\sqrt{a^2+2abx^4+b^2x^8-1}}{4b} & \text{for}\ b\neq 0 \\ \frac{x^4\,\mathrm{acosh}\,(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(b*x**4+a),x)
```

```
[Out] Piecewise((a*acosh(a + b*x**4)/(4*b) + x**4*acosh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 - 1)/(4*b), Ne(b, 0)), (x**4*acosh(a)/4, True))
```

Giac [B] time = 1.17469, size = 143, normalized size = 2.65

$$\frac{1}{4}x^4\log\left(bx^4+a+\sqrt{\left(bx^4+a\right)^2-1}\right)-\frac{1}{4}b\left(\frac{a\log\left(\left|-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,abx^4+a^2-}}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 - 1)) - 1/4*b*(a*log(abs(-a*b -
(x^4*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))*abs(b)))/(b*abs(b)) + sq
rt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/b^2)
```

3.293 $\int x^{-1+n} \cosh^{-1}(a + bx^n) \, dx$

Optimal. Leaf size=55

$$\frac{(a + bx^n)\cosh^{-1}(a + bx^n)}{bn} - \frac{\sqrt{a + bx^n - 1}\sqrt{a + bx^n + 1}}{bn}$$

```
[Out] -((Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n])/(b*n)) + ((a + b*x^n)*ArcCosh[
a + b*x^n])/(b*n)
```

Rubi [A] time = 0.0544168, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 5864, 5654, 74}

$$\frac{(a + bx^n)\cosh^{-1}(a + bx^n)}{bn} - \frac{\sqrt{a + bx^n - 1}\sqrt{a + bx^n + 1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCosh[a + b*x^n],x]
```

```
[Out] -((Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n])/(b*n)) + ((a + b*x^n)*ArcCosh[
a + b*x^n])/(b*n)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\cosh^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int\cosh^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\cosh^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\cosh^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int\frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx^n\right)}{bn}\\ &= -\frac{\sqrt{-1+a+bx^n}\sqrt{1+a+bx^n}}{bn}+\frac{(a+bx^n)\cosh^{-1}(a+bx^n)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0460324, size = 50, normalized size = 0.91

$$\frac{(a+bx^n)\cosh^{-1}(a+bx^n)-\sqrt{a+bx^n-1}\sqrt{a+bx^n+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCosh[a + b*x^n],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n]) + (a + b*x^n)*ArcCosh[a + b*x^
n])/(b*n)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x^{n-1}\text{arccosh}(a+bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arccosh(a+b*x^n),x)
```

```
[Out] int(x^(n-1)*arccosh(a+b*x^n),x)
```

Maxima [A] time = 0.971817, size = 53, normalized size = 0.96

$$\frac{(bx^n + a)\operatorname{arcosh}(bx^n + a) - \sqrt{(bx^n + a)^2 - 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arccosh(b*x^n + a) - sqrt((b*x^n + a)^2 - 1))/(b*n)
```

Fricas [B] time = 2.19117, size = 443, normalized size = 8.05

$$\frac{(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a)\log\left(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a + \sqrt{\frac{2\,ab+\left(a^2+b^2-1\right)\cosh(n\log(x))-(}{\cosh(n\log(x))-\sinh}}\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a^2
+ b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acosh(a+b*x**n),x)
```

[Out] Timed out

Giac [B] time = 1.18431, size = 154, normalized size = 2.8

$$-\frac{b\left(\frac{a\log\left(\left|-ab-\left(x^n|b|-\sqrt{b^2x^{2n}+2\,abx^n+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^{2n}+2\,abx^n+a^2-1}}{b^2}\right)-x^n\log\left(bx^n+a+\sqrt{(bx^n+a)^2-1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -(b*(a*log(abs(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)
)*abs(b)))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)/b^2) - x^n*
log(b*x^n + a + sqrt((b*x^n + a)^2 - 1)))/n
```

3.294 $\int \cosh^{-1}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=58

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\tan^{-1}\left(\sqrt{\frac{c\left(1-\frac{a}{c}\right)-bx}{a+bx+c}}\right)}{b}$$

```
[Out] ((a + b*x)*ArcSech[a/c + (b*x)/c])/b - (2*c*ArcTan[Sqrt[((1 - a/c)*c - b*x)
/(a + c + b*x)]])/b
```

Rubi [A] time = 0.09298, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {5893, 6313, 1961, 12, 203}

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\tan^{-1}\left(\sqrt{\frac{c\left(1-\frac{a}{c}\right)-bx}{a+bx+c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcSech[a/c + (b*x)/c])/b - (2*c*ArcTan[Sqrt[((1 - a/c)*c - b*x)
/(a + c + b*x)]])/b
```

Rule 5893

```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6313

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSech[c + d*
x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cosh^{-1}\left(\frac{c}{a+bx}\right)dx &= \int \operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)dx\\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\int \frac{\sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}}{1-\frac{a}{c}-\frac{bx}{c}}\,dx\\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{(4b)\,\text{Subst}\left(\int \frac{c^2}{2b^2(1+x^2)}\,dx, x, \sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}\right)}{c}\\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{(2c)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}\right)}{b}\\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\tan^{-1}\left(\sqrt{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}}\right)}{b}
\end{aligned}
$$

Mathematica [B] time = 0.42794, size = 175, normalized size = 3.02

$$
\frac{2(a+bx-c)^{3/2}\left(a\sqrt{b}\sqrt{a+bx+c}\tan^{-1}\left(\frac{\sqrt{a+bx-c}}{\sqrt{a+bx+c}}\right)-c\sqrt{bc}\sqrt{\frac{a+bx+c}{c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx-c}}{\sqrt{2}\sqrt{bc}}\right)\right)}{b^{3/2}(a+bx)^2\left(-\frac{a+bx-c}{a+bx}\right)^{3/2}\sqrt{\frac{a+bx+c}{a+bx}}}+x\cosh^{-1}\left(\frac{c}{a+bx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c/(a + b*x)],x]
```

```
[Out] x*ArcCosh[c/(a + b*x)] + (2*(a - c + b*x)^(3/2)*(-(c*Sqrt[b*c]*Sqrt[(a + c
+ b*x)/c]*ArcSinh[(Sqrt[b]*Sqrt[a - c + b*x])/(Sqrt[2]*Sqrt[b*c])]) + a*Sqr
t[b]*Sqrt[a + c + b*x]*ArcTan[Sqrt[a - c + b*x]/Sqrt[a + c + b*x]]))/(b^(3/
2)*(a + b*x)^2*(-((a - c + b*x)/(a + b*x)))^(3/2)*Sqrt[(a + c + b*x)/(a + b
*x)])
```

Maple [A] time = 0.043, size = 91, normalized size = 1.6

$$\operatorname{arccosh}\left(\frac{c}{bx+a}\right)x+\frac{a}{b}\operatorname{arccosh}\left(\frac{c}{bx+a}\right)+\frac{c}{b}\sqrt{\frac{c}{bx+a}-1}\sqrt{\frac{c}{bx+a}+1}\arctan\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c/(b*x+a)),x)
```

```
[Out] arccosh(c/(b*x+a))*x+1/b*arccosh(c/(b*x+a))*a+1/b*c*(c/(b*x+a)-1)^(1/2)*(c/
(b*x+a)+1)^(1/2)/(1/(b*x+a)^2*c^2-1)^(1/2)*arctan(1/(1/(b*x+a)^2*c^2-1)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bx\log\left(\sqrt{bx+a+c}\sqrt{-bx-a+c}bx+\sqrt{bx+a+c}\sqrt{-bx-a+c}a+(bx+a)c\right)-2\,bx\log\left(bx+a\right)+(a+c)\log\left(bx+a\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*log(sqrt(b*x + a + c)*sqrt(-b*x - a + c)*b*x + sqrt(b*x + a + c)
*sqrt(-b*x - a + c)*a + (b*x + a)*c) - 2*b*x*log(b*x + a) + (a + c)*log(b*x
 + a + c) - 2*(b*x + a)*log(b*x + a) + (a - c)*log(-b*x - a + c))/b + integ
rate((b^2*c*x^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c - c^3 + (b^2*x^2
```

```
+ 2*a*b*x + a^2 - c^2)*e^(1/2*log(b*x + a + c) + 1/2*log(-b*x - a + c)), x
)
```

Fricas [B] time = 2.46134, size = 583, normalized size = 10.05

$$\frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)-2\,c\arctan\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-c^2}\right)+a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+c}{x}\right)-a\log\left(\cdots\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 +
2*a*b*x + a^2)) + c)/(b*x + a)) - 2*c*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt
(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a
*b*x + a^2 - c^2)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)
/(b^2*x^2 + 2*a*b*x + a^2)) + c)/x) - a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - c)/x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acosh}\left(\frac{c}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c/(b*x+a)),x)
```

```
[Out] Integral(acosh(c/(a + b*x)), x)
```

Giac [B] time = 1.76533, size = 161, normalized size = 2.78

$$-\frac{c\arcsin\left(-\frac{bx+a}{c}\right)\operatorname{sgn}(b)\operatorname{sgn}(c)}{|b|}+x\log\left(\sqrt{\frac{c}{bx+a}+1}\sqrt{\frac{c}{bx+a}-1}+\frac{c}{bx+a}\right)+\frac{a\log\left(\frac{\left|-2\,bc-2\,\sqrt{-b^2x^2-2\,abx-a^2+c^2}|b|\right|}{\left|-2\,b^2x-2\,ab\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] -c*arcsin(-(b*x + a)/c)*sgn(b)*sgn(c)/abs(b) + x*log(sqrt(c/(b*x + a) + 1)*
sqrt(c/(b*x + a) - 1) + c/(b*x + a)) + a*log(abs(-2*b*c - 2*sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + c^2)*abs(b))/abs(-2*b^2*x - 2*a*b))/abs(b)
```

3.295 $\int \frac{\cosh^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

Rubi [A] time = 0.118908, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5895, 5676}

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5895

```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Dist[(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]])/(b*x), Subs
t[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
/; FreeQ[{b, n}, x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rubi steps

$$\int \frac{\cosh^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \frac{\left(\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\right)\text{Subst}\left(\int \frac{\cosh^{-1}(x)^n}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{1+bx^2}\right)}{bx}$$
$$= \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\cosh^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

Mathematica [A] time = 0.154314, size = 62, normalized size = 1.

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

Maple [F] time = 0.236, size = 0, normalized size = 0.

$$\int \left(\text{arccosh}\left(\sqrt{bx^2+1}\right)\right)^n \frac{1}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcosh}\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(sqrt(b*x^2 + 1))^n/sqrt(b*x^2 + 1), x)
```

Fricas [B] time = 2.22667, size = 282, normalized size = 4.55

$$\frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\sinh\left(n\log\left(\log\left(\sqrt{b\right.\right.\right.}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 + 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 + 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.296 $\int \frac{1}{\sqrt{1+bx^2}\cosh^{-1}\left(\sqrt{1+bx^2}\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
b*x^2]]])/(b*x)
```

Rubi [A] time = 0.108626, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5895, 5674}

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
b*x^2]]])/(b*x)
```

Rule 5895

```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Dist[(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]])/(b*x), Subs
t[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
/; FreeQ[{b, n}, x]
```

Rule 5674

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[Log[a + b*ArcCosh[c*x]]/(b*c*Sqrt[-
(d1*d2)]), x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && Eq
Q[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+bx^2}\cosh^{-1}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\left(\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)}\,dx, x, \sqrt{1+bx^2}\right)}{bx}$$
$$= \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\log\left(\cosh^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

Mathematica [A] time = 0.0894676, size = 54, normalized size = 1.

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
b*x^2]]])/(b*x)
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \left(\text{arccosh}\left(\sqrt{bx^2+1}\right)\right)^{-1} \frac{1}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

```
[Out] int(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+1}\,\text{arcosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccosh(sqrt(b*x^2 + 1))), x)
```

Fricas [A] time = 2.12005, size = 80, normalized size = 1.48

$$\frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+1}\operatorname{acosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh((b*x**2+1)**(1/2))/(b*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+1}\operatorname{arcosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccosh(sqrt(b*x^2 + 1))), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```