Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/7.1.5-Inverse-hyperbolic-sine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [371]. This is test number [188].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.73 (370)	% 0.27 (1)
Mathematica	% 99.73 (370)	% 0.27 (1)
Maple	% 67.12 (249)	% 32.88 (122)
Maxima	% 17.52 (65)	% 82.48 (306)
Fricas	% 39.62 (147)	% 60.38 (224)
Sympy	% 23.18 (86)	% 76.82 (285)
Giac	% 25.34 (94)	% 74.66 (277)

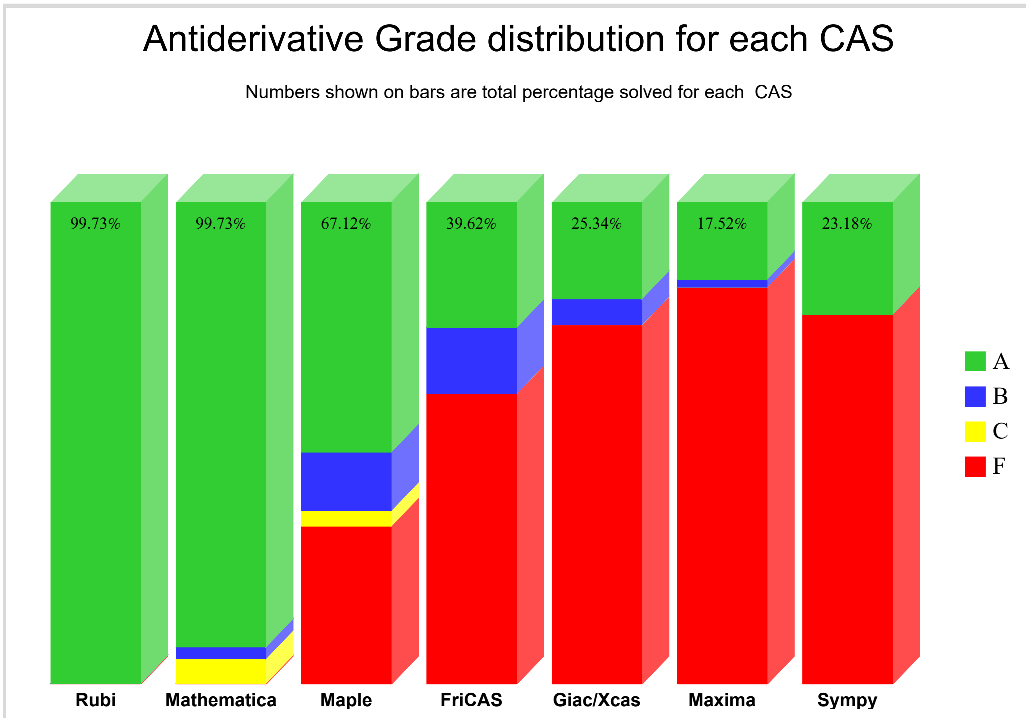
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

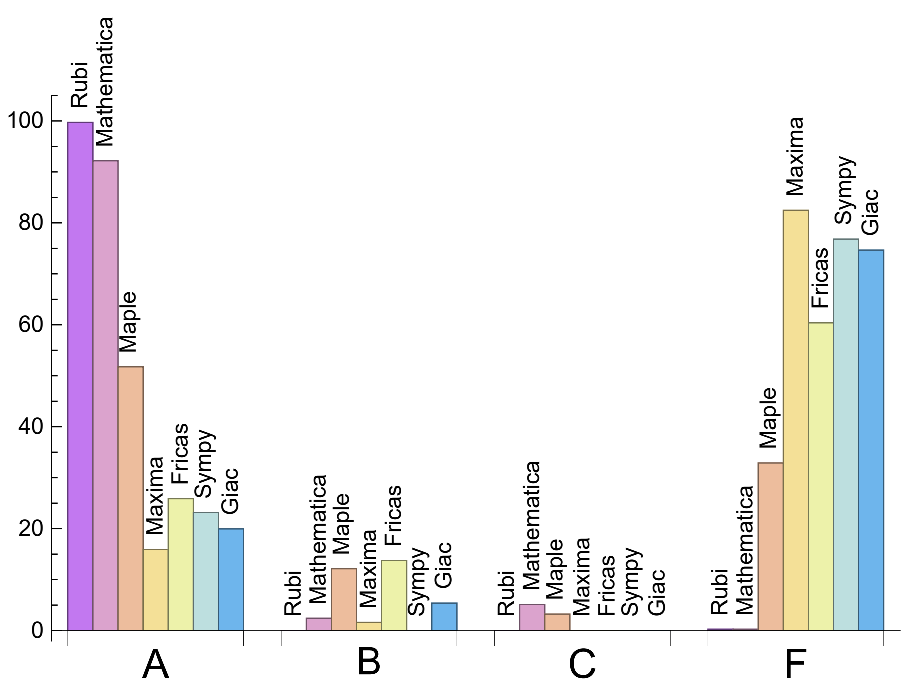
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.73	0.	0.	0.27
Mathematica	92.18	2.43	5.12	0.27
Maple	51.75	12.13	3.23	32.88
Maxima	15.9	1.62	0.	82.48
Fricas	25.88	13.75	0.	60.38
Sympy	23.18	0.	0.	76.82
Giac	19.95	5.39	0.	74.66

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	178.03	0.86	132.5	1.
Mathematica	3.58	202.52	0.89	115.	0.92
Maple	0.14	263.75	1.28	114.	1.15
Maxima	0.69	75.72	1.09	31.	0.87
Fricas	1.93	337.74	3.17	197.	2.87
Sympy	2.7	274.2	1.63	29.	1.18
Giac	0.69	69.97	1.23	0.	0.

1.4 list of integrals that has no closed form antiderivative

{23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {120, 132, 142, 144, 151, 153, 343, 344, 345}

Mathematica {37, 38, 42, 46, 51, 52, 74, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 120, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 278, 279, 325, 326, 327, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

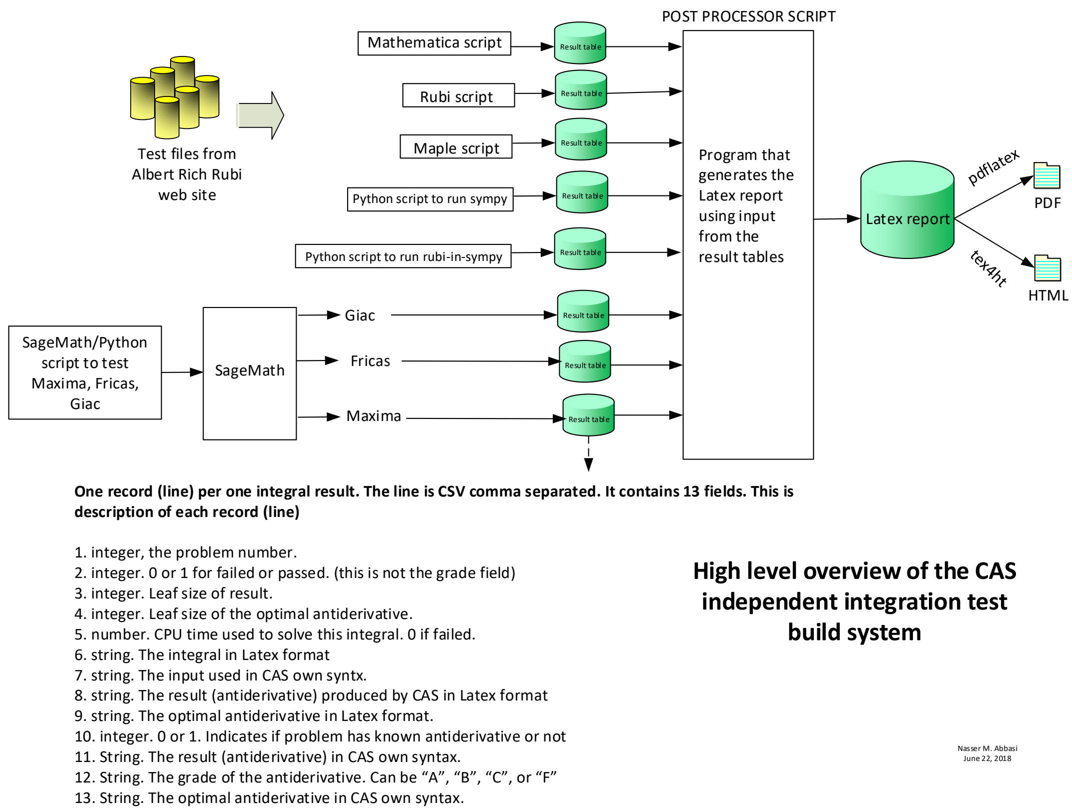
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371 }

B grade: { }

C grade: { }

F grade: { 369}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 287, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371 }

B grade: { 103, 104, 145, 152, 154, 196, 202, 302, 368 }

C grade: { 37, 38, 42, 46, 74, 125, 153, 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F grade: { 31}

2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 37, 39, 40, 41, 42, 43, 44, 45, 47, 49, 50, 53, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 135, 136, 137, 138, 139, 140, 141, 146, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 171, 172, 173, 177, 178, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 285, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 317, 324, 342, 345, 346, 347, 348, 349, 350, 351, 353, 362, 363, 364, 365, 366, 368 }

B grade: { 9, 10, 11, 18, 35, 36, 38, 46, 48, 51, 52, 62, 132, 134, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 162, 163, 168, 169, 170, 174, 175, 176, 266, 267, 268, 280, 343, 344, 352, 354, 355, 356, 357 }

C grade: { 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F grade: { 2, 3, 16, 31, 54, 55, 56, 71, 78, 79, 80, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 126, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 287, 291, 307, 308, 309, 310, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 358, 359, 360, 361, 367, 369, 370, 371 }

2.1.4 Maxima

A grade: { 4, 5, 6, 7, 12, 13, 14, 15, 23, 24, 28, 29, 32, 33, 57, 61, 84, 88, 92, 93, 97, 119, 155, 161, 167, 185, 191, 197, 203, 209, 215, 221, 227, 281, 282, 285, 289, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 306, 317, 324, 342, 346, 347, 362, 363, 366, 367, 369

B grade: { 122, 124, 134, 276, 283, 305 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

2.1.5 FriCAS

A grade: { 4, 5, 6, 7, 12, 13, 14, 23, 24, 28, 29, 30, 32, 33, 53, 57, 58, 59, 60, 61, 67, 68, 69, 75, 76, 77, 84, 88, 92, 93, 97, 118, 119, 136, 146, 155, 161, 167, 173, 179, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 268, 281, 282, 283, 285, 292, 293, 294, 296, 297, 298, 299, 301, 304, 305, 306, 315, 317, 322, 324, 342, 346, 347, 349, 350, 351, 353, 355, 356, 357, 362, 363, 366, 369, 371 }

B grade: { 9, 10, 11, 15, 63, 64, 65, 66, 70, 115, 116, 117, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 134, 137, 138, 139, 140, 141, 147, 148, 149, 150, 272, 273, 274, 275, 276, 277, 280, 289, 300, 302, 314, 316, 321, 323, 352, 354, 367, 368, 370 }

C grade: { }

F grade: { 1, 2, 3, 8, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 126, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 269, 270, 271, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 318, 319, 320, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365 }

2.1.6 Sympy

A grade: { 4, 5, 6, 7, 12, 13, 14, 15, 23, 24, 28, 29, 30, 32, 33, 57, 58, 59, 60, 61, 67, 68, 69, 70, 75, 76, 77, 84, 88, 92, 93, 97, 115, 116, 117, 118, 119, 127, 128, 129, 130, 131, 136, 137, 138, 139, 140, 141, 147, 148, 149, 150, 155, 161, 167, 185, 191, 209, 215, 221, 247, 248, 249, 250, 255, 256, 257, 258, 266, 267, 268, 272, 273, 274, 275, 276, 277, 281, 282, 283, 285, 294, 304, 362, 363, 366 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 142, 143, 144, 145, 146, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 367, 368, 369, 370, 371 }

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A grade: { 4, 5, 6, 7, 23, 24, 28, 29, 30, 32, 33, 53, 57, 58, 59, 60, 84, 88, 92, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 280, 281, 282, 283, 285, 292, 293, 294, 296, 297, 298, 299, 300, 301, 304, 306, 342, 346, 347, 349, 350, 351, 362, 363, 369 }

B grade: { 9, 15, 61, 116, 117, 118, 119, 121, 272, 273, 274, 275, 276, 277, 289, 302, 305, 352, 366, 367 }

C grade: { }

F grade: { 1, 2, 3, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 248, 256, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	168	263	0	0	0	0
normalized size	1	1.	0.99	1.55	0.	0.	0.	0.
time (sec)	N/A	0.264	0.01	0.076	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	240	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.405	0.13	0.112	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	322	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.433	0.043	0.113	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	166	259	343	468	316	400
normalized size	1	1.	0.94	1.47	1.95	2.66	1.8	2.27
time (sec)	N/A	0.17	0.138	0.008	1.049	2.435	2.127	1.667

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	121	189	219	324	190	265
normalized size	1	1.	0.98	1.52	1.77	2.61	1.53	2.14
time (sec)	N/A	0.096	0.085	0.005	1.057	2.544	0.964	1.561

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	96	127	197	99	169
normalized size	1	1.	0.94	0.99	1.31	2.03	1.02	1.74
time (sec)	N/A	0.052	0.039	0.006	1.078	2.374	0.403	1.587

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	41	95	26	55
normalized size	1	1.	1.	1.03	1.37	3.17	0.87	1.83
time (sec)	N/A	0.013	0.008	0.004	1.031	2.439	0.16	1.287

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	175	282	0	0	0	0
normalized size	1	1.	0.94	1.51	0.	0.	0.	0.
time (sec)	N/A	0.259	0.054	0.037	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	79	178	0	520	0	290
normalized size	1	1.	0.96	2.17	0.	6.34	0.	3.54
time (sec)	N/A	0.055	0.096	0.018	0.	2.741	0.	1.447

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	166	279	0	1146	0	0
normalized size	1	1.	1.3	2.18	0.	8.95	0.	0.
time (sec)	N/A	0.084	0.33	0.03	0.	3.441	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	205	516	0	1960	0	0
normalized size	1	1.	1.12	2.82	0.	10.71	0.	0.
time (sec)	N/A	0.138	0.388	0.012	0.	7.693	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	354	641	880	1010	743	0
normalized size	1	1.	0.96	1.74	2.39	2.74	2.02	0.
time (sec)	N/A	0.759	0.524	0.085	1.181	2.592	5.122	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	248	410	552	687	454	0
normalized size	1	1.	1.04	1.72	2.31	2.87	1.9	0.
time (sec)	N/A	0.512	0.368	0.061	1.385	2.578	2.325	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	142	193	338	406	233	0
normalized size	1	1.	1.01	1.38	2.41	2.9	1.66	0.
time (sec)	N/A	0.322	0.33	0.04	1.203	2.462	0.906	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	97	212	82	150
normalized size	1	1.	1.61	1.57	2.11	4.61	1.78	3.26
time (sec)	N/A	0.063	0.059	0.002	1.076	2.351	0.319	1.579

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	273	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.469	0.202	0.026	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	191	529	0	0	0	0
normalized size	1	1.	0.73	2.01	0.	0.	0.	0.
time (sec)	N/A	0.472	0.21	0.155	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	270	1013	0	0	0	0
normalized size	1	1.	0.77	2.9	0.	0.	0.	0.
time (sec)	N/A	0.609	0.874	0.262	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	305	394	0	0	0	0
normalized size	1	1.	0.77	1.	0.	0.	0.	0.
time (sec)	N/A	1.172	0.67	0.195	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	188	254	0	0	0	0
normalized size	1	1.	0.77	1.04	0.	0.	0.	0.
time (sec)	N/A	0.702	0.4	0.128	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	98	120	0	0	0	0
normalized size	1	1.	0.84	1.03	0.	0.	0.	0.
time (sec)	N/A	0.318	0.16	0.085	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	45	56	0	0	0	0
normalized size	1	1.	0.83	1.04	0.	0.	0.	0.
time (sec)	N/A	0.074	0.019	0.	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.201	0.158	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.376	0.23	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	351	288	616	0	0	0	0
normalized size	1	0.98	0.8	1.72	0.	0.	0.	0.
time (sec)	N/A	0.688	1.581	0.2	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	176	150	272	0	0	0	0
normalized size	1	0.98	0.83	1.51	0.	0.	0.	0.
time (sec)	N/A	0.337	0.734	0.125	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	81	71	118	0	0	0	0
normalized size	1	0.95	0.84	1.39	0.	0.	0.	0.
time (sec)	N/A	0.179	0.198	0.	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	3.076	0.171	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	5.081	0.25	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.261	3.938	2.824	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.043	2.781	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.342	0.891	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.738	0.93	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	640	413	1119	0	0	0	0
normalized size	1	1.	0.65	1.75	0.	0.	0.	0.
time (sec)	N/A	0.687	1.419	0.463	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	301	791	0	0	0	0
normalized size	1	1.	0.7	1.84	0.	0.	0.	0.
time (sec)	N/A	0.532	0.927	0.355	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	208	423	0	0	0	0
normalized size	1	1.	0.92	1.86	0.	0.	0.	0.
time (sec)	N/A	0.25	1.168	0.27	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	664	664	1353	992	0	0	0	0
normalized size	1	1.	2.04	1.49	0.	0.	0.	0.
time (sec)	N/A	1.65	6.045	0.247	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	781	781	1398	1814	0	0	0	0
normalized size	1	1.	1.79	2.32	0.	0.	0.	0.
time (sec)	N/A	2.547	9.769	0.324	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	918	918	779	1510	0	0	0	0
normalized size	1	1.	0.85	1.64	0.	0.	0.	0.
time (sec)	N/A	0.932	4.245	0.493	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	651	651	546	1087	0	0	0	0
normalized size	1	1.	0.84	1.67	0.	0.	0.	0.
time (sec)	N/A	0.731	2.193	0.388	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	392	601	0	0	0	0
normalized size	1	1.	1.11	1.7	0.	0.	0.	0.
time (sec)	N/A	0.335	1.206	0.284	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	984	984	2889	1838	0	0	0	0
normalized size	1	1.	2.94	1.87	0.	0.	0.	0.
time (sec)	N/A	1.889	13.539	0.22	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1228	1228	1899	1954	0	0	0	0
normalized size	1	1.	1.55	1.59	0.	0.	0.	0.
time (sec)	N/A	1.139	7.051	0.575	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	901	901	1047	1424	0	0	0	0
normalized size	1	1.	1.16	1.58	0.	0.	0.	0.
time (sec)	N/A	0.926	2.715	0.475	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	656	805	0	0	0	0
normalized size	1	1.	1.33	1.63	0.	0.	0.	0.
time (sec)	N/A	0.397	1.293	0.362	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1536	1536	7163	3928	0	0	0	0
normalized size	1	1.	4.66	2.56	0.	0.	0.	0.
time (sec)	N/A	2.448	25.453	0.303	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	304	751	0	0	0	0
normalized size	1	1.	0.71	1.75	0.	0.	0.	0.
time (sec)	N/A	0.575	0.921	0.348	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	233	486	0	0	0	0
normalized size	1	1.	0.9	1.88	0.	0.	0.	0.
time (sec)	N/A	0.43	0.58	0.266	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	158	209	0	0	0	0
normalized size	1	1.	1.32	1.74	0.	0.	0.	0.
time (sec)	N/A	0.217	0.254	0.178	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	48	77	0	0	0	0
normalized size	1	1.	1.02	1.64	0.	0.	0.	0.
time (sec)	N/A	0.057	0.036	0.006	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	256	678	0	0	0	0
normalized size	1	1.	0.79	2.09	0.	0.	0.	0.
time (sec)	N/A	0.548	0.623	0.132	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	448	1770	0	0	0	0
normalized size	1	1.	1.01	3.99	0.	0.	0.	0.
time (sec)	N/A	0.661	2.205	0.31	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.131	1.526	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	397	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.727	0.253	1.591	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	303	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.559	0.305	1.241	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	206	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.018	0.148	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	0.266	0.843	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	95	200	0	262	255	219
normalized size	1	1.	0.73	1.53	0.	2.	1.95	1.67
time (sec)	N/A	0.172	0.082	0.014	0.	2.83	1.814	1.293

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	74	130	0	216	170	177
normalized size	1	1.	0.82	1.44	0.	2.4	1.89	1.97
time (sec)	N/A	0.113	0.056	0.004	0.	2.567	0.844	1.304

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	60	74	0	178	104	150
normalized size	1	1.	0.79	0.97	0.	2.34	1.37	1.97
time (sec)	N/A	0.066	0.04	0.004	0.	2.52	0.365	1.297

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	40	31	41	135	46	124
normalized size	1	1.	1.18	0.91	1.21	3.97	1.35	3.65
time (sec)	N/A	0.014	0.027	0.001	1.08	2.47	0.184	1.285

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	153	388	0	0	0	0
normalized size	1	1.	1.17	2.96	0.	0.	0.	0.
time (sec)	N/A	0.243	0.012	0.082	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	71	0	405	0	0
normalized size	1	1.	1.	1.25	0.	7.11	0.	0.
time (sec)	N/A	0.07	0.039	0.006	0.	2.777	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	110	106	0	564	0	0
normalized size	1	1.	1.2	1.15	0.	6.13	0.	0.
time (sec)	N/A	0.102	0.166	0.008	0.	2.708	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	149	203	0	672	0	0
normalized size	1	1.	1.16	1.57	0.	5.21	0.	0.
time (sec)	N/A	0.154	0.217	0.009	0.	2.902	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	179	275	0	801	0	0
normalized size	1	1.	1.07	1.65	0.	4.8	0.	0.
time (sec)	N/A	0.228	0.201	0.007	0.	3.057	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	145	387	0	439	366	0
normalized size	1	1.	0.44	1.17	0.	1.33	1.11	0.
time (sec)	N/A	0.547	0.179	0.072	0.	2.495	4.164	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	107	219	0	352	243	0
normalized size	1	1.	0.51	1.04	0.	1.67	1.15	0.
time (sec)	N/A	0.375	0.137	0.056	0.	2.51	1.736	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	79	113	0	277	138	0
normalized size	1	1.	0.63	0.9	0.	2.2	1.1	0.
time (sec)	N/A	0.236	0.08	0.04	0.	2.543	0.785	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	46	0	217	63	0
normalized size	1	1.	1.04	1.02	0.	4.82	1.4	0.
time (sec)	N/A	0.051	0.022	0.027	0.	2.426	0.263	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	251	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.35	0.03	0.11	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	178	217	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.383	0.112	0.136	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	279	384	0	0	0	0
normalized size	1	1.	1.19	1.63	0.	0.	0.	0.
time (sec)	N/A	0.486	0.124	0.224	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	478	478	1830	730	0	0	0	0
normalized size	1	1.	3.83	1.53	0.	0.	0.	0.
time (sec)	N/A	1.573	10.313	0.39	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	175	326	0	556	432	0
normalized size	1	1.	0.49	0.92	0.	1.57	1.22	0.
time (sec)	N/A	0.45	0.198	0.06	0.	2.556	3.914	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	129	169	0	444	248	0
normalized size	1	1.	0.64	0.83	0.	2.19	1.22	0.
time (sec)	N/A	0.303	0.127	0.042	0.	2.468	1.656	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	67	0	346	109	0
normalized size	1	1.	0.9	0.86	0.	4.44	1.4	0.
time (sec)	N/A	0.074	0.029	0.028	0.	2.387	0.727	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	346	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.398	0.033	0.106	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	259	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.58	0.112	0.177	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	514	524	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.883	0.236	0.299	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	49	0	0	0	0
normalized size	1	1.	0.73	0.82	0.	0.	0.	0.
time (sec)	N/A	0.527	0.136	0.04	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	0	0	0
normalized size	1	1.	1.	0.9	0.	0.	0.	0.
time (sec)	N/A	0.211	0.051	0.03	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	0	0
normalized size	1	1.	1.	1.09	0.	0.	0.	0.
time (sec)	N/A	0.023	0.007	0.023	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.19	0.091	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	83	146	0	0	0	0
normalized size	1	1.	0.54	0.95	0.	0.	0.	0.
time (sec)	N/A	0.216	0.436	0.051	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	62	73	0	0	0	0
normalized size	1	1.	0.74	0.87	0.	0.	0.	0.
time (sec)	N/A	0.132	0.162	0.036	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	34	0	0	0	0
normalized size	1	1.	0.92	0.89	0.	0.	0.	0.
time (sec)	N/A	0.073	0.022	0.027	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	2.043	0.088	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	110	215	0	0	0	0
normalized size	1	1.	0.43	0.84	0.	0.	0.	0.
time (sec)	N/A	0.497	0.391	0.053	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	117	107	0	0	0	0
normalized size	1	1.	0.8	0.73	0.	0.	0.	0.
time (sec)	N/A	0.25	0.102	0.037	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	51	0	0	0	0
normalized size	1	1.	0.84	0.81	0.	0.	0.	0.
time (sec)	N/A	0.079	0.07	0.024	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.025	0.101	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.419	0.132	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	345	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	1.152	0.967	0.216	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	228	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.483	0.18	0.151	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	109	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.113	0.104	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.171	0.122	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	656	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.847	1.807	0.28	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	251	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.7	1.766	0.161	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.074	0.079	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	582	0	0	0	0	0
normalized size	1	1.	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.965	5.199	0.159	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	272	0	0	0	0	0
normalized size	1	1.	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.141	0.083	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	389	389	939	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.126	9.256	0.166	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	458	0	0	0	0	0
normalized size	1	1.	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.388	1.432	0.086	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	471	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.857	1.041	0.264	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	217	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.392	0.96	0.167	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	111	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.108	0.092	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	301	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.542	2.554	0.158	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	155	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	0.063	0.087	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	375	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.883	2.606	0.165	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	207	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	0.553	0.085	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	445	445	429	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	1.047	2.504	0.156	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	238	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.411	0.113	0.091	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	79	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.044	1.579	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	71	93	0	608	527	0
normalized size	1	1.	0.71	0.93	0.	6.08	5.27	0.
time (sec)	N/A	0.079	0.098	0.01	0.	2.433	4.663	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	86	0	495	394	807
normalized size	1	1.	0.79	0.82	0.	4.71	3.75	7.69
time (sec)	N/A	0.07	0.073	0.005	0.	2.343	2.329	2.721

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	73	0	365	258	548
normalized size	1	1.	0.84	0.96	0.	4.8	3.39	7.21
time (sec)	N/A	0.066	0.044	0.006	0.	2.395	1.211	2.287

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	57	62	0	257	148	331
normalized size	1	1.	0.84	0.91	0.	3.78	2.18	4.87
time (sec)	N/A	0.039	0.062	0.004	0.	2.401	0.437	1.964

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	36	47	154	51	134
normalized size	1	1.	1.28	0.92	1.21	3.95	1.31	3.44
time (sec)	N/A	0.023	0.028	0.002	1.118	2.424	0.183	1.396

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	70	159	0	0	0	0
normalized size	1	1.	0.86	1.96	0.	0.	0.	0.
time (sec)	N/A	0.112	0.024	0.033	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	54	0	410	0	177
normalized size	1	1.	0.9	1.1	0.	8.37	0.	3.61
time (sec)	N/A	0.054	0.032	0.005	0.	2.738	0.	1.353

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	47	60	158	257	0	0
normalized size	1	1.	0.8	1.02	2.68	4.36	0.	0.
time (sec)	N/A	0.051	0.05	0.003	1.139	2.636	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	74	74	0	760	0	0
normalized size	1	1.	0.88	0.88	0.	9.05	0.	0.
time (sec)	N/A	0.071	0.087	0.006	0.	2.828	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	61	80	348	448	0	0
normalized size	1	1.	0.68	0.89	3.87	4.98	0.	0.
time (sec)	N/A	0.067	0.057	0.004	1.236	2.812	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	64	94	0	1116	0	0
normalized size	1	1.	0.56	0.82	0.	9.7	0.	0.
time (sec)	N/A	0.086	0.034	0.006	0.	3.353	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	155	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.13	1.458	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	192	282	0	1331	1268	0
normalized size	1	1.	0.97	1.43	0.	6.76	6.44	0.
time (sec)	N/A	0.307	0.248	0.039	0.	2.694	11.104	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	170	229	0	1034	916	0
normalized size	1	1.	0.99	1.33	0.	6.01	5.33	0.
time (sec)	N/A	0.258	0.19	0.043	0.	2.789	6.293	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	147	192	0	767	610	0
normalized size	1	1.	1.08	1.41	0.	5.64	4.49	0.
time (sec)	N/A	0.205	0.166	0.033	0.	2.896	3.183	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	120	135	0	533	335	0
normalized size	1	1.	1.17	1.31	0.	5.17	3.25	0.
time (sec)	N/A	0.145	0.177	0.027	0.	2.969	1.094	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	87	90	0	333	143	0
normalized size	1	1.	1.53	1.58	0.	5.84	2.51	0.
time (sec)	N/A	0.07	0.096	0.028	0.	2.667	0.401	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	116	115	100	404	0	0	0	0
normalized size	1	0.99	0.86	3.48	0.	0.	0.	0.
time (sec)	N/A	0.198	0.046	0.031	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	154	229	0	0	0	0
normalized size	1	1.	1.54	2.29	0.	0.	0.	0.
time (sec)	N/A	0.176	0.591	0.046	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	120	180	311	714	0	0
normalized size	1	1.	1.41	2.12	3.66	8.4	0.	0.
time (sec)	N/A	0.137	0.238	0.062	1.296	3.043	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	212	310	0	0	0	0
normalized size	1	1.	1.25	1.83	0.	0.	0.	0.
time (sec)	N/A	0.247	1.679	0.08	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.173	3.502	1.48	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	355	548	0	2325	2518	0
normalized size	1	1.	1.09	1.68	0.	7.13	7.72	0.
time (sec)	N/A	0.474	0.466	0.05	0.	3.17	25.807	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	303	433	0	1773	1828	0
normalized size	1	1.	1.09	1.55	0.	6.35	6.55	0.
time (sec)	N/A	0.386	0.384	0.041	0.	3.051	12.647	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	258	360	0	1285	1173	0
normalized size	1	1.	1.14	1.59	0.	5.66	5.17	0.
time (sec)	N/A	0.3	0.311	0.037	0.	2.878	7.2	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	200	243	0	891	685	0
normalized size	1	1.	1.24	1.51	0.	5.53	4.25	0.
time (sec)	N/A	0.213	0.214	0.032	0.	2.78	2.8	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	147	160	0	548	282	0
normalized size	1	1.	1.47	1.6	0.	5.48	2.82	0.
time (sec)	N/A	0.108	0.157	0.029	0.	2.738	1.06	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	128	736	0	0	0	0
normalized size	1	1.	0.83	4.75	0.	0.	0.	0.
time (sec)	N/A	0.224	0.047	0.053	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	315	481	0	0	0	0
normalized size	1	1.	1.9	2.9	0.	0.	0.	0.
time (sec)	N/A	0.255	0.779	0.046	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	229	409	0	0	0	0
normalized size	1	1.	1.46	2.61	0.	0.	0.	0.
time (sec)	N/A	0.261	0.784	0.089	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	694	651	0	0	0	0
normalized size	1	1.	2.66	2.49	0.	0.	0.	0.
time (sec)	N/A	0.41	7.533	0.094	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	1.808	1.5	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	475	683	0	2631	2876	0
normalized size	1	1.	1.36	1.96	0.	7.54	8.24	0.
time (sec)	N/A	0.673	0.593	0.065	0.	3.345	28.844	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	412	567	0	1894	1889	0
normalized size	1	1.	1.47	2.02	0.	6.74	6.72	0.
time (sec)	N/A	0.485	0.441	0.04	0.	2.998	13.905	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	300	371	0	1287	1027	0
normalized size	1	1.	1.54	1.9	0.	6.6	5.27	0.
time (sec)	N/A	0.321	0.31	0.069	0.	2.97	6.312	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	226	245	0	784	444	0
normalized size	1	1.	1.97	2.13	0.	6.82	3.86	0.
time (sec)	N/A	0.16	0.25	0.03	0.	2.751	2.2	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	157	1153	0	0	0	0
normalized size	1	1.	0.84	6.2	0.	0.	0.	0.
time (sec)	N/A	0.259	0.064	0.06	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	501	820	0	0	0	0
normalized size	1	1.	2.14	3.5	0.	0.	0.	0.
time (sec)	N/A	0.317	1.764	0.047	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	360	723	0	0	0	0
normalized size	1	1.	1.94	3.89	0.	0.	0.	0.
time (sec)	N/A	0.328	1.232	0.066	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	1182	1202	0	0	0	0
normalized size	1	1.	3.07	3.12	0.	0.	0.	0.
time (sec)	N/A	0.565	8.928	0.126	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	1.115	0.622	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	209	151	194	0	0	0	0
normalized size	1	0.98	0.71	0.91	0.	0.	0.	0.
time (sec)	N/A	0.469	0.302	0.164	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	109	134	0	0	0	0
normalized size	1	1.	0.75	0.92	0.	0.	0.	0.
time (sec)	N/A	0.34	0.212	0.079	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	137	102	130	0	0	0	0
normalized size	1	0.97	0.72	0.92	0.	0.	0.	0.
time (sec)	N/A	0.293	0.178	0.071	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	61	66	0	0	0	0
normalized size	1	1.	0.88	0.96	0.	0.	0.	0.
time (sec)	N/A	0.161	0.087	0.036	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	49	60	0	0	0	0
normalized size	1	1.	0.84	1.03	0.	0.	0.	0.
time (sec)	N/A	0.089	0.022	0.027	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.815	0.135	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	252	281	602	0	0	0	0
normalized size	1	0.98	1.1	2.35	0.	0.	0.	0.
time (sec)	N/A	0.409	1.086	0.207	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	193	388	0	0	0	0
normalized size	1	1.	1.03	2.06	0.	0.	0.	0.
time (sec)	N/A	0.311	0.861	0.105	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	180	138	342	0	0	0	0
normalized size	1	0.98	0.75	1.86	0.	0.	0.	0.
time (sec)	N/A	0.285	0.733	0.093	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	97	160	0	0	0	0
normalized size	1	1.	0.94	1.55	0.	0.	0.	0.
time (sec)	N/A	0.148	0.295	0.049	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	87	179	128	0	0	0	0
normalized size	1	0.96	1.97	1.41	0.	0.	0.	0.
time (sec)	N/A	0.174	0.272	0.038	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.406	0.125	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	316	316	896	0	0	0	0
normalized size	1	0.99	0.99	2.8	0.	0.	0.	0.
time (sec)	N/A	0.892	1.238	0.224	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	179	579	0	0	0	0
normalized size	1	1.	0.72	2.34	0.	0.	0.	0.
time (sec)	N/A	0.682	0.661	0.127	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	305	216	507	0	0	0	0
normalized size	1	1.24	0.88	2.06	0.	0.	0.	0.
time (sec)	N/A	0.612	0.729	0.114	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	120	239	0	0	0	0
normalized size	1	1.	0.77	1.53	0.	0.	0.	0.
time (sec)	N/A	0.331	0.318	0.061	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	100	190	0	0	0	0
normalized size	1	1.	0.8	1.52	0.	0.	0.	0.
time (sec)	N/A	0.177	0.275	0.053	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	1.02	0.142	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	406	410	1244	0	0	0	0
normalized size	1	0.99	1.	3.03	0.	0.	0.	0.
time (sec)	N/A	0.878	1.914	0.258	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	318	800	0	0	0	0
normalized size	1	1.	0.94	2.35	0.	0.	0.	0.
time (sec)	N/A	0.696	1.12	0.151	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	327	258	709	0	0	0	0
normalized size	1	0.99	0.78	2.14	0.	0.	0.	0.
time (sec)	N/A	0.675	0.791	0.14	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	181	333	0	0	0	0
normalized size	1	1.	0.89	1.63	0.	0.	0.	0.
time (sec)	N/A	0.341	0.791	0.072	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	156	130	272	0	0	0	0
normalized size	1	0.98	0.81	1.7	0.	0.	0.	0.
time (sec)	N/A	0.267	0.436	0.062	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	3.366	0.161	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	342	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.875	0.629	0.329	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	223	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.665	0.282	0.195	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.65	0.39	0.209	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	140	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.429	0.098	0.092	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	0.082	0.	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	2.03	0.141	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	601	601	343	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.662	0.468	0.332	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	225	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	1.059	0.287	0.196	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	238	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.882	0.29	0.218	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	142	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.483	0.105	0.137	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	272	0	0	0	0	0
normalized size	1	1.	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.254	0.182	0.	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	1.415	0.158	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	701	701	342	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	2.203	0.698	0.323	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	455	223	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.52	0.314	0.194	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	394	394	238	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.244	0.413	0.203	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	126	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.731	0.076	0.093	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	458	0	0	0	0	0
normalized size	1	1.	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.402	1.329	0.	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	1.072	0.154	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	835	835	343	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	3.289	0.501	0.329	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	547	547	225	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	2.143	0.313	0.196	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	481	481	238	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.68	0.328	0.204	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	125	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.796	0.079	0.098	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	698	0	0	0	0	0
normalized size	1	1.	3.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.419	4.536	0.082	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	1.076	0.153	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	320	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.637	0.359	0.37	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	205	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.455	0.225	0.219	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	217	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.455	0.232	0.224	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	119	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.066	0.115	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	111	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.064	0.	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.065	0.171	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	490	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.651	0.634	0.354	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	253	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.453	0.444	0.224	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	327	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.465	0.359	0.247	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	147	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.112	0.105	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	155	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.114	0.	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.071	0.176	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	437	437	551	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	1.555	3.79	0.309	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	390	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	1.096	1.75	0.197	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	389	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.96	1.449	0.203	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	227	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.542	0.674	0.089	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	207	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.297	0.	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.076	0.158	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	531	531	701	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.467	2.585	0.298	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	420	420	429	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	1.098	2.061	0.192	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	410	410	474	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	1.11	1.443	0.208	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	235	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.562	0.946	0.099	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	238	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.465	0.189	0.	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.081	0.167	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	113	238	0	0	0	0
normalized size	1	1.	0.38	0.8	0.	0.	0.	0.
time (sec)	N/A	0.332	0.209	0.045	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	113	212	0	0	0	0
normalized size	1	1.	0.64	1.2	0.	0.	0.	0.
time (sec)	N/A	0.159	0.179	0.01	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	87	205	0	0	0	0
normalized size	1	1.	0.33	0.79	0.	0.	0.	0.
time (sec)	N/A	0.243	0.047	0.01	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	87	179	0	0	0	0
normalized size	1	1.	0.61	1.26	0.	0.	0.	0.
time (sec)	N/A	0.126	0.031	0.01	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	61	161	0	0	0	0
normalized size	1	1.	0.27	0.72	0.	0.	0.	0.
time (sec)	N/A	0.212	0.03	0.011	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	56	140	0	0	0	0
normalized size	1	1.	0.53	1.32	0.	0.	0.	0.
time (sec)	N/A	0.113	0.025	0.009	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	58	202	0	0	0	0
normalized size	1	1.	0.22	0.76	0.	0.	0.	0.
time (sec)	N/A	0.238	0.029	0.011	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	61	176	0	0	0	0
normalized size	1	1.	0.42	1.21	0.	0.	0.	0.
time (sec)	N/A	0.136	0.036	0.014	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.133	0.275	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	0.12	0.259	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	0.101	0.257	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.085	0.275	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.062	0.337	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	109	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.098	0.249	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	106	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.075	0.247	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.086	0.25	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	85.427	0.25	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	99.817	0.255	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	67.719	0.258	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	89.133	0.263	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	8.703	0.19	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	18.941	0.251	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	21.077	0.25	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	69.217	0.25	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	93.713	0.257	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	122.473	0.261	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	79.35	0.26	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	113.666	0.271	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	8.669	0.199	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	35.136	0.251	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	40.104	0.26	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	102.925	0.259	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	127	204	0	495	0	0
normalized size	1	1.	0.97	1.56	0.	3.78	0.	0.
time (sec)	N/A	0.189	0.108	0.084	0.	2.714	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	110	167	0	401	0	0
normalized size	1	1.	1.03	1.56	0.	3.75	0.	0.
time (sec)	N/A	0.119	0.085	0.052	0.	2.674	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	91	0	240	0	0
normalized size	1	1.	1.	1.49	0.	3.93	0.	0.
time (sec)	N/A	0.068	0.055	0.047	0.	2.607	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	28	0	0	0	0
normalized size	1	1.	0.77	0.9	0.	0.	0.	0.
time (sec)	N/A	0.119	0.063	0.052	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	47	44	0	0	0	0
normalized size	1	1.	1.31	1.22	0.	0.	0.	0.
time (sec)	N/A	0.115	0.058	0.049	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	85	63	0	0	0	0
normalized size	1	1.	1.23	0.91	0.	0.	0.	0.
time (sec)	N/A	0.106	0.267	0.054	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	266	592	0	792	694	0
normalized size	1	1.	1.13	2.52	0.	3.37	2.95	0.
time (sec)	N/A	0.309	0.228	0.066	0.	2.874	33.8	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	211	479	0	612	568	0
normalized size	1	1.	1.12	2.53	0.	3.24	3.01	0.
time (sec)	N/A	0.19	0.156	0.063	0.	2.819	16.214	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	124	262	0	374	298	0
normalized size	1	1.	1.17	2.47	0.	3.53	2.81	0.
time (sec)	N/A	0.104	0.091	0.055	0.	2.691	7.344	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	42	0	0	0	0
normalized size	1	1.	0.79	0.89	0.	0.	0.	0.
time (sec)	N/A	0.144	0.286	0.048	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	70	72	0	0	0	0
normalized size	1	1.	1.3	1.33	0.	0.	0.	0.
time (sec)	N/A	0.157	0.265	0.05	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	108	110	0	0	0	0
normalized size	1	1.	1.29	1.31	0.	0.	0.	0.
time (sec)	N/A	0.281	0.36	0.052	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	78	26	43
normalized size	1	1.	1.	0.93	0.	5.2	1.73	2.87
time (sec)	N/A	0.069	0.019	0.049	0.	2.562	1.408	1.713

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	78	26	43
normalized size	1	1.	1.	0.93	0.	5.2	1.73	2.87
time (sec)	N/A	0.068	0.018	0.04	0.	2.58	0.989	1.505

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	78	24	43
normalized size	1	1.	1.	0.93	0.	5.2	1.6	2.87
time (sec)	N/A	0.042	0.015	0.045	0.	2.706	0.821	1.391

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	77	22	42
normalized size	1	1.	1.	1.09	0.	7.	2.	3.82
time (sec)	N/A	0.075	0.03	0.043	0.	2.635	1.178	1.285

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	203	77	26	43
normalized size	1	1.	1.	1.08	15.62	5.92	2.	3.31
time (sec)	N/A	0.075	0.014	0.042	1.321	2.678	1.734	1.375

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	82	29	43
normalized size	1	1.	1.	0.93	0.	5.47	1.93	2.87
time (sec)	N/A	0.07	0.013	0.043	0.	2.613	2.688	1.402

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	128	203	0	0	0	0
normalized size	1	1.	1.11	1.77	0.	0.	0.	0.
time (sec)	N/A	0.206	0.57	0.1	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	98	168	0	0	0	0
normalized size	1	1.	1.14	1.95	0.	0.	0.	0.
time (sec)	N/A	0.16	0.382	0.096	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	62	131	0	274	0	103
normalized size	1	1.	1.35	2.85	0.	5.96	0.	2.24
time (sec)	N/A	0.058	0.089	0.094	0.	2.806	0.	1.319

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.551	0.113	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	54	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	2.696	0.097	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	67	138	115	42	100
normalized size	1	1.	0.88	1.34	2.76	2.3	0.84	2.
time (sec)	N/A	0.037	0.018	0.03	1.166	2.589	1.243	1.325

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	89	0	0	0	0
normalized size	1	1.	0.74	0.88	0.	0.	0.	0.
time (sec)	N/A	0.044	0.121	0.016	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	41	89	27	54
normalized size	1	1.	1.	0.91	1.21	2.62	0.79	1.59
time (sec)	N/A	0.022	0.015	0.001	1.148	2.655	0.252	1.3

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	35	77	0	0	0	0
normalized size	1	1.	0.22	0.48	0.	0.	0.	0.
time (sec)	N/A	0.054	0.005	0.005	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.007	0.048	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	42	66	0	0	0	0
normalized size	1	1.	0.56	0.88	0.	0.	0.	0.
time (sec)	N/A	0.023	0.037	0.006	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	62	242	0	78
normalized size	1	1.	1.	0.85	1.88	7.33	0.	2.36
time (sec)	N/A	0.026	0.006	0.01	1.111	3.029	0.	1.264

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	88	101	0	0	0	0
normalized size	1	1.	0.45	0.51	0.	0.	0.	0.
time (sec)	N/A	0.09	0.156	0.009	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.008	0.036	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	43	47	62	128	0	68
normalized size	1	1.	0.6	0.65	0.86	1.78	0.	0.94
time (sec)	N/A	0.025	0.021	0.01	1.697	2.708	0.	1.381

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	37	37	49	112	0	65
normalized size	1	1.	0.66	0.66	0.88	2.	0.	1.16
time (sec)	N/A	0.017	0.017	0.004	1.689	2.69	0.	1.314

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	24	31	92	29	54
normalized size	1	1.	0.94	0.69	0.89	2.63	0.83	1.54
time (sec)	N/A	0.011	0.038	0.002	1.752	2.613	0.359	1.579

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	78	0	0	0	0
normalized size	1	1.	1.	1.7	0.	0.	0.	0.
time (sec)	N/A	0.064	0.007	0.026	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	74	0	47
normalized size	1	1.	1.	0.81	1.04	2.85	0.	1.81
time (sec)	N/A	0.013	0.009	0.005	1.612	2.668	0.	1.213

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	41	97	0	70
normalized size	1	1.	0.74	0.67	0.89	2.11	0.	1.52
time (sec)	N/A	0.017	0.012	0.004	1.66	2.849	0.	1.352

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	39	41	54	112	0	90
normalized size	1	1.	0.63	0.66	0.87	1.81	0.	1.45
time (sec)	N/A	0.022	0.015	0.006	1.661	3.056	0.	1.323

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	44	51	68	124	0	111
normalized size	1	1.	0.56	0.65	0.87	1.59	0.	1.42
time (sec)	N/A	0.027	0.017	0.006	1.809	3.185	0.	1.356

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	54	93	290	0	103
normalized size	1	1.	1.02	0.96	1.66	5.18	0.	1.84
time (sec)	N/A	0.04	0.038	0.017	1.13	2.952	0.	1.327

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	36	105	0	65
normalized size	1	1.	0.88	1.15	1.09	3.18	0.	1.97
time (sec)	N/A	0.017	0.022	0.005	1.044	2.68	0.	1.315

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	77	31	58	219	0	68
normalized size	1	1.	3.08	1.24	2.32	8.76	0.	2.72
time (sec)	N/A	0.017	0.089	0.006	1.135	2.752	0.	1.321

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	114	0	0	0	0
normalized size	1	1.	1.	2.19	0.	0.	0.	0.
time (sec)	N/A	0.062	0.007	0.006	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	31	41	101	20	53
normalized size	1	1.	1.	1.07	1.41	3.48	0.69	1.83
time (sec)	N/A	0.024	0.015	0.003	1.113	2.683	2.662	1.369

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	46	131	130	0	124
normalized size	1	1.	0.88	0.92	2.62	2.6	0.	2.48
time (sec)	N/A	0.034	0.022	0.004	1.1	2.682	0.	1.419

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	53	63	136	0	101
normalized size	1	1.	0.89	0.98	1.17	2.52	0.	1.87
time (sec)	N/A	0.04	0.028	0.006	1.209	2.756	0.	1.394

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	74	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.074	0.07	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.047	0.014	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.046	0.01	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.025	0.011	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	133	0	0	0	0
normalized size	1	1.	1.	2.22	0.	0.	0.	0.
time (sec)	N/A	0.066	0.008	0.003	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.053	0.01	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.044	0.011	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	648	0	0
normalized size	1	1.	0.97	0.	0.	4.24	0.	0.
time (sec)	N/A	0.036	0.117	0.159	0.	2.824	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	180	0	0	458	0	0
normalized size	1	1.	1.4	0.	0.	3.55	0.	0.
time (sec)	N/A	0.064	0.134	0.105	0.	2.729	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	288	0	0
normalized size	1	1.	1.	0.	0.	3.79	0.	0.
time (sec)	N/A	0.015	0.026	0.112	0.	2.645	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	47	59	138	0	0
normalized size	1	1.	0.96	0.94	1.18	2.76	0.	0.
time (sec)	N/A	0.04	0.024	0.014	1.214	2.677	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	150	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.711	0.066	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	197	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	1.322	0.064	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	229	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.552	0.066	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	648	0	0
normalized size	1	1.	0.97	0.	0.	4.24	0.	0.
time (sec)	N/A	0.032	0.117	0.125	0.	2.798	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	180	0	0	458	0	0
normalized size	1	1.	1.4	0.	0.	3.55	0.	0.
time (sec)	N/A	0.059	0.133	0.108	0.	2.796	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	288	0	0
normalized size	1	1.	1.	0.	0.	3.79	0.	0.
time (sec)	N/A	0.013	0.017	0.11	0.	2.618	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	48	59	138	0	0
normalized size	1	1.	0.96	0.96	1.18	2.76	0.	0.
time (sec)	N/A	0.039	0.027	0.015	1.192	2.649	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	146	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.64	0.058	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	196	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	1.367	0.068	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	227	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.72	0.06	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	337	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.271	0.102	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	258	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.216	0.058	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	259	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.052	0.065	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.004	0.063	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.366	0.062	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	308	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.774	0.059	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	365	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.921	0.061	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	337	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.278	0.066	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	255	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.253	0.059	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	259	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.052	0.062	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.004	0.063	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.357	0.065	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	308	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.787	0.06	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	370	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.914	0.062	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.089	0.46	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	244	1175	0	0	0	0
normalized size	1	1.	0.93	4.5	0.	0.	0.	0.
time (sec)	N/A	0.226	0.057	0.72	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	194	195	187	649	0	0	0	0
normalized size	1	1.01	0.96	3.35	0.	0.	0.	0.
time (sec)	N/A	0.216	0.06	0.007	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	127	263	0	0	0	0
normalized size	1	1.	0.95	1.98	0.	0.	0.	0.
time (sec)	N/A	0.122	0.028	0.006	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.093	0.24	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.774	0.237	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	166	0	0	0	0
normalized size	1	1.	1.	2.18	0.	0.	0.	0.
time (sec)	N/A	0.076	0.491	0.023	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	119	322	0	321	0	234
normalized size	1	1.	0.72	1.95	0.	1.95	0.	1.42
time (sec)	N/A	0.172	0.081	0.006	0.	2.85	0.	1.327

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	102	264	0	266	0	189
normalized size	1	1.	0.89	2.3	0.	2.31	0.	1.64
time (sec)	N/A	0.125	0.105	0.005	0.	2.891	0.	1.36

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	73	138	0	212	0	143
normalized size	1	1.	1.09	2.06	0.	3.16	0.	2.13
time (sec)	N/A	0.071	0.099	0.003	0.	2.606	0.	1.346

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	31	31	46	89	0	169	0	108
normalized size	1	1.	1.48	2.87	0.	5.45	0.	3.48
time (sec)	N/A	0.017	0.032	0.003	0.	2.587	0.	1.482

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	99	126	0	335	0	0
normalized size	1	1.	1.11	1.42	0.	3.76	0.	0.
time (sec)	N/A	0.121	0.074	0.004	0.	2.624	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	110	267	0	437	0	0
normalized size	1	1.	1.11	2.7	0.	4.41	0.	0.
time (sec)	N/A	0.105	0.159	0.009	0.	2.723	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	129	457	0	423	0	0
normalized size	1	1.	1.11	3.94	0.	3.65	0.	0.
time (sec)	N/A	0.086	0.194	0.009	0.	2.637	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	162	501	0	533	0	0
normalized size	1	1.	1.04	3.21	0.	3.42	0.	0.
time (sec)	N/A	0.108	0.107	0.009	0.	2.641	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	192	841	0	683	0	0
normalized size	1	1.	0.93	4.06	0.	3.3	0.	0.
time (sec)	N/A	0.171	0.739	0.01	0.	2.879	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	198	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.743	0.303	0.01	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	138	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.516	0.183	0.01	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.277	0.093	0.01	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.032	0.005	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.128	0.009	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.428	0.008	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	125	0	0	0	0
normalized size	1	1.	0.87	2.08	0.	0.	0.	0.
time (sec)	N/A	0.096	0.017	0.037	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	0	0	0
normalized size	1	1.	1.	1.33	0.	0.	0.	0.
time (sec)	N/A	0.06	0.058	0.047	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	38	50	151	61	142
normalized size	1	1.	0.91	0.84	1.11	3.36	1.36	3.16
time (sec)	N/A	0.049	0.025	0.003	1.112	2.429	1.13	1.39

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	41	0	53	443	0	153
normalized size	1	1.	0.89	0.	1.15	9.63	0.	3.33
time (sec)	N/A	0.05	0.037	0.067	1.135	2.781	0.	1.353

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	131	46	0	532	0	0
normalized size	1	1.	2.67	0.94	0.	10.86	0.	0.
time (sec)	N/A	0.032	0.118	0.02	0.	2.627	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	28	0	1	4	0	1
normalized size	1	0.	1.04	0.	0.04	0.15	0.	0.04
time (sec)	N/A	0.039	0.519	0.041	1.818	2.208	0.	1.352

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	282	0	0
normalized size	1	1.	1.	0.	0.	7.62	0.	0.
time (sec)	N/A	0.067	0.04	0.177	0.	2.789	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	0	0	80	0	0
normalized size	1	1.	0.83	0.	0.	2.76	0.	0.
time (sec)	N/A	0.062	0.021	0.146	0.	2.426	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [74] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	5	1.	12	0.417
2	A	10	6	1.	14	0.429
3	A	12	7	1.	14	0.5
4	A	5	5	1.	16	0.312
5	A	4	4	1.	16	0.25
6	A	4	4	1.	14	0.286
7	A	3	2	1.	8	0.25
8	A	8	5	1.	16	0.312
9	A	3	3	1.	16	0.188
10	A	4	4	1.	16	0.25
11	A	5	5	1.	16	0.312
12	A	18	7	1.	18	0.389
13	A	13	7	1.	18	0.389
14	A	9	7	1.	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	3	3	1.	10	0.3
16	A	10	6	1.	18	0.333
17	A	10	7	1.	18	0.389
18	A	13	10	1.	18	0.556
19	A	27	7	1.	18	0.389
20	A	17	6	1.	18	0.333
21	A	11	7	1.	16	0.438
22	A	4	4	1.	10	0.4
23	A	0	0	0.	0	0.
24	A	0	0	0.	0	0.
25	A	19	7	0.98	18	0.389
26	A	11	7	0.98	16	0.438
27	A	5	5	0.95	10	0.5
28	A	0	0	0.	0	0.
29	A	0	0	0.	0	0.
30	A	0	0	0.	0	0.
31	A	3	3	1.	16	0.188
32	A	0	0	0.	0	0.
33	A	0	0	0.	0	0.
34	A	16	12	1.	30	0.4
35	A	13	8	1.	30	0.267
36	A	8	6	1.	28	0.214
37	A	22	20	1.	30	0.667
38	A	35	22	1.	30	0.733
39	A	24	17	1.	30	0.567
40	A	20	12	1.	30	0.4
41	A	12	9	1.	28	0.321
42	A	29	24	1.	30	0.8
43	A	30	18	1.	30	0.6
44	A	26	15	1.	30	0.5
45	A	14	10	1.	28	0.357
46	A	37	29	1.	30	0.967
47	A	13	7	1.	30	0.233
48	A	9	7	1.	30	0.233
49	A	6	5	1.	28	0.179
50	A	2	2	1.	23	0.087
51	A	10	7	1.	30	0.233
52	A	13	10	1.	30	0.333
53	A	0	0	0.	0	0.
54	A	13	9	1.	34	0.265
55	A	11	8	1.	32	0.25
56	A	9	7	1.	24	0.292
57	A	0	0	0.	0	0.
58	A	6	6	1.	10	0.6

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	5	5	1.	10	0.5
60	A	5	5	1.	8	0.625
61	A	3	3	1.	6	0.5
62	A	9	6	1.	10	0.6
63	A	4	4	1.	10	0.4
64	A	5	5	1.	10	0.5
65	A	6	6	1.	10	0.6
66	A	7	7	1.	10	0.7
67	A	19	8	1.	12	0.667
68	A	14	8	1.	12	0.667
69	A	10	8	1.	10	0.8
70	A	4	4	1.	8	0.5
71	A	11	7	1.	12	0.583
72	A	11	8	1.	12	0.667
73	A	14	11	1.	12	0.917
74	A	40	16	1.	12	1.333
75	A	18	11	1.	12	0.917
76	A	12	10	1.	10	1.
77	A	5	4	1.	8	0.5
78	A	13	8	1.	12	0.667
79	A	13	9	1.	12	0.75
80	A	21	13	1.	12	1.083
81	A	14	8	1.	12	0.667
82	A	10	8	1.	10	0.8
83	A	3	3	1.	8	0.375
84	A	0	0	0.	0	0.
85	A	12	7	1.	12	0.583
86	A	8	7	1.	10	0.7
87	A	4	4	1.	8	0.5
88	A	0	0	0.	0	0.
89	A	24	12	1.	12	1.
90	A	14	12	1.	10	1.2
91	A	5	5	1.	8	0.625
92	A	0	0	0.	0	0.
93	A	0	0	0.	0	0.
94	A	22	9	1.	16	0.562
95	A	14	9	1.	14	0.643
96	A	5	4	1.	12	0.333
97	A	0	0	0.	0	0.
98	A	23	12	1.	18	0.667
99	A	14	10	1.	16	0.625
100	A	8	7	1.	14	0.5
101	A	16	10	1.	16	0.625
102	A	9	8	1.	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
103	A	18	10	1.	16	0.625
104	A	10	8	1.	14	0.571
105	A	20	9	1.	18	0.5
106	A	12	8	1.	16	0.5
107	A	7	6	1.	14	0.429
108	A	16	10	1.	16	0.625
109	A	8	7	1.	14	0.5
110	A	22	15	1.	16	0.938
111	A	9	8	1.	14	0.571
112	A	21	13	1.	16	0.812
113	A	10	8	1.	14	0.571
114	A	3	3	1.	21	0.143
115	A	6	5	1.	21	0.238
116	A	6	5	1.	21	0.238
117	A	6	5	1.	21	0.238
118	A	5	5	1.	19	0.263
119	A	4	3	1.	10	0.3
120	A	7	7	1.	21	0.333
121	A	6	6	1.	21	0.286
122	A	4	4	1.	21	0.19
123	A	7	7	1.	21	0.333
124	A	5	5	1.	21	0.238
125	A	8	7	1.	21	0.333
126	A	3	3	1.	23	0.13
127	A	9	7	1.	23	0.304
128	A	8	6	1.	23	0.261
129	A	7	7	1.	23	0.304
130	A	6	6	1.	21	0.286
131	A	4	4	1.	12	0.333
132	A	8	8	0.99	23	0.348
133	A	9	7	1.	23	0.304
134	A	5	5	1.	23	0.217
135	A	11	9	1.	23	0.391
136	A	0	0	0.	0	0.
137	A	17	9	1.	23	0.391
138	A	13	7	1.	23	0.304
139	A	12	9	1.	23	0.391
140	A	8	7	1.	21	0.333
141	A	6	4	1.	12	0.333
142	A	9	9	1.	23	0.391
143	A	11	8	1.	23	0.348
144	A	9	9	1.	23	0.391
145	A	16	12	1.	23	0.522
146	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
147	A	16	6	1.	23	0.261
148	A	13	8	1.	23	0.348
149	A	9	6	1.	21	0.286
150	A	6	4	1.	12	0.333
151	A	10	9	1.	23	0.391
152	A	13	9	1.	23	0.391
153	A	10	10	1.	23	0.435
154	A	21	12	1.	23	0.522
155	A	0	0	0.	0	0.
156	A	14	7	0.98	23	0.304
157	A	11	7	1.	23	0.304
158	A	11	7	0.97	23	0.304
159	A	8	7	1.	21	0.333
160	A	5	5	1.	12	0.417
161	A	0	0	0.	0	0.
162	A	13	6	0.98	23	0.261
163	A	10	6	1.	23	0.261
164	A	10	6	0.98	23	0.261
165	A	6	6	1.	21	0.286
166	A	6	6	0.96	12	0.5
167	A	0	0	0.	0	0.
168	A	26	9	0.99	23	0.391
169	A	20	9	1.	23	0.391
170	A	18	10	1.24	23	0.435
171	A	11	10	1.	21	0.476
172	A	7	7	1.	12	0.583
173	A	0	0	0.	0	0.
174	A	24	8	0.99	23	0.348
175	A	17	8	1.	23	0.348
176	A	18	10	0.99	23	0.435
177	A	9	9	1.	21	0.429
178	A	8	7	0.98	12	0.583
179	A	0	0	0.	0	0.
180	A	21	9	1.	25	0.36
181	A	16	9	1.	25	0.36
182	A	16	9	1.	25	0.36
183	A	11	9	1.	23	0.391
184	A	8	7	1.	14	0.5
185	A	0	0	0.	0	0.
186	A	43	12	1.	25	0.48
187	A	27	11	1.	25	0.44
188	A	24	12	1.	25	0.48
189	A	13	11	1.	23	0.478
190	A	9	8	1.	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
191	A	0	0	0.	0	0.
192	A	46	12	1.	25	0.48
193	A	29	11	1.	25	0.44
194	A	26	12	1.	25	0.48
195	A	14	11	1.	23	0.478
196	A	10	8	1.	14	0.571
197	A	0	0	0.	0	0.
198	A	77	13	1.	25	0.52
199	A	42	11	1.	25	0.44
200	A	35	13	1.	25	0.52
201	A	16	11	1.	23	0.478
202	A	11	8	1.	14	0.571
203	A	0	0	0.	0	0.
204	A	20	8	1.	25	0.32
205	A	15	8	1.	25	0.32
206	A	15	8	1.	25	0.32
207	A	10	8	1.	23	0.348
208	A	7	6	1.	14	0.429
209	A	0	0	0.	0	0.
210	A	19	7	1.	25	0.28
211	A	14	7	1.	25	0.28
212	A	14	7	1.	25	0.28
213	A	8	7	1.	23	0.304
214	A	8	7	1.	14	0.5
215	A	0	0	0.	0	0.
216	A	36	10	1.	25	0.4
217	A	26	10	1.	25	0.4
218	A	24	11	1.	25	0.44
219	A	13	11	1.	23	0.478
220	A	9	8	1.	14	0.571
221	A	0	0	0.	0	0.
222	A	34	9	1.	25	0.36
223	A	23	9	1.	25	0.36
224	A	24	11	1.	25	0.44
225	A	11	10	1.	23	0.435
226	A	10	8	1.	14	0.571
227	A	0	0	0.	0	0.
228	A	8	7	1.	23	0.304
229	A	6	5	1.	23	0.217
230	A	7	7	1.	23	0.304
231	A	5	5	1.	23	0.217
232	A	6	6	1.	23	0.261
233	A	4	4	1.	23	0.174
234	A	7	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	5	1.	23	0.217
236	A	3	3	1.	25	0.12
237	A	3	3	1.	25	0.12
238	A	3	3	1.	25	0.12
239	A	3	3	1.	25	0.12
240	A	3	3	1.	25	0.12
241	A	3	3	1.	25	0.12
242	A	3	3	1.	25	0.12
243	A	3	3	1.	25	0.12
244	A	0	0	0.	0	0.
245	A	0	0	0.	0	0.
246	A	0	0	0.	0	0.
247	A	0	0	0.	0	0.
248	A	0	0	0.	0	0.
249	A	0	0	0.	0	0.
250	A	0	0	0.	0	0.
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	0	0	0.	0	0.
254	A	0	0	0.	0	0.
255	A	0	0	0.	0	0.
256	A	0	0	0.	0	0.
257	A	0	0	0.	0	0.
258	A	0	0	0.	0	0.
259	A	0	0	0.	0	0.
260	A	7	6	1.	30	0.2
261	A	6	6	1.	30	0.2
262	A	4	4	1.	28	0.143
263	A	5	4	1.	30	0.133
264	A	6	6	1.	30	0.2
265	A	4	4	1.	30	0.133
266	A	15	9	1.	30	0.3
267	A	11	9	1.	30	0.3
268	A	7	6	1.	28	0.214
269	A	6	4	1.	30	0.133
270	A	7	5	1.	30	0.167
271	A	11	8	1.	30	0.267
272	A	2	2	1.	30	0.067
273	A	2	2	1.	30	0.067
274	A	2	2	1.	28	0.071
275	A	2	2	1.	30	0.067
276	A	2	2	1.	30	0.067
277	A	2	2	1.	30	0.067
278	A	8	8	1.	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
279	A	7	7	1.	30	0.233
280	A	3	3	1.	28	0.107
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	5	5	1.	10	0.5
284	A	4	4	1.	10	0.4
285	A	3	3	1.	8	0.375
286	A	5	5	1.	6	0.833
287	A	5	5	1.	10	0.5
288	A	3	3	1.	10	0.3
289	A	5	5	1.	10	0.5
290	A	6	6	1.	10	0.6
291	A	5	5	1.	10	0.5
292	A	7	5	1.	10	0.5
293	A	6	5	1.	8	0.625
294	A	6	6	1.	6	1.
295	A	5	5	1.	10	0.5
296	A	3	3	1.	10	0.3
297	A	4	4	1.	10	0.4
298	A	5	4	1.	10	0.4
299	A	6	4	1.	10	0.4
300	A	6	6	1.	10	0.6
301	A	3	3	1.	8	0.375
302	A	5	5	1.	6	0.833
303	A	5	5	1.	10	0.5
304	A	3	3	1.	10	0.3
305	A	5	5	1.	10	0.5
306	A	5	4	1.	10	0.4
307	A	3	3	1.	10	0.3
308	A	3	3	1.	10	0.3
309	A	3	3	1.	8	0.375
310	A	3	3	1.	6	0.5
311	A	5	5	1.	10	0.5
312	A	3	3	1.	10	0.3
313	A	3	3	1.	10	0.3
314	A	3	2	1.	20	0.1
315	A	5	4	1.	20	0.2
316	A	2	2	1.	20	0.1
317	A	4	3	1.	18	0.167
318	A	1	1	1.	20	0.05
319	A	1	1	1.	20	0.05
320	A	2	2	1.	20	0.1
321	A	3	2	1.	20	0.1
322	A	5	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	2	2	1.	20	0.1
324	A	4	3	1.	18	0.167
325	A	1	1	1.	20	0.05
326	A	1	1	1.	20	0.05
327	A	2	2	1.	20	0.1
328	A	2	2	1.	22	0.091
329	A	2	2	1.	22	0.091
330	A	1	1	1.	22	0.045
331	A	1	1	1.	22	0.045
332	A	1	1	1.	22	0.045
333	A	2	2	1.	22	0.091
334	A	2	2	1.	22	0.091
335	A	2	2	1.	22	0.091
336	A	2	2	1.	22	0.091
337	A	1	1	1.	22	0.045
338	A	1	1	1.	22	0.045
339	A	1	1	1.	22	0.045
340	A	2	2	1.	22	0.091
341	A	2	2	1.	22	0.091
342	A	0	0	0.	0	0.
343	A	8	8	1.	40	0.2
344	A	7	7	1.01	40	0.175
345	A	6	7	1.	38	0.184
346	A	0	0	0.	0	0.
347	A	0	0	0.	0	0.
348	A	6	6	1.	10	0.6
349	A	5	4	1.	12	0.333
350	A	5	4	1.	12	0.333
351	A	5	4	1.	10	0.4
352	A	5	4	1.	8	0.5
353	A	9	8	1.	12	0.667
354	A	9	8	1.	12	0.667
355	A	6	5	1.	12	0.417
356	A	7	6	1.	12	0.5
357	A	8	7	1.	12	0.583
358	A	37	8	1.	14	0.571
359	A	27	8	1.	14	0.571
360	A	17	8	1.	12	0.667
361	A	7	4	1.	10	0.4
362	A	0	0	0.	0	0.
363	A	0	0	0.	0	0.
364	A	7	7	1.	19	0.368
365	A	2	2	1.	15	0.133
366	A	4	4	1.	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	4	4	1.	14	0.286
368	A	6	6	1.	10	0.6
369	F	0	0	N/A	0	N/A
370	A	2	2	1.	26	0.077
371	A	2	2	1.	26	0.077

Chapter 3

Listing of integrals

3.1 $\int \frac{\sinh^{-1}(cx)}{d+ex} dx$

Optimal. Leaf size=170

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+\dots}\right.}{e}$$

```
[Out] -ArcSinh[c*x]^2/(2*e) + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Rubi [A] time = 0.264292, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5799, 5561, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+\dots}\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^2/(2*e) + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{x\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \text{Subst}\left(\int \frac{e^x x}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x x}{cd+\sqrt{c^2d^2+e^2}+e\ldots}\right.$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{\text{Subst}\left(\int \log\ldots\right.}{\ldots}$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{\log(\ldots}{\ldots}\right.}{\ldots}$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\text{Li}_2\left(-\frac{ee^{\sinh^{-1}\ldots}}{cd-\sqrt{c^2d^2\ldots}}\right.}{e}$$

Mathematica [A] time = 0.009979, size = 168, normalized size = 0.99

$$\frac{\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)}{e} + \frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\ldots\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^2/(2*e) + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 + e^2])]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Maple [A] time = 0.076, size = 263, normalized size = 1.6

$$-\frac{(\text{Arcsinh}(cx))^2}{2e}+\frac{\text{Arcsinh}(cx)}{e}\ln\left(\left(-\left(cx+\sqrt{c^2x^2+1}\right)e-cd+\sqrt{c^2d^2+e^2}\right)\left(-cd+\sqrt{c^2d^2+e^2}\right)^{-1}\right)+\frac{\text{Arcsinh}(c}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)/(e*x+d),x)
```

```
[Out] -1/2*arcsinh(c*x)^2/e+1/e*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(
c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+1/e*arcsinh(c*x)*ln(((c*x+(
c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))+1/e
*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^
2)^(1/2)))+1/e*dilog((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1/2))/(
-c*d+(c^2*d^2+e^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}(cx)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

3.2 $\int \frac{\sinh^{-1}(cx)^2}{d+ex}\,dx$

Optimal. Leaf size=260

$$\frac{2\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{2\text{PolyLog}\left(3,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\text{F}}{}$$

```
[Out] -ArcSinh[c*x]^3/(3*e) + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*
x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rubi [A] time = 0.404947, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5799, 5561, 2190, 2531, 2282, 6589}

$$\frac{2\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{2\text{PolyLog}\left(3,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\text{F}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^3/(3*e) + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*
x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$
$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x x^2}{cd+\sqrt{c^2d^2+e^2}+\dots}\right.$$
$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{2\,\text{Subst}\left(\int\dots\right.}{\dots}$$
$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\sinh^{-1}(c\dots}{\dots}$$
$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\sinh^{-1}(c\dots}{\dots}$$
$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\sinh^{-1}(c\dots}{\dots}$$

Mathematica [A] time = 0.129949, size = 240, normalized size = 0.92

$$-\frac{-6\sinh^{-1}(cx)\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right) - 6\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right) + 6\text{PolyLog}\left(3,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right) + 6\text{PolyI}\dots}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -(ArcSinh[c*x]^3 - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[
c^2*d^2 + e^2])] - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[
```

```
c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + S
qrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d
+ Sqrt[c^2*d^2 + e^2]))] + 6*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c
^2*d^2 + e^2])] + 6*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e
^2]))])/(3*e)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{Arcsinh}(cx))^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)^2/(e*x+d),x)
```

```
[Out] int(arcsinh(c*x)^2/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)^2/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)**2/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

3.3 $\int \frac{\sinh^{-1}(cx)^3}{d+ex}\,dx$

Optimal. Leaf size=348

$$\frac{3\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{3\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{6\sinh^{-1}(cx)\text{PolyLog}\left(3,-\frac{ee^{\sin}}{cd-\sqrt{}}\right.}{e}$$

```
[Out] -ArcSinh[c*x]^4/(4*e) + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^A
rcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*PolyLog[3,
-((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[
4, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rubi [A] time = 0.433145, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5799, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{3\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{3\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{6\sinh^{-1}(cx)\text{PolyLog}\left(3,-\frac{ee^{\sin}}{cd-\sqrt{}}\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^4/(4*e) + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^A
rcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*PolyLog[3,
-((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[
4, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)^3}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^3\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \text{Subst}\left(\int \frac{e^x x^3}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x x^3}{cd+\sqrt{c^2d^2+e}}\right.$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{3\,\text{Subst}}{}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\sinh^{-}}{}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\sinh^{-}}{}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\sinh^{-}}{}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3\sinh^{-}}{}$$

Mathematica [A] time = 0.0433466, size = 322, normalized size = 0.93

$$\frac{12\sinh^{-1}(cx)^2\text{PolyLog}\left(2, \frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right) + 12\sinh^{-1}(cx)^2\text{PolyLog}\left(2, -\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right) - 24\sinh^{-1}(cx)\text{PolyLog}\left(3, \frac{ee^{\sin}}{\sqrt{c^2d^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] (-ArcSinh[c*x]^4 + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[
c^2*d^2 + e^2])] + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[
c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
+ Sqrt[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d + Sqrt[c^2*d^2 + e^2]))] - 24*ArcSinh[c*x]*PolyLog[3, (e*E^ArcSinh[c*
x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 24*ArcSinh[c*x]*PolyLog[3, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 24*PolyLog[4, (e*E^ArcSinh[c*x])
/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 24*PolyLog[4, -((e*E^ArcSinh[c*x])/(c*d
+ Sqrt[c^2*d^2 + e^2]))])/(4*e)
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int \frac{(\text{Arcsinh}\,(cx))^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)^3/(e*x+d),x)
```

```
[Out] int(arcsinh(c*x)^3/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(cx)^3}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(cx)^3}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)^3/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(cx)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)**3/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)**3/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(cx)^3}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

3.4 $\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=176

$$\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{c^2x^2+1}\left(ex\left(26c^2d^2-9e^2\right)+4d\left(19c^2d^2-16e^2\right)\right)}{96c^3}-\frac{b\left(-24c^2d^2e^2+8c^4d^4+3e^4\right)\sin}{32c^4e}$$

```
[Out] (-7*b*d*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(48*c) - (b*(d + e*x)^3*Sqrt[1 + c^2
*x^2])/(16*c) - (b*(4*d*(19*c^2*d^2 - 16*e^2) + e*(26*c^2*d^2 - 9*e^2)*x)*S
qrt[1 + c^2*x^2])/(96*c^3) - (b*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSin
h[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e)
```

Rubi [A] time = 0.170185, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5801, 743, 833, 780, 215}

$$\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{c^2x^2+1}\left(ex\left(26c^2d^2-9e^2\right)+4d\left(19c^2d^2-16e^2\right)\right)}{96c^3}-\frac{b\left(-24c^2d^2e^2+8c^4d^4+3e^4\right)\sin}{32c^4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-7*b*d*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(48*c) - (b*(d + e*x)^3*Sqrt[1 + c^2
*x^2])/(16*c) - (b*(4*d*(19*c^2*d^2 - 16*e^2) + e*(26*c^2*d^2 - 9*e^2)*x)*S
qrt[1 + c^2*x^2])/(96*c^3) - (b*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSin
h[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e)
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\sinh^{-1}(cx)\right)dx = \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{\sqrt{1+c^2x^2}}dx}{4e}$$

$$= -\frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{b\int \frac{(d+ex)^2\left(4c^2d^2-3e^2+7c^2dex\right)}{\sqrt{1+c^2x^2}}dx}{16ce}$$

$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{b\int}{}$$

$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b\left(4d\left(19c^2d^2-16e^2\right)+e\left(26c^2d^2\right.\right.}{96c^3}$$

$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b\left(4d\left(19c^2d^2-16e^2\right)+e\left(26c^2d^2\right.\right.}{96c^3}$$

Mathematica [A] time = 0.137749, size = 166, normalized size = 0.94

$$\frac{24ac^4x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)-bc\sqrt{c^2x^2+1}\left(c^2\left(72d^2ex+96d^3+32de^2x^2+6e^3x^3\right)-e^2(64d+9ex)\right)+3b\sinh^{-}}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[1 + c^2*
x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e
^3*x^3)) + 3*b*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2
*x^2 + e^3*x^3))*ArcSinh[c*x])/(96*c^4)
```

Maple [A] time = 0.008, size = 259, normalized size = 1.5

$$\frac{1}{c}\left(\frac{(cex+cd)^4a}{4c^3e} + \frac{b}{c^3}\left(\frac{e^3\mathrm{Arcsinh}(cx)c^4x^4}{4} + e^2\mathrm{Arcsinh}(cx)c^4x^3d + \frac{3e\mathrm{Arcsinh}(cx)c^4x^2d^2}{2} + \mathrm{Arcsinh}(cx)c^4xd^3 + \frac{c^4}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4*e^3*arcsinh(c*x)*c^4*x^4+e^2*arcs
inh(c*x)*c^4*x^3*d+3/2*e*arcsinh(c*x)*c^4*x^2*d^2+arcsinh(c*x)*c^4*x*d^3+1/
4/e*arcsinh(c*x)*c^4*d^4-1/4/e*(e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2)-3/8*c*x*
```

```
(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))+4*c*d*e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2
)-2/3*(c^2*x^2+1)^(1/2))+6*c^2*d^2*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsi
nh(c*x))+4*c^3*d^3*e*(c^2*x^2+1)^(1/2)+c^4*d^4*arcsinh(c*x))))
```

Maxima [A] time = 1.04887, size = 343, normalized size = 1.95

$$\frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{4}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)bd^2e + \frac{1}{3}\left(3x^3\operatorname{arsinh}(cx) - c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d^
2*e + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x
^2 + 1)/c^4))*b*d*e^2 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3
/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^
4))*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A] time = 2.4354, size = 468, normalized size = 2.66

$$\frac{24\,ac^4e^3x^4 + 96\,ac^4de^2x^3 + 144\,ac^4d^2ex^2 + 96\,ac^4d^3x + 3\left(8\,bc^4e^3x^4 + 32\,bc^4de^2x^3 + 48\,bc^4d^2ex^2 + 32\,bc^4d^3x + 24\,bc^2\right.}{96\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c^
4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 + 32
*b*c^4*d^3*x + 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*
b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 - 64*b*c*d*e^2 + 9*(8*b*c
^3*d^2*e - b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 2.12706, size = 316, normalized size = 1.8

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{asinh}(cx) + \frac{3bd^2ex^2\operatorname{asinh}(cx)}{2} + bde^2x^3\operatorname{asinh}(cx) + \frac{be^3x^4\operatorname{asinh}(cx)}{4} - \frac{bd^3\sqrt{c^2x^2+1}}{c} - \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*asinh(c*x) + 3*b*d**2*e*x**2*asinh(c*x)/2 + b*d*e**2*x**3*asinh(c*x
) + b*e**3*x**4*asinh(c*x)/4 - b*d**3*sqrt(c**2*x**2 + 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 + 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 + 1)/(16*c) + 3*b*d**2*e*asinh(c*x)/(4*c**2) + 2*b*d*
```

```
e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*b*e**3*x*sqrt(c**2*x**2 + 1)/(32*c**3
) - 3*b*e**3*asinh(c*x)/(32*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2
+ d*e**2*x**3 + e**3*x**4/4), True))
```

Giac [A] time = 1.66671, size = 400, normalized size = 2.27

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)bd^3+ad^3x+\frac{1}{32}\left(8\,ax^4+\left(8\,x^4\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(\sqrt{c^2x^2+1}x\left(\frac{2\,x^2}{c^2}-\frac{3}{c^4}\right)-\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^3 + a*d^3*x + 1/
32*(8*a*x^4 + (8*x^4*log(c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*x*(2
*x^2/c^2 - 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4*abs(c)))
*c)*b)*e^3 + 1/3*(3*a*d*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x
^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*d)*e^2 + 3/4*(2*a*d^2*x^2 + (2*
x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + log(abs(-x*
abs(c) + sqrt(c^2*x^2 + 1)))/(c^2*abs(c))))*b*d^2)*e
```

3.5 $\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{3e}-\frac{b\sqrt{c^2x^2+1}\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)}{18c^3}-\frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh^{-1}(cx)}{6e}-\frac{b\sqrt{c^2x^2+1}(d+ex}{9c}$$

```
[Out] -(b*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(9*c) - (b*(4*(4*c^2*d^2 - e^2) + 5*c^2*
d*e*x)*Sqrt[1 + c^2*x^2])/(18*c^3) - (b*d*(2*d^2 - (3*e^2)/c^2)*ArcSinh[c*x
])/(6*e) + ((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e)
```

Rubi [A] time = 0.0957302, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5801, 743, 780, 215}

$$\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{3e}-\frac{b\sqrt{c^2x^2+1}\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)}{18c^3}-\frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh^{-1}(cx)}{6e}-\frac{b\sqrt{c^2x^2+1}(d+ex}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(9*c) - (b*(4*(4*c^2*d^2 - e^2) + 5*c^2*
d*e*x)*Sqrt[1 + c^2*x^2])/(18*c^3) - (b*d*(2*d^2 - (3*e^2)/c^2)*ArcSinh[c*x
])/(6*e) + ((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e)
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{\sqrt{1+c^2x^2}} dx}{3e}$$
$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} + \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)\left(3c^2d^2-2e^2+5c^2dex\right)}{\sqrt{1+c^2x^2}} dx}{9ce}$$
$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)\sqrt{1+c^2x^2}}{18c^3} + \frac{(d+ex)^3 \left(a+b\sin\right.}{3e}$$
$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)\sqrt{1+c^2x^2}}{18c^3} - \frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh}{6e}$$

Mathematica [A] time = 0.0854894, size = 121, normalized size = 0.98

$$\frac{6ac^3x\left(3d^2+3dex+e^2x^2\right)-b\sqrt{c^2x^2+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)-4e^2\right)+3bc\sinh^{-1}(cx)\left(6c^2d^2x+3d\left(2c^2ex^2+e\right)+2\right.}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*
(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*b*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*
(e + 2*c^2*e*x^2))*ArcSinh[c*x])/(18*c^3)
```

Maple [A] time = 0.005, size = 189, normalized size = 1.5

$$\frac{1}{c}\left(\frac{(cex+cd)^3 a}{3c^2e} + \frac{b}{c^2}\left(\frac{e^2\text{Arcsinh}(cx)c^3x^3}{3} + e\text{Arcsinh}(cx)c^3x^2d + \text{Arcsinh}(cx)c^3xd^2 + \frac{c^3d^3\text{Arcsinh}(cx)}{3e} - \frac{1}{3e}\left(e^3\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(1/3*e^2*arcsinh(c*x)*c^3*x^3+e*arcsin
h(c*x)*c^3*x^2*d+arcsinh(c*x)*c^3*x*d^2+1/3/e*arcsinh(c*x)*c^3*d^3-1/3/e*(e
^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*c*d*e^2*(1/2*c*x
*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+3*c^2*d^2*e*(c^2*x^2+1)^(1/2)+c^3*d^3*
arcsinh(c*x))))
```

Maxima [A] time = 1.05731, size = 219, normalized size = 1.77

$$\frac{1}{3}ae^2x^3 + adex^2 + \frac{1}{2}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)bde + \frac{1}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*
x/c^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d*e + 1/9*(3*x^3*arcsi
nh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*e^2 +
a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^2/c
```

Fricas [A] time = 2.54437, size = 324, normalized size = 2.61

$$\frac{6\,ac^3e^2x^3 + 18\,ac^3dex^2 + 18\,ac^3d^2x + 3\left(2\,bc^3e^2x^3 + 6\,bc^3dex^2 + 6\,bc^3d^2x + 3\,bcde\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(2\,bc^2e^2x^2 \cdots\right.}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2*
x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x + 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2 +
 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 - 4*b*e^2)*sqrt(c^2*
x^2 + 1))/c^3
```

Sympy [A] time = 0.963745, size = 190, normalized size = 1.53

$$\begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{asinh}(cx) + bdex^2\operatorname{asinh}(cx) + \frac{be^2x^3\operatorname{asinh}(cx)}{3} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{bdex\sqrt{c^2x^2+1}}{2c} - \frac{be^2x^2\sqrt{c^2x^2+1}}{9c} + \cdots \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asinh(c*x) + b*
d*e*x**2*asinh(c*x) + b*e**2*x**3*asinh(c*x)/3 - b*d**2*sqrt(c**2*x**2 + 1)
/c - b*d*e*x*sqrt(c**2*x**2 + 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 + 1)/(9
*c) + b*d*e*asinh(c*x)/(2*c**2) + 2*b*e**2*sqrt(c**2*x**2 + 1)/(9*c**3), Ne
(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [A] time = 1.56095, size = 265, normalized size = 2.14

$$\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)bd^2 + ad^2x + \frac{1}{9}\left(3\,ax^3 + \left(3\,x^3\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\left(c^2x^2+1\right)^{\frac{3}{2}} - 3\sqrt{c^2x^2+1} \cdots}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^2 + a*d^2*x + 1/
9*(3*a*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3
*sqrt(c^2*x^2 + 1))/c^3)*b)*e^2 + 1/2*(2*a*d*x^2 + (2*x^2*log(c*x + sqrt(c^
2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + log(abs(-x*abs(c) + sqrt(c^2*x^2
 + 1)))/(c^2*abs(c))))*b*d)*e
```

3.6 $\int (d+ex)\left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=97

$$\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}-\frac{b\sqrt{c^2x^2+1}(d+ex)}{4c}-\frac{3bd\sqrt{c^2x^2+1}}{4c}$$

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2])/(4*c) - (b*(d + e*x)*Sqrt[1 + c^2*x^2])/(4*c) -
(b*(2*d^2 - e^2/c^2)*ArcSinh[c*x])/(4*e) + ((d + e*x)^2*(a + b*ArcSinh[c*x]
))/(2*e)
```

Rubi [A] time = 0.0521902, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5801, 743, 641, 215}

$$\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}-\frac{b\sqrt{c^2x^2+1}(d+ex)}{4c}-\frac{3bd\sqrt{c^2x^2+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2])/(4*c) - (b*(d + e*x)*Sqrt[1 + c^2*x^2])/(4*c) -
(b*(2*d^2 - e^2/c^2)*ArcSinh[c*x])/(4*e) + ((d + e*x)^2*(a + b*ArcSinh[c*x]
))/(2*e)
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)dx = \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{\sqrt{1+c^2x^2}}dx}{2e}$$
$$= -\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} + \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e} - \frac{b\int \frac{2c^2d^2-e^2+3c^2dex}{\sqrt{1+c^2x^2}}dx}{4ce}$$
$$= -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} + \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e} - \frac{1}{4}\left(b\left(\frac{2cd^2}{e}\right.\right.$$
$$= -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} - \frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e} + \frac{(d+ex)^2\left(a+b\right.}{2e}$$

Mathematica [A] time = 0.0391368, size = 91, normalized size = 0.94

$$adx + \frac{1}{2}aex^2 - \frac{bd\sqrt{c^2x^2+1}}{c} - \frac{bex\sqrt{c^2x^2+1}}{4c} + \frac{be\sinh^{-1}(cx)}{4c^2} + bdx\sinh^{-1}(cx) + \frac{1}{2}bex^2\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[1 + c^2*x^2])/c - (b*e*x*Sqrt[1 + c^2*x^2])
/(4*c) + (b*e*ArcSinh[c*x])/(4*c^2) + b*d*x*ArcSinh[c*x] + (b*e*x^2*ArcSinh
[c*x])/2
```

Maple [A] time = 0.006, size = 96, normalized size = 1.

$$\frac{1}{c}\left(\frac{a}{c}\left(\frac{x^2c^2e}{2}+c^2dx\right)+\frac{b}{c}\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^2x^2e}{2}+\mathrm{Arcsinh}\,(cx)\,c^2xd-\frac{e}{2}\left(\frac{cx}{2}\sqrt{c^2x^2+1}-\frac{\mathrm{Arcsinh}\,(cx)}{2}\right)-cd\sqrt{c^2x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c*(1/2*x^2*c^2*e+c^2*d*x)+b/c*(1/2*arcsinh(c*x)*c^2*x^2*e+arcsinh(c*
x)*c^2*x*d-1/2*e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))-c*d*(c^2*x^2+
1)^(1/2)))
```

Maxima [A] time = 1.07767, size = 127, normalized size = 1.31

$$\frac{1}{2}aex^2 + \frac{1}{4}\left(2x^2\,\mathrm{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\mathrm{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)be + adx + \frac{\left(cx\,\mathrm{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsin
h(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt
(c^2*x^2 + 1))*b*d/c
```

Fricas [A] time = 2.37408, size = 197, normalized size = 2.03

$$\frac{2\,ac^2ex^2 + 4\,ac^2dx + \left(2\,bc^2ex^2 + 4\,bc^2dx + be\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right) - (bcex + 4\,bcd)\sqrt{c^2x^2 + 1}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x + b*e)*log(
c*x + sqrt(c^2*x^2 + 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [A] time = 0.403193, size = 99, normalized size = 1.02

$$\begin{cases} adx + \frac{aex^2}{2} + bdx\operatorname{asinh}(cx) + \frac{bex^2\operatorname{asinh}(cx)}{2} - \frac{bd\sqrt{c^2x^2+1}}{c} - \frac{bex\sqrt{c^2x^2+1}}{4c} + \frac{be\operatorname{asinh}(cx)}{4c^2} & \text{for}\ c \neq 0 \\ a\left(dx + \frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*asinh(c*x) + b*e*x**2*asinh(c*x)/2 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x*sqrt(c**2*x**2 + 1)/(4*c) + b*e*asinh(c*x
)/(4*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

Giac [A] time = 1.58733, size = 169, normalized size = 1.74

$$\left(x\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \frac{\sqrt{c^2x^2 + 1}}{c}\right)bd + adx + \frac{1}{4}\left(2\,ax^2 + \left(2\,x^2\log\left(cx + \sqrt{c^2x^2 + 1}\right) - c\left(\frac{\sqrt{c^2x^2 + 1}x}{c^2} + \frac{\log\left(\left|-x|c| + \sqrt{}\right.\right.}{c^2|c|}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d + a*d*x + 1/4*(2
*a*x^2 + (2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 +
 log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^2*abs(c))))*b)*e
```

3.7 $\int \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rubi [A] time = 0.0132673, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5653, 261}

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)\,dx &= ax+b\int \sinh^{-1}(cx)\,dx\\ &= ax+bx\sinh^{-1}(cx)-(bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx\\ &= ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)\end{aligned}$$

Mathematica [A] time = 0.007988, size = 30, normalized size = 1.

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Maple [A] time = 0.004, size = 31, normalized size = 1.

$$ax+\frac{b}{c}\left(\mathrm{Arcsinh}\left(cx\right)cx-\sqrt{c^{2}x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(c*x),x)
```

```
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.03064, size = 41, normalized size = 1.37

$$ax+\frac{\left(cx\,\mathrm{arsinh}\left(cx\right)-\sqrt{c^{2}x^{2}+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Fricas [A] time = 2.43868, size = 95, normalized size = 3.17

$$\frac{bcx\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+acx-\sqrt{c^{2}x^{2}+1}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A] time = 0.160008, size = 26, normalized size = 0.87

$$ax+b\left(\begin{cases}x\,\mathrm{asinh}\left(cx\right)-\frac{\sqrt{c^{2}x^{2}+1}}{c} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

Giac [A] time = 1.28654, size = 55, normalized size = 1.83

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

3.8 $\int \frac{a+b\sinh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=187

$$\frac{b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx\right.}{}$$

```
[Out] -(a + b*ArcSinh[c*x])^2/(2*b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 + (e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rubi [A] time = 0.258696, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5799, 5561, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^2/(2*b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 + (e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \text{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd}\right.$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{e}$$

Mathematica [A] time = 0.0535466, size = 175, normalized size = 0.94

$$\frac{2b^2\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)+2b^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)-\left(a+b\sinh^{-1}(cx)\right)\left(a-2b\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)-2b\,\text{l}\right.}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 2*b*Log[1 + (e*E^ArcSinh[c*x]
)/(c*d - Sqrt[c^2*d^2 + e^2])] - 2*b*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt
[c^2*d^2 + e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 + e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/(2*b*e)
```

Maple [A] time = 0.037, size = 282, normalized size = 1.5

$$\frac{a\ln(cex+cd)}{e} - \frac{b\left(\text{Arcsinh}(cx)\right)^2}{2e} + \frac{b\text{Arcsinh}(cx)}{e}\ln\left(\left(-\left(cx+\sqrt{c^2x^2+1}\right)e-cd+\sqrt{c^2d^2+e^2}\right)\left(-cd+\sqrt{c^2d^2+e^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e-1/2*b/e*arcsinh(c*x)^2+b/e*arcsinh(c*x)*ln((-(c*x+(c^2*x^
2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*arcs
```

```
inh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d
^2+e^2)^(1/2)))+b/e*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2
))/(c*d+(c^2*d^2+e^2)^(1/2)))+b/e*dilog((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^
2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{ex+d}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x) + a*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{asinh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d), x)
```

3.9 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

```
[Out] -((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^
2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

Rubi [A] time = 0.0545173, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5801, 725, 206}

$$-\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^
2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e}\\ &= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx,x,\frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{e}\\ &= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}\end{aligned}$$

Mathematica [A] time = 0.095708, size = 79, normalized size = 0.96

$$-\frac{\frac{a+b\sinh^{-1}(cx)}{d+ex}+\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{\sqrt{c^2d^2+e^2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -(((a + b*ArcSinh[c*x])/(d + e*x) + (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^
2 + e^2]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d^2 + e^2])/e)
```

Maple [B] time = 0.018, size = 178, normalized size = 2.2

$$-\frac{ca}{(cex+cd)\,e}-\frac{bc\text{Arcsinh}\,(cx)}{(cex+cd)\,e}-\frac{bc}{e^2}\ln\left(\left(2\,\frac{c^2d^2+e^2}{e^2}-2\,\frac{cd}{e}\left(cx+\frac{cd}{e}\right)+2\,\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{cd}{e}\right)^2-2\,\frac{cd}{e}\left(cx+\frac{cd}{e}\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arcsinh(c*x)-c*b/e^2/((c^2*d^2+e^2)/e^
2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^
(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d
/e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.74072, size = 520, normalized size = 6.34

$$-\frac{ac^2d^3 + ade^2 - \left(bc^2d^2e + be^3\right)x\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(bcdex + bcd^2\right)\sqrt{c^2d^2+e^2}\log\left(-\frac{c^3d^2x - cde + \sqrt{c^2d^2+e^2}\left(c^2dx - e\right) + \left(c^2d^2\right.}{ex+d}\right.}{c^2d^4e + d^2e^3 + \left(c^2d^3e^2 + de^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*e^2 - (b*c^2*d^2*e + b*e^3)*x*log(c*x + sqrt(c^2*x^2 + 1)
) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqr
t(c^2*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*
sqrt(c^2*x^2 + 1))/(e*x + d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3
)*x)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d
*e^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{asinh}(cx)}{(d+ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**2, x)
```

Giac [B] time = 1.44707, size = 290, normalized size = 3.54

$$\left(\left(\frac{e^{(-1)}\log\left(-c^2d + \sqrt{c^2d^2+e^2}|c|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{\sqrt{c^2d^2+e^2}} - \frac{e^{(-1)}\log\left(-c^2d + \sqrt{c^2d^2+e^2}\left(\sqrt{c^2 - \frac{2c^2d}{xe+d} + \frac{c^2d^2}{(xe+d)^2} + \frac{e^2}{(xe+d)^2}} + \frac{\sqrt{c^2d^2e^2+e}}{xe+d}\right.\right.}{\sqrt{c^2d^2+e^2}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] ((e^(-1)*log(-c^2*d + sqrt(c^2*d^2 + e^2)*abs(c))*sgn(1/(x*e + d))/sqrt(c^2
*d^2 + e^2) - e^(-1)*log(-c^2*d + sqrt(c^2*d^2 + e^2)*(sqrt(c^2 - 2*c^2*d/(
x*e + d) + c^2*d^2/(x*e + d)^2 + e^2/(x*e + d)^2) + sqrt(c^2*d^2*e^2 + e^4)
*e^(-1)/(x*e + d)))/(sqrt(c^2*d^2 + e^2)*sgn(1/(x*e + d))))*c - e^(-1)*log(
c*x + sqrt(c^2*x^2 + 1))/(x*e + d))*b - a*e^(-1)/(x*e + d)
```

3.10 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=128

$$-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*(c^2*d^2 + e^2)*(d + e*x)) - (a + b*ArcSinh[c*x
])/(2*e*(d + e*x)^2) - (b*c^3*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 + e^2]*
Sqrt[1 + c^2*x^2])])/(2*e*(c^2*d^2 + e^2)^(3/2))
```

Rubi [A] time = 0.0837947, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5801, 731, 725, 206}

$$-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*(c^2*d^2 + e^2)*(d + e*x)) - (a + b*ArcSinh[c*x
])/(2*e*(d + e*x)^2) - (b*c^3*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 + e^2]*
Sqrt[1 + c^2*x^2])])/(2*e*(c^2*d^2 + e^2)^(3/2))
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc)\int \frac{1}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{2e}$$
$$= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{2e\left(c^2d^2+e^2\right)}$$
$$= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{\left(bc^3d\right)\text{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx,x,\frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)}$$
$$= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

Mathematica [A] time = 0.330422, size = 166, normalized size = 1.3

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\log\left(\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}+c^2(-d)x+e\right)}{e\left(c^2d^2+e^2\right)^{3/2}}+\frac{bc^3d\log(d+ex)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{b\sinh^{-1}(\cdots}{e(d+e\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x))
 - (b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*(c^2*d^2 +
e^2)^(3/2)) - (b*c^3*d*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x
^2]])/(e*(c^2*d^2 + e^2)^(3/2)))/2
```

Maple [B] time = 0.03, size = 279, normalized size = 2.2

$$-\frac{c^2a}{2\,(cex+cd)^2\,e}-\frac{c^2b\text{Arcsinh}\,(cx)}{2\,(cex+cd)^2\,e}-\frac{c^2b}{2\,e\left(c^2d^2+e^2\right)}\sqrt{\left(cx+\frac{cd}{e}\right)^2-2\,\frac{cd}{e}\left(cx+\frac{cd}{e}\right)+\frac{c^2d^2+e^2}{e^2}}\left(cx+\frac{cd}{e}\right)^{-1}-\frac{b\cdots}{2\,e^2\,(c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arcsinh(c*x)-1/2*c^2*b
/e/(c^2*d^2+e^2)/(c*x+c*d/e)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^
2)/e^2)^(1/2)-1/2*c^3*b/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2
*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*
d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.44087, size = 1146, normalized size = 8.95

$$-\frac{(a+b)c^4d^6 + (2\,a+b)c^2d^4e^2 + ad^2e^4 + \left(bc^4d^4e^2 + bc^2d^2e^4\right)x^2 - \left(bc^3d^3e^2x^2 + 2\,bc^3d^4ex + bc^3d^5\right)\sqrt{c^2d^2+e^2}\log\left(-\frac{c^3d^2x-}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*c^4*d^6 + (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
+ b*c^2*d^2*e^4)*x^2 - (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*sq
rt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqrt(c^2*d^2 + e^2)*(c^2*d*x -
e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x^2 + 1))/(e*x + d)
) + 2*(b*c^4*d^5*e + b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 +
 b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2
+ 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5
)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (b*c^3*d^5*e + b*c*d^3*e^3 + (b*c^3*d^
4*e^2 + b*c*d^2*e^4)*x)*sqrt(c^2*x^2 + 1))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4
*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2
*d^5*e^4 + d^3*e^6)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{asinh}(cx)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx) + a}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^3, x)
```

3.11 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3d\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc\sqrt{c^2x^2+1}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*(c^2*d^2 + e^2)*(d + e*x)^2) - (b*c^3*d*Sqrt[1
+ c^2*x^2])/(2*(c^2*d^2 + e^2)^2*(d + e*x)) - (a + b*ArcSinh[c*x])/(3*e*(d
+ e*x)^3) - (b*c^3*(2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 +
e^2]*Sqrt[1 + c^2*x^2])])/(6*e*(c^2*d^2 + e^2)^(5/2))
```

Rubi [A] time = 0.138058, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5801, 745, 807, 725, 206}

$$-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3d\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc\sqrt{c^2x^2+1}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*(c^2*d^2 + e^2)*(d + e*x)^2) - (b*c^3*d*Sqrt[1
+ c^2*x^2])/(2*(c^2*d^2 + e^2)^2*(d + e*x)) - (a + b*ArcSinh[c*x])/(3*e*(d
+ e*x)^3) - (b*c^3*(2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 +
e^2]*Sqrt[1 + c^2*x^2])])/(6*e*(c^2*d^2 + e^2)^(5/2))
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc)\int \frac{1}{(d+ex)^3\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{\left(bc^3\right)\int \frac{-2d+ex}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{6e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(bc^3\left(2c^2d^2-e^2\right)\right)\int \frac{}{(d+e}}{6e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{\left(bc^3\left(2c^2d^2-e^2\right)\right)\mathrm{Subs}}{6e\left(c^2\right.}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}}{6e\left(c^2d^2+e\right.}$$

Mathematica [A] time = 0.388277, size = 205, normalized size = 1.12

$$\frac{1}{6}\left(-\frac{2a}{e(d+ex)^3}-\frac{bc\sqrt{c^2x^2+1}\left(c^2d(4d+3ex)+e^2\right)}{\left(c^2d^2+e^2\right)^2(d+ex)^2}+\frac{bc^3\left(e^2-2c^2d^2\right)\log\left(\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}+c^2(-d)x+e\right)}{e\left(c^2d^2+e^2\right)^{5/2}}-\frac{bc^3\left(e^2\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] ((-2*a)/(e*(d + e*x)^3) - (b*c*Sqrt[1 + c^2*x^2]*(e^2 + c^2*d*(4*d + 3*e*x)
))/((c^2*d^2 + e^2)^2*(d + e*x)^2) - (2*b*ArcSinh[c*x])/(e*(d + e*x)^3) - (
b*c^3*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(e*(c^2*d^2 + e^2)^(5/2)) + (b*c^3*(
-2*c^2*d^2 + e^2)*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x^2]])
/(e*(c^2*d^2 + e^2)^(5/2)))/6
```

Maple [B] time = 0.012, size = 516, normalized size = 2.8

$$-\frac{c^3a}{3\left(cex+cd\right)^3e}-\frac{c^3b\mathrm{Arcsinh}\left(cx\right)}{3\left(cex+cd\right)^3e}-\frac{c^3b}{6e^2\left(c^2d^2+e^2\right)}\sqrt{\left(cx+\frac{cd}{e}\right)^2-2\frac{cd}{e}\left(cx+\frac{cd}{e}\right)+\frac{c^2d^2+e^2}{e^2}}\left(cx+\frac{cd}{e}\right)^{-2}-\frac{bc^4}{2e\left(c^2d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3*a/(c*e*x+c*d)^3/e-1/3*c^3*b/(c*e*x+c*d)^3/e*arcsinh(c*x)-1/6*c^3*b
/e^2/(c^2*d^2+e^2)/(c*x+c*d/e)^2*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^
2+e^2)/e^2)^(1/2)-1/2*c^4*b/e*d/(c^2*d^2+e^2)^2/(c*x+c*d/e)*((c*x+c*d/e)^2-
2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2)-1/2*c^5*b/e^2*d^2/(c^2*d^2+e^2
)^2/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2
*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)
/e^2)^(1/2))/(c*x+c*d/e))+1/6*c^3*b/e^2/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(
1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2
)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 + 3*c*d^2*e^2*x^2 + c
*d^3*e*x + (3*c^3*d^2*e^2 + c*e^4)*x^4 + (c^3*d^3*e + 3*c*d*e^3)*x^3 + (c^2
*e^4*x^5 + 3*c^2*d*e^3*x^4 + 3*d^2*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)*x^
3 + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(c^6*d^3 - 3*c^4*
d*e^2)*log(e*x + d)/(c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7) + (3*
c^6*d^6 + 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e^2 - c^2*e^6)*x^2 + (5*
c^6*d^5*e + 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x + (c^6*d^6 - 3*c^4*d^4*e^2 + (c^
6*d^3*e^3 - 3*c^4*d*e^5)*x^3 + 3*(c^6*d^4*e^2 - 3*c^4*d^2*e^4)*x^2 + 3*(c^6
*d^5*e - 3*c^4*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*(c^6*d^6 + 3*c^4*d^4*e^2 +
3*c^2*d^2*e^4 + e^6)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^6*d^9*e + 3*c^4*d^7*e
^3 + 3*c^2*d^5*e^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*c^2*d^2*e^8
 + e^10)*x^3 + 3*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 + d*e^9)*x^2
+ 3*(c^6*d^8*e^2 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x) - I*(3*c^6*d
^2 - c^4*e^2)*(log(I*c*x + 1) - log(-I*c*x + 1))/((c^6*d^6 + 3*c^4*d^4*e^2
+ 3*c^2*d^2*e^4 + e^6)*c))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x +
 d^3*e)
```

Fricas [B] time = 7.69333, size = 1960, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*((2*a + 3*b)*c^6*d^9 + 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 + 2*a
*d^3*e^6 + 3*(b*c^6*d^6*e^3 + b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 + b*c^4
*d^5*e^4)*x^2 + (2*b*c^5*d^8 - b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 - b*c^3*d^3
*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 - b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e - b*
c^3*d^5*e^3)*x)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e - sqrt(c^2*d^2
+ e^2)*(c^2*d*x - e) + (c^2*d^2 - sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x
^2 + 1))/(e*x + d)) + 9*(b*c^6*d^8*e + b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^3
 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 + 3*b*
c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*b*c^4*d^6
```

```
*e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(b*
c^6*d^9 + 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 + b*d^3*e^6 + (b*c^6*d^6*e^3 +
3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 + 3*b*c^4
*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*b*c^4*d^6*e^
3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (4*b*c^
5*d^8*e + 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 + b*c^3*d^4*e^5)
*x^2 + (7*b*c^5*d^7*e^2 + 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 +
1))/(c^6*d^12*e + 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 + d^6*e^7 + (c^6*d^9*e^4 +
 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 + d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 + 3*c^4*d^
8*e^5 + 3*c^2*d^6*e^7 + d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 + 3*c^4*d^9*e^4 + 3*
c^2*d^7*e^6 + d^5*e^8)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**4,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(ex + d)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^4, x)
```

3.12 $\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=368

$$-\frac{3bd^2ex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{3d^2e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4bde^2\sqrt{c^2x^2}}{}$$

```
[Out] 2*b^2*d^3*x - (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 - (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*
c) + (3*b*e^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^3) - (2*b*d*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) - (d^4*(a + b*ArcSinh[c*x])^2)/(4*e) +
 (3*d^2*e*(a + b*ArcSinh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcSinh[c*x])^2)/
(32*c^4) + ((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e)
```

Rubi [A] time = 0.759384, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{3bd^2ex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{3d^2e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4bde^2\sqrt{c^2x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] 2*b^2*d^3*x - (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 - (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*
c) + (3*b*e^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^3) - (2*b*d*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) - (d^4*(a + b*ArcSinh[c*x])^2)/(4*e) +
 (3*d^2*e*(a + b*ArcSinh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcSinh[c*x])^2)/
(32*c^4) + ((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e)
```


Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e}-\frac{(bc)\int\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{2e}$$

$$=\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e}-\frac{(bc)\int\left(\frac{d^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{4d^3ex\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{6d^2e^2x^2}{}\right.}{}$$

$$=\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e}-\left(2bcd^3\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx-\frac{\left(bcd^4\right)\int\frac{a+b\sin}{\sqrt{1+}}}{2e}$$

$$=-\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{3bd^2ex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c}-\frac{2bde^2x}{}$$

$$=2b^2d^3x+\frac{3}{4}b^2d^2ex^2+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+$$

$$=2b^2d^3x-\frac{4b^2de^2x}{3c^2}+\frac{3}{4}b^2d^2ex^2-\frac{3b^2e^3x^2}{32c^2}+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{1+c^2x^2}}{}$$

Mathematica [A] time = 0.524257, size = 354, normalized size = 0.96

$$\frac{c\left(72a^2c^3x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)-6ab\sqrt{c^2x^2+1}\left(c^2\left(72d^2ex+96d^3+32de^2x^2+6e^3x^3\right)-e^2(64d+9ex)\right)+b^2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[1
 + c^2*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^
2 + 6*e^3*x^3)) + b^2*c*x*(-3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*
e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*(-3*a*(24*c^2*d^2*e - 3*e^3 + 8*c^4
*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + b*c*Sqrt[1 + c^2*x^2]*(-(
e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))
)*ArcSinh[c*x] + 9*b^2*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x +
 4*d*e^2*x^2 + e^3*x^3))*ArcSinh[c*x]^2)/(288*c^4)
```

Maple [A] time = 0.085, size = 641, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a^2/c^3/e+b^2/c^3*(c^3*d^3*(arcsinh(c*x)^2*c*x-2*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+3/4*c^2*d^2*e*(2*arcsinh(c*x)^2*c^2*x^2-
2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1)+1/9*c*d*e^2*
(9*arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+27*arcsi
nh(c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)+1/32*e^3*
(8*arcsinh(c*x)^2*c^4*x^4-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3+16*arcsi
nh(c*x)^2*c^2*x^2+c^4*x^4-10*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+5*arcsinh(c
*x)^2+5*c^2*x^2+4)-3*c*d*e^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)+2*c*x)-1/4*e^3*(2*arcsinh(c*x)^2*c^2*x^2-2*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1))+2*a*b/c^3*(1/4*e^3*arcsinh(c*x)*c^4*x^
4+e^2*arcsinh(c*x)*c^4*x^3*d+3/2*e*arcsinh(c*x)*c^4*x^2*d^2+arcsinh(c*x)*c^
4*x*d^3+1/4/e*arcsinh(c*x)*c^4*d^4-1/4/e*(e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2
)-3/8*c*x*(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))+4*c*d*e^3*(1/3*c^2*x^2*(c^2*x
^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+6*c^2*d^2*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)
-1/2*arcsinh(c*x))+4*c^3*d^3*e*(c^2*x^2+1)^(1/2)+c^4*d^4*arcsinh(c*x))))
```

Maxima [A] time = 1.18135, size = 880, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e^3*x^4*arcsinh(c*x)^2 + b^2*d*e^2*x^3*arcsinh(c*x)^2 + 1/4*a^2*e^3
*x^4 + 3/2*b^2*d^2*e*x^2*arcsinh(c*x)^2 + a^2*d*e^2*x^3 + b^2*d^3*x*arcsinh
(c*x)^2 + 3/2*a^2*d^2*e*x^2 + 3/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b*d^2*e + 3/4*(c^2*(
x^2/c^2 - log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x
^2 + 1)*x/c^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2))*arcsinh(c*x))*b^2
*d^2*e + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^
2*x^2 + 1)/c^4))*a*b*d*e^2 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c
^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d*e^2 + 1/16*(8*x^
```

```
4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 +
 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*a*b*e^3 + 1/32*((x^4/c^2 -
3*x^2/c^4 + 3*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sq
rt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(
c^2))/(sqrt(c^2)*c^4))*c*arcsinh(c*x))*b^2*e^3 + 2*b^2*d^3*(x - sqrt(c^2*x^
2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1
))*a*b*d^3/c
```

Fricas [A] time = 2.59181, size = 1010, normalized size = 2.74

$$\frac{9\left(8\,a^2+b^2\right)c^4e^3x^4+32\left(9\,a^2+2\,b^2\right)c^4de^2x^3+27\left(8\left(2\,a^2+b^2\right)c^4d^2e-b^2c^2e^3\right)x^2+9\left(8\,b^2c^4e^3x^4+32\,b^2c^4de^2x^3+48\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27*
(8*(2*a^2 + b^2)*c^4*d^2*e - b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32*b
^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x + 24*b^2*c^2*d^2
*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 96*(3*(a^2 + 2*b^2)*c^4*d^
3 - 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3 + 144
*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x + 72*a*b*c^2*d^2*e - 9*a*b*e^3 - (6*b
^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 - 64*b^2*c*d*e^2 + 9
*(8*b^2*c^3*d^2*e - b^2*c*e^3)*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*d^3 - 64*
a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e - a*b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 5.12166, size = 743, normalized size = 2.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*asinh(c*x) + 3*a*b*d**2*e*x**2*asinh(c*x) + 2*a*b*d
*e**2*x**3*asinh(c*x) + a*b*e**3*x**4*asinh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 + 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 + 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 + 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 + 1)/(8*c) + 3*a*b*
d**2*e*asinh(c*x)/(2*c**2) + 4*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*
a*b*e**3*x*sqrt(c**2*x**2 + 1)/(16*c**3) - 3*a*b*e**3*asinh(c*x)/(16*c**4)
+ b**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*asinh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*asinh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*asinh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(8*c) + 3*b**2*d**2*e*asinh(c
*x)**2/(4*c**2) - 4*b**2*d*e**2*x/(3*c**2) - 3*b**2*e**3*x**2/(32*c**2) + 4
*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 3*b**2*e**3*x*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(16*c**3) - 3*b**2*e**3*asinh(c*x)**2/(32*c**4), N
e(c, 0)), (a**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), Tru
```

```
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3(b \operatorname{arsinh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arcsinh(c*x) + a)^2, x)
```

3.13 $\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=239

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{de\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2}+\frac{4be^2\sqrt{c^2x^2+1}\left(a+b\right.}{9c^3}$$

```
[Out] 2*b^2*d^2*x - (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e^2*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/c - (2*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c)
 - (d^3*(a + b*ArcSinh[c*x])^2)/(3*e) + (d*e*(a + b*ArcSinh[c*x])^2)/(2*c^2
) + ((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e)
```

Rubi [A] time = 0.511892, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{de\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2}+\frac{4be^2\sqrt{c^2x^2+1}\left(a+b\right.}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] 2*b^2*d^2*x - (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e^2*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/c - (2*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c)
 - (d^3*(a + b*ArcSinh[c*x])^2)/(3*e) + (d*e*(a + b*ArcSinh[c*x])^2)/(2*c^2
) + ((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e)
```


Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int\left(\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{3d^2ex\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \ldots\right.}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e} - \left(2bcd^2\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx - \frac{\left(2bcd^3\right)\int \ldots}{\ldots}$$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bdex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{2be^2x\ldots}{\ldots}$$

$$= 2b^2d^2x + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + \frac{4be^2\sqrt{1+\ldots}}{\ldots}$$

$$= 2b^2d^2x - \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + \ldots$$

Mathematica [A] time = 0.367827, size = 248, normalized size = 1.04

$$\frac{18a^2c^3x\left(3d^2+3dex+e^2x^2\right) - 6ab\sqrt{c^2x^2+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)-4e^2\right) - 6b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(c^2\left(18d^2+\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (18*a^2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-4*e^2
 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + b^2*c*x*(-24*e^2 + c^2*(108*d^2 +
27*d*e*x + 4*e^2*x^2)) - 6*b*(-3*a*(3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^
2*x^2)) + b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))
)*ArcSinh[c*x] + 9*b^2*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*(e + 2*c^2*e*x^
2))*ArcSinh[c*x]^2)/(54*c^3)
```

Maple [A] time = 0.061, size = 410, normalized size = 1.7

$$\frac{1}{c}\left(\frac{(cex+cd)^3 a^2}{3c^2e}+\frac{b^2}{c^2}\left(c^2d^2\left((\mathrm{Arcsinh}(cx))^2 cx-2\,\mathrm{Arcsinh}(cx)\sqrt{c^2x^2+1}+2\,cx\right)+\frac{cde}{2}\left(2\,(\mathrm{Arcsinh}(cx))^2c^2x^2-2\,\mathrm{A}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a^2/c^2/e+b^2/c^2*(c^2*d^2*(arcsinh(c*x)^2*c*x-2*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/2*c*d*e*(2*arcsinh(c*x)^2*c^2*x^2-2*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1)+1/27*e^2*(9*arcs
inh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+27*arcsinh(c*x)
^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-e^2*(arcsinh(c*x
)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3*e^2*arcsinh
(c*x)*c^3*x^3+e*arcsinh(c*x)*c^3*x^2*d+arcsinh(c*x)*c^3*x*d^2+1/3/e*arcsinh
(c*x)*c^3*d^3-1/3/e*(e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/
2))+3*c*d*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+3*c^2*d^2*e*(c^2
*x^2+1)^(1/2)+c^3*d^3*arcsinh(c*x))))
```

Maxima [A] time = 1.38478, size = 552, normalized size = 2.31

$$\frac{1}{3}b^2e^2x^3\,\mathrm{arsinh}(cx)^2+b^2dex^2\,\mathrm{arsinh}(cx)^2+\frac{1}{3}a^2e^2x^3+b^2d^2x\,\mathrm{arsinh}(cx)^2+a^2dex^2+\left(2\,x^2\,\mathrm{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e^2*x^3*arcsinh(c*x)^2 + b^2*d*e*x^2*arcsinh(c*x)^2 + 1/3*a^2*e^2*x
^3 + b^2*d^2*x*arcsinh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arcsinh(c*x) - c*(sqrt
(c^2*x^2 + 1)*x/c^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b*d*e +
1/2*(c^2*(x^2/c^2 - log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(
sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2))*arcsinh
(c*x))*b^2*d*e + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2
*sqrt(c^2*x^2 + 1)/c^4))*a*b*e^2 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2
*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e^2 + 2*b^2
*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*
x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

Fricas [A] time = 2.57816, size = 687, normalized size = 2.87

$$\frac{2\left(9\,a^2+2\,b^2\right)c^3e^2x^3+27\left(2\,a^2+b^2\right)c^3dex^2+9\left(2\,b^2c^3e^2x^3+6\,b^2c^3dex^2+6\,b^2c^3d^2x+3\,b^2cde\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*b
^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x + 3*b^2*c*d*e)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 - 4*b^2*c*e^2)*x + 6*(
6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x + 9*a*b*c*d*e - (
2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 - 4*b^2*e^2)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b*c^2*d
*e*x + 18*a*b*c^2*d^2 - 4*a*b*e^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 2.32498, size = 454, normalized size = 1.9

$$\begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\operatorname{asinh}(cx) + 2abdex^2\operatorname{asinh}(cx) + \frac{2abe^2x^3\operatorname{asinh}(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{abdex\sqrt{c^2x^2+1}}{c} - 2a \\ a^2\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*as
inh(c*x) + 2*a*b*d*e*x**2*asinh(c*x) + 2*a*b*e**2*x**3*asinh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 + 1)/c - a*b*d*e*x*sqrt(c**2*x**2 + 1)/c - 2*a*b*e**2*
x**2*sqrt(c**2*x**2 + 1)/(9*c) + a*b*d*e*asinh(c*x)/c**2 + 4*a*b*e**2*sqrt(
c**2*x**2 + 1)/(9*c**3) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*asinh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*asinh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 +
1)*asinh(c*x)/(9*c) + b**2*d*e*asinh(c*x)**2/(2*c**2) - 4*b**2*e**2*x/(9*c*
*2) + 4*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3), Ne(c, 0)), (a**2
*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2(b\operatorname{arsinh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arcsinh(c*x) + a)^2, x)
```

3.14 $\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=140

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e}+$$

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c - (b*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c) - (d^2*(a + b*ArcS
inh[c*x])^2)/(2*e) + (e*(a + b*ArcSinh[c*x])^2)/(4*c^2) + ((d + e*x)^2*(a +
 b*ArcSinh[c*x])^2)/(2*e)
```

Rubi [A] time = 0.322187, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c - (b*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c) - (d^2*(a + b*ArcS
inh[c*x])^2)/(2*e) + (e*(a + b*ArcSinh[c*x])^2)/(4*c^2) + ((d + e*x)^2*(a +
 b*ArcSinh[c*x])^2)/(2*e)
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx}{e}$$

$$= \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int\left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{2dex\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{e^2x^2(}{}\right.}{e}$$

$$= \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - (2bcd)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx - \frac{\left(bcd^2\right)\int \frac{a+b\sinh}{\sqrt{1}}}{e}$$

$$= -\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c} - \frac{d^2\left(a+b\right.}{}$$

$$= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.}{2c}$$

Mathematica [A] time = 0.330025, size = 142, normalized size = 1.01

$$\frac{c\left(2a^2cx(2d+ex) - 2ab\sqrt{c^2x^2+1}(4d+ex) + b^2cx(8d+ex)\right) + 2b\sinh^{-1}(cx)\left(a\left(4c^2dx+2c^2ex^2+e\right) - bc\sqrt{c^2x^2+1}(4\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) + b^2*c*x*(8*d + e*x) - 2*a*b*(4*d + e*x)*Sqrt[1
+ c^2*x^2]) + 2*b*(-(b*c*(4*d + e*x)*Sqrt[1 + c^2*x^2]) + a*(e + 4*c^2*d*x
+ 2*c^2*e*x^2))*ArcSinh[c*x] + b^2*(e + 4*c^2*d*x + 2*c^2*e*x^2)*ArcSinh[c*
x]^2)/(4*c^2)
```

Maple [A] time = 0.04, size = 193, normalized size = 1.4

$$\frac{1}{c}\left(\frac{a^2}{c}\left(\frac{x^2c^2e}{2}+c^2dx\right)+\frac{b^2}{c}\left(\frac{e}{4}\left(2\,(\mathrm{Arcsinh}\,(cx))^2\,c^2x^2-2\,\mathrm{Arcsinh}\,(cx)\,\sqrt{c^2x^2+1}cx+(\mathrm{Arcsinh}\,(cx))^2+c^2x^2+1\right)+cd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c*(1/2*x^2*c^2*e+c^2*d*x)+b^2/c*(1/4*e*(2*arcsinh(c*x)^2*c^2*x^2-2
*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1)+c*d*(arcsinh(
c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c*(1/2*arcsinh(c*
x)*c^2*x^2*e+arcsinh(c*x)*c^2*x*d-1/2*e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcs
inh(c*x))-c*d*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.20272, size = 338, normalized size = 2.41

$$\frac{1}{2}\,b^2ex^2\,\mathrm{arsinh}\,(cx)^2+b^2dx\,\mathrm{arsinh}\,(cx)^2+\frac{1}{2}\,a^2ex^2+\frac{1}{2}\left(2\,x^2\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\mathrm{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^2}\right)\right)abe+\frac{1}{4}\left(c^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*x^2*arcsinh(c*x)^2 + b^2*d*x*arcsinh(c*x)^2 + 1/2*a^2*e*x^2 + 1/2
*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c^2*x/sqrt(c^2)
)/(sqrt(c^2)*c^2)))*a*b*e + 1/4*(c^2*(x^2/c^2 - log(c^2*x/sqrt(c^2) + sqrt(
c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c^2*x/sqrt(c^
2))/(sqrt(c^2)*c^2))*arcsinh(c*x))*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 + 1)*a
rcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d/c
```

Fricas [A] time = 2.4621, size = 406, normalized size = 2.9

$$\frac{\left(2\,a^2+b^2\right)c^2ex^2+4\left(a^2+2\,b^2\right)c^2dx+\left(2\,b^2c^2ex^2+4\,b^2c^2dx+b^2e\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(2\,abc^2ex^2+4\,abc^2dx+ab\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2 +
 4*b^2*c^2*d*x + b^2*e)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(2*a*b*c^2*e*x^2
 + 4*a*b*c^2*d*x + a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [A] time = 0.905639, size = 233, normalized size = 1.66

$$\begin{cases} a^2dx+\frac{a^2ex^2}{2}+2abdx\,\mathrm{asinh}\,(cx)+abex^2\,\mathrm{asinh}\,(cx)-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{abex\sqrt{c^2x^2+1}}{2c}+\frac{abe\,\mathrm{asinh}\,(cx)}{2c^2}+b^2dx\,\mathrm{asinh}^2\,(cx)+2b^2dx \\ a^2\left(dx+\frac{ex^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*asinh(c*x) + a*b*e*x**2*asi
nh(c*x) - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - a*b*e*x*sqrt(c**2*x**2 + 1)/(2*c)
 + a*b*e*asinh(c*x)/(2*c**2) + b**2*d*x*asinh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*asinh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c) + b**2*e*asinh(c*x)*
*2/(4*c**2), Ne(c, 0)), (a**2*(d*x + e*x**2/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)(b\operatorname{arsinh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arcsinh(c*x) + a)^2, x)
```

3.15 $\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rubi [A] time = 0.0633947, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5653, 5717, 8}

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx &= x\left(a+b\sinh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx\\ &= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.0585261, size = 74, normalized size = 1.61

$$x\left(a^2+2b^2\right)-\frac{2ab\sqrt{c^2x^2+1}}{c}+\frac{2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\right)}{c}+b^2x\sinh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Maple [A] time = 0.002, size = 72, normalized size = 1.6

$$\frac{1}{c}\left(cxa^2+b^2\left((\mathrm{Arcsinh}\,(cx))^2\,cx-2\,\mathrm{Arcsinh}\,(cx)\,\sqrt{c^2x^2+1}+2\,cx\right)+2\,ab\left(\mathrm{Arcsinh}\,(cx)\,cx-\sqrt{c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(c*x*a^2+b^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.07637, size = 97, normalized size = 2.11

$$b^2x\,\mathrm{arsinh}\,(cx)^2+2\,b^2\left(x-\frac{\sqrt{c^2x^2+1}\,\mathrm{arsinh}\,(cx)}{c}\right)+a^2x+\frac{2\left(cx\,\mathrm{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Fricas [B] time = 2.35053, size = 212, normalized size = 4.61

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(a^2+2\,b^2\right)cx-2\,\sqrt{c^2x^2+1}ab+2\left(abcx-\sqrt{c^2x^2+1}b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

Sympy [A] time = 0.319157, size = 82, normalized size = 1.78

$$\begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

Giac [B] time = 1.57865, size = 150, normalized size = 3.26

$$2\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)ab + \left(x\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2 + a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

3.16 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=291

$$\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{2b^2\text{PolyLog}\left(3\right.}{e}$$

```
[Out] -(a + b*ArcSinh[c*x])^3/(3*b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log[1 +
 (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (2*b*(a + b*ArcSinh[c
*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*
b*(a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 +
 e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 +
e^2]))])/e
```

Rubi [A] time = 0.469464, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5799, 5561, 2190, 2531, 2282, 6589}

$$\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{2b^2\text{PolyLog}\left(3\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```


```
[Out] -(a + b*ArcSinh[c*x])^3/(3*b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log[1 +
 (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (2*b*(a + b*ArcSinh[c
*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*
b*(a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 +
 e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 +
e^2]))])/e
```


Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd\ldots}\right.$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\ldots\right.}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\ldots\right.}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\ldots\right.}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\ldots\right.}{e}$$

Mathematica [A] time = 0.201623, size = 273, normalized size = 0.94

$$\frac{6b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)+6b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)-6b^2\text{PolyLog}\left(3,\frac{ee^{\sin\ldots}}{\sqrt{c^2d\ldots}}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^3/b) + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*ArcSinh[c*x])*Pol
yLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*Arc
Sinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(3*e)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(ex + d)}{e} + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{ex + d} + \frac{2\,ab \log\left(cx + \sqrt{c^2x^2 + 1}\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x + d)
 + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d), x)
```

3.17 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=263

$$\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e\sqrt{c^2d^2+e^2}}+\frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2bc}{}$$

```
[Out] -((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcSinh[c*x])*Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2])
 - (2*b*c*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d
^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2 + e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2
+ e^2])
```

Rubi [A] time = 0.471534, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5801, 5831, 3322, 2264, 2190, 2279, 2391}

$$\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e\sqrt{c^2d^2+e^2}}+\frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```


```
[Out] -((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcSinh[c*x])*Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2])
 - (2*b*c*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d
^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2 + e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2
+ e^2])
```


Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\,\text{Subst}\left(\int \frac{a+bx}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\text{Subst}\left(\int \frac{e^x(a+bx)}{-e+2cde^x+ee^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\text{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2+e^2}+2ee^x}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{c^2d^2+e^2}} - \frac{(4bc)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{e\sqrt{c^2d}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{e\sqrt{c^2d}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{e\sqrt{c^2d}}$$

Mathematica [A] time = 0.210305, size = 191, normalized size = 0.73

$$\frac{\frac{2bc\left(b\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)-b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)+\left(a+b\sinh^{-1}(cx)\right)\left(\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)-\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)\right)\right)}{\sqrt{c^2d^2+e^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^2/(d + e*x)) + (2*b*c*((a + b*ArcSinh[c*x])*(Log[1
+ (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*
x])/(c*d + Sqrt[c^2*d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^
2*d^2 + e^2]))]))/Sqrt[c^2*d^2 + e^2])/e
```

Maple [A] time = 0.155, size = 529, normalized size = 2.

$$-\frac{ca^2}{(cex+cd)e}-\frac{cb^2(\text{Arcsinh}(cx))^2}{(cex+cd)e}+2\frac{cb^2\text{Arcsinh}(cx)}{e\sqrt{c^2d^2+e^2}}\ln\left(\frac{-\left(cx+\sqrt{c^2x^2+1}\right)e-cd+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)-2\frac{cb^2\text{Arcsinh}}{e\sqrt{c^2d^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^2,x)
```

```
[Out] -c*a^2/(c*e*x+c*d)/e-c*b^2*arcsinh(c*x)^2/e/(c*e*x+c*d)+2*c*b^2/e*arcsinh(c
*x)/(c^2*d^2+e^2)^(1/2)*ln((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1
/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2*c*b^2/e*arcsinh(c*x)/(c^2*d^2+e^2)^(1/2)
*ln(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^
(1/2)))+2*c*b^2/e/(c^2*d^2+e^2)^(1/2)*dilog((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d
+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2*c*b^2/e/(c^2*d^2+e^2)^(
1/2)*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^
2+e^2)^(1/2)))-2*c*a*b/(c*e*x+c*d)/e*arcsinh(c*x)-2*c*a*b/e^2/((c^2*d^2+e^2
)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e
^2)^(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x
+c*d/e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}(cx)^2+2\,ab\,\text{arsinh}(cx)+a^2}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^2*x^2 + 2*d*e*x
 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^2, x)
```

3.18 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=349

$$\frac{b^2c^3d\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{b^2c^3d\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}+\frac{bc^3d\left(a+b\,\text{si}\right.}{}$$

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*x)))
 - (a + b*ArcSinh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d*(a + b*ArcSinh[c*x])
*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2
)^(3/2)) - (b*c^3*d*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2)^(3/2)) + (b^2*c^2*Log[d + e*x])/(
e*(c^2*d^2 + e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2)) - (b^2*c^3*d*PolyLog[2, -((e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2))
```

Rubi [A] time = 0.60894, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5801, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{b^2c^3d\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{b^2c^3d\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}+\frac{bc^3d\left(a+b\,\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*x)))
 - (a + b*ArcSinh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d*(a + b*ArcSinh[c*x])
*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2
)^(3/2)) - (b*c^3*d*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2)^(3/2)) + (b^2*c^2*Log[d + e*x])/(
e*(c^2*d^2 + e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2)) - (b^2*c^3*d*PolyLog[2, -((e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2))
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{a+bx}{(cd+e\sinh(x))^2}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(b^2c^2\right)\text{Subst}\left(\int \frac{\cosh(x)}{cd+e\sinh(x)}\,dx,\right.}{c^2d^2+e^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{cd+x}\,dx, x, cex\right.}{e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2+e^2\right)} + \frac{\left(2bc^3d\right)\text{Sul}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{e\left(c^2d^2+e^2\right)^{3/2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{e\left(c^2d^2+e^2\right)^{3/2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{e\left(c^2d^2+e^2\right)^{3/2}}$$

Mathematica [A] time = 0.874181, size = 270, normalized size = 0.77

$$\frac{\frac{2bc^3d\left(b\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)-b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)+\left(a+b\sinh^{-1}(cx)\right)\left(\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)-\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)\right)\right)}{\left(c^2d^2+e^2\right)^{3/2}} - \frac{2bce\sqrt{c^2x^2+1}\left(a+b\sinh\right.}{\left(c^2d^2+e^2\right)(d+ex)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] ((-2*b*c*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*
x)) - (a + b*ArcSinh[c*x])^2/(d + e*x)^2 + (2*b^2*c^2*Log[d + e*x])/(c^2*d^
2 + e^2) + (2*b*c^3*d*((a + b*ArcSinh[c*x])*(Log[1 + (e*E^ArcSinh[c*x])/(c*
d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])]
 - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]))/(c^2*d
^2 + e^2)^(3/2))/(2*e)
```

Maple [B] time = 0.262, size = 1013, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a^2/(c*e*x+c*d)^2/e-1/2*c^4*b^2*arcsinh(c*x)^2/e/(c*e*x+c*d)^2/(c^
2*d^2+e^2)*d^2-c^3*b^2*arcsinh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2+e^2)*(c^2*x^2+
1)^(1/2)*x-c^3*b^2*arcsinh(c*x)/(c*e*x+c*d)^2/(c^2*d^2+e^2)*d*(c^2*x^2+1)^(
1/2)+c^4*b^2*arcsinh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2+e^2)*x^2+2*c^4*b^2*arcsi
nh(c*x)/(c*e*x+c*d)^2/(c^2*d^2+e^2)*d*x+c^4*b^2*arcsinh(c*x)/e/(c*e*x+c*d)^
2/(c^2*d^2+e^2)*d^2-1/2*c^2*b^2*arcsinh(c*x)^2*e/(c*e*x+c*d)^2/(c^2*d^2+e^2
)+c^2*b^2/e/(c^2*d^2+e^2)*ln(2*c*d*(c*x+(c^2*x^2+1)^(1/2))+(c*x+(c^2*x^2+1)
^(1/2))^2*e-e)-2*c^2*b^2/e/(c^2*d^2+e^2)*ln(c*x+(c^2*x^2+1)^(1/2))+c^3*b^2/
e/(c^2*d^2+e^2)^(3/2)*d*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^
2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-c^3*b^2/e/(c^2*d^2+e^2)^(3/2)
*d*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d
+(c^2*d^2+e^2)^(1/2)))+c^3*b^2/e/(c^2*d^2+e^2)^(3/2)*d*dilog((-(c*x+(c^2*x^
2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-c^3*b^2/
e/(c^2*d^2+e^2)^(3/2)*d*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^
(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))-c^2*a*b/(c*e*x+c*d)^2/e*arcsinh(c*x)-c^2*
a*b/e/(c^2*d^2+e^2)/(c*x+c*d/e)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2
+e^2)/e^2)^(1/2)-c^3*a*b/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((
2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c
*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^3, x)
```

3.19 $\int \frac{(d+ex)^3}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=394

$$-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}-\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}$$

```
[Out] (d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (3*d*e^2*Cosh[a/b]
*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*Cosh[(3*a)/b]*CoshI
ntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*a)/
b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (e^3*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (e^3*CoshIntegral[(4*a)/b + 4*Ar
cSinh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) - (d^3*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(b*c) + (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(
4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(
2*b*c^2) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b*
c^4) - (3*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*
c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^4)
```

Rubi [A] time = 1.17237, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5805, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}-\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```


```
[Out] (d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (3*d*e^2*Cosh[a/b]
*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*Cosh[(3*a)/b]*CoshI
ntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*a)/
b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (e^3*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (e^3*CoshIntegral[(4*a)/b + 4*Ar
cSinh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) - (d^3*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(b*c) + (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(
4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(
2*b*c^2) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b*
c^4) - (3*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*
c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^4)
```


Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^3}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^3d^3\cosh(x)}{a+bx}+\frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx}+\frac{3cde^2\cosh(x)\sinh^2(x)}{a+bx}+\frac{e^3\cosh(x)\sinh^3(x)}{a+bx}\right)dx, x, \sinh \ldots}{c^4}$$

$$= \frac{d^3\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \ldots$$

$$= \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{\left(3de^2\right)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sin \ldots}{c^3}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\sin \ldots}{a \ldots}\right.}{\ldots}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{\left(3de^2\cosh\left(\frac{a}{b}\right)\right)\text{Sub} \ldots}{\ldots}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{3de^2\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{C} \ldots}{\ldots}$$

Mathematica [A] time = 0.670153, size = 305, normalized size = 0.77

$$-\frac{3d^2e\left(\sinh\left(\frac{2a}{b}\right)\textbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)-\cosh\left(\frac{2a}{b}\right)\textbf{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\right)}{2bc^2} + \frac{3de^2\left(-\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c \ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]]))/(b*c) + (3*d*e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSi
nh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*
SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])]))/(4*b*c^3) + (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[
(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcSinh[c*x])]))/(8*b*c^4) - (3*d^2*e*(CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]
]))/(2*b*c^2)
```

Maple [A] time = 0.195, size = 394, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e^3}{16\,c^3b}\mathrm{e}^{-4\,\frac{a}{b}}\mathrm{Ei}\left(1,-4\,\mathrm{Arcsinh}\,(cx)-4\,\frac{a}{b}\right)+\frac{e^3}{16\,c^3b}\mathrm{e}^{4\,\frac{a}{b}}\mathrm{Ei}\left(1,4\,\mathrm{Arcsinh}\,(cx)+4\,\frac{a}{b}\right)+\frac{3\,d^2e}{4\,bc}\mathrm{e}^{2\,\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{Arcsinh}\,(cx)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/16/c^3*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+1/16/c^3*e^3/b
*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arcsinh(
c*x)+2*a/b)*d^2-1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-3/4/c*e
/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*d^2+1/8/c^3*e^3/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-
3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^3+3/8/c^2*d/b*exp(a/b)*Ei(1,
arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2*d
/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2-3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*
arcsinh(c*x)+3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

3.20 $\int \frac{(d+ex)^2}{a+b\sinh^{-1}(cx)} dx$

Optimal. Leaf size=245

$$-\frac{de\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{bc^2}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{de}{}$$

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e^2*Cosh[a/b]*Cos
hIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e^2*Cosh[(3*a)/b]*CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]])/(b*c) + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) +
 (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b*c^2) - (e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3)
```

Rubi [A] time = 0.70211, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5805, 6742, 3303, 3298, 3301, 5448}

$$-\frac{de\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{bc^2}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{de}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```


```
[Out] (d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e^2*Cosh[a/b]*Cos
hIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e^2*Cosh[(3*a)/b]*CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]])/(b*c) + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) +
 (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b*c^2) - (e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3)
```


Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^2}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^2d^2\cosh(x)}{a+bx}+\frac{e^2\cosh(x)\sinh^2(x)}{a+bx}+\frac{cde\sinh(2x)}{a+bx}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{(de)\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{e^2\,\text{Subst}}{}$$

$$= \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,\right.}{c}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} - \frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}}{b}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} - \frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}}{b}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}}{4bc}$$

Mathematica [A] time = 0.399842, size = 188, normalized size = 0.77

$$\frac{\cosh\left(\frac{a}{b}\right)\left(4c^2d^2-e^2\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4c^2d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4cde\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d^2 - e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c*d*e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 4*c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcS
inh[c*x]] + e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 4*c*d*e*Cosh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - e^2*Sinh[(3*a)/b]*SinhIntegr
```

```
al[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Maple [A] time = 0.128, size = 254, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e^2}{8c^2b}e^{-3\frac{a}{b}}\mathrm{Ei}\left(1,-3\,\mathrm{Arcsinh}\,(cx)-3\,\frac{a}{b}\right)-\frac{e^2}{8c^2b}e^{3\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\,(cx)+3\,\frac{a}{b}\right)-\frac{d^2}{2b}e^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8/c^2*e^2/b*e
xp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)*d^2+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2-1/2/c*d*
e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/2/c*d*e/b*exp(2*a/b)*Ei(1,2*a
rcsinh(c*x)+2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

3.21 $\int \frac{d+ex}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=116

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\sinh}{}$$

```
[Out] (d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (d*Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcS
inh[c*x]])/(2*b*c^2)
```

Rubi [A] time = 0.318003, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5805, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (d*Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcS
inh[c*x]])/(2*b*c^2)
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{d+ex}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{cd\cosh(x)}{a+bx}+\frac{e\cosh(x)\sinh(x)}{a+bx}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{d\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} -$$
$$= \frac{d\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx\right.}{2c^2}$$
$$= \frac{d\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\right.}{}$$
$$= \frac{d\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{e\textbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} - \frac{d\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}\right.}{bc}$$

Mathematica [A] time = 0.159526, size = 98, normalized size = 0.84

$$\frac{2cd\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - e\sinh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2cd\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + e\cos}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 2*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^2)
```

Maple [A] time = 0.085, size = 120, normalized size = 1.

$$\frac{1}{c}\left(-\frac{d}{2b}e^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\left(cx\right)+\frac{a}{b}\right)-\frac{d}{2b}e^{-\frac{a}{b}}\mathrm{Ei}\left(1,-\mathrm{Arcsinh}\left(cx\right)-\frac{a}{b}\right)-\frac{e}{4bc}e^{-2\frac{a}{b}}\mathrm{Ei}\left(1,-2\,\mathrm{Arcsinh}\left(cx\right)-2\frac{a}{b}\right)+\frac{e}{4bc}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d-1/2/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)*d-1/4/c*e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/4/c*e/b*ex
p(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex+d}{b\,\mathrm{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

3.22 $\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rubi [A] time = 0.073575, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\,\text{s}\right.}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A] time = 0.0191837, size = 45, normalized size = 0.83

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
ArcSinh[c*x]])/(b*c)
```

Maple [A] time = 0., size = 56, normalized size = 1.

$$\frac{1}{c}\left(-\frac{1}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{Arcsinh}(cx)+\frac{a}{b}\right)-\frac{1}{2b}e^{-\frac{a}{b}}\text{Ei}\left(1,-\text{Arcsinh}(cx)-\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\,\text{arsinh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsinh}(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

3.23 $\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.0419735, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.201066, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)\left(a+b\text{Arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{aex+ad+(bex+bd)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

3.24 $\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.0392964, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.376284, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.23, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2\,(a+b\text{Arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{ae^2x^2+2\,adex+ad^2+\left(be^2x^2+2\,bdex+bd^2\right)\mathrm{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2)
*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

3.25 $\int \frac{(d+ex)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=359

$$\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{2de\sinh}{}$$

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 + c
^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a +
 b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(b^2*c^2) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b
^2*c) + (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b^2*c^3) -
(3*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b^2*c^3)
+ (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e^2*Cosh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/
b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/
b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Rubi [A] time = 0.688232, antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5803, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 + c
^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a +
 b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c
*x]])/(b^2*c^2) - (d^2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b^2*c)
+ (e^2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b^2*c^3) - (3*e^2*Cos
hIntegral[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b^2*c^3) + (d^2*Cosh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (e^2*Cosh[a/b]*SinhIntegra
l[a/b + ArcSinh[c*x]])/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/b]*SinhIntegral[(2*a
)/b + 2*ArcSinh[c*x]])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/
b + 3*ArcSinh[c*x]])/(4*b^2*c^3)
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d^2}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{2dex}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{e^2x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d^2\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+(2de)\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+e^2\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(cd^2\right)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\right.}}{b}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\right.}{b}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2de\cosh\left(\frac{2a}{b}\right)\mathbf{Chi}}{b}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2de\cosh\left(\frac{2a}{b}\right)\mathbf{Chi}}{b}$$

Mathematica [A] time = 1.58072, size = 288, normalized size = 0.8

$$-\frac{\sinh\left(\frac{a}{b}\right)\left(4c^2d^2-e^2\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4c^2d^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\frac{4bc^2d^2\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}+\frac{8bc^2dex\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((4*b*c^2*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (8*b*c^2*d*e*x*Sqr
t[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e^2*x^2*Sqrt[1 + c^2*x^2])/
(a + b*ArcSinh[c*x]) - 8*c*d*e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[
c*x])] + (4*c^2*d^2 - e^2)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*e
^2*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d^2*Cosh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + ArcSin
h[c*x]] + 8*c*d*e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 3*e^
2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b^2*c^3)
```

Maple [A] time = 0.2, size = 616, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e^
2/c^2/b/(a+b*arcsinh(c*x))+3/8*e^2/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3
*a/b)-1/8*e^2/c^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1
)^(1/2))/(a+b*arcsinh(c*x))-3/8*e^2/c^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x
)-3*a/b)+1/2*(c*x-(c^2*x^2+1)^(1/2))*d^2/b/(a+b*arcsinh(c*x))+1/2*d^2/b^2*e
xp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8*(c*x-(c^2*x^2+1)^(1/2))*e^2/c^2/b/(a+b*a
rcsinh(c*x))-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x
+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(
c*x)-a/b)+1/8/c^2/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c^2/
b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/2*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^
(1/2)+1)*d*e/c/(a+b*arcsinh(c*x))/b-d/c*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x
)+2*a/b)-1/2*d/c*e/b*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x
))-d/c*e/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3e^2x^5+2\,c^3dex^4+2\,cdex^2+cd^2x+\left(c^3d^2+ce^2\right)x^3+\left(c^2e^2x^4+2\,c^2dex^3+2\,dex+\left(c^2d^2+e^2\right)x^2+d^2\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^5 + 2*c^3*d*e*x^4 + 2*c*d*e*x^2 + c*d^2*x + (c^3*d^2 + c*e^2)*x
^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 + 2*d*e*x + (c^2*d^2 + e^2)*x^2 + d^2)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*
```

```
c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))
) + integrate((3*c^5*e^2*x^6 + 4*c^5*d*e*x^5 + 8*c^3*d*e*x^3 + (c^5*d^2 + 6
*c^3*e^2)*x^4 + 4*c*d*e*x + c*d^2 + (2*c^3*d^2 + 3*c*e^2)*x^2 + (3*c^3*e^2*
x^4 + 4*c^3*d*e*x^3 - c*d^2 + (c^3*d^2 + c*e^2)*x^2)*(c^2*x^2 + 1) + (6*c^4
*e^2*x^5 + 8*c^4*d*e*x^4 + 8*c^2*d*e*x^2 + (2*c^4*d^2 + 7*c^2*e^2)*x^3 + 2*
d*e + (c^2*d^2 + 2*e^2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^2 + 2\,dex + d^2}{b^2\,\text{arsinh}\,(cx)^2 + 2\,ab\,\text{arsinh}\,(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x)
 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{(a + b\,\text{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2}{(b\,\text{arsinh}\,(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

3.26 $\int \frac{d+ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=180

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}}{b}$$

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^2
])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*Arc
Sinh[c*x]))/b])/(b^2*c^2) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/
b])/(b^2*c) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) -
(e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2)
```

Rubi [A] time = 0.33673, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5803, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}+d\cos$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^2
])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*A
rcSinh[c*x]])/(b^2*c^2) - (d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b
^2*c) + (d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (e*Sinh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b^2*c^2)
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + e\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b} + \frac{e\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc} + \frac{(e\,c}{}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2} - \frac{e\,\mathrm{si}}{}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2} - \frac{d\mathrm{C}}{}$$

Mathematica [A] time = 0.734094, size = 150, normalized size = 0.83

$$-\frac{\frac{bcd\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)} + \frac{bcex\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)} + cd\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - cd\cosh\left(\frac{a}{b}\right)\mathrm{Sh}}{b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(((b*c*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (b*c*e*x*Sqrt[1 + c^2*x
^2])/(a + b*ArcSinh[c*x]) - e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c
*x])] + c*d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - c*d*Cosh[a/b]*Sinh
Integral[a/b + ArcSinh[c*x]] + e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c*x])])/(b^2*c^2))
```

Maple [A] time = 0.125, size = 272, normalized size = 1.5

$$\frac{1}{c}\left(\frac{d}{2\,b\,(a+b\mathrm{Arcsinh}\,(cx))}\left(cx-\sqrt{c^2x^2+1}\right)+\frac{d}{2\,b^2}e^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right)-\frac{d}{2\,b\,(a+b\mathrm{Arcsinh}\,(cx))}\left(cx+\sqrt{c^2x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(c*x-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(c*x))+1/2*d/b^2*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)-1/2/b*d*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1
/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)
^(1/2)+1)*e/c/(a+b*arcsinh(c*x))/b-1/2/c*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*
x)+2*a/b)-1/4/c*e/b*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x)
)-1/2/c*e/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3ex^4+c^3dx^3+cex^2+cdx+\left(c^2ex^3+c^2dx^2+ex+d\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{2}{abc^5x^4+\left(c^2x^2+1\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^4 + c^3*d*x^3 + c*e*x^2 + c*d*x + (c^2*e*x^3 + c^2*d*x^2 + e*x +
d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
+ 1))) + integrate((2*c^5*e*x^5 + c^5*d*x^4 + 4*c^3*e*x^3 + 2*c^3*d*x^2 + 2
*c*e*x + (2*c^3*e*x^3 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (4*c^4*e*x^4
 + 2*c^4*d*x^3 + 4*c^2*e*x^2 + c^2*d*x + e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4
 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*
x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*
x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex+d}{b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a)^2, x)
```

3.27 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(a + b*ArcS
inh[c*x])/b]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c)
```

Rubi [A] time = 0.179424, antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5655, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[a/b + ArcSi
nh[c*x]]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(b^2*c)
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}\\ &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}\\ &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\right.}{}\\ &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}\end{aligned}$$

Mathematica [A] time = 0.198078, size = 71, normalized size = 0.84

$$\frac{-\frac{b\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Maple [A] time = 0., size = 118, normalized size = 1.4

$$\frac{1}{c}\left(\frac{1}{2\,b\,(a+b\text{Arcsinh}\,(cx))}\left(cx-\sqrt{c^2x^2+1}\right)+\frac{1}{2\,b^2}e^{\frac{a}{b}}\text{Ei}\left(1,\text{Arcsinh}\,(cx)+\frac{a}{b}\right)-\frac{1}{2\,b\,(a+b\text{Arcsinh}\,(cx))}\left(cx+\sqrt{c^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(c*x-(c^2*x^2+1)^(1/2))/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(1
,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2
*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^4x^4+\left(c^2x^2+1\right)abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

3.28 $\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0314303, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 3.07591, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.171, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)\,(a+b\text{Arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3 + cx + \left(c^2x^2 + 1\right)^{\frac{3}{2}}}{abc^3ex^3 + abc^3dx^2 + abcex + abcd + \left(b^2c^3ex^3 + b^2c^3dx^2 + b^2cex + b^2cd + \left(b^2c^2ex^2 + b^2c^2dx\right)\sqrt{c^2x^2 + 1}\right)\log\left(cx + \sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^3 + a*b*c^3*d*x^2 + a*b
*c*e*x + a*b*c*d + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (
b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) + integrate((c^5*d*x^4
 + 2*c^3*d*x^2 + (c^3*d*x^2 - 2*c*e*x - c*d)*(c^2*x^2 + 1) + c*d + (2*c^4*d
*x^3 - 2*c^2*e*x^2 + c^2*d*x - e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^6 + 2*a
*b*c^5*d*e*x^5 + 4*a*b*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*a*b*x^4 + 2*a*b*
c*d*e*x + a*b*c*d^2 + (2*c^3*d^2 + c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^4 + 2*a*
b*c^3*d*e*x^3 + a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^6 + 2*b^2*c
^5*d*e*x^5 + 4*b^2*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*b^2*x^4 + 2*b^2*c*d*
e*x + b^2*c*d^2 + (2*c^3*d^2 + c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4 + 2*b^2*c^
3*d*e*x^3 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^2*x^5 + 2*b^2*c^4
*d*e*x^4 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x + (c^4*d^2 + c^2*e^2)*b^2*x^3)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^2*x^5 + 2*a
*b*c^4*d*e*x^4 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x + (c^4*d^2 + c^2*e^2)*a*
b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2ex + a^2d + \left(b^2ex + b^2d\right)\operatorname{arsinh}(cx)^2 + 2\left(abex + abd\right)\operatorname{arsinh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*e*x
 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{asinh}(cx)\right)^2 (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)^2), x)
```

3.29 $\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0291871, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2} \, dx = \int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$$

Mathematica [A] time = 5.08055, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2 (a+b\text{Arcsinh}(cx))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^4 + 2*a*b*c^3*d*e*x^3
 + 2*a*b*c*d*e*x + a*b*c*d^2 + (c^3*d^2 + c*e^2)*a*b*x^2 + (b^2*c^3*e^2*x^4
 + 2*b^2*c^3*d*e*x^3 + 2*b^2*c*d*e*x + b^2*c*d^2 + (c^3*d^2 + c*e^2)*b^2*x^
2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2 + a*b
*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^5 - c^5*d*x^4 + 2*c^3*e
*x^3 - 2*c^3*d*x^2 + c*e*x + (c^3*e*x^3 - c^3*d*x^2 + 3*c*e*x + c*d)*(c^2*x
^2 + 1) - c*d + (2*c^4*e*x^4 - 2*c^4*d*x^3 + 5*c^2*e*x^2 - c^2*d*x + 2*e)*s
qrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*d^2*e + 2
*c^3*e^3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*a*b*x^4 + a*b
*c*d^3 + (6*c^3*d^2*e + c*e^3)*a*b*x^3 + (2*c^3*d^3 + 3*c*d*e^2)*a*b*x^2 +
(a*b*c^3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 + a*b*c^3*d^3*
x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^5*d*e^2*x^6 + (3*c^5*d^2*e
+ 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*b^2*x^4 +
b^2*c*d^3 + (6*c^3*d^2*e + c*e^3)*b^2*x^3 + (2*c^3*d^3 + 3*c*d*e^2)*b^2*x^2
 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^2*c^3*d^2*e*x^3 + b^2*c^3*d
^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^3*x^6 + 3*b^2*c^4*d*e^2*x^5 + 3*b^2*c^
2*d^2*e*x^2 + b^2*c^2*d^3*x + (3*c^4*d^2*e + c^2*e^3)*b^2*x^4 + (c^4*d^3 +
3*c^2*d*e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(
a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 + 3*a*b*c^2*d^2*e*x^2 + a*b*c^2*d^3*x
 + (3*c^4*d^2*e + c^2*e^3)*a*b*x^4 + (c^4*d^3 + 3*c^2*d*e^2)*a*b*x^3)*sqrt(
c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2e^2x^2 + 2\,a^2dex + a^2d^2 + \left(b^2e^2x^2 + 2\,b^2dex + b^2d^2\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abe^2x^2 + 2\,abdex + abd^2\right)\operatorname{arsinh}\left(cx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*
x + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arcsi
nh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\operatorname{asinh}\left(cx\right)\right)^2\left(d + ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)^2(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

3.30 $\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=74

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)^2}{e(m+1)}-\frac{2bc\text{Unintegrable}\left(\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}},x\right)}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x])^2)/(e*(1 + m)) - (2*b*c*Unintegrabl
e[((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2], x])/(e*(1 + m
))
```

Rubi [A] time = 0.261283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2], x])/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx=\frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)^2}{e(1+m)}-\frac{(2bc)\int \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{e(1+m)}$$

Mathematica [A] time = 3.93758, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 2.824, size = 0, normalized size = 0.

$$\int (ex+d)^m (a+b\text{Arcsinh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2\right)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*(e*x + d)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asinh}(cx))^2 (d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*(d + e*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(cx) + a)^2 (ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*(e*x + d)^m, x)
```

3.31 $\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=179

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

```
[Out] -((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (d
 + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d -
 e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt[1
+ c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

Rubi [A] time = 0.100638, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5801, 760, 133}

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (d
 + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d -
 e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt[1
+ c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{1+c^2x^2}} dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{e(1+m)} - \frac{\left(bc\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-c^2}e}{c^2}}}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{1-\frac{\cdots}{d-\cdots}}}\cdots\right.}{e^2(1+m)\sqrt{1+c^2x^2}\cdots}$$

$$= -\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}F_1\left(2+m;\frac{1}{2},\frac{1}{2};3+m;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}} - \cdots$$

Mathematica [F] time = 0.0432109, size = 0, normalized size = 0.

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]), x]
```

Maple [F] time = 2.781, size = 0, normalized size = 0.

$$\int (ex+d)^m (a+b\text{Arcsinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\,\text{arsinh}(cx)+a)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{asinh}(cx))(d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(d + e*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(cx) + a)(ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

3.32 $\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(d+ex)^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Rubi [A] time = 0.0289984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.341616, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Maple [A] time = 0.891, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{a+b\text{Arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b \operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{b \operatorname{arsinh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{a+b \operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{b \operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

3.33 $\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [A] time = 0.0308005, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]

[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A] time = 0.738448, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]

[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.93, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(a+b\text{Arcsinh}\,(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}(ex+d)^m+\left(c^3x^3+cx\right)(ex+d)^m}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5ex^5+abc^5dx^4+2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*(e*x + d)^m + (c^3*x^3 + c*x)*(e*x + d)^m)/(a*b*c^3*x
^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)
*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*e*(m +
1)*x^3 + c^3*d*x^2 + c*e*(m - 1)*x - c*d)*(c^2*x^2 + 1)*(e*x + d)^m + (2*c^
4*e*(m + 1)*x^4 + 2*c^4*d*x^3 + c^2*e*(3*m + 1)*x^2 + c^2*d*x + e*m)*sqrt(c
^2*x^2 + 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 + 2*c^3*e*(m + 1)*
x^3 + 2*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5 + a*b*
c^5*d*x^4 + 2*a*b*c^3*e*x^3 + 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d + (a*b*
c^3*e*x^3 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*x^5 + b^2*c^5*d*x^4 +
 2*b^2*c^3*e*x^3 + 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (b^2*c^3*e*x^3 +
 b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^4 + b^2*c^4*d*x^3 + b^2*c^2*
e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a
*b*c^4*e*x^4 + a*b*c^4*d*x^3 + a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^m}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a)^2, x)
```

3.34 $\int (f+gx)^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=640

$$\frac{f^2g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}$$

```
[Out] -((b*f^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2])) + (2*b*g^3*x*Sqrt[
d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*c*f^3*x^2*Sqrt[d + c^2*d*x^
2])/(4*Sqrt[1 + c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[
1 + c^2*x^2]) - (b*c*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d + c^2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (3*b*c*f*g^2*x^
4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c*g^3*x^5*Sqrt[d + c^2*d
*x^2])/(25*Sqrt[1 + c^2*x^2]) + (f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/2 + (3*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (3
*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 + c^2*x^
2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (f^3*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2]) - (3*f*g^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.686775, antiderivative size = 640, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5835, 5821, 5682, 5675, 30, 5717, 5742, 5758, 266, 43, 5732, 12}

$$\frac{f^2g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -((b*f^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2])) + (2*b*g^3*x*Sqrt[
d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*c*f^3*x^2*Sqrt[d + c^2*d*x^
2])/(4*Sqrt[1 + c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[
1 + c^2*x^2]) - (b*c*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d + c^2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (3*b*c*f*g^2*x^
4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c*g^3*x^5*Sqrt[d + c^2*d
*x^2])/(25*Sqrt[1 + c^2*x^2]) + (f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/2 + (3*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (3
*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 + c^2*x^
2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (f^3*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2]) - (3*f*g^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\sqrt{d+c^2dx^2}\int (f+gx)^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\sqrt{d+c^2dx^2}\int\left(f^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+3f^2gx\sqrt{1+c^2x^2}\left(a+b\right.\right.}{}$$

$$=\frac{\left(f^3\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}+\frac{\left(3f^2g\sqrt{d+c^2dx^2}\right)}{}$$

$$=\frac{1}{2}f^3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.$$

$$=-\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{3bcfg}{16}$$

$$=-\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}+\frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{3bfg^2x^2}{16c}$$

Mathematica [A] time = 1.41868, size = 413, normalized size = 0.65

$$\frac{240a\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(6c^4x\left(20f^2gx+10f^3+15fg^2x^2+4g^3x^3\right)+c^2g\left(120f^2+45fgx+8g^2x^2\right)-16g^3\right)+3600ac}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-16*g^3 + c^2*g*(120*f^2 + 45
*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x
```

```
^3)) - 9600*b*c^2*f^2*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x
^2)^(3/2)*ArcSinh[c*x]) - 128*b*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x
^2 + 9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c
*x]) + 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*Log[c*d*x +
 Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3600*b*c^3*f^3*Sqrt[d + c^2*d*x^2]*(Cosh[2*
ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 675
*b*c*f*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4
*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(28800*c^4*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.463, size = 1119, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/5*a*g^3*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-3/8*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(1/
2)+a*f^2*g/c^2/d*(c^2*d*x^2+d)^(3/2)+4/15*b*(d*(c^2*x^2+1))^(1/2)*g^3/(c^2*
x^2+1)*arcsinh(c*x)*x^4+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3/(c^2*x^2+1)*arcsinh
(c*x)*x+1/4*b*(d*(c^2*x^2+1))^(1/2)*f^3*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c-
2/15*b*(d*(c^2*x^2+1))^(1/2)*g^3/c^4/(c^2*x^2+1)*arcsinh(c*x)-1/25*b*(d*(c^
2*x^2+1))^(1/2)*g^3*c/(c^2*x^2+1)^(1/2)*x^5-1/45*b*(d*(c^2*x^2+1))^(1/2)*g^
3/c/(c^2*x^2+1)^(1/2)*x^3+2/15*b*(d*(c^2*x^2+1))^(1/2)*g^3/c^3/(c^2*x^2+1)^
(1/2)*x-3/128*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/c^3/(c^2*x^2+1)^(1/2)-1/4*b*(d*
(c^2*x^2+1))^(1/2)*f^3*c/(c^2*x^2+1)^(1/2)*x^2-2/15*a*g^3/d/c^4*(c^2*d*x^2+
d)^(3/2)+3/4*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5
+3/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+b*(d*(c^2
*x^2+1))^(1/2)*g*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*f^2+9/8*b*(d*(c^2*x^2+1))
^(1/2)*f*g^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+2*b*(d*(c^2*x^2+1))^(1/2)*g/(c^2*
x^2+1)*arcsinh(c*x)*x^2*f^2+1/2*a*f^3*x*(c^2*d*x^2+d)^(1/2)-1/8*b*(d*(c^2*x
^2+1))^(1/2)*f^3/c/(c^2*x^2+1)^(1/2)+1/2*a*f^3*d*ln(x*c^2*d/(c^2*d)^(1/2)+(
c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+3/4*a*f*g^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-3
/8*a*f*g^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2
)-1/3*b*(d*(c^2*x^2+1))^(1/2)*g*c/(c^2*x^2+1)^(1/2)*x^3*f^2-b*(d*(c^2*x^2+1
))^(1/2)*g/c/(c^2*x^2+1)^(1/2)*x*f^2-3/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*c/(
c^2*x^2+1)^(1/2)*x^4-3/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/c/(c^2*x^2+1)^(1/2)
*x^2-3/16*b*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c^3*g^
2+1/5*b*(d*(c^2*x^2+1))^(1/2)*g^3*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^6-1/15*b*(
d*(c^2*x^2+1))^(1/2)*g^3/c^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+b*(d*(c^2*x^2+1))
^(1/2)*g/c^2/(c^2*x^2+1)*arcsinh(c*x)*f^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*c
^2/(c^2*x^2+1)*arcsinh(c*x)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\left(c^2x^2 + 1\right)}\left(a + b\,\text{asinh}\left(cx\right)\right)\left(f + gx\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2 + d}\left(gx + f\right)^3\left(b\,\text{arsinh}\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(g*x + f)^3*(b*arcsinh(c*x) + a), x)
```

3.35 $\int (f+gx)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=431

$$\frac{1}{2}f^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{2fg\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(c\right.}{3c^2}$$

```
[Out] (-2*b*f*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f^2*x^2*Sqr
t[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*g^2*x^2*Sqrt[d + c^2*d*x^2])/(
16*c*Sqrt[1 + c^2*x^2]) - (2*b*c*f*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt[1 + c
^2*x^2]) - (b*c*g^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (f^2*
x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/4 + (2*f*g*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(3*c^2) + (f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2]) - (g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt
[1 + c^2*x^2])
```

Rubi [A] time = 0.532257, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5835, 5821, 5682, 5675, 30, 5717, 5742, 5758}

$$\frac{1}{2}f^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{2fg\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(c\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-2*b*f*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f^2*x^2*Sqr
t[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*g^2*x^2*Sqrt[d + c^2*d*x^2])/(
16*c*Sqrt[1 + c^2*x^2]) - (2*b*c*f*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt[1 + c
^2*x^2]) - (b*c*g^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (f^2*
x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/4 + (2*f*g*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(3*c^2) + (f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2]) - (g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt
[1 + c^2*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
```

```
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (f+gx)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\sqrt{d+c^2dx^2}\int (f+gx)^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\int \left(f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+2fgx\sqrt{1+c^2x^2}\left(a+\right.\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(2fg\sqrt{d+c^2dx\right.}{}$$

$$= \frac{1}{2}f^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

$$= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} - \frac{bc}{}$$

$$= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}} - \frac{2bcf}{9}$$

Mathematica [A] time = 0.926528, size = 301, normalized size = 0.7

$$\frac{48ac\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(12c^2f^2x+16f\left(c^2gx^2+g\right)+3g^2x\left(2c^2x^2+1\right)\right)+144a\sqrt{d}\sqrt{c^2x^2+1}(2cf-g)(2cf+g)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(12*c^2*f^2*x + 3*g^2*x*(1 +
2*c^2*x^2) + 16*f*(g + c^2*g*x^2)) - 256*b*c*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x
 + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 144*a*Sqrt[d]*(2*c*f - g
)*(2*c*f + g)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] -
144*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(A
rcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 9*b*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSi
nh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(1
152*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.355, size = 791, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a*g^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a*g^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1
/8*a*g^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+
2/3*a*f*g/c^2/d*(c^2*d*x^2+d)^(3/2)+1/2*a*f^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f
^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/128*b*(d
*(c^2*x^2+1))^(1/2)*g^2/c^3/(c^2*x^2+1)^(1/2)-1/8*b*(d*(c^2*x^2+1))^(1/2)*f
^2/c/(c^2*x^2+1)^(1/2)+2/3*b*(d*(c^2*x^2+1))^(1/2)*f*g*c^2/(c^2*x^2+1)*arcs
inh(c*x)*x^4-2/9*b*(d*(c^2*x^2+1))^(1/2)*f*g*c/(c^2*x^2+1)^(1/2)*x^3+4/3*b*
(d*(c^2*x^2+1))^(1/2)*f*g/(c^2*x^2+1)*arcsinh(c*x)*x^2-2/3*b*(d*(c^2*x^2+1)
```

```
)^(1/2)*f*g/c/(c^2*x^2+1)^(1/2)*x+1/4*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^
2/(c^2*x^2+1)^(1/2)/c*f^2-1/16*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*
x^2+1)^(1/2)/c^3*g^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*g^2*c^2/(c^2*x^2+1)*arcsin
h(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*g^2*c/(c^2*x^2+1)^(1/2)*x^4+3/8*b*(
d*(c^2*x^2+1))^(1/2)*g^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/16*b*(d*(c^2*x^2+1)
)^(1/2)*g^2/c/(c^2*x^2+1)^(1/2)*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*g^2/c^2/(c^
2*x^2+1)*arcsinh(c*x)*x+2/3*b*(d*(c^2*x^2+1))^(1/2)*f*g/c^2/(c^2*x^2+1)*arc
sinh(c*x)+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-
1/4*b*(d*(c^2*x^2+1))^(1/2)*f^2*c/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1
))^(1/2)*f^2/(c^2*x^2+1)*arcsinh(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c^2dx^2 + d}\left(ag^2x^2 + 2\,afgx + af^2 + \left(bg^2x^2 + 2\,bfgx + bf^2\right)\text{arsinh}\left(cx\right)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
2*b*f*g*x + b*f^2)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\left(c^2x^2 + 1\right)}\left(a + b\,\text{asinh}\left(cx\right)\right)\left(f + gx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2 + d}\left(gx + f\right)^2\left(b\,\text{arsinh}\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(g*x + f)^2*(b*arcsinh(c*x) + a), x)
```

3.36 $\int (f+gx)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=227

$$\frac{1}{2}fx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^2}-\frac{bc}{}$$

```
[Out] -(b*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f*x^2*Sqrt[d +
c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*c*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt
[1 + c^2*x^2]) + (f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1 +
 c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.250144, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5835, 5821, 5682, 5675, 30, 5717}

$$\frac{1}{2}fx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^2}-\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f*x^2*Sqrt[d +
c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*c*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt
[1 + c^2*x^2]) + (f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1 +
 c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int (f+gx)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\sqrt{d+c^2dx^2}\int (f+gx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\int \left(f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+gx\sqrt{1+c^2x^2}\left(a+b\sin\right.\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(g\sqrt{d+c^2dx^2}\right)\int}{}$$

$$= \frac{1}{2}fx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{g\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{3c^2}$$

$$= -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{1}{2}fx\sqrt{d+}$$

Mathematica [A] time = 1.16786, size = 208, normalized size = 0.92

$$\frac{1}{6}a\sqrt{c^2dx^2+d}\left(\frac{2g}{c^2}+x(3f+2gx)\right)+\frac{a\sqrt{d}f\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{2c}+\frac{bf\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\text{s}\right.\right.}{8c\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2]*((2*g)/c^2 + x*(3*f + 2*g*x)))/6 - (b*g*Sqrt[d + c^2
*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*c^2*Sqrt
[1 + c^2*x^2]) + (a*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*
c) + (b*f*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(ArcS
inh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.27, size = 423, normalized size = 1.9

$$\frac{ag}{3c^2d}\left(c^2dx^2+d\right)^{\frac{3}{2}}+\frac{afx}{2}\sqrt{c^2dx^2+d}+\frac{afd}{2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{bg\mathrm{Arcsinh}\,(cx)}{3c^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-\frac{bf}{8c}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a*g/c^2/d*(c^2*d*x^2+d)^(3/2)+1/2*a*f*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f*d*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b*(d*(c^2*x^
2+1))^(1/2)*g/c^2/(c^2*x^2+1)*arcsinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)*f/c/
(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)*g*c^2/(c^2*x^2+1)*arcsinh(c*x
)*x^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*g*c/(c^2*x^2+1)^(1/2)*x^3+2/3*b*(d*(c^2*x
^2+1))^(1/2)*g/(c^2*x^2+1)*arcsinh(c*x)*x^2-1/3*b*(d*(c^2*x^2+1))^(1/2)*g/c
/(c^2*x^2+1)^(1/2)*x+1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^2/(c^2*x^2+1)*arcsinh(
c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^
2*x^2+1))^(1/2)*f/(c^2*x^2+1)*arcsinh(c*x)*x+1/4*b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\mathrm{arsinh}\,(cx)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\left(c^2x^2+1\right)}\left(a+b\,\mathrm{asinh}\,(cx)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2+d}(gx+f)(b\operatorname{arsinh}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(g*x + f)*(b*arcsinh(c*x) + a), x)
```

3.37 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=664

$$\frac{b\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\,(}{2b}$$

```
[Out] (a*Sqrt[d + c^2*d*x^2])/g - (b*c*x*Sqrt[d + c^2*d*x^2])/(g*Sqrt[1 + c^2*x^2
]) + (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g - (c*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x])^2)/(2*b*g*Sqrt[1 + c^2*x^2]) - ((1 + (c^2*f^2)/g^2)*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 + c^2*x^2]) +
 (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f +
 g*x)) - (a*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(
Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[
c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g
^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d
+ c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*PolyL
og[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^2*Sqrt[1 + c^2
*x^2])
```

Rubi [A] time = 1.64957, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 20, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5835, 5823, 683, 5815, 6742, 261, 725, 206, 5859, 1654, 12, 5857, 5717, 8, 5831, 3322, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\,(}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```


```
[Out] (a*Sqrt[d + c^2*d*x^2])/g - (b*c*x*Sqrt[d + c^2*d*x^2])/(g*Sqrt[1 + c^2*x^2
]) + (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g - (c*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x])^2)/(2*b*g*Sqrt[1 + c^2*x^2]) - ((1 + (c^2*f^2)/g^2)*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 + c^2*x^2]) +
 (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f +
 g*x)) - (a*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(
Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[
c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g
^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d
+ c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*PolyL
og[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^2*Sqrt[1 + c^2
*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 5815

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
```

```
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)} - \frac{\sqrt{d+c^2dx^2}\int \frac{\left(-g+2c^2fx+c^2gx^2\right)\left(a+b\sinh\right.}{(f+gx)^2}}{2bc\sqrt{1+c^2x^2}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)\sqrt{1+c^2x^2}} -$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)\sqrt{1+c^2x^2}} -$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)\sqrt{1+c^2x^2}} -$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\right.}{2bc(f+gx)\sqrt{1+c^2}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\right.}{2bc(f+gx)\sqrt{1+c^2}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{(1}{}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\right.}{2bg\sqrt{1+c}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\right.}{2bg\sqrt{1+c}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\right.}{2bg\sqrt{1+c}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\right.}{2bg\sqrt{1+c}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\right.}{2bg\sqrt{1+c}}$$

Mathematica [C] time = 6.04509, size = 1353, normalized size = 2.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (2*a*g*Sqrt[d + c^2*d*x^2] + 2*a*Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Log[f + g*x] -
 2*a*c*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 2*a*Sqrt[d]*Sqr
t[c^2*f^2 + g^2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d +
 c^2*d*x^2]] + b*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*g*Ar
cSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*(c^2*f^2 + g^2)*((
(-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[
c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*
I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*Ar
cTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]
] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi
+ (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(
c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (Ar
cCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)
*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*
c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt
[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2
])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f
^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2
*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
- g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi
 + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi
+ (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2
])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))
/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
)] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^
2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c
^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))]))/Sqrt[-(c^2*f^2) - g^2]
))/Sqrt[1 + c^2*x^2]))/(2*g^2)
```

Maple [A] time = 0.247, size = 992, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x)
```

```
[Out] a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-a/g^2*c
^2*d*f*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d
*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^3*d/(d*(c^2*f^2+
g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2
+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^
(1/2))/(x+f/g))*c^2*f^2-a/g*d/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+
g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d
-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c/g^2+b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/g*c*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)/g*arcsinh(c*x)+b*(d*(c^2*x^2+1
))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x)*ln((-(c*x+(
c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*
(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x
)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)
^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
```

```
^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g
^2*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+
g^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

3.38 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=781

$$-\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}+\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}-\frac{\sqrt{c^2dx^2+d}\left(g-c^2fx\right)^2}{2bc\sqrt{c^2x^2+1}\left(c^2f^2\right.}$$

```
[Out] -((a*Sqrt[d + c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c
*x])/(g*(f + g*x)) + (a*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(g^2*(c^2
*f^2 + g^2)*Sqrt[1 + c^2*x^2]) + (b*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x
]^2)/(2*g^2*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]) - ((g - c^2*f*x)^2*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(c^2*f^2 + g^2)*(f + g*x)^2*Sqrt[
1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(f + g*x)^2) + (a*c^2*f*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x
)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1
 + c^2*x^2]) - (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh
[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^
2*x^2]) + (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]
*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2
]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 + c^2*x^2]) - (b*c^
2*f*Sqrt[d + c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
 + g^2]))])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (b*c^2*f*Sqrt[d +
 c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/
(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 2.54735, antiderivative size = 781, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 22, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {5835, 5823, 37, 5813, 12, 1651, 844, 215, 725, 206, 5859, 5857, 5675, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}+\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}-\frac{\sqrt{c^2dx^2+d}\left(g-c^2fx\right)^2}{2bc\sqrt{c^2x^2+1}\left(c^2f^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d + c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c
*x])/(g*(f + g*x)) + (a*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(g^2*(c^2
*f^2 + g^2)*Sqrt[1 + c^2*x^2]) + (b*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x
]^2)/(2*g^2*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]) - ((g - c^2*f*x)^2*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(c^2*f^2 + g^2)*(f + g*x)^2*Sqrt[
1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(f + g*x)^2) + (a*c^2*f*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x
)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1
 + c^2*x^2]) - (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh
[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^
2*x^2]) + (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]
*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2
]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 + c^2*x^2]) - (b*c^
2*f*Sqrt[d + c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
 + g^2]))])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (b*c^2*f*Sqrt[d +
```

```
 c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]])/
(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5813

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && L
tQ[m + p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)^2} - \frac{\sqrt{d+c^2dx^2}\int \frac{\left(-2g+2c^2fx\right)\left(a+b\sinh^{-1}\cdots\right.}{(f+gx)^3}}{2bc\sqrt{1+c^2x^2}}\cdots$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\cdots\right.}{2bc(f+gx)^2}\cdots$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\cdots\right.}{2bc(f+gx)^2}\cdots$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\cdots\right.}{2bc(f+gx)^2}\cdots$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\cdots\right.}{2bc(f+gx)^2}\cdots$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d\cdots}}{\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d\cdots}}{\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}\cdots$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc\cdots}{2\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc\cdots}{2\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc\cdots}{2\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc\cdots}{2\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc\cdots}{2\cdots}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc\cdots}{2\cdots}$$

Mathematica [C] time = 9.76935, size = 1398, normalized size = 1.79

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d*(1 + c^2*x^2)])/(g*(f + g*x))) - (a*c^2*Sqrt[d]*f*Log[f + g*x])
/(g^2*Sqrt[c^2*f^2 + g^2]) + (a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c
^2*x^2)]])/g^2 + (a*c^2*Sqrt[d]*f*Log[d*g - c^2*d*f*x + Sqrt[d]*Sqrt[c^2*f^
2 + g^2]*Sqrt[d*(1 + c^2*x^2)]])/(g^2*Sqrt[c^2*f^2 + g^2]) + (b*c*Sqrt[d*(1
 + c^2*x^2)]*((-2*g*ArcSinh[c*x])/(c*f + c*g*x) + ArcSinh[c*x]^2/Sqrt[1 + c
^2*x^2] + ((2*I)*c*f*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^
2 + g^2]])/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (2*Log[1 + (g*x)/f])/S
qrt[1 + c^2*x^2] + (2*c*f*(2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[
(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh
[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^
2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (
2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)
*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2
)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*f + c*g*x
])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*A
rcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (
2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSin
h[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] - (ArcC
os[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])
/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f
^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqr
t[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f
)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(
c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I
+ Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f
^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c
*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[
c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g +
Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I
*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))])))/(Sqrt[-(c^2*
f^2) - g^2]*Sqrt[1 + c^2*x^2])))/(2*g^2)
```

Maple [B] time = 0.324, size = 1814, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2+g^2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g
)+d*(c^2*f^2+g^2)/g^2)^(1/2)+a/g^2*c^4*f^2/(c^2*f^2+g^2)*d*ln((-c^2*d*f/g+c
^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2
+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)+a/g^3*c^4*f^3/(c^2*f^2+g^2)*d/(d*(c^2*f^2+g
^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+
g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(
```

```
1/2))/(x+f/g))+a/g*c^2*f/(c^2*f^2+g^2)*d/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*
d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f
/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/
(c^2*f^2+g^2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/
2)*x+a*c^2/(c^2*f^2+g^2)*d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+
f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)+
1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c/g^2+b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g^2/(g*x+f)*x^3*c^4*f-b*(d*(c^2*x^
2+1))^(1/2)*arcsinh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(d*(c^2*x^2+1))^(1/2)*arcsin
h(c*x)/(c^2*x^2+1)/g/(g*x+f)*x^2*c^2+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(
c^2*x^2+1)^(1/2)/g/(g*x+f)*x*c+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^
2+1)/g^2/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)^(
1/2)/g^2/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g/(g*
x+f)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*c^2*
f*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*
f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^2/(c^2*
f^2+g^2)^(1/2)*c^2*f*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^
2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/g^2/(c^2*f^2+g^2)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*f^2+b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)*c^3*ln((c*x+(c^2*x^2+1)^(1/2
))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*f^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*c^2*f*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-
c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*c^2*f*dilog(((c*x+(c^2*x^2+1)^(
1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)*c*ln(c*x+(c^2*x^2+1)^(1/2))+b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)*c*ln((c*x+(c^2*x^2+1)^(
1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2 + d}(b\,\text{arsinh}\,(cx) + a)}{g^2x^2 + 2\,fgx + f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(f+gx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^{2}dx^{2}+d}(b\operatorname{arsinh}\left(cx\right)+a)}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f)^2, x)
```

3.39 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=918

$$-\frac{bc^3dg^3\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}-\frac{bc^3dfg^2\sqrt{c^2dx^2+d}x^6}{12\sqrt{c^2x^2+1}}-\frac{8bcdg^3\sqrt{c^2dx^2+d}x^5}{175\sqrt{c^2x^2+1}}-\frac{3bc^3df^2g\sqrt{c^2dx^2+d}x^5}{25\sqrt{c^2x^2+1}}-\frac{bc^3df^3\sqrt{c^2dx^2+\cdots}}{16\sqrt{c^2x^2+1}}$$

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) + (2*b*d*g^3*x
*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^3*x^2*Sqrt[d
+ c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d + c^2*d*x^2]
)/(32*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(5*Sqr
t[1 + c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d + c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (7*b*c*
d*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(32*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*f^2*g*
x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (8*b*c*d*g^3*x^5*Sqrt[d +
 c^2*d*x^2])/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d + c^2*d*x^
2])/(12*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt
[1 + c^2*x^2]) + (3*d*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (
3*d*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*d*f*g^2
*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f^3*x*(1 + c^2*x^2)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*f*g^2*x^3*(1 + c^2*x^2)*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (3*d*f^2*g*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (d*g^3*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (d*g^3*(1 + c^2*x^2)^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (3*d*f^3*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2]) - (3*d*f*g^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.932396, antiderivative size = 918, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.567, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 5717, 194, 5744, 5742, 5758, 266, 43, 5732, 12, 373}

$$-\frac{bc^3dg^3\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}-\frac{bc^3dfg^2\sqrt{c^2dx^2+d}x^6}{12\sqrt{c^2x^2+1}}-\frac{8bcdg^3\sqrt{c^2dx^2+d}x^5}{175\sqrt{c^2x^2+1}}-\frac{3bc^3df^2g\sqrt{c^2dx^2+d}x^5}{25\sqrt{c^2x^2+1}}-\frac{bc^3df^3\sqrt{c^2dx^2+\cdots}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) + (2*b*d*g^3*x
*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^3*x^2*Sqrt[d
+ c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d + c^2*d*x^2]
)/(32*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(5*Sqr
t[1 + c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d + c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (7*b*c*
d*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(32*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*f^2*g*
x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (8*b*c*d*g^3*x^5*Sqrt[d +
 c^2*d*x^2])/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d + c^2*d*x^
2])/(12*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt
[1 + c^2*x^2]) + (3*d*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (
3*d*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*d*f*g^2
*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f^3*x*(1 + c^2*x^2)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*f*g^2*x^3*(1 + c^2*x^2)*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (3*d*f^2*g*(1 + c^2*x^2)^2*Sqrt[
```

```
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (d*g^3*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (d*g^3*(1 + c^2*x^2)^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (3*d*f^3*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2]) - (3*d*f*g^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(f^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+3f^2gx\left(1+c^2\right.\right.}{}$$

$$=\frac{\left(df^3\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}+\frac{\left(3df^2g\sqrt{d\right.}{}$$

$$=\frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}dfg^2x^3\left(1+c^2x^2\right)$$

$$=\frac{3}{8}df^3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d+c^2dx^2}\left(a+b\sin\right.$$

$$=-\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{2bcdf^2gx^3\sqrt{d+c^2dx}}{5\sqrt{1+c^2x^2}}$$

$$=-\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}+\frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-$$

Mathematica [A] time = 4.24489, size = 779, normalized size = 0.85

$$\frac{529200acd^{3/2}f\sqrt{c^2x^2+1}\left(2c^2f^2-g^2\right)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+5040ad\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(4c^6x^3\left(84f^2gx+35f^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (5040*a*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-32*g^3 + c^2*g*(336*f^2 +
 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*x + 70*f*g^2*x^2 +
20*g^3*x^3) + 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2*x^2 + 64*g^3*x^3))
 - 940800*b*c^2*d*f^2*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x
```

```
^2)^(3/2)*ArcSinh[c*x]) - 37632*b*c^2*d*f^2*g*Sqrt[d + c^2*d*x^2]*(c*x*(-30
 + 5*c^2*x^2 + 9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)
*ArcSinh[c*x]) - 12544*b*d*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 +
9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x])
- 256*b*d*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(840 - 140*c^2*x^2 + 63*c^4*x^4 + 22
5*c^6*x^6) - 105*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4 + 15*c^6*x^6)
*ArcSinh[c*x]) + 529200*a*c*d^(3/2)*f*(2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 352800*b*c^3*d*f^3*Sqrt[d + c^2*d
*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh
[c*x]])) - 22050*b*c^3*d*f^3*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4
*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - 66150*b*c*d*f*g^2*S
qrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x
]*Sinh[4*ArcSinh[c*x]]) + 3675*b*c*d*f*g^2*Sqrt[d + c^2*d*x^2]*(72*ArcSinh[
c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSin
h[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]
 + Sinh[6*ArcSinh[c*x]])))/(2822400*c^4*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.493, size = 1510, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] -3/5*b*(d*(c^2*x^2+1))^(1/2)*g*d/c/(c^2*x^2+1)^(1/2)*x*f^2-1/12*b*(d*(c^2*x
^2+1))^(1/2)*f*g^2*d*c^3/(c^2*x^2+1)^(1/2)*x^6-1/35*b*(d*(c^2*x^2+1))^(1/2)
*g^3*d/c^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+3/5*b*(d*(c^2*x^2+1))^(1/2)*g*d/c^2
/(c^2*x^2+1)*arcsinh(c*x)*f^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*f^3*d*c^4/(c^2*x^
2+1)*arcsinh(c*x)*x^5+7/8*b*(d*(c^2*x^2+1))^(1/2)*f^3*d*c^2/(c^2*x^2+1)*arc
sinh(c*x)*x^3-3/32*b*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2*d/(c^2*x^2+1)^(
1/2)/c^3*g^2+1/7*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c^4/(c^2*x^2+1)*arcsinh(c*x)
*x^8+13/35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^6+1
7/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d/(c^2*x^2+1)*arcsinh(c*x)*x^3+9/5*b*(d*
(c^2*x^2+1))^(1/2)*g*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*f^2-7/32*b*(d*(c^2*x^2+
1))^(1/2)*f*g^2*d*c/(c^2*x^2+1)^(1/2)*x^4-3/32*b*(d*(c^2*x^2+1))^(1/2)*f*g^
2*d/c/(c^2*x^2+1)^(1/2)*x^2-3/25*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^3/(c^2*x^2+1
)^(1/2)*x^5*f^2-2/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c/(c^2*x^2+1)^(1/2)*x^3*f^2
+3/8*a*f^3*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+
1/4*a*f^3*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f^3*d*x*(c^2*d*x^2+d)^(1/2)-2/35*a*g^
3/d/c^4*(c^2*d*x^2+d)^(5/2)+3/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d/c^2/(c^2*x
^2+1)*arcsinh(c*x)*x+3/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^4/(c^2*x^2+1)*arcsin
h(c*x)*x^6*f^2+9/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^2/(c^2*x^2+1)*arcsinh(c*x)
*x^4*f^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x
^7+11/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5+2/
35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/c^3/(c^2*x^2+1)^(1/2)*x+7/768*b*(d*(c^2*x^
2+1))^(1/2)*f*g^2*d/c^3/(c^2*x^2+1)^(1/2)-1/16*b*(d*(c^2*x^2+1))^(1/2)*f^3*
d*c^3/(c^2*x^2+1)^(1/2)*x^4-5/16*b*(d*(c^2*x^2+1))^(1/2)*f^3*d*c/(c^2*x^2+1
)^(1/2)*x^2-1/49*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c^3/(c^2*x^2+1)^(1/2)*x^7-3/
16*a*f*g^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-3/16*a*f*g^2/c^2*d^2*ln(x*c^2*d/(c^2
*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*a*f*g^2*x*(c^2*d*x^2+d)^(5
/2)/c^2/d-8/175*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c/(c^2*x^2+1)^(1/2)*x^5-1/105
*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/c/(c^2*x^2+1)^(1/2)*x^3+3/16*b*(d*(c^2*x^2+1
))^(1/2)*f^3*arcsinh(c*x)^2*d/(c^2*x^2+1)^(1/2)/c-2/35*b*(d*(c^2*x^2+1))^(1
/2)*g^3*d/c^4/(c^2*x^2+1)*arcsinh(c*x)+9/35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/(
c^2*x^2+1)*arcsinh(c*x)*x^4+5/8*b*(d*(c^2*x^2+1))^(1/2)*f^3*d/(c^2*x^2+1)*a
rcsinh(c*x)*x-1/8*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(3/2)+3/5*a*f^2*g/c^2/d*(c^2*
```

```
d*x^2+d)^(5/2)+1/7*a*g^3*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-17/128*b*(d*(c^2*x^2
+1))^(1/2)*f^3*d/c/(c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^2dg^3x^5 + 3\,ac^2dfg^2x^4 + 3\,adf^2gx + adf^3 + \left(3\,ac^2df^2g + adg^3\right)x^3 + \left(ac^2df^3 + 3\,adfg^2\right)x^2 + \left(bc^2dg^3x^5 + 3\,b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 + 3*a*d*f^2*g*x + a*d*f^3 +
 (3*a*c^2*d*f^2*g + a*d*g^3)*x^3 + (a*c^2*d*f^3 + 3*a*d*f*g^2)*x^2 + (b*c^2
*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 + 3*b*d*f^2*g*x + b*d*f^3 + (3*b*c^2*d*f^2
*g + b*d*g^3)*x^3 + (b*c^2*d*f^3 + 3*b*d*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2
*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}}\left(gx + f\right)^3\left(b\,\text{arsinh}\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(g*x + f)^3*(b*arcsinh(c*x) + a), x)
```

3.40 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=651

$$\frac{3}{8}df^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}df^2x\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df^2\sqrt{c^2dx^2+d}\left(a+b\sinh\right.}{16bc\sqrt{c^2x^2+1}}$$

```
[Out] (-2*b*d*f*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^2*x
^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d + c^2*
d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (4*b*c*d*f*g*x^3*Sqrt[d + c^2*d*x^2])/(1
5*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c
^2*x^2]) - (7*b*c*d*g^2*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d + c^2*d*x^2])/(36*Sqrt[1 + c^2*x^2]) + (3*d*f^2*x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8
 + (d*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*
g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (2*d*f*
g*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*
f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
- (d*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2
*x^2])
```

Rubi [A] time = 0.730868, antiderivative size = 651, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 5717, 194, 5744, 5742, 5758}

$$\frac{3}{8}df^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}df^2x\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df^2\sqrt{c^2dx^2+d}\left(a+b\sinh\right.}{16bc\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-2*b*d*f*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^2*x
^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d + c^2*
d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (4*b*c*d*f*g*x^3*Sqrt[d + c^2*d*x^2])/(1
5*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c
^2*x^2]) - (7*b*c*d*g^2*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d + c^2*d*x^2])/(36*Sqrt[1 + c^2*x^2]) + (3*d*f^2*x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8
 + (d*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*
g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (2*d*f*
g*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*
f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
- (d*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2
*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
```

```
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (f+gx)^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+2fgx\left(1+c\right.\right.}{\sqrt{1}}$$

$$= \frac{\left(df^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(2dfg\sqrt{}\right.}{}$$

$$= \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dg^2x^3\left(1+c^2x\right.$$

$$= \frac{3}{8}df^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{s}\right.$$

$$= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{4bcdfgx^3\sqrt{d+c^2d}}{15\sqrt{1+c^2x^2}}$$

$$= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bdg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 2.19319, size = 546, normalized size = 0.84

$$\frac{3600ad^{3/2}\sqrt{c^2x^2+1}\left(6c^2f^2-g^2\right)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+240acd\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(30c^2f^2x\left(2c^2x^2+5\right)+96fg\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(96*f*g*(1 + c^2*x^2)^2 +
30*c^2*f^2*x*(5 + 2*c^2*x^2) + 5*g^2*x*(3 + 14*c^2*x^2 + 8*c^4*x^4)) - 1280
0*b*c*d*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*Ar
cSinh[c*x]) - 512*b*c*d*f*g*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 + 9*c
^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) + 3
600*a*d^(3/2)*(6*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[
d + c^2*d*x^2]] - 7200*b*c^2*d*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]
] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 450*b*c^2*d*f^2
*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c
*x]*Sinh[4*ArcSinh[c*x]]) - 450*b*d*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]
^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + 25*b*d*g
^2*Sqrt[d + c^2*d*x^2]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cos
h[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSinh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*Arc
Sinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcSinh[c*x]])))/(57600*c^3*Sq
rt[1 + c^2*x^2])
```

Maple [A] time = 0.388, size = 1087, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 17/48*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/(c^2*x^2+1)*arcsinh(c*x)*x^3+5/8*b*(d*(
c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x*f^2-1/32*b*(d*(c^2*x^2+1))^(
1/2)*arcsinh(c*x)^2*d/(c^2*x^2+1)^(1/2)/c^3*g^2+3/16*b*(d*(c^2*x^2+1))^(1/2
)*arcsinh(c*x)^2*d/(c^2*x^2+1)^(1/2)/c*f^2-1/36*b*(d*(c^2*x^2+1))^(1/2)*g^2
*d*c^3/(c^2*x^2+1)^(1/2)*x^6-7/96*b*(d*(c^2*x^2+1))^(1/2)*g^2*d*c/(c^2*x^2+
1)^(1/2)*x^4-1/32*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/c/(c^2*x^2+1)^(1/2)*x^2-1/1
6*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*x^4*f^2-5/16*b*(d*(c^2*x^
2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^2*f^2-4/15*b*(d*(c^2*x^2+1))^(1/2)*f*g*
d*c/(c^2*x^2+1)^(1/2)*x^3+6/5*b*(d*(c^2*x^2+1))^(1/2)*f*g*d/(c^2*x^2+1)*arc
sinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)*g^2*d*c^4/(c^2*x^2+1)*arcsinh(c*x
)*x^7+11/24*b*(d*(c^2*x^2+1))^(1/2)*g^2*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5+
1/16*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+2/5*b*(d*
(c^2*x^2+1))^(1/2)*f*g*d/c^2/(c^2*x^2+1)*arcsinh(c*x)+1/4*b*(d*(c^2*x^2+1))
^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5*f^2+7/8*b*(d*(c^2*x^2+1))^(1/2)*d
*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3*f^2-2/5*b*(d*(c^2*x^2+1))^(1/2)*f*g*d/c/(
c^2*x^2+1)^(1/2)*x-2/25*b*(d*(c^2*x^2+1))^(1/2)*f*g*d*c^3/(c^2*x^2+1)^(1/2)
*x^5+2/5*b*(d*(c^2*x^2+1))^(1/2)*f*g*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^6+6/5
*b*(d*(c^2*x^2+1))^(1/2)*f*g*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/24*a*g^2/
c^2*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*f^2*d^2*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*a*f^2*x*(c^2
*d*x^2+d)^(3/2)+7/2304*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/c^3/(c^2*x^2+1)^(1/2)-
17/128*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)*f^2+1/6*a*g^2*x*(c^2*d
*x^2+d)^(5/2)/c^2/d-1/16*a*g^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a*g^2/c^2*d
```

```
^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2/5*a*f*g/c^
2/d*(c^2*d*x^2+d)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\Big(\big(ac^2dg^2x^4 + 2\,ac^2dfgx^3 + 2\,adfgx + adf^2 + \big(ac^2df^2 + adg^2\big)x^2 + \big(bc^2dg^2x^4 + 2\,bc^2dfgx^3 + 2\,bdfgx + bdf$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 + 2*a*d*f*g*x + a*d*f^2 + (a*
c^2*d*f^2 + a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 + 2*b*d*f*g
*x + b*d*f^2 + (b*c^2*d*f^2 + b*d*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 +
d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}} \left(gx + f\right)^2 (b\,\mathrm{arsinh}\,(cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(g*x + f)^2*(b*arcsinh(c*x) + a), x)
```

3.41 $\int (f+gx)\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=353

$$\frac{3}{8}dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dfx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16bc\sqrt{c^2x^2+1}}$$

```
[Out] -(b*d*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f*x^2*Sqr
t[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (2*b*c*d*g*x^3*Sqrt[d + c^2*d*x^
2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) +
 (3*d*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f*x*(1 + c^2*x^2
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*g*(1 + c^2*x^2)^2*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*f*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.335457, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 5717, 194}

$$\frac{3}{8}dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dfx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16bc\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f*x^2*Sqr
t[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (2*b*c*d*g*x^3*Sqrt[d + c^2*d*x^
2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) +
 (3*d*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f*x*(1 + c^2*x^2
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*g*(1 + c^2*x^2)^2*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*f*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+gx\left(1+c^2x^2\right)^{3/2}\right.}{\sqrt{1+c^2x^2}}$$
$$= \frac{\left(df\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(dg\sqrt{d+c^2dx\right.}{}$$
$$= \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{dg\left(1+c^2x^2\right)^2\sqrt{d+}}{}$$
$$= \frac{3}{8}dfx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a\right.$$
$$= -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{2bcdgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} - \frac{bc^3}{}$$

Mathematica [A] time = 1.20627, size = 392, normalized size = 1.11

$$\frac{3600acd^{3/2}f\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+240ad\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(5c^2fx\left(2c^2x^2+5\right)+8g\left(c^2x^2+1\right)^2\right)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-640*b*c*d*g*x*(3 + c^2*x^2)*Sqrt[d + c^2*d*x^2] - 128*b*c^3*d*g*x^3*(5 +
3*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 240*a*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x
^2]*(8*g*(1 + c^2*x^2)^2 + 5*c^2*f*x*(5 + 2*c^2*x^2)) + 3200*b*d*g*(1 + c^2
*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x] + 640*b*d*g*(1 + c^2*x^2)^(3/2
)*(-2 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x] - 1200*b*c*d*f*Sqrt[d +
 c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 3600*a*c*d^(3/2)*f*Sqrt[1 + c^2*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2400*b*c*d*f*Sqrt[d + c^2*d*x^2]*Ar
cSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]]) - 75*b*c*d*f*Sqrt[d + c^2*
d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*Arc
Sinh[c*x]]))/(9600*c^2*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.284, size = 601, normalized size = 1.7

$$\frac{ag}{5c^2d}\left(c^2dx^2+d\right)^{\frac{5}{2}} + \frac{afx}{4}\left(c^2dx^2+d\right)^{\frac{3}{2}} + \frac{3afdx}{8}\sqrt{c^2dx^2+d} + \frac{3afd^2}{8}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}} + \frac{3bf(\mathrm{Arcsi}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/5*a*g/c^2/d*(c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x
*(c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)+3/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsin
h(c*x)^2*d+1/5*b*(d*(c^2*x^2+1))^(1/2)*g*d/c^2/(c^2*x^2+1)*arcsinh(c*x)-17/
128*b*(d*(c^2*x^2+1))^(1/2)*f*d/c/(c^2*x^2+1)^(1/2)-1/5*b*(d*(c^2*x^2+1))^(
```

```
1/2)*g*d/c/(c^2*x^2+1)^(1/2)*x+1/4*b*(d*(c^2*x^2+1))^(1/2)*f*d*c^4/(c^2*x^2
+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*f*d*c^3/(c^2*x^2+1)^(1/2)
*x^4+7/8*b*(d*(c^2*x^2+1))^(1/2)*f*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-5/16*
b*(d*(c^2*x^2+1))^(1/2)*f*d*c/(c^2*x^2+1)^(1/2)*x^2+5/8*b*(d*(c^2*x^2+1))^(
1/2)*f*d/(c^2*x^2+1)*arcsinh(c*x)*x+1/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^4/(c^
2*x^2+1)*arcsinh(c*x)*x^6-1/25*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^3/(c^2*x^2+1)^
(1/2)*x^5+3/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-
2/15*b*(d*(c^2*x^2+1))^(1/2)*g*d*c/(c^2*x^2+1)^(1/2)*x^3+3/5*b*(d*(c^2*x^2+
1))^(1/2)*g*d/(c^2*x^2+1)*arcsinh(c*x)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^2dgx^3 + ac^2dfx^2 + adgx + adf + \left(bc^2dgx^3 + bc^2dfx^2 + bdgx + bdf\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*g*x^3 + a*c^2*d*f*x^2 + a*d*g*x + a*d*f + (b*c^2*d*g*x^3
+ b*c^2*d*f*x^2 + b*d*g*x + b*d*f)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}} (a + b\operatorname{asinh}(cx))\left(f + gx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}}\left(gx + f\right)(b\operatorname{arsinh}(cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(g*x + f)*(b*arcsinh(c*x) + a), x)
```

3.42 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{f+gx}\,dx$

Optimal. Leaf size=984

$$-\frac{bdx^3\sqrt{c^2dx^2+d}c^3}{9g\sqrt{c^2x^2+1}}+\frac{bdfx^2\sqrt{c^2dx^2+d}c^3}{4g^2\sqrt{c^2x^2+1}}-\frac{dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)c^2}{2g^2}-\frac{d\left(c^2f^2+g^2\right)x\sqrt{c^2dx^2+d}\left(a+b\right.}{2bg^3\sqrt{c^2x^2+1}}$$

```
[Out] (a*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])/g^3 - (b*c*d*x*Sqrt[d + c^2*d*x^2
])/(3*g*Sqrt[1 + c^2*x^2]) - (b*c*d*(c^2*f^2 + g^2)*x*Sqrt[d + c^2*d*x^2])/
(g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d + c^2*d*x^2])/(4*g^2*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d + c^2*d*x^2])/(9*g*Sqrt[1 + c^2*x^2]) +
(b*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g^3 - (c^2*d*f*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + (d*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) - (c*d*f*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2)/(4*b*g^2*Sqrt[1 + c^2*x^2]) - (c*d*(c^2*f^2 + g^2)*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^3*Sqrt[1 + c^2*x^2]) - (d*(c^
2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f +
g*x)*Sqrt[1 + c^2*x^2]) + (d*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*d*(c^2*f^2 + g^2
)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt
[1 + c^2*x^2])])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[
d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
+ g^2])])/(g^4*Sqrt[1 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1
 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]*PolyLog[2, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 1.88879, antiderivative size = 984, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 24, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5835, 5825, 5682, 5675, 30, 5717, 5823, 683, 5815, 6742, 261, 725, 206, 5859, 1654, 12, 5857, 8, 5831, 3322, 2264, 2190, 2279, 2391}

$$-\frac{bdx^3\sqrt{c^2dx^2+d}c^3}{9g\sqrt{c^2x^2+1}}+\frac{bdfx^2\sqrt{c^2dx^2+d}c^3}{4g^2\sqrt{c^2x^2+1}}-\frac{dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)c^2}{2g^2}-\frac{d\left(c^2f^2+g^2\right)x\sqrt{c^2dx^2+d}\left(a+b\right.}{2bg^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```


```
[Out] (a*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])/g^3 - (b*c*d*x*Sqrt[d + c^2*d*x^2
])/(3*g*Sqrt[1 + c^2*x^2]) - (b*c*d*(c^2*f^2 + g^2)*x*Sqrt[d + c^2*d*x^2])/
(g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d + c^2*d*x^2])/(4*g^2*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d + c^2*d*x^2])/(9*g*Sqrt[1 + c^2*x^2]) +
(b*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g^3 - (c^2*d*f*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + (d*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) - (c*d*f*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2)/(4*b*g^2*Sqrt[1 + c^2*x^2]) - (c*d*(c^2*f^2 + g^2)*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^3*Sqrt[1 + c^2*x^2]) - (d*(c^
2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f +
g*x)*Sqrt[1 + c^2*x^2]) + (d*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*d*(c^2*f^2 + g^2
```

```
)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt
[1 + c^2*x^2])])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[
d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
+ g^2])])/(g^4*Sqrt[1 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1
 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]*PolyLog[2, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5825

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
```

```
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 5815

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
```

```
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(-\frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{g^2}+\frac{c^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{g}+\frac{\left(c^2f^2+g^2\right.}{}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2df\sqrt{d+c^2dx^2}\right)\int\sqrt{1}}{g^2\sqrt{}}$$

$$= -\frac{c^2dfx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2g^2}+\frac{d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{3g}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}}-\frac{c^2dfx\sqrt{d+c^2}}{}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}}-\frac{c^2dfx\sqrt{d+c^2}}{}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}}-\frac{c^2dfx\sqrt{d+c^2}}{}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx^3}{9g\sqrt{}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx^3}{9g\sqrt{}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx^3}{9g\sqrt{}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}-\frac{bcd\left(c^2f^2+g^2\right)x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}}-$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}-\frac{bcd\left(c^2f^2+g^2\right)x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}}-$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}-\frac{bcd\left(c^2f^2+g^2\right)x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}}-$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}-\frac{bcd\left(c^2f^2+g^2\right)x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}}-$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}-\frac{bcd\left(c^2f^2+g^2\right)x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}}-$$

Mathematica [C] time = 13.5387, size = 2889, normalized size = 2.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((a*d*(3*c^2*f^2 + 4*g^2))/(3*g^3) - (a*c^2*d*f*x)/(2
*g^2) + (a*c^2*d*x^2)/(3*g)) + (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[f + g*x
])/g^4 - (a*c*d^(3/2)*f*(2*c^2*f^2 + 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 +
 c^2*x^2)]])/(2*g^4) - (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[d*g - c^2*d*f*x
 + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d*(1 + c^2*x^2)]])/g^4 + (b*d*Sqrt[d*(1
 + c^2*x^2)]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*g*ArcSinh[c*x] - (c*f*ArcSin
h[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*(c^2*f^2 + g^2)*(((-I)*Pi*ArcTanh[(-g + c*
f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcC
os[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqr
t[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(
Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g
] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2
*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4]
)/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(Arc
Sinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] + (ArcCos[((-I)*c*f)/g] + (2*
I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
- g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2
*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])
/(Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[
((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log
[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I
)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*
I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g
)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4
]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]
))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-
(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[
-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f +
I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi +
(2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi +
 (2*I)*ArcSinh[c*x])/4]))]))/Sqrt[-(c^2*f^2) - g^2]))/Sqrt[1 + c^2*x^2]))/(
2*g^2) + (b*d*Sqrt[d*(1 + c^2*x^2)]*((-9*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sq
rt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2
 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))
/Sqrt[c^2*f^2 + g^2] + (-18*c*g*(4*c^2*f^2 + g^2)*x + 18*g*(4*c^2*f^2 + g^2
)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 18*c*f*(2*c^2*f^2 + g^2)*ArcSinh[c*x]^2
+ 9*c*f*g^2*Cosh[2*ArcSinh[c*x]] + 6*g^3*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]
+ 9*(8*c^4*f^4 + 8*c^2*f^2*g^2 + g^4)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcS
inh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*
f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2
) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*
ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*A
rcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2
]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^
2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2
)*Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh
[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + A
rcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2
]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)
```

```
*g]*Sqrt[c*f + c*g*x])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g
)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f +
g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c
*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[
c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi +
(2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f -
g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f
 - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(Po
lyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) -
 g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2)
- g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^
2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSi
nh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4]))]))/Sqrt[-(c^2*f^2) - g^2]) - 18*c*f*g^2*ArcSinh[c*x]*Sinh[2*
ArcSinh[c*x]] - 2*g^3*Sinh[3*ArcSinh[c*x]])/g^4))/(72*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.22, size = 1838, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x)
```

```
[Out] -1/2*a/g^2*c^2*d*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2
)^(1/2)*x-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g*c*x+1/8*b*(d*(c
^2*x^2+1))^(1/2)*f*c*d/(c^2*x^2+1)^(1/2)/g^2-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/
(c^2*x^2+1)^(1/2)/g*c^3*x^3+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/g^4*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1
/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2
)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^4*d/(c^
2*x^2+1)/g^2*arcsinh(c*x)*x^3-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^2*d/(c^2*x^2+
1)/g^2*arcsinh(c*x)*x+b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g^3*arcsinh(c*x
)*x^2*c^4*f^2+a/g*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^
2)^(1/2)+1/3*a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^
(3/2)+a/g^3*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/
2)*c^2*f^2+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)+1/4*b*(
d*(c^2*x^2+1))^(1/2)*f*c^3*d/(c^2*x^2+1)^(1/2)/g^2*x^2-b*(d*(c^2*x^2+1))^(1
/2)*d/(c^2*x^2+1)^(1/2)/g^3*x*c^3*f^2+b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)
/g^3*arcsinh(c*x)*c^2*f^2+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2
+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*
g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*f^3*arcsinh(c*x)^2*c^3*d/g^4-3/4*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c*d/g^2+1/3*b*(d*(c^2*x^2+1))^(1/
2)*d/(c^2*x^2+1)/g*arcsinh(c*x)*x^4*c^4+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*
x^2+1)/g*arcsinh(c*x)*x^2*c^2-a/g*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(
c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)
^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-3/2*a/g^2
*c^2*d^2*f*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c
^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^4*d^2*c^4*f^
3*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*
(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^5*d^2/(d*(c^2*f^2+g^2
)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^
2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/
```

```
2))/(x+f/g))*c^4*f^4-2*a/g^3*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f
^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^
2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}\left(a + b\operatorname{asinh}\left(cx\right)\right)}{f + gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right) + a\right)}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

3.43 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=1228

result too large to display

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) + (2*b*d^2*g
^3*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f^3*x^2*
Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d +
c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d + c^2*d
*x^2])/(7*Sqrt[1 + c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d + c^2*d*x^2])/(189*c*S
qrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 +
c^2*x^2]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(256*Sqrt[1 + c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^2*x^2]) -
(b*c*d^2*g^3*x^5*Sqrt[d + c^2*d*x^2])/(21*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d + c^2*d*x^2])/(441*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*g^3*x^9*Sqrt[d + c^2*d*x
^2])/(81*Sqrt[1 + c^2*x^2]) - (b*d^2*f^3*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (15*d^2*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(128*c^2) + (15*
d^2*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*d^2*f^3*x*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (5*d^2*f*g^2*x^
3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (d^2*f^3*x*(
1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (3*d^2*f*g^2*x
^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*d^2*f^2
*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) - (d^2
*g^3*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (d
^2*g^3*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*c^4) +
(5*d^2*f^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2
*x^2]) - (15*d^2*f*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c
^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 1.13905, antiderivative size = 1228, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 261, 5717, 194, 5744, 5742, 5758, 266, 43, 5732, 12, 373}

$$-\frac{bc^5d^2g^3\sqrt{c^2dx^2+d}x^9}{81\sqrt{c^2x^2+1}}-\frac{3bc^5d^2fg^2\sqrt{c^2dx^2+d}x^8}{64\sqrt{c^2x^2+1}}-\frac{19bc^3d^2g^3\sqrt{c^2dx^2+d}x^7}{441\sqrt{c^2x^2+1}}-\frac{3bc^5d^2f^2g\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3d^2fg^2\sqrt{}}{96\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) + (2*b*d^2*g
^3*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f^3*x^2*
Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d +
c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d + c^2*d
*x^2])/(7*Sqrt[1 + c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d + c^2*d*x^2])/(189*c*S
qrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 +
c^2*x^2]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(256*Sqrt[1 + c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^2*x^2]) -
(b*c*d^2*g^3*x^5*Sqrt[d + c^2*d*x^2])/(21*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f^2*
```

```
g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d + c^2*d*x^2])/(441*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*g^3*x^9*Sqrt[d + c^2*d*x
^2])/(81*Sqrt[1 + c^2*x^2]) - (b*d^2*f^3*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (15*d^2*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(128*c^2) + (15*
d^2*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*d^2*f^3*x*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (5*d^2*f*g^2*x^
3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (d^2*f^3*x*(
1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (3*d^2*f*g^2*x
^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*d^2*f^2
*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) - (d^2
*g^3*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (d
^2*g^3*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*c^4) +
(5*d^2*f^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2
*x^2]) - (15*d^2*f*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c
^3*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^3 \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+3f^2gx\left(1+c^2\right.\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) dx}{\sqrt{1+c^2x^2}} + \frac{\left(3d^2f^2g\sqrt{\right.}{}$$

$$= \frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}d^2fg^2x^3\left(1+c^2\right.$$

$$= -\frac{bd^2f^3\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a\right.$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{9bc^3d^2f^2gx^5\sqrt{d+}}{35\sqrt{1+c^2x}}$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{25bcd^2f^3x^2\sqrt{d+c^2d}}{96\sqrt{1+c^2x^2}}$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{25bcd^2f^3x^2\sqrt{d+c^2d}}{96\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 7.05114, size = 1899, normalized size = 1.55

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-(a*d^2*g*(-27*c^2*f^2 + 2*g^2))/(63*c^4) + (a*d^2*f
*(88*c^2*f^2 + 15*g^2)*x)/(128*c^2) + (a*d^2*g*(81*c^2*f^2 + g^2)*x^2)/(63*
c^2) + (a*d^2*f*(104*c^2*f^2 + 177*g^2)*x^3)/192 + (a*d^2*g*(27*c^2*f^2 + 5
*g^2)*x^4)/21 + (a*c^2*d^2*f*(8*c^2*f^2 + 51*g^2)*x^5)/48 + (a*c^2*d^2*g*(2
7*c^2*f^2 + 19*g^2)*x^6)/63 + (3*a*c^4*d^2*f*g^2*x^7)/8 + (a*c^4*d^2*g^3*x^
8)/9) + (3*b*d^2*f^2*g*(-(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(9*Sqrt[
1 + c^2*x^2]) + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3))/c^2
+ (6*b*d^2*f^2*g*((2*c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(45*Sqrt[1 +
c^2*x^2]) - (c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(75*Sqrt[1 + c^
2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(-2 + 3*c^2*x^2)*ArcSinh[c*x])/(15*d)))/
c^2 + (b*d^2*g^3*((2*c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(45*Sqrt[1 +
c^2*x^2]) - (c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(75*Sqrt[1 + c^
2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(-2 + 3*c^2*x^2)*ArcSinh[c*x])/(15*d)))/
c^4 + (3*b*d^2*f^2*g*((-8*c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(315*Sqr
t[1 + c^2*x^2]) + (4*c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(525*Sq
rt[1 + c^2*x^2]) - (c^5*x^5*Sqrt[d*(1 + c^2*x^2)]*(7 + 5*c^2*x^2))/(245*Sqr
t[1 + c^2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(8 - 12*c^2*x^2 + 15*c^4*x^4)*Ar
cSinh[c*x])/(105*d)))/c^2 + (2*b*d^2*g^3*((-8*c*x*Sqrt[d*(1 + c^2*x^2)]*(3
+ c^2*x^2))/(315*Sqrt[1 + c^2*x^2]) + (4*c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 +
 3*c^2*x^2))/(525*Sqrt[1 + c^2*x^2]) - (c^5*x^5*Sqrt[d*(1 + c^2*x^2)]*(7 +
5*c^2*x^2))/(245*Sqrt[1 + c^2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(8 - 12*c^2*
x^2 + 15*c^4*x^4)*ArcSinh[c*x])/(105*d)))/c^4 + (b*d^2*g^3*((16*c*x*Sqrt[d*
(1 + c^2*x^2)]*(3 + c^2*x^2))/(945*Sqrt[1 + c^2*x^2]) - (8*c^3*x^3*Sqrt[d*(
```

```
1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(1575*Sqrt[1 + c^2*x^2]) + (2*c^5*x^5*Sqrt[d
*(1 + c^2*x^2)]*(7 + 5*c^2*x^2))/(735*Sqrt[1 + c^2*x^2]) - (c^7*x^7*Sqrt[d*
(1 + c^2*x^2)]*(9 + 7*c^2*x^2))/(567*Sqrt[1 + c^2*x^2]) + ((d*(1 + c^2*x^2)
)^(3/2)*(-16 + 24*c^2*x^2 - 30*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x])/(315*d))
)/c^4 + (5*a*d^(5/2)*f*(8*c^2*f^2 - 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 +
c^2*x^2)]])/(128*c^3) + (b*d^2*f^3*Sqrt[d*(1 + c^2*x^2)]*(-Cosh[2*ArcSinh[c
*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 +
 c^2*x^2]) - (b*d^2*f^3*Sqrt[d*(1 + c^2*x^2)]*(8*ArcSinh[c*x]^2 + Cosh[4*Ar
cSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(64*c*Sqrt[1 + c^2*x^2]
) - (3*b*d^2*f*g^2*Sqrt[d*(1 + c^2*x^2)]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh
[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c^3*Sqrt[1 + c^2*x^2])
+ (b*d^2*f^3*Sqrt[d*(1 + c^2*x^2)]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c
*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSinh[c*x]] - 36*ArcSinh[c*x]*Si
nh[2*ArcSinh[c*x]] - 36*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] + 12*ArcSinh[c*x]
*Sinh[6*ArcSinh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2]) + (b*d^2*f*g^2*Sqrt[d*(1
 + c^2*x^2)]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSin
h[c*x]] - 2*Cosh[6*ArcSinh[c*x]] - 36*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 3
6*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] + 12*ArcSinh[c*x]*Sinh[6*ArcSinh[c*x]])
)/(384*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*f*g^2*Sqrt[d*(1 + c^2*x^2)]*(1440*Ar
cSinh[c*x]^2 + 576*Cosh[2*ArcSinh[c*x]] + 144*Cosh[4*ArcSinh[c*x]] - 64*Cos
h[6*ArcSinh[c*x]] + 9*Cosh[8*ArcSinh[c*x]] - 1152*ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]] - 576*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] + 384*ArcSinh[c*x]*Sinh[6*
ArcSinh[c*x]] - 72*ArcSinh[c*x]*Sinh[8*ArcSinh[c*x]]))/(24576*c^3*Sqrt[1 +
c^2*x^2])
```

Maple [A] time = 0.575, size = 1954, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 359/24576*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2/c^3/(c^2*x^2+1)^(1/2)-1/36*b*(d
*(c^2*x^2+1))^(1/2)*f^3*d^2*c^5/(c^2*x^2+1)^(1/2)*x^6-13/96*b*(d*(c^2*x^2+1
))^(1/2)*f^3*d^2*c^3/(c^2*x^2+1)^(1/2)*x^4-11/32*b*(d*(c^2*x^2+1))^(1/2)*f^
3*d^2*c/(c^2*x^2+1)^(1/2)*x^2-1/81*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c^5/(c^2
*x^2+1)^(1/2)*x^9-19/441*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c^3/(c^2*x^2+1)^(1
/2)*x^7-1/21*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c/(c^2*x^2+1)^(1/2)*x^5-1/189*
b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2/c/(c^2*x^2+1)^(1/2)*x^3+2/63*b*(d*(c^2*x^2+
1))^(1/2)*g^3*d^2/c^3/(c^2*x^2+1)^(1/2)*x+5/32*b*(d*(c^2*x^2+1))^(1/2)*f^3*
arcsinh(c*x)^2*d^2/(c^2*x^2+1)^(1/2)/c-2/63*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2
/c^4/(c^2*x^2+1)*arcsinh(c*x)+11/16*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2/(c^2*x^
2+1)*arcsinh(c*x)*x+16/63*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2/(c^2*x^2+1)*arcsi
nh(c*x)*x^4+3/8*a*f*g^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-5/64*a*f*g^2/c^2*d*x*(c
^2*d*x^2+d)^(3/2)-15/128*a*f*g^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-15/128*a*f*g
^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/9*
a*g^3*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-1/16*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(5/2)+
3/7*a*f^2*g/c^2/d*(c^2*d*x^2+d)^(7/2)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*f^3*
d^2/c/(c^2*x^2+1)^(1/2)+3/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^6/(c^2*x^2+1)*a
rcsinh(c*x)*x^8*f^2+12/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^4/(c^2*x^2+1)*arcs
inh(c*x)*x^6*f^2+18/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^2/(c^2*x^2+1)*arcsinh
(c*x)*x^4*f^2+3/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^6/(c^2*x^2+1)*arcsinh
(c*x)*x^9+23/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^4/(c^2*x^2+1)*arcsinh(c
*x)*x^7+127/64*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^2/(c^2*x^2+1)*arcsinh(c*
x)*x^5+15/128*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x
)*x-2/63*a*g^3/d/c^4*(c^2*d*x^2+d)^(7/2)-17/96*b*(d*(c^2*x^2+1))^(1/2)*f*g^
```

```
2*d^2*c^3/(c^2*x^2+1)^(1/2)*x^6-59/256*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c/
(c^2*x^2+1)^(1/2)*x^4-15/256*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2/c/(c^2*x^2+1
)^(1/2)*x^2+3/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*
f^2+1/6*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7+17
/24*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5+59/48*
b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+12/7*b*(d*
(c^2*x^2+1))^(1/2)*g*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*f^2+133/128*b*(d*(c^2
*x^2+1))^(1/2)*f*g^2*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-15/256*b*(d*(c^2*x^2+
1))^(1/2)*f*arcsinh(c*x)^2*d^2/(c^2*x^2+1)^(1/2)/c^3*g^2+1/9*b*(d*(c^2*x^2+
1))^(1/2)*g^3*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^10+26/63*b*(d*(c^2*x^2+1))
^(1/2)*g^3*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^8+34/63*b*(d*(c^2*x^2+1))^(1/
2)*g^3*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^6-1/63*b*(d*(c^2*x^2+1))^(1/2)*g^
3*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-3/49*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c
^5/(c^2*x^2+1)^(1/2)*x^7*f^2-9/35*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^3/(c^2*x^
2+1)^(1/2)*x^5*f^2-3/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c/(c^2*x^2+1)^(1/2)*x^
3*f^2-3/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2/c/(c^2*x^2+1)^(1/2)*x*f^2-3/64*b*(d
*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^5/(c^2*x^2+1)^(1/2)*x^8+1/6*a*f^3*x*(c^2*d*
x^2+d)^(5/2)+5/16*a*f^3*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f^3*d^3*ln(x*c^2*d
/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+5/24*a*f^3*d*x*(c^2*d*x^2
+d)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ac^4d^2g^3x^7 + 3\,ac^4d^2fg^2x^6 + 3\,ad^2f^2gx + ad^2f^3 + \left(3\,ac^4d^2f^2g + 2\,ac^2d^2g^3\right)x^5 + \left(ac^4d^2f^3 + 6\,ac^2d^2fg^2\right)x^4 + \left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g + 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 + 6*a*c
^2*d^2*f*g^2)*x^4 + (6*a*c^2*d^2*f^2*g + a*d^2*g^3)*x^3 + (2*a*c^2*d^2*f^3
+ 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g + 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 + 6*b*c^2*d^2*f*g^2)*x^4 + (6*b*c^2*d^2*f^2*g + b*d^2*g^3)*x^3 + (2*
b*c^2*d^2*f^3 + 3*b*d^2*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.44 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=901

$$-\frac{bc^5d^2g^2\sqrt{c^2dx^2+d}x^8}{64\sqrt{c^2x^2+1}}-\frac{2bc^5d^2fg\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3d^2g^2\sqrt{c^2dx^2+d}x^6}{288\sqrt{c^2x^2+1}}-\frac{6bc^3d^2fg\sqrt{c^2dx^2+d}x^5}{35\sqrt{c^2x^2+1}}-\frac{5bc^3d^2f^2\sqrt{c^2dx}}{96\sqrt{c^2x^2+}}$$

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*
f^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*f*g*x^3*Sqrt[d + c^
2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d + c^2*d*x^2])
/(96*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*g^2*x^4*Sqrt[d + c^2*d*x^2])/(768*Sqr
t[1 + c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (b*c
^5*d^2*g^2*x^8*Sqrt[d + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*d^2*f^2*(1
+ c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*d^2*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/24 + (5*d^2*g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/48 + (d^2*f^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/6 + (d^2*g^2*x^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/8 + (2*d^2*f*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*d^2*f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c
*Sqrt[1 + c^2*x^2]) - (5*d^2*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2
)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.925864, antiderivative size = 901, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 261, 5717, 194, 5744, 5742, 5758, 266, 43}

$$-\frac{bc^5d^2g^2\sqrt{c^2dx^2+d}x^8}{64\sqrt{c^2x^2+1}}-\frac{2bc^5d^2fg\sqrt{c^2dx^2+d}x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3d^2g^2\sqrt{c^2dx^2+d}x^6}{288\sqrt{c^2x^2+1}}-\frac{6bc^3d^2fg\sqrt{c^2dx^2+d}x^5}{35\sqrt{c^2x^2+1}}-\frac{5bc^3d^2f^2\sqrt{c^2dx}}{96\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*
f^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*f*g*x^3*Sqrt[d + c^
2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d + c^2*d*x^2])
/(96*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*g^2*x^4*Sqrt[d + c^2*d*x^2])/(768*Sqr
t[1 + c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (b*c
^5*d^2*g^2*x^8*Sqrt[d + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*d^2*f^2*(1
+ c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*d^2*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/24 + (5*d^2*g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/48 + (d^2*f^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
```

```
x]))/6 + (d^2*g^2*x^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/8 + (2*d^2*f*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*d^2*f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c
*Sqrt[1 + c^2*x^2]) - (5*d^2*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2
)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+2fgx\left(1+\right.\right.}{\sqrt{1}}$$

$$= \frac{\left(d^2f^2\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) dx}{\sqrt{1+c^2x^2}} + \frac{\left(2d^2fg\right.}{}$$

$$= \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}d^2g^2x^3\left(1+\right.$$

$$= -\frac{bd^2f^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{6bc^3d^2fgx^5\sqrt{d+}}{35\sqrt{1+c^2x}}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx^3\sqrt{d+}}{7\sqrt{1+c^2x}}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bd^2g^2x^2\sqrt{d+}}{256c\sqrt{1+c^2}}$$

Mathematica [A] time = 2.71485, size = 1047, normalized size = 1.16

$$\frac{d^2\left(-737280bfgx^7\sqrt{c^2dx^2+d}c^8+2257920ag^2x^7\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}c^7+5160960afgx^6\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}c^7+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(-5160960*b*c^2*f*g*x*Sqrt[d + c^2*d*x^2] - 5160960*b*c^4*f*g*x^3*Sqrt
[d + c^2*d*x^2] - 3096576*b*c^6*f*g*x^5*Sqrt[d + c^2*d*x^2] - 737280*b*c^8*
f*g*x^7*Sqrt[d + c^2*d*x^2] + 5160960*a*c*f*g*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2] + 12418560*a*c^3*f^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 705
600*a*c*g^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 15482880*a*c^3*f*g*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 9784320*a*c^5*f^2*x^3*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2] + 5550720*a*c^3*g^2*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 15482880*a*c^5*f*g*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
] + 3010560*a*c^7*f^2*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 6397440*a
*c^5*g^2*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 5160960*a*c^7*f*g*x^6*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2257920*a*c^7*g^2*x^7*Sqrt[1 + c^2*
x^2]*Sqrt[d + c^2*d*x^2] + 352800*b*(8*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*A
rcSinh[c*x]^2 - 141120*b*(15*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcS
inh[c*x]] - 211680*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 352
80*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 15680*b*c^2*f^2*Sqrt[d
```

```
+ c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 15680*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[6*
ArcSinh[c*x]] - 2205*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] + 56448
00*a*c^2*Sqrt[d]*f^2*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] - 705600*a*Sqrt[d]*g^2*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]] + 840*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(6144*c*f*g*Sqrt[1 + c
^2*x^2] + 18432*c^3*f*g*x^2*Sqrt[1 + c^2*x^2] + 18432*c^5*f*g*x^4*Sqrt[1 +
c^2*x^2] + 6144*c^7*f*g*x^6*Sqrt[1 + c^2*x^2] + 336*(15*c^2*f^2 - g^2)*Sinh
[2*ArcSinh[c*x]] + 168*(6*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 112*c^2*f^2
*Sinh[6*ArcSinh[c*x]] + 112*g^2*Sinh[6*ArcSinh[c*x]] + 21*g^2*Sinh[8*ArcSin
h[c*x]])))/(18063360*c^3*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.475, size = 1424, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)+1/6*b*(d*(
c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7*f^2+17/24*b*(d*(c^2*
x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5*f^2+59/48*b*(d*(c^2*x^2+
1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3*f^2+8/7*b*(d*(c^2*x^2+1))^(1
/2)*f*g*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^
2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^9+23/48*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2*c^
4/(c^2*x^2+1)*arcsinh(c*x)*x^7+127/192*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2*c^2/
(c^2*x^2+1)*arcsinh(c*x)*x^5+2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^6/(c^2*x
^2+1)*arcsinh(c*x)*x^8+8/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^4/(c^2*x^2+1)*
arcsinh(c*x)*x^6+12/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^2/(c^2*x^2+1)*arcsi
nh(c*x)*x^4-1/48*a*g^2/c^2*x*(c^2*d*x^2+d)^(5/2)-2/49*b*(d*(c^2*x^2+1))^(1/
2)*f*g*d^2*c^5/(c^2*x^2+1)^(1/2)*x^7-6/35*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c
^3/(c^2*x^2+1)^(1/2)*x^5-2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c/(c^2*x^2+1)^
(1/2)*x^3-2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2/c/(c^2*x^2+1)^(1/2)*x+5/128*b
*(d*(c^2*x^2+1))^(1/2)*g^2*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+5/24*a*f^2*d*
x*(c^2*d*x^2+d)^(3/2)+5/16*a*f^2*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f^2*d^3*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/6*a*f^2*x*(c^2
*d*x^2+d)^(5/2)-5/192*a*g^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a*g^2/c^2*d^2
*x*(c^2*d*x^2+d)^(1/2)-5/128*a*g^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*
x^2+d)^(1/2))/(c^2*d)^(1/2)+2/7*a*f*g/c^2/d*(c^2*d*x^2+d)^(7/2)+1/8*a*g^2*x
*(c^2*d*x^2+d)^(7/2)/c^2/d+359/73728*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2/c^3/(c
^2*x^2+1)^(1/2)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)*f^
2-1/36*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^6*f^2-13/96*b*(d
*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^4*f^2-11/32*b*(d*(c^2*x^2+1
))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^2*f^2-1/64*b*(d*(c^2*x^2+1))^(1/2)*g^2*d
^2*c^5/(c^2*x^2+1)^(1/2)*x^8-17/288*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2*c^3/(c^
2*x^2+1)^(1/2)*x^6-59/768*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2*c/(c^2*x^2+1)^(1/
2)*x^4-5/256*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2/c/(c^2*x^2+1)^(1/2)*x^2+11/16*
b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x*f^2+5/32*b*(d*(c^2*x
^2+1))^(1/2)*arcsinh(c*x)^2*d^2/(c^2*x^2+1)^(1/2)/c*f^2-5/256*b*(d*(c^2*x^2
+1))^(1/2)*arcsinh(c*x)^2*d^2/(c^2*x^2+1)^(1/2)/c^3*g^2+133/384*b*(d*(c^2*x
^2+1))^(1/2)*g^2*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\big(ac^4d^2g^2x^6 + 2\,ac^4d^2fgx^5 + 4\,ac^2d^2fgx^3 + 2\,ad^2fgx + ad^2f^2 + \big(ac^4d^2f^2 + 2\,ac^2d^2g^2\big)x^4 + \big(2\,ac^2d^2f^2 + ad^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 + 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 + 2*a*c^2*d^2*g^2)*x^4 + (2*a*c^2
*d^2*f^2 + a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 + 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 + 2*b*c^2*d^2*
g^2)*x^4 + (2*b*c^2*d^2*f^2 + b*d^2*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2
+ d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}}\left(gx + f\right)^2\left(b\,\text{arsinh}\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(g*x + f)^2*(b*arcsinh(c*x) + a), x)
```

3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=494

$$\frac{1}{6}d^2fx\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(c^2x^2+1\right)\sqrt{c^2dx^2+}$$

```
[Out] -(b*d^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f*x^
2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c*d^2*g*x^3*Sqrt[d + c^2
*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d + c^2*d*x^2])/(9
6*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 +
 c^2*x^2]) - (b*c^5*d^2*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) -
 (b*d^2*f*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*f*x*(1 + c^2*x^2)*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (d^2*f*x*(1 + c^2*x^2)^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (d^2*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.397254, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 261, 5717, 194}

$$\frac{1}{6}d^2fx\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(c^2x^2+1\right)\sqrt{c^2dx^2+}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -(b*d^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f*x^
2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c*d^2*g*x^3*Sqrt[d + c^2
*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d + c^2*d*x^2])/(9
6*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 +
 c^2*x^2]) - (b*c^5*d^2*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) -
 (b*d^2*f*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*f*x*(1 + c^2*x^2)*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (d^2*f*x*(1 + c^2*x^2)^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (d^2*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
&& EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+gx\left(1+c^2x^2\right)^{5/2}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2f\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(d^2g\sqrt{d+c^2\right.}{}$$

$$= \frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{d^2g\left(1+c^2x^2\right)^3\sqrt{d}}{}$$

$$= -\frac{bd^2f\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{24}d^2fx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+\right.$$

$$= -\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{3bc^3d^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}} - b$$

$$= -\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - 5$$

Mathematica [A] time = 1.29327, size = 656, normalized size = 1.33

$$\frac{d^2\left(94080ac^6fx^5\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+305760ac^4fx^3\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+388080ac^2fx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+176\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(-80640*b*c*g*x*Sqrt[d + c^2*d*x^2] - 80640*b*c^3*g*x^3*Sqrt[d + c^2*d
*x^2] - 48384*b*c^5*g*x^5*Sqrt[d + c^2*d*x^2] - 11520*b*c^7*g*x^7*Sqrt[d +
c^2*d*x^2] + 80640*a*g*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 388080*a*c^2
*f*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 241920*a*c^2*g*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2] + 305760*a*c^4*f*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d +
c^2*d*x^2] + 241920*a*c^4*g*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 940
80*a*c^6*f*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 80640*a*c^6*g*x^6*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 88200*b*c*f*Sqrt[d + c^2*d*x^2]*ArcSi
nh[c*x]^2 - 66150*b*c*f*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 6615*b*c
*f*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 490*b*c*f*Sqrt[d + c^2*d*x^2]
*Cosh[6*ArcSinh[c*x]] + 176400*a*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*Log[c*d*x +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 420*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(192*
g*Sqrt[1 + c^2*x^2] + 576*c^2*g*x^2*Sqrt[1 + c^2*x^2] + 576*c^4*g*x^4*Sqrt[
1 + c^2*x^2] + 192*c^6*g*x^6*Sqrt[1 + c^2*x^2] + 315*c*f*Sinh[2*ArcSinh[c*x
]] + 63*c*f*Sinh[4*ArcSinh[c*x]] + 7*c*f*Sinh[6*ArcSinh[c*x]])))/(564480*c^
2*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.362, size = 805, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/7*a*g/c^2/d*(c^2*d*x^2+d)^(7/2)+1/6*a*f*x*(c^2*d*x^2+d)^(5/2)+5/24*a*f*d*
x*(c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3*ln(x*
c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/7*b*(d*(c^2*x^2+1)
)^(1/2)*g*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)-299/2304*b*(d*(c^2*x^2+1))^(1/2)
*f*d^2/c/(c^2*x^2+1)^(1/2)+1/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^6/(c^2*x^2+1
)*arcsinh(c*x)*x^8-1/49*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^5/(c^2*x^2+1)^(1/2)
*x^7+4/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^6-3/3
5*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^3/(c^2*x^2+1)^(1/2)*x^5+6/7*b*(d*(c^2*x^2
+1))^(1/2)*g*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/7*b*(d*(c^2*x^2+1))^(1/
2)*g*d^2*c/(c^2*x^2+1)^(1/2)*x^3+4/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2/(c^2*x^2
+1)*arcsinh(c*x)*x^2-1/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2/c/(c^2*x^2+1)^(1/2)*
x+1/6*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7-1/36*b
*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^5/(c^2*x^2+1)^(1/2)*x^6+17/24*b*(d*(c^2*x^2+
1))^(1/2)*f*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-13/96*b*(d*(c^2*x^2+1))^(1
/2)*f*d^2*c^3/(c^2*x^2+1)^(1/2)*x^4+59/48*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^2
/(c^2*x^2+1)*arcsinh(c*x)*x^3-11/32*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c/(c^2*x^
2+1)^(1/2)*x^2+11/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2/(c^2*x^2+1)*arcsinh(c*x)
*x+5/32*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(ac^4d^2gx^5 + ac^4d^2fx^4 + 2\,ac^2d^2gx^3 + 2\,ac^2d^2fx^2 + ad^2gx + ad^2f + \left(bc^4d^2gx^5 + bc^4d^2fx^4 + 2\,bc^2d^2gx^3 + 2\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 + 2*a*c^2*d^2*g*x^3 + 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 + 2*b*c
^2*d^2*g*x^3 + 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arcsinh(c*x))*sqrt(
c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}\left(gx+f\right)\left(b\operatorname{arsinh}\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(g*x + f)*(b*arcsinh(c*x) + a), x)
```

3.46 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1536

result too large to display

```
[Out] (a*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2])/g^5 + (2*b*c*d^2*x*Sqrt[d + c
^2*d*x^2])/(15*g*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d +
 c^2*d*x^2])/(g^5*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + 2*g^2)*x*Sqrt[d
+ c^2*d*x^2])/(3*g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d + c^2*d*x
^2])/(16*g^2*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*(c^2*f^2 + 2*g^2)*x^2*Sqrt[d
 + c^2*d*x^2])/(4*g^4*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d + c^2*d*x^
2])/(45*g*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*(c^2*f^2 + 2*g^2)*x^3*Sqrt[d + c^
2*d*x^2])/(9*g^3*Sqrt[1 + c^2*x^2]) + (b*c^5*d^2*f*x^4*Sqrt[d + c^2*d*x^2])
/(16*g^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c^2*d*x^2])/(25*g*Sqr
t[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]
)/g^5 - (c^2*d^2*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*g^2) - (c
^2*d^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g
^4) - (c^4*d^2*f*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - (d
^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) + (d^2*(c^
2*f^2 + 2*g^2)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g
^3) + (d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*g)
+ (c*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*g^2*Sqrt[1 + c
^2*x^2]) - (c*d^2*f*(c^2*f^2 + 2*g^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*g^4*Sqrt[1 + c^2*x^2]) - (c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^5*Sqrt[1 + c^2*x^2]) - (d^2*(c^2*f^
2 + g^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^6*(f + g*x)
*Sqrt[1 + c^2*x^2]) + (d^2*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 + g
^2)^(5/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sq
rt[1 + c^2*x^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*S
qrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*
f^2 + g^2])])/(g^6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 +
 g^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^
2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^
6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^2*d*x^2]*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^6*Sqrt[1 + c
^2*x^2])
```

Rubi [A] time = 2.44762, antiderivative size = 1536, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 29, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.967, Rules used = {5835, 5825, 5682, 5675, 30, 5717, 5742, 5758, 266, 43, 5732, 12, 5823, 683, 5815, 6742, 261, 725, 206, 5859, 1654, 5857, 8, 5831, 3322, 2264, 2190, 2279, 2391}

$$-\frac{bd^2x^5\sqrt{c^2dx^2+d}c^5}{25g\sqrt{c^2x^2+1}}+\frac{bd^2fx^4\sqrt{c^2dx^2+d}c^5}{16g^2\sqrt{c^2x^2+1}}-\frac{d^2fx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)c^4}{4g^2}-\frac{bd^2\left(c^2f^2+2g^2\right)x^3\sqrt{c^2dx^2+}}{9g^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2])/g^5 + (2*b*c*d^2*x*Sqrt[d + c
^2*d*x^2])/(15*g*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d +
```

```
 c^2*d*x^2])/(g^5*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + 2*g^2)*x*Sqrt[d
+ c^2*d*x^2])/(3*g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d + c^2*d*x
^2])/(16*g^2*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*(c^2*f^2 + 2*g^2)*x^2*Sqrt[d
 + c^2*d*x^2])/(4*g^4*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d + c^2*d*x^
2])/(45*g*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*(c^2*f^2 + 2*g^2)*x^3*Sqrt[d + c^
2*d*x^2])/(9*g^3*Sqrt[1 + c^2*x^2]) + (b*c^5*d^2*f*x^4*Sqrt[d + c^2*d*x^2])
/(16*g^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c^2*d*x^2])/(25*g*Sqr
t[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]
)/g^5 - (c^2*d^2*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*g^2) - (c
^2*d^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g
^4) - (c^4*d^2*f*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - (d
^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) + (d^2*(c^
2*f^2 + 2*g^2)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g
^3) + (d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*g)
+ (c*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*g^2*Sqrt[1 + c
^2*x^2]) - (c*d^2*f*(c^2*f^2 + 2*g^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*g^4*Sqrt[1 + c^2*x^2]) - (c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^5*Sqrt[1 + c^2*x^2]) - (d^2*(c^2*f^
2 + g^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^6*(f + g*x)
*Sqrt[1 + c^2*x^2]) + (d^2*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 + g
^2)^(5/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sq
rt[1 + c^2*x^2])]])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*S
qrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*
f^2 + g^2])]])/(g^6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 +
 g^2])]])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^
2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^
6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^2*d*x^2]*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^6*Sqrt[1 + c
^2*x^2])
```

<u>Rule 5835</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

<u>Rule 5825</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

<u>Rule 5682</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

<u>Rule 5675</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
```

```
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 5815

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
```

```
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\left(\frac{\left(-c^4f^3-2c^2fg^2\right)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{g^4} + \frac{c^2\left(c^2f^2+2g^2\right)x\sqrt{1+c^2x^2}\left(a+\right.}{g^3}\right.}{}$$

$$= -\frac{\left(c^4d^2f\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{g^2\sqrt{1+c^2x^2}} + \frac{\left(c^4d^2\sqrt{d+c^2d\right.}{}$$

$$= -\frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2g^4} - \frac{c^4d^2fx^3\sqrt{d+c^2dx^2}}{4g}$$

$$= -\frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}} - \frac{bc^3}{}$$

$$= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}}$$

$$= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d}}{3g^3\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d}}{3g^3\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d}}{3g^3\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d}}{g^5\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d}}{g^5\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d}}{g^5\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d}}{g^5\sqrt{1+c^2x^2}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^2x\sqrt{d}}{g^5\sqrt{1+c^2x^2}}$$

Mathematica [C] time = 25.4527, size = 7163, normalized size = 4.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.303, size = 3928, normalized size = 2.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x)
```

```
[Out] -9/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^2/(c^2*x^2+1)/g^2*arcsinh(c*x)*x+1/3*b
*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^4*c^6*f^2+8/3*b*(
d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^2*c^4*f^2-1/2*b*(d*
(c^2*x^2+1))^(1/2)*f^3*d^2*c^6/(c^2*x^2+1)/g^4*arcsinh(c*x)*x^3-1/2*b*(d*(c
^2*x^2+1))^(1/2)*f^3*d^2*c^4/(c^2*x^2+1)/g^4*arcsinh(c*x)*x+b*(d*(c^2*x^2+1
))^(1/2)*d^2/(c^2*x^2+1)/g^5*arcsinh(c*x)*x^2*c^6*f^4+b*d^2*(d*(c^2*x^2+1))
^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*dilog((-(c*x+(c^2*x^2+1)^(
1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*c^4*f^4-b*d^2*
(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*dilog(((c*x
+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*c
^4*f^4+2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/
g^4*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f
^2+g^2)^(1/2)))*c^2*f^2-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(
c^2*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/
2))/(c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^
6/(c^2*x^2+1)/g^2*arcsinh(c*x)*x^5-11/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^4/(
c^2*x^2+1)/g^2*arcsinh(c*x)*x^3+1/5*a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g
)+d*(c^2*f^2+g^2)/g^2)^(5/2)-a/g*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c
^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^
2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-1/9*b*(d*(
c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^3*x^3*c^5*f^2-7/3*b*(d*(c^2*x^2+1
))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^3*x*c^3*f^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*f^
3*d^2*c^5/(c^2*x^2+1)^(1/2)/g^4*x^2-b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)
^(1/2)/g^5*x*c^5*f^4+7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsin
h(c*x)*c^2*f^2+b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^5*arcsinh(c*x)*c^4
*f^4+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*
arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+
(c^2*f^2+g^2)^(1/2)))-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*
x^2+1)^(1/2)/g^2*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^
2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*f^5*arcsinh(c*x)^2*d^2*c^5/g^6-5/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)*f^3*arcsinh(c*x)^2*d^2*c^3/g^4-15/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)*f*arcsinh(c*x)^2*d^2*c/g^2+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c
^2*x^2+1)/g*arcsinh(c*x)*x^6*c^6+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2
+1)/g*arcsinh(c*x)*x^2*c^2+1/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^5/(c^2*x^2+
1)^(1/2)/g^2*x^4+9/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^3/(c^2*x^2+1)^(1/2)/g
^2*x^2+14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^4*c^4
+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*arcs
```

```
inh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2
*f^2+g^2)^(1/2)))*c^4*f^4+1/3*a/g*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*
(c^2*f^2+g^2)/g^2)^(3/2)+a/g*d^2*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^
2*f^2+g^2)/g^2)^(1/2)-5/2*a/g^4*d^3*c^4*f^3*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(
c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2
))/(c^2*d)^(1/2)-a/g^6*d^3*c^6*f^5*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1
/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d
)^(1/2)-a/g^7*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c
^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g
*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^6*f^6-3*a/g^5*d^3/(d*(c^2*f
^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*
f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^
2)^(1/2))/(x+f/g))*c^4*f^4-3*a/g^3*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*
(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g
)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2-
1/2*a/g^4*d^2*c^4*f^3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/
g^2)^(1/2)*x-1/4*a/g^2*c^2*d*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*
f^2+g^2)/g^2)^(3/2)*x-7/8*a/g^2*c^2*d^2*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f
/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*x-15/8*a/g^2*c^2*d^3*f*ln((-c^2*d*f/g+c^2*d*
(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)
/g^2)^(1/2))/(c^2*d)^(1/2)-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/
2)/g*x^5*c^5-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*c*x+33/1
28*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c/(c^2*x^2+1)^(1/2)/g^2+1/8*b*(d*(c^2*x^2+
1))^(1/2)*f^3*d^2*c^3/(c^2*x^2+1)^(1/2)/g^4+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^
2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*
f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*d^2*(d*(c^2*x^2+1))^(1
/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*dilog(((c*x+(c^2*x^2+1)^(1/2)
)*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-11/45*b*(d*(c^2*x^2
+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*c^3*x^3-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2
*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/
2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*c^4*f^4+2*b*d^2*(
d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)
*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2
)^(1/2)))*c^2*f^2-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^
2+1)^(1/2)/g^4*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)
^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2+2*a/g^3*d^2*((x+f/g)^2*c^2*d-2*c
^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*c^2*f^2+23/15*b*(d*(c^2*x^2+1))
^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)+1/3*a/g^3*d*((x+f/g)^2*c^2*d-2*c^2*d*
f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)*c^2*f^2+a/g^5*d^2*((x+f/g)^2*c^2*d-2
*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*c^4*f^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 + d\right)^{\frac{5}{2}} \left(b \operatorname{arsinh}\left(cx\right) + a\right)}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

3.47 $\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=430

$$\frac{3f^2g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}-\frac{3fg^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}+\frac{3fg^2x}{}$$

```
[Out] (-3*b*f^2*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b*g^3*x*Sqrt[
1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 + c^2*x^2
])/(4*c*Sqrt[d + c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 + c^2*x^2])/(9*c*Sqrt[d +
c^2*d*x^2]) + (3*f^2*g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^
2*d*x^2]) - (2*g^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^4*Sqrt[d + c^2*
d*x^2]) + (3*f*g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*c^2*Sqrt[d + c^
2*d*x^2]) + (g^3*x^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^2*Sqrt[d + c^
2*d*x^2]) + (f^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d +
c^2*d*x^2]) - (3*f*g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*S
qrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.575307, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5835, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{3f^2g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}-\frac{3fg^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}+\frac{3fg^2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```



```
[Out] (-3*b*f^2*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b*g^3*x*Sqrt[
1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 + c^2*x^2
])/(4*c*Sqrt[d + c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 + c^2*x^2])/(9*c*Sqrt[d +
c^2*d*x^2]) + (3*f^2*g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^
2*d*x^2]) - (2*g^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^4*Sqrt[d + c^2*
d*x^2]) + (3*f*g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*c^2*Sqrt[d + c^
2*d*x^2]) + (g^3*x^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^2*Sqrt[d + c^
2*d*x^2]) + (f^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d +
c^2*d*x^2]) - (3*f*g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*S
qrt[d + c^2*d*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
```

```
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{3f^2gx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{3fg^2x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{g^3x^3\left(a+b\,\mathrm{si}\right.}{\sqrt{1+c}}\right.}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}+\frac{\left(3f^2g\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}+\frac{\left(3fg^2\sqrt{\right.}{}$$

$$= \frac{3f^2g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{3fg^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}+\frac{g^3x^2\left(1\right.}{}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}-\frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}-\frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}+\frac{3f^2g\left(1+c^2x^2\right)\left(a+\right.}{c^2\sqrt{d+c^2}}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}-\frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}+\frac{3}{}$$

Mathematica [A] time = 0.920952, size = 304, normalized size = 0.71

$$\frac{4\sqrt{d}g\left(3a\left(c^2x^2+1\right)\left(c^2\left(18f^2+9fgx+2g^2x^2\right)-4g^2\right)-2bcx\sqrt{c^2x^2+1}\left(c^2\left(27f^2+g^2x^2\right)-6g^2\right)\right)+36acf\sqrt{c^2dx^2+d}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*Sqrt[d]*g*(-2*b*c*x*Sqrt[1 + c^2*x^2]*(-6*g^2 + c^2*(27*f^2 + g^2*x^2))
+ 3*a*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 12*b*S
qrt[d]*g*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))*ArcSin
h[c*x] + 18*b*c*Sqrt[d]*f*(2*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
]^2 - 27*b*c*Sqrt[d]*f*g^2*Sqrt[1 + c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 36*a*c*
f*(2*c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/(72*c^4*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [A] time = 0.348, size = 751, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a*g^3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(c^2*d*x^2+d)^(1/2)
+3/2*a*f*g^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-3/2*a*f*g^2/c^2*ln(x*c^2*d/(c^2*d)
^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+3*a*f^2*g/c^2/d*(c^2*d*x^2+d)^(1/
2)+a*f^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+3/2*b*
(d*(c^2*x^2+1))^(1/2)*f*g^2/d/(c^2*x^2+1)*arcsinh(c*x)*x^3-3/4*b*(d*(c^2*x^
2+1))^(1/2)*f*g^2/c/d/(c^2*x^2+1)^(1/2)*x^2+3/2*b*(d*(c^2*x^2+1))^(1/2)*f*g
^2/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*x+3*b*(d*(c^2*x^2+1))^(1/2)*g/d/(c^2*x^2+
1)*arcsinh(c*x)*x^2*f^2-3*b*(d*(c^2*x^2+1))^(1/2)*g/c/d/(c^2*x^2+1)^(1/2)*x
*f^2-1/9*b*(d*(c^2*x^2+1))^(1/2)*g^3/c/d/(c^2*x^2+1)^(1/2)*x^3-1/3*b*(d*(c^
2*x^2+1))^(1/2)*g^3/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*x^2+2/3*b*(d*(c^2*x^2+1)
)^(1/2)*g^3/c^3/d/(c^2*x^2+1)^(1/2)*x-3/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/c^3
/d/(c^2*x^2+1)^(1/2)+3*b*(d*(c^2*x^2+1))^(1/2)*g/c^2/d/(c^2*x^2+1)*arcsinh(
c*x)*f^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c
/d-2/3*b*(d*(c^2*x^2+1))^(1/2)*g^3/c^4/d/(c^2*x^2+1)*arcsinh(c*x)-3/4*b*(d*
(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c^3/d*g^2+1/3*b*(d*(c
^2*x^2+1))^(1/2)*g^3/d/(c^2*x^2+1)*arcsinh(c*x)*x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\operatorname{arsinh}\left(cx\right)}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\operatorname{asinh}\left(cx\right)\right)\left(f + gx\right)^3}{\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**3/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx + f\right)^3\left(b\operatorname{arsinh}\left(cx\right) + a\right)}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^3*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.48 $\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=258

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{2fg\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{g^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}+\frac{g^2x\left(c^2x^2\right.}{2}$$

```
[Out] (-2*b*f*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
+ c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (2*f*g*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
]))/(2*c^2*Sqrt[d + c^2*d*x^2]) + (f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/(2*b*c*Sqrt[d + c^2*d*x^2]) - (g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.429551, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5835, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{2fg\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{g^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}+\frac{g^2x\left(c^2x^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```


```
[Out] (-2*b*f*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
+ c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (2*f*g*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
]))/(2*c^2*Sqrt[d + c^2*d*x^2]) + (f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/(2*b*c*Sqrt[d + c^2*d*x^2]) - (g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```


Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\
&= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{2fgx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{g^2x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+c^2dx^2}}\\
&= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}+\frac{\left(2fg\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}+\frac{\left(g^2\sqrt{1+\right.}{}\\
&= \frac{2fg\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{g^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}+\frac{f^2\sqrt{1+c^2}}{2b}\\
&= -\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}-\frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{2fg\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{g^2x\left(1\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.57982, size = 233, normalized size = 0.9

$$\frac{4c\sqrt{d}g\left(a\left(c^2x^2+1\right)(4f+gx)-4bcfx\sqrt{c^2x^2+1}\right)+4a\sqrt{c^2dx^2+d}\left(2c^2f^2-g^2\right)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+2b\sqrt{d}\sqrt{c^2x^2}}{8c^3\sqrt{d}\sqrt{c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*c*Sqrt[d]*g*(-4*b*c*f*x*Sqrt[1 + c^2*x^2] + a*(4*f + g*x)*(1 + c^2*x^2))
 + 4*b*c*Sqrt[d]*g*(4*f + g*x)*(1 + c^2*x^2)*ArcSinh[c*x] + 2*b*Sqrt[d]*(2*
c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[d]*g^2*Sqrt[1 + c^
2*x^2]*Cosh[2*ArcSinh[c*x]] + 4*a*(2*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c^3*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.266, size = 486, normalized size = 1.9

$$\frac{ag^2x}{2\,c^2d}\sqrt{c^2dx^2+d}-\frac{ag^2}{2\,c^2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+2\,\frac{afg\sqrt{c^2dx^2+d}}{c^2d}+af^2\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*g^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a*g^2/c^2*ln(x*c^2*d/(c^2*d)^(1/2
)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2*a*f*g/c^2/d*(c^2*d*x^2+d)^(1/2)+a*f^
2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)*g^2/d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1/
2)*g^2/c/d/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*g^2/c^2/d/(c^2
*x^2+1)*arcsinh(c*x)*x+2*b*(d*(c^2*x^2+1))^(1/2)*f*g/c^2/d/(c^2*x^2+1)*arcs
inh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)*g^2/c^3/d/(c^2*x^2+1)^(1/2)+1/2*b*(d*(
c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c/d*f^2-1/4*b*(d*(c^2*x^
2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c^3/d*g^2-2*b*(d*(c^2*x^2+1))^
(1/2)*f*g/c/d/(c^2*x^2+1)^(1/2)*x+2*b*(d*(c^2*x^2+1))^(1/2)*f*g/d/(c^2*x^2+
1)*arcsinh(c*x)*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arsinh}\left(cx\right)}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*a
rcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{asinh}(cx))\left(f + gx\right)^2}{\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx + f\right)^2(b\operatorname{arsinh}(cx) + a)}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^2*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.49 $\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=120

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{bgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

```
[Out] -((b*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (g*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (f*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.217123, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5835, 5821, 5675, 5717, 8}

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{bgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (g*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (f*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{gx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(g\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}} - \frac{\left(bg\sqrt{1+c^2x^2}\right)\int}{c\sqrt{d+c^2dx^2}}$$
$$= -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.253685, size = 158, normalized size = 1.32

$$\frac{2\sqrt{d}g\left(ac^2x^2+a-bcx\sqrt{c^2x^2+1}\right)+2acf\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+bc\sqrt{d}f\sqrt{c^2x^2+1}\sinh^{-1}(cx)^2+2b\sqrt{d}g}{2c^2\sqrt{d}\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*Sqrt[d]*g*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]) + 2*b*Sqrt[d]*g*(1 +
 c^2*x^2)*ArcSinh[c*x] + b*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + 2
*a*c*f*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*c^2
*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [A] time = 0.178, size = 209, normalized size = 1.7

$$\frac{ag}{c^2d}\sqrt{c^2dx^2+d}+af\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{bf\left(\mathrm{Arcsinh}\,(cx)\right)^2}{2\,cd}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+\frac{bg\mathrm{Arcsinh}\,(cx}{d\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*g/c^2/d*(c^2*d*x^2+d)^(1/2)+a*f*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1
/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*f*arcs
inh(c*x)^2+b*(d*(c^2*x^2+1))^(1/2)*g/d/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c
```

```
^2*x^2+1))^(1/2)*g/c/d/(c^2*x^2+1)^(1/2)*x+b*(d*(c^2*x^2+1))^(1/2)*g/c^2/d/
(c^2*x^2+1)*arcsinh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{agx + af + \left(bgx + bf\right)\operatorname{arsinh}\left(cx\right)}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{asinh}(cx))\left(f + gx\right)}{\sqrt{d\left(c^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx + f\right)(b\operatorname{arsinh}(cx) + a)}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((g*x + f)*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.50 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.0567424, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.036389, size = 48, normalized size = 1.02

$$\frac{\sqrt{c^2x^2+1}\sinh^{-1}(cx)\left(2a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(2*a + b*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d
*x^2])
```

Maple [A] time = 0.006, size = 77, normalized size = 1.6

$$a\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{2\,cd}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\left(cx\right)+a}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{asinh}\left(cx\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.51 $\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=325

$$\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\right.}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+}}$$

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - S
qrt[c^2*f^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f
^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^
2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2
*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^Arc
Sinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c
^2*d*x^2])
```

Rubi [A] time = 0.548057, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5835, 5831, 3322, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\right.}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - S
qrt[c^2*f^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f
^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^
2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2
*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^Arc
Sinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c
^2*d*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf-g+e^{2x}g}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(2g\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} - \frac{\left(2g\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\right.}{\sqrt{c^2f}}$$

$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx}}$$

$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx}}$$

$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1\right.}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx}}$$

Mathematica [A] time = 0.623303, size = 256, normalized size = 0.79

$$\frac{\frac{b\sqrt{c^2x^2+1}\left(\text{PolyLog}\left(2,\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}-cf}\right)-\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)+\sinh^{-1}(cx)\left(\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)-\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)\right)\right)}{\sqrt{c^2dx^2+d}} - \frac{a\log\left(\sqrt{d}\sqrt{c^2dx^2+d}\sqrt{c^2f^2}\right.}{\sqrt{d}}}{\sqrt{c^2f^2+g^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 +
 g^2]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(L
og[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c
*f) + Sqrt[c^2*f^2 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))])))/Sqrt[d + c^2*d*x^2])/Sqrt[c^2*f^2 + g^2]
```

Maple [B] time = 0.132, size = 678, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/g/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f
/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c
```

```
^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2
)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*ln
((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(
1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4
*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*
g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/
2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2
+g^2)*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2
*f^2+g^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(
1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*dilog(((c*x+(c^2*x^2+1)^(1/2))
*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)}{c^2dgx^3 + c^2dfx^2 + dgx + df}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x^
2 + d*g*x + d*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2 + 1\right)}\left(f + gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

3.52 $\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=444

$$\frac{bc^2f\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{bc^2f\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)(f+gx}$$

```
[Out] -((g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((c^2*f^2 + g^2)*(f + g*x)*Sqrt[d
+ c^2*d*x^2])) + (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d
+ c^2*d*x^2]) - (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d +
 c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[f + g*x])/((c^2*f^2 + g^2)*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
 - (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.660769, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5835, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bc^2f\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{bc^2f\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)(f+gx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((c^2*f^2 + g^2)*(f + g*x)*Sqrt[d
+ c^2*d*x^2])) + (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d
+ c^2*d*x^2]) - (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d +
 c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[f + g*x])/((c^2*f^2 + g^2)*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
 - (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
```

```
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\sinh(x))^2}\,dx,x,\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(c^2f\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx,x,\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{cf+x}\,dx,x,cgx\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} + \frac{\left(2c^2f\sqrt{1+\right.}{}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} + \frac{\left(2c^2fg\sqrt{1+c^2x^2}\right)\text{Subst}\left(\right.}{\left(c^2f^2+\right.}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 2.2051, size = 448, normalized size = 1.01

$$-bd\sqrt{c^2x^2+1}\left(-c^2f(f+gx)\text{PolyLog}\left(2,\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}-cf}\right)+c^2f(f+gx)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)+g\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-(a*g*Sqrt[c^2*f^2 + g^2]*(d + c^2*d*x^2)) + a*c^2*Sqrt[d]*f*(f + g*x)*Sqr
t[d + c^2*d*x^2]*Log[f + g*x] - a*c^2*Sqrt[d]*f*(f + g*x)*Sqrt[d + c^2*d*x^
2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]] -
 b*d*Sqrt[1 + c^2*x^2]*(g*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] - c^2*f*(f + g*x)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 + g^2])] + c^2*f*(f + g*x)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])] - c*Sqrt[c^2*f^2 + g^2]*(f + g*x)*Log[c*(f + g*x)
] - c^2*f*(f + g*x)*PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 +
g^2])] + c^2*f*(f + g*x)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))]))/(d*(c^2*f^2 + g^2)^(3/2)*(f + g*x)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.31, size = 1770, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2+g^2)/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d
*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^3*c^4*f
-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*f^2+g^2)/(g*x+f)*x*c^2*f-b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^2*c^2
*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(
g*x+f)*x*c*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^
2)/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)
/(c^2*f^2+g^2)/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+
1)/(c^2*f^2+g^2)/(g*x+f)*g+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6
*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*arc
sinh(c*x)*(c^2*f^2+g^2)^(1/2)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g
^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^
4)*c^2*f*arcsinh(c*x)*(c^2*f^2+g^2)^(1/2)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f
+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^
2*f^2*g^2+g^4)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*f^2+b*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*
f^2*g^2+g^4)*c^3*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2
))-g)*f^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*(c^2*f^2+g^2)^(1/2)*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*
f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*(c^2*f^2+g^2)^(1/2
)*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g
^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^
4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln(c*x+(c^2*x^2+1)^(
1/2))*g^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln((c*x+(c^2*x^2+1)^(1/2
))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*g^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2 d x^2 + d}\,(gx + f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{c^2dg^2x^4+2\,c^2dfgx^3+2\,dfgx+df^2+\left(c^2df^2+dg^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d*
f*g*x^3 + 2*d*f*g*x + d*f^2 + (c^2*d*f^2 + d*g^2)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2+1\right)}\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError
```

3.53 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}},x\right)$$

```
[Out] Unintegrable[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],
 x]
```

Rubi [A] time = 0.185919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] Defer[Int][((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x
]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

Mathematica [A] time = 0.130909, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x]
```

Maple [A] time = 1.526, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^n \ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \operatorname{arsinh}(cx) + a)^n \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] sage0*x
```

3.54 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2 \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=438

$$-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}+\frac{2bm\left(a+b\right.}{}$$

```
[Out] (m*(a + b*ArcSinh[c*x])^4)/(12*b^2*c) - (m*(a + b*ArcSinh[c*x])^3*Log[1 + (
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(3*b*c) - (m*(a + b*ArcSinh
[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(3*b*c) +
 ((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (m*(a + b*ArcSinh[c*
x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m
*(a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3,
-((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4,
 -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4
, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rubi [A] time = 0.72654, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {5675, 5838, 5799, 5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}+\frac{2bm\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^4)/(12*b^2*c) - (m*(a + b*ArcSinh[c*x])^3*Log[1 + (
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(3*b*c) - (m*(a + b*ArcSinh
[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(3*b*c) +
 ((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (m*(a + b*ArcSinh[c*
x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m
*(a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3,
-((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4,
 -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4
, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5838

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(Log[h*(f + g*x)^m]
*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(g*m)/(b*c*
```

```
Sqrt[d]*(n + 1)), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{f+gx}\,dx}{3bc}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\,\text{Subst}\left(\int\frac{(a+bx)^3\cosh(x)}{cf+g\sinh(x)}\,dx,\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} + \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\,\text{S}}{}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} - $$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} - $$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} - $$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} - $$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} - $$

Mathematica [A] time = 0.253453, size = 397, normalized size = 0.91

$$-\frac{3bm\left(-2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(3,\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}-cf}\right)+\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}-cf}\right)+2b^2\text{PolyLo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(-(m*(a + b*ArcSinh[c*x])^4)/(4*b) + m*(a + b*ArcSinh[c*x])^3*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^3*Log[
1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh[c*x])^
3*Log[h*(f + g*x)^m] + 3*b*m*((a + b*ArcSinh[c*x])^2*PolyLog[2, (E^ArcSinh[
c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - 2*b*(a + b*ArcSinh[c*x])*PolyLog[
3, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] + 2*b^2*PolyLog[4, (E
^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 3*b*m*((a + b*ArcSinh[c
*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] - 2*b*
(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 +
g^2]))] + 2*b^2*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
)]))/(3*b*c)
```

Maple [F] time = 1.591, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2\right) \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*log((g*x + f)^m*h)
/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2 \log\left(h\left(f + gx\right)^m\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.55 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=332

$$-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}+\frac{bm\text{PolyLog}\left(3,\cdot\right.}{c}$$

```
[Out] (m*(a + b*ArcSinh[c*x])^3)/(6*b^2*c) - (m*(a + b*ArcSinh[c*x])^2*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(2*b*c) - (m*(a + b*ArcSinh[
c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(2*b*c) +
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (m*(a + b*ArcSinh[c*x
])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*(a
 + b*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])
)])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])
/c
```

Rubi [A] time = 0.558748, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5675, 5838, 5799, 5561, 2190, 2531, 2282, 6589}

$$-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}+\frac{bm\text{PolyLog}\left(3,\cdot\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^3)/(6*b^2*c) - (m*(a + b*ArcSinh[c*x])^2*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(2*b*c) - (m*(a + b*ArcSinh[
c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(2*b*c) +
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (m*(a + b*ArcSinh[c*x
])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*(a
 + b*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])
)])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])
/c
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5838

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(Log[h*(f + g*x)^m]
*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(g*m)/(b*c*
Sqrt[d]*(n + 1)), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{f+gx}dx}{2bc}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\,\text{Subst}\left(\int\frac{(a+bx)^2\cosh(x)}{cf+g\sinh(x)}dx, x, \text{si}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\,\text{Subst}}{}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} - \frac{m\left(a\right.}{}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} - \frac{m\left(a\right.}{}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} - \frac{m\left(a\right.}{}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} - \frac{m\left(a\right.}{}$$

Mathematica [A] time = 0.30484, size = 303, normalized size = 0.91

$$-\frac{2bm\left(\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}-cf}\right)-b\text{PolyLog}\left(3,\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}-cf}\right)\right)+2bm\left(\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(-(m*(a + b*ArcSinh[c*x])^3)/(3*b) + m*(a + b*ArcSinh[c*x])^2*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^2*Log[
1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh[c*x])^
2*Log[h*(f + g*x)^m] + 2*b*m*((a + b*ArcSinh[c*x])*PolyLog[2, (E^ArcSinh[c*
x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - b*PolyLog[3, (E^ArcSinh[c*x]*g)/(-(
c*f) + Sqrt[c^2*f^2 + g^2])]) + 2*b*m*((a + b*ArcSinh[c*x])*PolyLog[2, -((E
^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] - b*PolyLog[3, -((E^ArcSinh[
c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/(2*b*c)
```

Maple [F] time = 1.241, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\ln\left(h\left(gx+f\right)^m\right)\frac{1}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a) \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \operatorname{arsinh}(cx) + a) \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx)) \log\left(h\left(f + gx\right)^m\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a) \log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.56 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=197

$$-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{c}-\frac{m\sinh^{-1}(cx)\log}{c}$$

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f
- Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (m*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*PolyLog[
2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rubi [A] time = 0.300469, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {215, 2404, 5799, 5561, 2190, 2279, 2391}

$$-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{c}-\frac{m\sinh^{-1}(cx)\log}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f
- Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (m*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*PolyLog[
2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sinh^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{x\cosh(x)}{c^2f+cg\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} + \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{e^x x}{c^2f+ce^xg-c\sqrt{c^2f^2+g^2}}\right.$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} -$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} -$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} -$$

Mathematica [A] time = 0.0179448, size = 206, normalized size = 1.05

$$-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(\frac{cge^{\sinh^{-1}(cx)}}{c^2f-c\sqrt{c^2f^2+g^2}}+1\right)}{c} - \frac{m\sinh^{-1}(cx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]*g)/(c^
2*f - c*Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
]*g)/(c^2*f + c*Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m]
```

```
)/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c -
 (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \ln\left(h\left(gx+f\right)^{m}\right)\frac{1}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)

[Out] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(h\left(f+gx\right)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)

[Out] Integral(log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.57 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x
]
```

Rubi [A] time = 0.194929, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.266206, size = 0, normalized size = 0.

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.843, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{a+b\text{Arcsinh}\,(cx)}\frac{1}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\log\left(\left(gx+f\right)^m h\right)}{ac^2x^2+\left(bc^2x^2+b\right)\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 + b)*
arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(h\left(f+gx\right)^m\right)}{\left(a+b\,\mathrm{asinh}\,(cx)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.58 $\int x^3 \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=131

$$-\frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{(a+bx)^2+1}}{96b^4}-\frac{\left(8a^4-24a^2+3\right)\sinh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{(a+bx)^2+1}}{48b^2}-x$$

```
[Out] (7*a*x^2*Sqrt[1 + (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 + (a + b*x)^2])/(16*
b) - ((4*a*(16 - 19*a^2) - (9 - 26*a^2)*(a + b*x))*Sqrt[1 + (a + b*x)^2])/(
96*b^4) - ((3 - 24*a^2 + 8*a^4)*ArcSinh[a + b*x])/(32*b^4) + (x^4*ArcSinh[a
 + b*x])/4
```

Rubi [A] time = 0.1722, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5865, 5801, 743, 833, 780, 215}

$$-\frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{(a+bx)^2+1}}{96b^4}-\frac{\left(8a^4-24a^2+3\right)\sinh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{(a+bx)^2+1}}{48b^2}-x$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[1 + (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 + (a + b*x)^2])/(16*
b) - ((4*a*(16 - 19*a^2) - (9 - 26*a^2)*(a + b*x))*Sqrt[1 + (a + b*x)^2])/(
96*b^4) - ((3 - 24*a^2 + 8*a^4)*ArcSinh[a + b*x])/(32*b^4) + (x^4*ArcSinh[a
 + b*x])/4
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^3 \sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\sinh^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4\sinh^{-1}(a+bx) - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-\frac{3-4a^2}{b^2}-\frac{7ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}\,dx, x, a+b$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4\sinh^{-1}(a+bx) - \frac{1}{48}\text{Subst}\left(\int \frac{\left(\frac{a(23-12a^2)}{b^3} - \frac{(9}{}\right.}{\sqrt{1}}\right.$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{1+(a+bx}}{96b^4}$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{1+(a+bx}}{96b^4}$$

Mathematica [A] time = 0.0817239, size = 95, normalized size = 0.73

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(-26a^2bx+50a^3+a\left(14b^2x^2-55\right)-6b^3x^3+9bx\right)-3\left(8a^4-24a^2-8b^4x^4+3\right)\sinh^{-1}(a+bx)}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*x^3
 + a*(-55 + 14*b^2*x^2)) - 3*(3 - 24*a^2 + 8*a^4 - 8*b^4*x^4)*ArcSinh[a + b
*x])/(96*b^4)
```

Maple [A] time = 0.014, size = 200, normalized size = 1.5

$$\frac{1}{b^4}\left(\frac{\text{Arcsinh}(bx+a)(bx+a)^4}{4} - \text{Arcsinh}(bx+a)(bx+a)^3 a + \frac{3\,\text{Arcsinh}(bx+a)(bx+a)^2 a^2}{2} - \text{Arcsinh}(bx+a)(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*arcsinh(b*x+a)*(b*x+a)^4-arcsinh(b*x+a)*(b*x+a)^3*a+3/2*arcsinh(
b*x+a)*(b*x+a)^2*a^2-arcsinh(b*x+a)*(b*x+a)*a^3-1/16*(b*x+a)^3*(1+(b*x+a)^2
)^(1/2)+3/32*(b*x+a)*(1+(b*x+a)^2)^(1/2)-3/32*arcsinh(b*x+a)+a*(1/3*(b*x+a)
^2*(1+(b*x+a)^2)^(1/2)-2/3*(1+(b*x+a)^2)^(1/2))-3/2*a^2*(1/2*(b*x+a)*(1+(b*
x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))+a^3*(1+(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8298, size = 262, normalized size = 2.

$$\frac{3\left(8\,b^4x^4-8\,a^4+24\,a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\left(6\,b^3x^3-14\,ab^2x^2-50\,a^3+\left(26\,a^2-9\right)bx+55\,a\right.}{96\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b*x + 55
*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

Sympy [A] time = 1.81439, size = 255, normalized size = 1.95

$$\begin{cases} -\frac{a^4\operatorname{asinh}(a+bx)}{4b^4}+\frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}}{48b^4}-\frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}}{48b^3}+\frac{3a^2\operatorname{asinh}(a+bx)}{4b^4}+\frac{7ax^2\sqrt{a^2+2abx+b^2x^2+1}}{48b^2}-\frac{55a\sqrt{a^2+2abx+b^2x^2+1}}{96b^4} \\ \frac{x^4\operatorname{asinh}(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(48
*b**3) + 3*a**2*asinh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**2) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(96*b**4
) + x**4*asinh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(16*b
) + 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(32*b**3) - 3*asinh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

Giac [A] time = 1.29298, size = 219, normalized size = 1.67

$$\frac{1}{4}\,x^4\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{96}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x\left(\frac{3\,x}{b^2}-\frac{7\,a}{b^3}\right)+\frac{26\,a^2b^3-9\,b^3}{b^7}\right)x-\frac{5\left(10\,a^3b^2-11\,a\right.}{b^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 - 9*b^3)/b^7)*x - 5*(10
*a^3*b^2 - 11*a*b^2)/b^7) - 3*(8*a^4 - 24*a^2 + 3)*log(-a*b - (x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^4*abs(b)))*b
```

3.59 $\int x^2 \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{\left(-11a^2+5abx+4\right)\sqrt{(a+bx)^2+1}}{18b^3}-\frac{a\left(3-2a^2\right)\sinh^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{(a+bx)^2+1}}{9b}+\frac{1}{3}x^3\sinh^{-1}(a+bx)$$

```
[Out] -(x^2*Sqrt[1 + (a + b*x)^2])/(9*b) + ((4 - 11*a^2 + 5*a*b*x)*Sqrt[1 + (a +
b*x)^2])/(18*b^3) - (a*(3 - 2*a^2)*ArcSinh[a + b*x])/(6*b^3) + (x^3*ArcSinh
[a + b*x])/3
```

Rubi [A] time = 0.112533, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5865, 5801, 743, 780, 215}

$$\frac{\left(-11a^2+5abx+4\right)\sqrt{(a+bx)^2+1}}{18b^3}-\frac{a\left(3-2a^2\right)\sinh^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{(a+bx)^2+1}}{9b}+\frac{1}{3}x^3\sinh^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x],x]
```

```
[Out] -(x^2*Sqrt[1 + (a + b*x)^2])/(9*b) + ((4 - 11*a^2 + 5*a*b*x)*Sqrt[1 + (a +
b*x)^2])/(18*b^3) - (a*(3 - 2*a^2)*ArcSinh[a + b*x])/(6*b^3) + (x^3*ArcSinh
[a + b*x])/3
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^2 \sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{1}{3}x^3\sinh^{-1}(a+bx) - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$
$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{1}{3}x^3\sinh^{-1}(a+bx) - \frac{1}{9}\text{Subst}\left(\int \frac{\left(-\frac{2-3a^2}{b^2}-\frac{5ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)}{\sqrt{1+x^2}}\,dx, x, a+bx\right.$$
$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{\left(4-11a^2+5abx\right)\sqrt{1+(a+bx)^2}}{18b^3} + \frac{1}{3}x^3\sinh^{-1}(a+bx) - \frac{\left(a\left(3-2a^2\right)\right)\,\text{S}}{}$$
$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{\left(4-11a^2+5abx\right)\sqrt{1+(a+bx)^2}}{18b^3} - \frac{a\left(3-2a^2\right)\sinh^{-1}(a+bx)}{6b^3} + \frac{1}{3}x^3\sin$$

Mathematica [A] time = 0.0562991, size = 74, normalized size = 0.82

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(-11a^2+5abx-2b^2x^2+4\right)+\left(6a^3-9a+6b^3x^3\right)\sinh^{-1}(a+bx)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x],x]
```

```
[Out] ((4 - 11*a^2 + 5*a*b*x - 2*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-9
*a + 6*a^3 + 6*b^3*x^3)*ArcSinh[a + b*x])/(18*b^3)
```

Maple [A] time = 0.004, size = 130, normalized size = 1.4

$$\frac{1}{b^3}\left(\frac{\text{Arcsinh}\,(bx+a)\,(bx+a)^3}{3} - \text{Arcsinh}\,(bx+a)\,(bx+a)^2\,a + \text{Arcsinh}\,(bx+a)\,(bx+a)\,a^2 - \frac{(bx+a)^2}{9}\sqrt{1+(bx+a)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*arcsinh(b*x+a)*(b*x+a)^3-arcsinh(b*x+a)*(b*x+a)^2*a+arcsinh(b*x+
a)*(b*x+a)*a^2-1/9*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/9*(1+(b*x+a)^2)^(1/2)+a*
(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))-a^2*(1+(b*x+a)^2)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.56749, size = 216, normalized size = 2.4

$$\frac{3\left(2b^3x^3+2a^3-3a\right)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)-\left(2b^2x^2-5abx+11a^2-4\right)\sqrt{b^2x^2+2abx+a^2+1}}{18b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))/b^3
```

Sympy [A] time = 0.844151, size = 170, normalized size = 1.89

$$\begin{cases}\frac{a^3\operatorname{asinh}(a+bx)}{3b^3}-\frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}}{18b^3}+\frac{5ax\sqrt{a^2+2abx+b^2x^2+1}}{18b^2}-\frac{a\operatorname{asinh}(a+bx)}{2b^3}+\frac{x^3\operatorname{asinh}(a+bx)}{3}-\frac{x^2\sqrt{a^2+2abx+b^2x^2+1}}{9b}+\frac{2\sqrt{a^2+2abx+}}{9b^3} \\ \frac{x^3\operatorname{asinh}(a)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a),x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(18*b**2
) - a*asinh(a + b*x)/(2*b**3) + x**3*asinh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)/(9*b) + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(9*b*
*3), Ne(b, 0)), (x**3*asinh(a)/3, True))
```

Giac [A] time = 1.30377, size = 177, normalized size = 1.97

$$\frac{1}{3}x^3\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{18}\left(\sqrt{b^2x^2+2abx+a^2+1}\left(x\left(\frac{2x}{b^2}-\frac{5a}{b^3}\right)+\frac{11a^2b-4b}{b^5}\right)+\frac{3\left(2a^3-3a\right)\log\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b - 4*b)/b^5) + 3*(2*a^3 - 3*a
)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*ab
s(b)))*b
```

3.60 $\int x \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{(a+bx)^2+1}}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{x\sqrt{(a+bx)^2+1}}{4b}$$

```
[Out] (3*a*Sqrt[1 + (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 + (a + b*x)^2])/(4*b) + ((1
 - 2*a^2)*ArcSinh[a + b*x])/(4*b^2) + (x^2*ArcSinh[a + b*x])/2
```

Rubi [A] time = 0.0661435, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5801, 743, 641, 215}

$$\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{(a+bx)^2+1}}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{x\sqrt{(a+bx)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[1 + (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 + (a + b*x)^2])/(4*b) + ((1
 - 2*a^2)*ArcSinh[a + b*x])/(4*b^2) + (x^2*ArcSinh[a + b*x])/2
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}\,dx,x,a+bx\right)\\
&= -\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{-\frac{1-2a^2}{b^2}-\frac{3ax}{b^2}}{\sqrt{1+x^2}}\,dx,x,a+bx\right)\\
&= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{1}{2}x^2\sinh^{-1}(a+bx)+\frac{\left(1-2a^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}\,dx,x
\end{aligned}$$

$$= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{\left(1-2a^2\right)\sinh^{-1}(a+bx)}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)$$

Mathematica [A] time = 0.0404693, size = 60, normalized size = 0.79

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}(3a-bx)+\left(-2a^2+2b^2x^2+1\right)\sinh^{-1}(a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 - 2*a^2 + 2*b^2*x^2)*Ar
cSinh[a + b*x])/(4*b^2)
```

Maple [A] time = 0.004, size = 74, normalized size = 1.

$$\frac{1}{b^2}\left(\frac{\text{Arcsinh}\,(bx+a)\,(bx+a)^2}{2}-\text{Arcsinh}\,(bx+a)\,a\,(bx+a)-\frac{bx+a}{4}\sqrt{1+(bx+a)^2}+\frac{\text{Arcsinh}\,(bx+a)}{4}+a\sqrt{1+(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)*(b*x+a)^2-arcsinh(b*x+a)*a*(b*x+a)-1/4*(b*x+a)*(1
+(b*x+a)^2)^(1/2)+1/4*arcsinh(b*x+a)+a*(1+(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52007, size = 178, normalized size = 2.34

$$\frac{\left(2\,b^{2}x^{2}-2\,a^{2}+1\right)\log\left(bx+a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)-\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}(bx-3\,a)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^2
```

Sympy [A] time = 0.365054, size = 104, normalized size = 1.37

$$\begin{cases} -\frac{a^{2}\operatorname{asinh}(a+bx)}{2b^{2}}+\frac{3a\sqrt{a^{2}+2abx+b^{2}x^{2}+1}}{4b^{2}}+\frac{x^{2}\operatorname{asinh}(a+bx)}{2}-\frac{x\sqrt{a^{2}+2abx+b^{2}x^{2}+1}}{4b}+\frac{\operatorname{asinh}(a+bx)}{4b^{2}} & \text{for}\ b\neq 0\\ \frac{x^{2}\operatorname{asinh}(a)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(4*b**2) + x**2*asinh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(4*b) + asinh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*asinh(a)/2, True)
)
```

Giac [A] time = 1.29715, size = 150, normalized size = 1.97

$$\frac{1}{2}\,x^{2}\log\left(bx+a+\sqrt{(bx+a)^{2}+1}\right)-\frac{1}{4}\left(\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\left(\frac{x}{b^{2}}-\frac{3\,a}{b^{3}}\right)-\frac{\left(2\,a^{2}-1\right)\log\left(-ab-\left(x|b|-\sqrt{b^{2}x^{2}+2\,abx\right.\right.}{b^{2}|b|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(x/b^2 - 3*a/b^3) - (2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b)))*b
```

3.61 $\int \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b}$$

```
[Out] -(Sqrt[1 + (a + b*x)^2]/b) + ((a + b*x)*ArcSinh[a + b*x])/b
```

Rubi [A] time = 0.0139822, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5653, 261}

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x],x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/b) + ((a + b*x)*ArcSinh[a + b*x])/b
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.026631, size = 40, normalized size = 1.18

$$\frac{(a+bx)\sinh^{-1}(a+bx)-\sqrt{a^2+2abx+b^2x^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x],x]
```

```
[Out] (-Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (a + b*x)*ArcSinh[a + b*x])/b
```

Maple [A] time = 0.001, size = 31, normalized size = 0.9

$$\frac{1}{b}\left((bx+a)\operatorname{Arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))
```

Maxima [A] time = 1.08046, size = 41, normalized size = 1.21

$$\frac{(bx+a)\operatorname{arsinh}(bx+a)-\sqrt{(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arcsinh(b*x + a) - sqrt((b*x + a)^2 + 1))/b
```

Fricas [A] time = 2.4697, size = 135, normalized size = 3.97

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))/b
```

Sympy [A] time = 0.183704, size = 46, normalized size = 1.35

$$\begin{cases}\frac{a\operatorname{asinh}(a+bx)}{b}+x\operatorname{asinh}(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{for } b\neq 0\\ x\operatorname{asinh}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a),x)
```

```
[Out] Piecewise((a*asinh(a + b*x)/b + x*asinh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a), True))
```

Giac [B] time = 1.28512, size = 124, normalized size = 3.65

$$-b\left(\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}\right)+x\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b*
abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2) + x*log(b*x + a + sqrt((b*
x + a)^2 + 1))
```

3.62 $\int \frac{\sinh^{-1}(a+bx)}{x} dx$

Optimal. Leaf size=131

$$\text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right) + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + \sinh^{-1}(a+bx)\log\left(1-\right.$$

```
[Out] -ArcSinh[a + b*x]^2/2 + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sq
rt[1 + a^2])] + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])] + PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + PolyLog[2, E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rubi [A] time = 0.243267, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5865, 5799, 5561, 2190, 2279, 2391}

$$\text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right) + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + \sinh^{-1}(a+bx)\log\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x,x]
```

```
[Out] -ArcSinh[a + b*x]^2/2 + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sq
rt[1 + a^2])] + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])] + PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + PolyLog[2, E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\right.}{b}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(}}{a+\sqrt{1}}\right.$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(}}{a+\sqrt{1}}\right.$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(}}{a+\sqrt{1}}\right.$$

Mathematica [A] time = 0.0121277, size = 153, normalized size = 1.17

$$\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}-a}\right)+\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)+\sinh^{-1}(a+bx)\log\left(\frac{e^{\sinh^{-1}(a+bx)}}{b\left(-\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)}+1\right)+\sinh^{-1}(a+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x,x]
```

```
[Out] -ArcSinh[a + b*x]^2/2 + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((-(a/b
) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((-(
a/b) + Sqrt[1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcSinh[a + b*x]/(-a + Sqrt[1
+ a^2]))] + PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Maple [B] time = 0.082, size = 388, normalized size = 3.

$$-\frac{(\operatorname{Arcsinh}(bx+a))^2}{2}+\frac{\operatorname{Arcsinh}(bx+a)}{a^2+1}\left(a^2+1+\sqrt{a^2+1}a\right)\left(2\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)a^2+\ln\left(\left(\sqrt{a^2+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x,x)
```

```
[Out] -1/2*arcsinh(b*x+a)^2+(a^2+1+(a^2+1)^(1/2)*a)/(a^2+1)*arcsinh(b*x+a)*(2*ln(
((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a^2+ln(((a^2+1)^
(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+ln(((a^2+1)^(1/2)+b*x+(1+
(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-2*(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b*x
-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a)+dilog(((a^2+1)^(1/2)-b*x-(1+(b*
x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1
/2))/(-a+(a^2+1)^(1/2)))+a*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b
*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-a*arcsinh(b*x+a)/(a^2+1)^(1/2)*l
n(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x,x)
```

```
[Out] Integral(asinh(a + b*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/x, x)
```

3.63 $\int \frac{\sinh^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{b\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}}-\frac{\sinh^{-1}(a+bx)}{x}$$

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Rubi [A] time = 0.0704265, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5865, 5801, 725, 206}

$$-\frac{b\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}}-\frac{\sinh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} + \operatorname{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} - \operatorname{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} - \frac{b\tanh^{-1}\left(\frac{b\left(\frac{1}{b}+\frac{a(a+bx)}{b}\right)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

Mathematica [A] time = 0.0385993, size = 57, normalized size = 1.

$$-\frac{b\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}} - \frac{\sinh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Maple [A] time = 0.006, size = 71, normalized size = 1.3

$$-\frac{\operatorname{Arcsinh}(bx+a)}{x} - b\ln\left(\frac{1}{bx}\left(2a^2+2+2xab+2\sqrt{a^2+1}\sqrt{b^2x^2+2xab+a^2+1}\right)\right)\frac{1}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^2,x)
```

```
[Out] -arcsinh(b*x+a)/x-b/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/b/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.77731, size = 405, normalized size = 7.11

$$\frac{\sqrt{a^2+1}bx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)+\left(a^2+1\right)x\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+}\right.}{\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + (a^
2 + 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (a^2 - (a^2 +
1)*x + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/((a^2 + 1)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.64 $\int \frac{\sinh^{-1}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=92

$$\frac{ab^2\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{b\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}$$

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(2*(1 + a^2)*x) - ArcSinh[a + b*x]/(2*x^2) + (a*
b^2*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(2*(1
 + a^2)^(3/2))
```

Rubi [A] time = 0.102091, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5865, 5801, 731, 725, 206}

$$\frac{ab^2\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{b\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(2*(1 + a^2)*x) - ArcSinh[a + b*x]/(2*x^2) + (a*
b^2*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(2*(1
 + a^2)^(3/2))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{x^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx\right)}{b}\\
&= -\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx,x,a+bx\right)\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx,x,\frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)}\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{ab^2\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.166296, size = 110, normalized size = 1.2

$$-\frac{\frac{bx\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}-abx\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)+abx\log(x)\right)}{\left(a^2+1\right)^{3/2}}+\sinh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -(ArcSinh[a + b*x] + (b*x*(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]
+ a*b*x*Log[x] - a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2
*a*b*x + b^2*x^2]]))/(1 + a^2)^(3/2))/(2*x^2)
```

Maple [A] time = 0.008, size = 106, normalized size = 1.2

$$-\frac{\text{Arcsinh}\,(bx+a)}{2\,x^2}-\frac{b}{\left(2\,a^2+2\right)x}\sqrt{b^2x^2+2\,xab+a^2+1}+\frac{b^2a}{2}\ln\left(\frac{1}{bx}\left(2\,a^2+2+2\,xab+2\,\sqrt{a^2+1}\sqrt{b^2x^2+2\,xab+a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^3,x)
```

```
[Out] -1/2*arcsinh(b*x+a)/x^2-1/2*b/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*b
^2*a/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/b/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.708, size = 564, normalized size = 6.13

$$\sqrt{a^2+1}ab^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-\left(a^2+1\right)b^2x^2+\left(a^4+2\,a^2+1\right)x^2\log\left(-b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*a*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a
)/x) - (a^2 + 1)*b^2*x^2 + (a^4 + 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*b*x -
 (a^4 - (a^4 + 2*a^2 + 1)*x^2 + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)))/((a^4 + 2*a^2 + 1)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.65 $\int \frac{\sinh^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=129

$$\frac{ab^2\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)^2 x}+\frac{\left(1-2a^2\right)b^3\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{6\left(a^2+1\right)^{5/2}}-\frac{b\sqrt{(a+bx)^2+1}}{6\left(a^2+1\right)x^2}-\frac{\sinh^{-1}(a+bx)}{3x^3}$$

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(6*(1 + a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]
)/(2*(1 + a^2)^2*x) - ArcSinh[a + b*x]/(3*x^3) + ((1 - 2*a^2)*b^3*ArcTanh[(
1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(6*(1 + a^2)^(5/2)
)
```

Rubi [A] time = 0.154325, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5865, 5801, 745, 807, 725, 206}

$$\frac{ab^2\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)^2 x}+\frac{\left(1-2a^2\right)b^3\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{6\left(a^2+1\right)^{5/2}}-\frac{b\sqrt{(a+bx)^2+1}}{6\left(a^2+1\right)x^2}-\frac{\sinh^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(6*(1 + a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]
)/(2*(1 + a^2)^2*x) - ArcSinh[a + b*x]/(3*x^3) + ((1 - 2*a^2)*b^3*ArcTanh[(
1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(6*(1 + a^2)^(5/2)
)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} - \frac{\sinh^{-1}(a+bx)}{3x^3} - \frac{b^2\,\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)}{6\left(1+a^2\right)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\sinh^{-1}(a+bx)}{3x^3} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1}}\right.}{6\left(1+a^2\right)^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\right.}{6\left(1+a^2\right)^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2 x} - \frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{\left(1-2a^2\right)b^3\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+}}\right.}{6\left(1+a^2\right)^{5/2}}$$

Mathematica [A] time = 0.217009, size = 149, normalized size = 1.16

$$\frac{-\sqrt{a^2+1}bx\left(a^2-3abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}+\left(2a^2-1\right)b^3x^3\log(x)+\left(1-2a^2\right)b^3x^3\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx-}\right.}{6\left(a^2+1\right)^{5/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] (-(Sqrt[1 + a^2]*b*x*(1 + a^2 - 3*a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
 - 2*(1 + a^2)^(5/2)*ArcSinh[a + b*x] + (-1 + 2*a^2)*b^3*x^3*Log[x] + (1 -
```

```
2*a^2)*b^3*x^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x +
 b^2*x^2]])/(6*(1 + a^2)^(5/2)*x^3)
```

Maple [A] time = 0.009, size = 203, normalized size = 1.6

$$-\frac{\text{Arcsinh}(bx+a)}{3x^3}-\frac{b}{(6a^2+6)x^2}\sqrt{b^2x^2+2xab+a^2+1}+\frac{b^2a}{2(a^2+1)^2x}\sqrt{b^2x^2+2xab+a^2+1}-\frac{b^3a^2}{2}\ln\left(\frac{1}{bx}\left(2a^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^4,x)
```

```
[Out] -1/3*arcsinh(b*x+a)/x^3-1/6*b/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2
*b^2*a/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*b^3*a^2/(a^2+1)^(5/2)*
ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x)+1/6
*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/b/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.90232, size = 672, normalized size = 5.21

$$\frac{(2a^2-1)\sqrt{a^2+1}b^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+3(a^3+a)b^3x^3+2(a^6+3a^4+3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2 - 1)*sqrt(a^2 + 1)*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^
2 + 1) + a)/x) + 3*(a^3 + a)*b^3*x^3 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*x^3*log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(a^6 + 3*a^4 - (a^6 + 3*a
^4 + 3*a^2 + 1)*x^3 + 3*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)) + (3*(a^3 + a)*b^2*x^2 - (a^4 + 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))/((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**4,x)
```

```
[Out] Integral(asinh(a + b*x)/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.66 $\int \frac{\sinh^{-1}(a+bx)}{x^5}\,dx$

Optimal. Leaf size=167

$$\frac{5ab^2\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^2x^2}+\frac{\left(4-11a^2\right)b^3\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^3x}-\frac{a\left(3-2a^2\right)b^4\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{8\left(a^2+1\right)^{7/2}}-\frac{b\sqrt{(a+bx)^2+1}}{12\left(a^2+1\right)x^3}-\sinh^{-}$$

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(12*(1 + a^2)*x^3) + (5*a*b^2*Sqrt[1 + (a + b*x)
^2])/(24*(1 + a^2)^2*x^2) + ((4 - 11*a^2)*b^3*Sqrt[1 + (a + b*x)^2])/(24*(1
 + a^2)^3*x) - ArcSinh[a + b*x]/(4*x^4) - (a*(3 - 2*a^2)*b^4*ArcTanh[(1 + a
*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(8*(1 + a^2)^(7/2))
```

Rubi [A] time = 0.228429, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5865, 5801, 745, 835, 807, 725, 206}

$$\frac{5ab^2\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^2x^2}+\frac{\left(4-11a^2\right)b^3\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^3x}-\frac{a\left(3-2a^2\right)b^4\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{8\left(a^2+1\right)^{7/2}}-\frac{b\sqrt{(a+bx)^2+1}}{12\left(a^2+1\right)x^3}-\sinh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(12*(1 + a^2)*x^3) + (5*a*b^2*Sqrt[1 + (a + b*x)
^2])/(24*(1 + a^2)^2*x^2) + ((4 - 11*a^2)*b^3*Sqrt[1 + (a + b*x)^2])/(24*(1
 + a^2)^3*x) - ArcSinh[a + b*x]/(4*x^4) - (a*(3 - 2*a^2)*b^4*ArcTanh[(1 + a
*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(8*(1 + a^2)^(7/2))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^5}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^5}\,dx,x,a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4\sqrt{1+x^2}}\,dx,x,a+bx\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3}-\frac{\sinh^{-1}(a+bx)}{4x^4}-\frac{b^2\,\text{Subst}\left(\int \frac{\frac{3a}{b}+\frac{2x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx,x,a+bx\right)}{12\left(1+a^2\right)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3}+\frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2}-\frac{\sinh^{-1}(a+bx)}{4x^4}+\frac{b^4\,\text{Subst}\left(\int \frac{-\frac{2\left(2-3a^2\right)}{b^2}+\frac{5ax}{b^2}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx,x,\right.}{24\left(1+a^2\right)^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3}+\frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2}+\frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x}-\frac{\sinh^{-1}(a+bx)}{4x^4}+$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3}+\frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2}+\frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x}-\frac{\sinh^{-1}(a+bx)}{4x^4}-$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3}+\frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2}+\frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x}-\frac{\sinh^{-1}(a+bx)}{4x^4}-$$

Mathematica [A] time = 0.200879, size = 179, normalized size = 1.07

$$\frac{1}{8}\left(-\frac{b\sqrt{a^2+2abx+b^2x^2+1}\left(a^2\left(11b^2x^2+4\right)-5a^3bx+2a^4-5abx-4b^2x^2+2\right)}{3\left(a^2+1\right)^3x^3}+\frac{a\left(2a^2-3\right)b^4\log\left(\sqrt{a^2+1}\sqrt{a^2+2ab\right.}{\left(a^2+1\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] (-(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 - 5*a*b*x - 5*a^3*b*x - 4
*b^2*x^2 + a^2*(4 + 11*b^2*x^2)))/(3*(1 + a^2)^3*x^3) - (2*ArcSinh[a + b*x]
)/x^4 - (a*(-3 + 2*a^2)*b^4*Log[x])/(1 + a^2)^(7/2) + (a*(-3 + 2*a^2)*b^4*L
og[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 +
 a^2)^(7/2))/8
```

Maple [A] time = 0.007, size = 275, normalized size = 1.7

$$-\frac{\mathrm{Arcsinh}\,(bx+a)}{4x^4}-\frac{b}{\left(12a^2+12\right)x^3}\sqrt{b^2x^2+2xab+a^2+1}+\frac{5b^2a}{24\left(a^2+1\right)^2x^2}\sqrt{b^2x^2+2xab+a^2+1}-\frac{5b^3a^2}{8\left(a^2+1\right)^3x}\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^5,x)
```

```
[Out] -1/4*arcsinh(b*x+a)/x^4-1/12*b/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/
24*b^2*a/(a^2+1)^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/8*b^3*a^2/(a^2+1)^3/
x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/8*b^4*a^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*x*a
*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x)-3/8*b^4*a/(a^2+1)^(5
/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x)
+1/6*b^3/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.0567, size = 801, normalized size = 4.8

$$\frac{3\left(2a^3-3a\right)\sqrt{a^2+1}b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-\left(11a^4+7a^2-4\right)b^4x^4+6\left(a^8+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(2*a^3 - 3*a)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sq
rt(a^2 + 1) + a)/x) - (11*a^4 + 7*a^2 - 4)*b^4*x^4 + 6*(a^8 + 4*a^6 + 6*a^4
 + 4*a^2 + 1)*x^4*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*(a^
8 + 4*a^6 - (a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4 + 6*a^4 + 4*a^2 + 1)*log(
b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - ((11*a^4 + 7*a^2 - 4)*b^3*x^
3 - 5*(a^5 + 2*a^3 + a)*b^2*x^2 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*b*x)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(a+bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**5,x)
```

```
[Out] Integral(asinh(a + b*x)/x**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.67 $\int x^3 \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=331

$$-\frac{2a^3x}{b^3}+\frac{3a^2(a+bx)^2}{4b^4}-\frac{a^4\sinh^{-1}(a+bx)^2}{4b^4}+\frac{2a^3\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{b^4}+\frac{3a^2\sinh^{-1}(a+bx)^2}{4b^4}-\frac{3a^2(a+bx)\sqrt{(}}{}$$

```
[Out] (4*a*x)/(3*b^3) - (2*a^3*x)/b^3 - (3*(a + b*x)^2)/(32*b^4) + (3*a^2*(a + b*
x)^2)/(4*b^4) - (2*a*(a + b*x)^3)/(9*b^4) + (a + b*x)^4/(32*b^4) - (4*a*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) + (2*a^3*Sqrt[1 + (a + b*x)^2]
*ArcSinh[a + b*x])/b^4 + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(16*b^4) - (3*a^2*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b
^4) + (2*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) - ((
a + b*x)^3*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(8*b^4) - (3*ArcSinh[a +
 b*x]^2)/(32*b^4) + (3*a^2*ArcSinh[a + b*x]^2)/(4*b^4) - (a^4*ArcSinh[a + b
*x]^2)/(4*b^4) + (x^4*ArcSinh[a + b*x]^2)/4
```

Rubi [A] time = 0.546772, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{2a^3x}{b^3}+\frac{3a^2(a+bx)^2}{4b^4}-\frac{a^4\sinh^{-1}(a+bx)^2}{4b^4}+\frac{2a^3\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{b^4}+\frac{3a^2\sinh^{-1}(a+bx)^2}{4b^4}-\frac{3a^2(a+bx)\sqrt{(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x]^2,x]
```



```
[Out] (4*a*x)/(3*b^3) - (2*a^3*x)/b^3 - (3*(a + b*x)^2)/(32*b^4) + (3*a^2*(a + b*
x)^2)/(4*b^4) - (2*a*(a + b*x)^3)/(9*b^4) + (a + b*x)^4/(32*b^4) - (4*a*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) + (2*a^3*Sqrt[1 + (a + b*x)^2]
*ArcSinh[a + b*x])/b^4 + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(16*b^4) - (3*a^2*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b
^4) + (2*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) - ((
a + b*x)^3*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(8*b^4) - (3*ArcSinh[a +
 b*x]^2)/(32*b^4) + (3*a^2*ArcSinh[a + b*x]^2)/(4*b^4) - (a^4*ArcSinh[a + b
*x]^2)/(4*b^4) + (x^4*ArcSinh[a + b*x]^2)/4
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4 \sinh^{-1}(x)}{b^4\sqrt{1+x^2}} - \frac{4a^3 x \sinh^{-1}(x)}{b^4\sqrt{1+x^2}} + \frac{6a^2x^2 \sinh^{-1}(x)}{b^4\sqrt{1+x^2}} - \frac{4ax}{b^4}\right.\right.$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{x^4 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^4} + \frac{(2a)\,\text{Subst}\left(\int \frac{x^3 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a-\right.}{b^4}$$

$$= \frac{2a^3\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^4} - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b^4} + \frac{2a(a+bx)^2}{}$$

$$= -\frac{2a^3x}{b^3} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3b^4} + \frac{2a^3\sqrt{}}{}$$

$$= \frac{4ax}{3b^3} - \frac{2a^3x}{b^3} - \frac{3(a+bx)^2}{32b^4} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\sinh}{3b^4}$$

Mathematica [A] time = 0.179018, size = 145, normalized size = 0.44

$$\frac{bx\left(78a^2bx - 300a^3 + a\left(330 - 28b^2x^2\right) + 9bx\left(b^2x^2 - 3\right)\right) - 9\left(8a^4 - 24a^2 - 8b^4x^4 + 3\right)\sinh^{-1}(a+bx)^2 + 6\sqrt{a^2+2abx+}}{288b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-300*a^3 + 78*a^2*b*x + a*(330 - 28*b^2*x^2) + 9*b*x*(-3 + b^2*x^2))
+ 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*
x^3 + a*(-55 + 14*b^2*x^2))*ArcSinh[a + b*x] - 9*(3 - 24*a^2 + 8*a^4 - 8*b^
4*x^4)*ArcSinh[a + b*x]^2)/(288*b^4)
```

Maple [A] time = 0.072, size = 387, normalized size = 1.2

$$\frac{1}{b^4}\left(-a^3\left((\text{Arcsinh}(bx+a))^2(bx+a) - 2\,\text{Arcsinh}(bx+a)\sqrt{1+(bx+a)^2} + 2\,bx + 2\,a\right) + \frac{3\,a^2}{4}\left(2\,(\text{Arcsinh}(bx+a))^2(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(-a^3*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+
2*b*x+2*a)+3/4*a^2*(2*arcsinh(b*x+a)^2*(b*x+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a
)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)-1/9*a*(9*arcsinh(b*x+a)^2*
(b*x+a)^3-6*arcsinh(b*x+a)*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+27*arcsinh(b*x+a)^
2*(b*x+a)+2*(b*x+a)^3-42*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+42*b*x+42*a)+1/
4*arcsinh(b*x+a)^2*(b*x+a)^2*(1+(b*x+a)^2)-1/4*arcsinh(b*x+a)^2*(1+(b*x+a)^
2)-1/8*arcsinh(b*x+a)*(b*x+a)*(1+(b*x+a)^2)^(3/2)+5/16*arcsinh(b*x+a)*(1+(b
*x+a)^2)^(1/2)*(b*x+a)+5/32*arcsinh(b*x+a)^2+1/32*(b*x+a)^2*(1+(b*x+a)^2)-1
/8*(b*x+a)^2-1/8+3*a*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^
2)^(1/2)+2*b*x+2*a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49499, size = 439, normalized size = 1.33

$$9\,b^4x^4 - 28\,ab^3x^3 + 3\left(26\,a^2 - 9\right)b^2x^2 - 30\left(10\,a^3 - 11\,a\right)bx + 9\left(8\,b^4x^4 - 8\,a^4 + 24\,a^2 - 3\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*b^4*x^4 - 28*a*b^3*x^3 + 3*(26*a^2 - 9)*b^2*x^2 - 30*(10*a^3 - 11*
a)*b*x + 9*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2 - 6*(6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b
*x + 55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)))/b^4
```

Sympy [A] time = 4.16391, size = 366, normalized size = 1.11

$$\begin{cases} -\frac{a^4\operatorname{asinh}^2(a+bx)}{4b^4} - \frac{25a^3x}{24b^3} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^4} + \frac{13a^2x^2}{48b^2} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^3} + \frac{3a^2\operatorname{asinh}^2(a+bx)}{4b^4} - \frac{7}{7} \\ \frac{x^4\operatorname{asinh}^2(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)**2/(4*b**4) - 25*a**3*x/(24*b**3) + 25*a**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**4) + 13*a**2*x*
*2/(48*b**2) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)/(24*b**3) + 3*a**2*asinh(a + b*x)**2/(4*b**4) - 7*a*x**3/(72*b) + 7*a*x**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**2) + 55*a*x/(4
8*b**3) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(48*b**4
) + x**4*asinh(a + b*x)**2/4 + x**4/32 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*asinh(a + b*x)/(8*b) - 3*x**2/(32*b**2) + 3*x*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)*asinh(a + b*x)/(16*b**3) - 3*asinh(a + b*x)**2/(32*b**4), N
e(b, 0)), (x**4*asinh(a)**2/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsinh}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(b*x + a)^2, x)
```

3.68 $\int x^2 \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=211

$$\frac{2a^2x}{b^2}+\frac{a^3\sinh^{-1}(a+bx)^2}{3b^3}-\frac{2a^2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{b^3}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}-\frac{a\sinh^{-1}(a+bx)^2}{2b^3}+\frac{a(a$$

```
[Out] (-4*x)/(9*b^2) + (2*a^2*x)/b^2 - (a*(a + b*x)^2)/(2*b^3) + (2*(a + b*x)^3)/
(27*b^3) + (4*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(9*b^3) - (2*a^2*Sqrt
[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b^3 + (a*(a + b*x)*Sqrt[1 + (a + b*x)^2
]*ArcSinh[a + b*x])/b^3 - (2*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
b*x])/(9*b^3) - (a*ArcSinh[a + b*x]^2)/(2*b^3) + (a^3*ArcSinh[a + b*x]^2)/(
3*b^3) + (x^3*ArcSinh[a + b*x]^2)/3
```

Rubi [A] time = 0.374804, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5801, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{2a^2x}{b^2}+\frac{a^3\sinh^{-1}(a+bx)^2}{3b^3}-\frac{2a^2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{b^3}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}-\frac{a\sinh^{-1}(a+bx)^2}{2b^3}+\frac{a(a$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-4*x)/(9*b^2) + (2*a^2*x)/b^2 - (a*(a + b*x)^2)/(2*b^3) + (2*(a + b*x)^3)/
(27*b^3) + (4*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(9*b^3) - (2*a^2*Sqrt
[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b^3 + (a*(a + b*x)*Sqrt[1 + (a + b*x)^2
]*ArcSinh[a + b*x])/b^3 - (2*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
b*x])/(9*b^3) - (a*ArcSinh[a + b*x]^2)/(2*b^3) + (a^3*ArcSinh[a + b*x]^2)/(
3*b^3) + (x^3*ArcSinh[a + b*x]^2)/3
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \left(-\frac{a^3\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} + \frac{3a^2x\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} - \frac{3ax^2\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} + \frac{x^3 \text{s}}{b^3}\right.\right.$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int \frac{x^3\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{3b^3} + \frac{(2a)\,\text{Subst}\left(\int \frac{x^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a\right.}{b^3}$$

$$= -\frac{2a^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^3} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^3} - \frac{2(a+bx)^2\sqrt{}}{}$$

$$= \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{9b^3} - \frac{2a^2\sqrt{1+(a+bx)^2}\sinh}{b^3}$$

$$= -\frac{4x}{9b^2} + \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{9b^3} - \frac{2a^2\sqrt{1+(a+b}}{}$$

Mathematica [A] time = 0.137144, size = 107, normalized size = 0.51

$$\frac{bx\left(66a^2-15abx+4b^2x^2-24\right)+9\left(2a^3-3a+2b^3x^3\right)\sinh^{-1}(a+bx)^2-6\sqrt{a^2+2abx+b^2x^2+1}\left(11a^2-5abx+2b^2x^2\right.}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-24 + 66*a^2 - 15*a*b*x + 4*b^2*x^2) - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2
*x^2]*(-4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x] + 9*(-3*a + 2*a^
3 + 2*b^3*x^3)*ArcSinh[a + b*x]^2)/(54*b^3)
```

Maple [A] time = 0.056, size = 219, normalized size = 1.

$$\frac{1}{b^3}\left(-\frac{a}{2}\left(2\left(\operatorname{Arcsinh}(bx+a)\right)^2(bx+a)^2-2\operatorname{Arcsinh}(bx+a)\sqrt{1+(bx+a)^2}(bx+a)+\left(\operatorname{Arcsinh}(bx+a)\right)^2+(bx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(-1/2*a*(2*arcsinh(b*x+a)^2*(b*x+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)-1/3*arcsinh(b*x+a)^2*(b*x+a)+1/
3*arcsinh(b*x+a)^2*(b*x+a)*(1+(b*x+a)^2)+4/9*arcsinh(b*x+a)*(1+(b*x+a)^2)^(
1/2)-14/27*b*x-14/27*a-2/9*arcsinh(b*x+a)*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/2
7*(1+(b*x+a)^2)*(b*x+a)+a^2*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(
b*x+a)^2)^(1/2)+2*b*x+2*a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50995, size = 352, normalized size = 1.67

$$\frac{4\,b^3x^3-15\,ab^2x^2+6\left(11\,a^2-4\right)bx+9\left(2\,b^3x^3+2\,a^3-3\,a\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-6\left(2\,b^2x^2-5\,ab\right.}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*b^3*x^3 - 15*a*b^2*x^2 + 6*(11*a^2 - 4)*b*x + 9*(2*b^3*x^3 + 2*a^3
- 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^2*x^2 -
5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^3
```

Sympy [A] time = 1.73613, size = 243, normalized size = 1.15

$$\begin{cases} \frac{a^3 \operatorname{asinh}^2(a+bx)}{3b^3} + \frac{11a^2x}{9b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^3} - \frac{5ax^2}{18b} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^2} - \frac{a\operatorname{asinh}^2(a+bx)}{2b^3} + \frac{x^3\operatorname{asinh}^2(a+b \\ \frac{x^3\operatorname{asinh}^2(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**2/(3*b**3) + 11*a**2*x/(9*b**2) - 11*a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3) - 5*a*x**2/(18*
b) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**2) - a
*asinh(a + b*x)**2/(2*b**3) + x**3*asinh(a + b*x)**2/3 + 2*x**3/27 - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b) - 4*x/(9*b**2) +
 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3), Ne(b, 0)),
 (x**3*asinh(a)**2/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^2, x)
```

3.69 $\int x \sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=126

$$-\frac{a^2\sinh^{-1}(a+bx)^2}{2b^2}+\frac{(a+bx)^2}{4b^2}-\frac{\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)}{2b^2}+\frac{\sinh^{-1}(a+bx)^2}{4b^2}+\frac{2a\sqrt{(a+bx)^2+1}\,\text{sinl}}{b^2}$$

```
[Out] (-2*a*x)/b + (a + b*x)^2/(4*b^2) + (2*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b
*x])/b^2 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b^2) + Arc
Sinh[a + b*x]^2/(4*b^2) - (a^2*ArcSinh[a + b*x]^2)/(2*b^2) + (x^2*ArcSinh[a
 + b*x]^2)/2
```

Rubi [A] time = 0.236425, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5865, 5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{a^2\sinh^{-1}(a+bx)^2}{2b^2}+\frac{(a+bx)^2}{4b^2}-\frac{\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)}{2b^2}+\frac{\sinh^{-1}(a+bx)^2}{4b^2}+\frac{2a\sqrt{(a+bx)^2+1}\,\text{sinl}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x]^2,x]
```


```
[Out] (-2*a*x)/b + (a + b*x)^2/(4*b^2) + (2*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b
*x])/b^2 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b^2) + Arc
Sinh[a + b*x]^2/(4*b^2) - (a^2*ArcSinh[a + b*x]^2)/(2*b^2) + (x^2*ArcSinh[a
 + b*x]^2)/2
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)^2\,dx = \frac{\operatorname{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\operatorname{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\operatorname{Subst}\left(\int\left(\frac{a^2\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}-\frac{2ax\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}+\frac{x^2\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}\right)dx,x,a+bx$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\frac{\operatorname{Subst}\left(\int\frac{x^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b^2}+\frac{(2a)\operatorname{Subst}\left(\int\frac{x\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+\right.}{b^2}$$

$$= \frac{2a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b^2}-\frac{a^2\sinh^{-1}(a+bx)}{2b^2}$$

$$= -\frac{2ax}{b}+\frac{(a+bx)^2}{4b^2}+\frac{2a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx}{2b^2}$$

Mathematica [A] time = 0.0795007, size = 79, normalized size = 0.63

$$\frac{\left(-2a^2+2b^2x^2+1\right)\sinh^{-1}(a+bx)^2+2(3a-bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)+bx(bx-6a)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-6*a + b*x) + 2*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + (1 - 2*a^2 + 2*b^2*x^2)*ArcSinh[a + b*x]^2)/(4*b^2)
```

Maple [A] time = 0.04, size = 113, normalized size = 0.9

$$\frac{1}{b^2}\left(\frac{(\operatorname{Arcsinh}(bx+a))^2\left(1+(bx+a)^2\right)}{2}-\frac{\operatorname{Arcsinh}(bx+a)(bx+a)}{2}\sqrt{1+(bx+a)^2}-\frac{(\operatorname{Arcsinh}(bx+a))^2}{4}+\frac{(bx+a)}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)-1/2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^2+1/4*(b*x+a)^2+1/4-a*(arcsinh(b*x+a)^2*(b
*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54278, size = 277, normalized size = 2.2

$$\frac{b^2x^2-6\,abx+\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)\log\left(bx\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2 - 6*a*b*x + (2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^2 - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*
log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^2
```

Sympy [A] time = 0.785134, size = 138, normalized size = 1.1

$$\begin{cases} -\frac{a^2\operatorname{asinh}^2(a+bx)}{2b^2}-\frac{3ax}{2b}+\frac{3a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b^2}+\frac{x^2\operatorname{asinh}^2(a+bx)}{2}+\frac{x^2}{4}-\frac{x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b}+\frac{\operatorname{asinh}^2(a+bx)}{4b^2} \\ \frac{x^2\operatorname{asinh}^2(a)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**2/(2*b**2) - 3*a*x/(2*b) + 3*a*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b**2) + x**2*asinh(a + b*x)**2/
2 + x**2/4 - x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b) +
asinh(a + b*x)**2/(4*b**2), Ne(b, 0)), (x**2*asinh(a)**2/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^2, x)
```

3.70 $\int \sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=45

$$\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}-\frac{2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b}+2x$$

```
[Out] 2*x - (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b + ((a + b*x)*ArcSinh[a +
 b*x]^2)/b
```

Rubi [A] time = 0.0506684, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5653, 5717, 8}

$$\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}-\frac{2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b}+2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b + ((a + b*x)*ArcSinh[a +
 b*x]^2)/b
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b} - \frac{2\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b} + \frac{2\,\text{Subst}(\int 1\,dx, x, a+bx)}{b}$$
$$= 2x - \frac{2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}$$

Mathematica [A] time = 0.0217076, size = 47, normalized size = 1.04

$$\frac{2(a+bx)+(a+bx)\sinh^{-1}(a+bx)^2-2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) - 2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + (a + b*x)*ArcSinh
[a + b*x]^2)/b
```

Maple [A] time = 0.027, size = 46, normalized size = 1.

$$\frac{1}{b}\left((\text{Arcsinh}\,(bx+a))^2\,(bx+a)-2\,\text{Arcsinh}\,(bx+a)\sqrt{1+(bx+a)^2}+2\,bx+2\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*
a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42553, size = 217, normalized size = 4.82

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,bx-2\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 2*b*x - 2*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)))/b
```

Sympy [A] time = 0.262953, size = 63, normalized size = 1.4

$$\begin{cases} \frac{a \operatorname{asinh}^{2}{\left (a + b x \right)}}{b} + x \operatorname{asinh}^{2}{\left (a + b x \right)} + 2 x - \frac{2 \sqrt{a^{2} + 2 a b x + b^{2} x^{2} + 1} \operatorname{asinh}{\left (a + b x \right)}}{b} & \text{for}\: b \neq 0 \\ x \operatorname{asinh}^{2}{\left (a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**2/b + x*asinh(a + b*x)**2 + 2*x - 2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/b, Ne(b, 0)), (x*asinh(a)**2, Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}\left(bx + a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2, x)
```

3.71 $\int \frac{\sinh^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=205

$$2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-$$

```
[Out] -ArcSinh[a + b*x]^3/3 + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 +
a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rubi [A] time = 0.350385, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5865, 5799, 5561, 2190, 2531, 2282, 6589}

$$2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -ArcSinh[a + b*x]^3/3 + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 +
a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\right.}{b}$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh}}{a+}\right.$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh}}{a+}\right.$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh}}{a+}\right.$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh}}{a+}\right.$$

Mathematica [A] time = 0.0298745, size = 251, normalized size = 1.22

$$2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{b\left(-\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)}\right)+2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{b\left(\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)}\right)-2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(}}{a-\sqrt{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -ArcSinh[a + b*x]^3/3 + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x]/((-(a
/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x]/
((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 2*ArcSinh[a + b*x]*PolyLog[2, -(E^ArcSinh
[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 2*ArcSinh[a + b*x]*PolyLog[2,
-(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 2*PolyLog[3, E^ArcS
inh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sq
rt[1 + a^2])]
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int \frac{(\text{Arcsinh}\,(bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x,x)
```

```
[Out] int(arcsinh(b*x+a)^2/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\,(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

3.72 $\int \frac{\sinh^{-1}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=178

$$-\frac{2b\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}-\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\sinh^{-1}(a+bx}{\sqrt{}}$$

```
[Out] -(ArcSinh[a + b*x]^2/x) - (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/
(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (2*b*PolyLog[2, E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*PolyLog[2, E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2]
```

Rubi [A] time = 0.382659, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5801, 5831, 3322, 2264, 2190, 2279, 2391}

$$-\frac{2b\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}-\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\sinh^{-1}(a+bx}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^2/x) - (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/
(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (2*b*PolyLog[2, E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*PolyLog[2, E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + 2\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + 2\,\text{Subst}\left(\int \frac{x}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + 4\,\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + \frac{4\,\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{\sqrt{1+a^2}} - \frac{4\,\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}+\frac{2\sqrt{1+a^2}}{b}+\cdots}\right.}{\sqrt{1\cdots}}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(\cdots}}{a+\sqrt{1+\cdots}}\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(\cdots}}{a+\sqrt{1+\cdots}}\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(\cdots}}{a+\sqrt{1+\cdots}}\right.}{\sqrt{1+a^2}}$$

Mathematica [A] time = 0.112402, size = 178, normalized size = 1.

$$\frac{-2bx\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2bx\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\sinh^{-1}(a+bx)\left(\sqrt{a^2+1}\sinh^{-1}(a+bx)+2bx\left(\log\left(\frac{\sqrt{a^2+\cdots}}{\cdots}\right.\right.\right.}{\sqrt{a^2+1}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] (-(ArcSinh[a + b*x]*(Sqrt[1 + a^2]*ArcSinh[a + b*x] + 2*b*x*(-Log[(a + Sqrt
[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + Log[(-a + Sqrt[1 + a
^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])]))) - 2*b*x*PolyLog[2, E^Arc
Sinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*b*x*PolyLog[2, E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x)
```

Maple [A] time = 0.136, size = 217, normalized size = 1.2

$$-\frac{(\text{Arcsinh}\,(bx+a))^2}{x}+2\,\frac{b\text{Arcsinh}\,(bx+a)}{\sqrt{a^2+1}}\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-2\,\frac{b\text{Arcsinh}\,(bx+a)}{\sqrt{a^2+1}}\ln\left(\frac{\sqrt{a^2+1}+b\cdots}{-a\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^2,x)
```

```
[Out] -arcsinh(b*x+a)^2/x+2*b*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b*x-
(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*b*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln
(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2*b/(a^2+1)^(1
/2)*dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*b/(a
^2+1)^(1/2)*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^2, x)
```

3.73 $\int \frac{\sinh^{-1}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=235

$$\frac{ab^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{ab^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{3/2}}+\frac{b^2\log(x)}{a^2+1}+\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-$$

```
[Out] -((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x)) - ArcSinh[a + b
*x]^2/(2*x^2) + (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqr
t[1 + a^2])])/(1 + a^2)^(3/2) - (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (b^2*Log[x])/(1 + a^2) + (a
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) -
(a*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2)
```

Rubi [A] time = 0.485757, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5865, 5801, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{ab^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{ab^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{3/2}}+\frac{b^2\log(x)}{a^2+1}+\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^3,x]
```


```
[Out] -((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x)) - ArcSinh[a + b
*x]^2/(2*x^2) + (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqr
t[1 + a^2])])/(1 + a^2)^(3/2) - (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (b^2*Log[x])/(1 + a^2) + (a
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) -
(a*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
```

```
[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a\right.}{1+a^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} - \frac{(2ab)\,\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh\right.}{1+a^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{1+a^2} - \frac{(2ab)\,\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}}\right.}{(1+}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx}}{a-\sqrt{1+a^2}}\right.}{(1+a^2)^{3/2}}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx}}{a-\sqrt{1+a^2}}\right.}{(1+a^2)^{3/2}}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx}}{a-\sqrt{1+a^2}}\right.}{(1+a^2)^{3/2}}$$

Mathematica [A] time = 0.123856, size = 279, normalized size = 1.19

$$-\frac{-2ab^2x^2\text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + 2ab^2x^2\text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right) - 2\sqrt{a^2+1}b^2x^2\log(x) + 2ab^2x^2\sinh^{-1}(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -(2*Sqrt[1 + a^2]*b*x*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + Sqrt[1 + a^2
]*ArcSinh[a + b*x]^2 + a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^2 + 2*a*b^2*x^2*A
rcSinh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 +
a^2])] - 2*a*b^2*x^2*ArcSinh[a + b*x]*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a
 + b*x])/(-a + Sqrt[1 + a^2])] - 2*Sqrt[1 + a^2]*b^2*x^2*Log[x] - 2*a*b^2*x
^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*a*b^2*x^2*PolyLog
```

```
[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)*x^2)
```

Maple [A] time = 0.224, size = 384, normalized size = 1.6

$$\frac{b^2 \mathrm{Arcsinh}\,(bx+a)}{a^2+1} - \frac{(\mathrm{Arcsinh}\,(bx+a))^2 a^2}{(2a^2+2)\,x^2} - \frac{b\mathrm{Arcsinh}\,(bx+a)}{(a^2+1)\,x}\sqrt{1+(bx+a)^2} - \frac{(\mathrm{Arcsinh}\,(bx+a))^2}{(2a^2+2)\,x^2} - b^2 a\mathrm{Arcsinh}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^3,x)
```

```
[Out] b^2*arcsinh(b*x+a)/(a^2+1)-1/2*arcsinh(b*x+a)^2/(a^2+1)/x^2*a^2-b*arcsinh(b
*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x-1/2*arcsinh(b*x+a)^2/(a^2+1)/x^2-b^2/(a
^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+
(a^2+1)^(1/2)))+b^2/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1
+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-b^2/(a^2+1)^(3/2)*dilog(((a^2+1)^(1/
2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a+b^2/(a^2+1)^(3/2)*dilog(((
a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))*a-2*b^2/(a^2+1)*l
n(b*x+a+(1+(b*x+a)^2)^(1/2))+b^2/(a^2+1)*ln((b*x+a+(1+(b*x+a)^2)^(1/2))^2-2
*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}\,(bx+a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{asinh}^2\,(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^3, x)
```

3.74 $\int \frac{\sinh^{-1}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=478

$$\frac{b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}} - \frac{a^2 b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}} - \frac{b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{3\left(a^2+1\right)^{3/2}} + \frac{a^2 b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{5/2}}$$

```
[Out] -b^2/(3*(1 + a^2)*x) - (b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*(1 + a
^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)^2*x) -
 ArcSinh[a + b*x]^2/(3*x^3) - (a^2*b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*ArcSinh[a + b*x]*Log[1
 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*
ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)
^(5/2) - (b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(3*(1 + a^2)^(3/2)) - (a*b^3*Log[x])/(1 + a^2)^2 - (a^2*b^3*PolyLog[2,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*PolyLog[2,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*Po
lyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(5/2) - (b^3*Po
lyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2))
```

Rubi [A] time = 1.57321, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {5865, 5801, 5831, 3325, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31, 6741, 12, 6742, 32}

$$\frac{b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}} - \frac{a^2 b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}} - \frac{b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{3\left(a^2+1\right)^{3/2}} + \frac{a^2 b^3 \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] -b^2/(3*(1 + a^2)*x) - (b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*(1 + a
^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)^2*x) -
 ArcSinh[a + b*x]^2/(3*x^3) - (a^2*b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*ArcSinh[a + b*x]*Log[1
 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*
ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)
^(5/2) - (b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(3*(1 + a^2)^(3/2)) - (a*b^3*Log[x])/(1 + a^2)^2 - (a^2*b^3*PolyLog[2,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*PolyLog[2,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*Po
lyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(5/2) - (b^3*Po
lyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3325

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> -Simp[(b*(c + d*x)^m*Cos[e + f*x]*(a + b*Sin[e + f*x])^(n + 1
))/(f*(n + 1)*(a^2 - b^2)), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a +
b*Sin[e + f*x])^(n + 1), x], x] - Dist[(b*(n + 2))/((n + 1)*(a^2 - b^2)), I
nt[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[(b*
d*m)/(f*(n + 1)*(a^2 - b^2)), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{2}{3}\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{2}{3}\,\text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} - \frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(\right.}{3\left(1+a^2\right)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2 x} - \frac{\sinh^{-1}(a+bx)^2}{3x^3} -$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2 x} - \text{sinl}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2 x} - \text{sinl}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2 x} - \text{sinl}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2 x} - \text{sinh}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2 x} - \text{sinh}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2 x} - \text{sinh}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2 x} - \text{sinh}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2 x} - \text{sinh}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2 x} - \text{sinh}$$

Mathematica [C] time = 10.3128, size = 1830, normalized size = 3.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] (-((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x^2)) - ArcSinh[a
+ b*x]^2/x^3 - (b^2*(1 + a^2 - 3*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]))
/((1 + a^2)^2*x) + (I*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSinh[a + b*x]/2])/Sqrt
[1 + a^2]])/(1 + a^2)^(5/2) - ((2*I)*a^2*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSin
h[a + b*x]/2])/Sqrt[1 + a^2]])/(1 + a^2)^(5/2) - (3*a*b^3*Log[-((b*x)/a)])/
(1 + a^2)^2 + (b^3*(-2*ArcCos[I*a]*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSin
h[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)*ArcSinh[a + b*x])*ArcTanh[((I
 + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (ArcCos[I*a]
+ (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a
^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1
 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcSinh[a + b*x]/2)*Sqrt[b*x])] +
(ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4
])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])
/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(ArcSinh[a + b*x]/2))/(Sqrt[2]*S
qrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a + I*(-1 + Sqrt[-1 - a^2]))*
(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(I + a - Sqrt[-1 - a^2]*Cot[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a] - (2*I)*ArcTanh[((-I + a)*Co
t[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a - I*(1
 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a +
Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] + I*(PolyLog[2, -((((
-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a +
 b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))
] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2
*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])])))/(-1 - a^2)^(5/2) - (2*a^2*b^3*(-2*ArcCos[I*a]*ArcTanh[((
-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)
*ArcSinh[a + b*x])*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/S
qrt[-1 - a^2]] + (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)
*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcS
inh[a + b*x]/2)*Sqrt[b*x])] + (ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(
Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(A
rcSinh[a + b*x]/2))/(Sqrt[2]*Sqrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-
I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*
(a + I*(-1 + Sqrt[-1 - a^2]))*(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(
I + a - Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a
] - (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 -
 a^2]])*Log[((I + a)*(a - I*(1 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*
x])/4])] + I*(PolyLog[2, -((((-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^
2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi
+ (2*I)*ArcSinh[a + b*x])/4]))] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a
 + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1
- a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])])))/(-1 - a^2)^(5/2))/3
```

Maple [A] time = 0.39, size = 730, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^4,x)
```

```
[Out] -b^3/(a^2+1)^2*arcsinh(b*x+a)*a+a*b^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a
^2+1)^2/x-1/3/(a^2+1)^2/x^3*arcsinh(b*x+a)^2*a^4-1/3*b/(a^2+1)^2/x^2*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^2-1/3*b^2/(a^2+1)^2/x*a^2-2/3/(a^2+1)^2/x^3*
arcsinh(b*x+a)^2*a^2-1/3*b/(a^2+1)^2/x^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)
-1/3*b^2/(a^2+1)^2/x-1/3/(a^2+1)^2/x^3*arcsinh(b*x+a)^2+2*b^3/(a^2+1)^2*a*l
n(b*x+a+(1+(b*x+a)^2)^(1/2))-b^3/(a^2+1)^2*a*ln((b*x+a+(1+(b*x+a)^2)^(1/2))
^2-2*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-1)-1/3*b^3/(a^2+1)^(5/2)*arcsinh(b*x+a)*
ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/3*b^3/(a^2+
1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2
+1)^(1/2)))-1/3*b^3/(a^2+1)^(5/2)*dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1
/2))/(a+(a^2+1)^(1/2)))+1/3*b^3/(a^2+1)^(5/2)*dilog(((a^2+1)^(1/2)+b*x+(1+(
b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3*b^3/(a^2+1)^(5/2)*a^2*arcsinh(b*x+
a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3*b^3/(a
^2+1)^(5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(
-a+(a^2+1)^(1/2)))+2/3*b^3/(a^2+1)^(5/2)*a^2*dilog(((a^2+1)^(1/2)-b*x-(1+(b
*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3*b^3/(a^2+1)^(5/2)*a^2*dilog(((a^2+1)
^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^4, x)
```

3.75 $\int x^2 \sinh^{-1}(a+bx)^3\, dx$

Optimal. Leaf size=355

$$-\frac{6a^2\sqrt{(a+bx)^2+1}}{b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}+\frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}-\frac{3a^2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b^3}+\frac{3a\sqrt{(}}{}$$

```
[Out] (14*Sqrt[1 + (a + b*x)^2])/(9*b^3) - (6*a^2*Sqrt[1 + (a + b*x)^2])/b^3 + (3
*a*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b^3) - (2*(1 + (a + b*x)^2)^(3/2))/(
27*b^3) - (3*a*ArcSinh[a + b*x])/(4*b^3) - (4*(a + b*x)*ArcSinh[a + b*x])/(
3*b^3) + (6*a^2*(a + b*x)*ArcSinh[a + b*x])/b^3 - (3*a*(a + b*x)^2*ArcSinh[
a + b*x])/(2*b^3) + (2*(a + b*x)^3*ArcSinh[a + b*x])/(9*b^3) + (2*Sqrt[1 +
(a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^3) - (3*a^2*Sqrt[1 + (a + b*x)^2]*Arc
Sinh[a + b*x]^2)/b^3 + (3*a*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
]^2)/(2*b^3) - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^
3) - (a*ArcSinh[a + b*x]^3)/(2*b^3) + (a^3*ArcSinh[a + b*x]^3)/(3*b^3) + (x
^3*ArcSinh[a + b*x]^3)/3
```

Rubi [A] time = 0.449703, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5865, 5801, 5831, 3317, 3296, 2638, 3311, 30, 2635, 8, 2633}

$$-\frac{6a^2\sqrt{(a+bx)^2+1}}{b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}+\frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}-\frac{3a^2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b^3}+\frac{3a\sqrt{(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] (14*Sqrt[1 + (a + b*x)^2])/(9*b^3) - (6*a^2*Sqrt[1 + (a + b*x)^2])/b^3 + (3
*a*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b^3) - (2*(1 + (a + b*x)^2)^(3/2))/(
27*b^3) - (3*a*ArcSinh[a + b*x])/(4*b^3) - (4*(a + b*x)*ArcSinh[a + b*x])/(
3*b^3) + (6*a^2*(a + b*x)*ArcSinh[a + b*x])/b^3 - (3*a*(a + b*x)^2*ArcSinh[
a + b*x])/(2*b^3) + (2*(a + b*x)^3*ArcSinh[a + b*x])/(9*b^3) + (2*Sqrt[1 +
(a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^3) - (3*a^2*Sqrt[1 + (a + b*x)^2]*Arc
Sinh[a + b*x]^2)/b^3 + (3*a*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
]^2)/(2*b^3) - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^
3) - (a*ArcSinh[a + b*x]^3)/(2*b^3) + (a^3*ArcSinh[a + b*x]^3)/(3*b^3) + (x
^3*ArcSinh[a + b*x]^3)/3
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x^2\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)^3\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx,x,a+bx\right)$$
$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3\,dx,x,\sinh^{-1}(a+bx)\right)$$
$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int\left(-\frac{a^3x^2}{b^3}+\frac{3a^2x^2\sinh(x)}{b^3}-\frac{3ax^2\sinh^2(x)}{b^3}+\frac{x^2\sinh^3(x)}{b^3}\right.\right.$$
$$= \frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)^3-\frac{\text{Subst}\left(\int x^2\sinh^3(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^3}+$$
$$= -\frac{3a(a+bx)^2\sinh^{-1}(a+bx)}{2b^3}+\frac{2(a+bx)^3\sinh^{-1}(a+bx)}{9b^3}-\frac{3a^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+}{b^3}$$
$$= \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}-\frac{3a(a+bx)^2\sinh^{-1}(a+bx)}{2b^3}+$$
$$= \frac{2\sqrt{1+(a+bx)^2}}{9b^3}-\frac{6a^2\sqrt{1+(a+bx)^2}}{b^3}+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}-\frac{2\left(1+(a+bx)^2\right)^{3/2}}{27b^3}-$$
$$= \frac{14\sqrt{1+(a+bx)^2}}{9b^3}-\frac{6a^2\sqrt{1+(a+bx)^2}}{b^3}+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}-\frac{2\left(1+(a+bx)^2\right)^{3/2}}{27b^3}$$

Mathematica [A] time = 0.1985, size = 175, normalized size = 0.49

$$\left(-575a^2+65abx-8b^2x^2+160\right)\sqrt{a^2+2abx+b^2x^2+1}+18\left(2a^3-3a+2b^3x^3\right)\sinh^{-1}(a+bx)^3-18\sqrt{a^2+2abx+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] ((160 - 575*a^2 + 65*a*b*x - 8*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] +
 3*(170*a^3 + 132*a^2*b*x + 8*b*x*(-6 + b^2*x^2) - 15*a*(5 + 2*b^2*x^2))*Ar
cSinh[a + b*x] - 18*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4 + 11*a^2 - 5*a*b*
x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 18*(-3*a + 2*a^3 + 2*b^3*x^3)*ArcSinh[a
 + b*x]^3)/(108*b^3)
```

Maple [A] time = 0.06, size = 326, normalized size = 0.9

$$\frac{1}{b^3}\left(-\frac{a}{4}\left(4\,(\text{Arcsinh}\,(bx+a))^3\,(bx+a)^2-6\,(\text{Arcsinh}\,(bx+a))^2\sqrt{1+(bx+a)^2}\,(bx+a)+2\,(\text{Arcsinh}\,(bx+a))^3+6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(-1/4*a*(4*arcsinh(b*x+a)^3*(b*x+a)^2-6*arcsinh(b*x+a)^2*(1+(b*x+a)^2
)^(1/2)*(b*x+a)+2*arcsinh(b*x+a)^3+6*arcsinh(b*x+a)*(b*x+a)^2-3*(b*x+a)*(1+
(b*x+a)^2)^(1/2)+3*arcsinh(b*x+a))-1/3*arcsinh(b*x+a)^3*(b*x+a)+1/3*arcsinh
(b*x+a)^3*(b*x+a)*(1+(b*x+a)^2)+2/3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-14
/9*(b*x+a)*arcsinh(b*x+a)+40/27*(1+(b*x+a)^2)^(1/2)-1/3*arcsinh(b*x+a)^2*(b
```

```
*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/9*(b*x+a)*(1+(b*x+a)^2)*arcsinh(b*x+a)-2/27*(
b*x+a)^2*(1+(b*x+a)^2)^(1/2)+a^2*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)
^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-6*(1+(b*x+a)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.55595, size = 556, normalized size = 1.57

$$18\left(2\,b^3x^3+2\,a^3-3\,a\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-18\left(2\,b^2x^2-5\,abx+11\,a^2-4\right)\sqrt{b^2x^2+2\,abx+a^2+1}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(18*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))^3 - 18*(2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(8*b^3*x^
3 - 30*a*b^2*x^2 + 170*a^3 + 12*(11*a^2 - 4)*b*x - 75*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (8*b^2*x^2 - 65*a*b*x + 575*a^2 - 160)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A] time = 3.91407, size = 432, normalized size = 1.22

$$\begin{cases}\frac{a^3\operatorname{asinh}^3(a+bx)}{3b^3}+\frac{85a^3\operatorname{asinh}(a+bx)}{18b^3}+\frac{11a^2x\operatorname{asinh}(a+bx)}{3b^2}-\frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{6b^3}-\frac{575a^2\sqrt{a^2+2abx+b^2x^2+1}}{108b^3}-\frac{5ax^2\operatorname{asinh}(a+bx)}{6b} \\ \frac{x^3\operatorname{asinh}^3(a)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**3/(3*b**3) + 85*a**3*asinh(a + b*x)/(18*b**
3) + 11*a**2*x*asinh(a + b*x)/(3*b**2) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)*asinh(a + b*x)**2/(6*b**3) - 575*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(108*b**3) - 5*a*x**2*asinh(a + b*x)/(6*b) + 5*a*x*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(6*b**2) + 65*a*x*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)/(108*b**2) - a*asinh(a + b*x)**3/(2*b**3) - 25*a*asin
h(a + b*x)/(12*b**3) + x**3*asinh(a + b*x)**3/3 + 2*x**3*asinh(a + b*x)/9 -
 x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b) - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b) - 4*x*asinh(a + b*x)/(3*b**2)
+ 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b**3) + 40*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b**3), Ne(b, 0)), (x**3*asinh(a)**3/
```

```
3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^3, x)
```

3.76 $\int x \sinh^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=203

$$-\frac{a^2 \sinh^{-1}(a+bx)^3}{2b^2} - \frac{3(a+bx)\sqrt{(a+bx)^2+1}}{8b^2} + \frac{6a\sqrt{(a+bx)^2+1}}{b^2} + \frac{\sinh^{-1}(a+bx)^3}{4b^2} - \frac{3(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}}{4b^2}$$

```
[Out] (6*a*Sqrt[1 + (a + b*x)^2])/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(8*b^
2) + (3*ArcSinh[a + b*x])/(8*b^2) - (6*a*(a + b*x)*ArcSinh[a + b*x])/b^2 +
(3*(a + b*x)^2*ArcSinh[a + b*x])/(4*b^2) + (3*a*Sqrt[1 + (a + b*x)^2]*ArcSi
nh[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)
/(4*b^2) + ArcSinh[a + b*x]^3/(4*b^2) - (a^2*ArcSinh[a + b*x]^3)/(2*b^2) +
(x^2*ArcSinh[a + b*x]^3)/2
```

Rubi [A] time = 0.30306, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5865, 5801, 5831, 3317, 3296, 2638, 3311, 30, 2635, 8}

$$-\frac{a^2 \sinh^{-1}(a+bx)^3}{2b^2} - \frac{3(a+bx)\sqrt{(a+bx)^2+1}}{8b^2} + \frac{6a\sqrt{(a+bx)^2+1}}{b^2} + \frac{\sinh^{-1}(a+bx)^3}{4b^2} - \frac{3(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (6*a*Sqrt[1 + (a + b*x)^2])/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(8*b^
2) + (3*ArcSinh[a + b*x])/(8*b^2) - (6*a*(a + b*x)*ArcSinh[a + b*x])/b^2 +
(3*(a + b*x)^2*ArcSinh[a + b*x])/(4*b^2) + (3*a*Sqrt[1 + (a + b*x)^2]*ArcSi
nh[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)
/(4*b^2) + ArcSinh[a + b*x]^3/(4*b^2) - (a^2*ArcSinh[a + b*x]^3)/(2*b^2) +
(x^2*ArcSinh[a + b*x]^3)/2
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^3\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)^2}{\sqrt{1+x^2}}dx,x,a+bx\right)$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2dx,x,\sinh^{-1}(a+bx)\right)$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int\left(\frac{a^2x^2}{b^2}-\frac{2ax^2\sinh(x)}{b^2}+\frac{x^2\sinh^2(x)}{b^2}\right)dx,x,\sinh^{-1}(a+bx\right)$$
$$= -\frac{a^2\sinh^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)^3-\frac{3\,\text{Subst}\left(\int x^2\sinh^2(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{2b^2}+$$
$$= \frac{3(a+bx)^2\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{b^2}-\frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\text{sinh}}{4b^2}$$
$$= -\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}-\frac{6a(a+bx)\sinh^{-1}(a+bx)}{b^2}+\frac{3(a+bx)^2\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{1+}}{}$$
$$= \frac{6a\sqrt{1+(a+bx)^2}}{b^2}-\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}+\frac{3\sinh^{-1}(a+bx)}{8b^2}-\frac{6a(a+bx)\sinh^{-1}(a+bx)}{b^2}$$

Mathematica [A] time = 0.126947, size = 129, normalized size = 0.64

$$\frac{3(15a-bx)\sqrt{a^2+2abx+b^2x^2+1}+\left(-4a^2+4b^2x^2+2\right)\sinh^{-1}(a+bx)^3+6(3a-bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (3*(15*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3 - 42*a^2 - 36*a*b*x
+ 6*b^2*x^2)*ArcSinh[a + b*x] + 6*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*
x^2]*ArcSinh[a + b*x]^2 + (2 - 4*a^2 + 4*b^2*x^2)*ArcSinh[a + b*x]^3)/(8*b^
2)
```

Maple [A] time = 0.042, size = 169, normalized size = 0.8

$$\frac{1}{b^2}\left(\frac{(\text{Arcsinh}\,(bx+a))^3\left(1+(bx+a)^2\right)}{2}-\frac{3\,(\text{Arcsinh}\,(bx+a))^2\,(bx+a)}{4}\sqrt{1+(bx+a)^2}-\frac{(\text{Arcsinh}\,(bx+a))^3}{4}+\frac{3\,\text{Arcs}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)^3*(1+(b*x+a)^2)-3/4*arcsinh(b*x+a)^2*(1+(b*x+a)^2
)^(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^3+3/4*arcsinh(b*x+a)*(1+(b*x+a)^2)-3/8*(
b*x+a)*(1+(b*x+a)^2)^(1/2)-3/8*arcsinh(b*x+a)-a*(arcsinh(b*x+a)^3*(b*x+a)-3
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-6*(1+(b*x+a)
^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.46764, size = 444, normalized size = 2.19

$$\frac{2\left(2b^2x^2-2a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)^3-6\sqrt{b^2x^2+2abx+a^2+1}(bx-3a)\log\left(bx+a+\sqrt{b^2x^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^3 - 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(2*b^2*x^2 - 12*a*b*x - 14*a^2 + 1)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(b*x - 15*a))/b^2
```

Sympy [A] time = 1.65634, size = 248, normalized size = 1.22

$$\begin{cases} -\frac{a^2\operatorname{asinh}^3(a+bx)}{2b^2}-\frac{21a^2\operatorname{asinh}(a+bx)}{4b^2}-\frac{9ax\operatorname{asinh}(a+bx)}{2b}+\frac{9a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b^2}+\frac{45a\sqrt{a^2+2abx+b^2x^2+1}}{8b^2}+\frac{x^2\operatorname{asinh}^3(a+bx)}{2} \\ \frac{x^2\operatorname{asinh}^3(a)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**3/(2*b**2) - 21*a**2*asinh(a + b*x)/(4*b**
2) - 9*a*x*asinh(a + b*x)/(2*b) + 9*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*
asinh(a + b*x)**2/(4*b**2) + 45*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(8*b
**2) + x**2*asinh(a + b*x)**3/2 + 3*x**2*asinh(a + b*x)/4 - 3*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) - 3*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(8*b) + asinh(a + b*x)**3/(4*b**2) + 3*asinh(a + b*x)/(8*
b**2), Ne(b, 0)), (x**2*asinh(a)**3/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{arsinh}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^3, x)
```

3.77 $\int \sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=78

$$-\frac{6\sqrt{(a+bx)^2+1}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b}-\frac{3\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b}+\frac{6(a+bx)\sinh^{-1}(a+bx)}{b}$$

```
[Out] (-6*Sqrt[1 + (a + b*x)^2])/b + (6*(a + b*x)*ArcSinh[a + b*x])/b - (3*Sqrt[1
 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/b + ((a + b*x)*ArcSinh[a + b*x]^3)/b
```

Rubi [A] time = 0.0738137, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5653, 5717, 261}

$$-\frac{6\sqrt{(a+bx)^2+1}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b}-\frac{3\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b}+\frac{6(a+bx)\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + (a + b*x)^2])/b + (6*(a + b*x)*ArcSinh[a + b*x])/b - (3*Sqrt[1
 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/b + ((a + b*x)*ArcSinh[a + b*x]^3)/b
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} - \frac{3\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{3\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} + \frac{6\,\text{Subst}\left(\int \sinh^{-1}(x)\,dx, x, \ldots\right.}{b}$$
$$= \frac{6(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} - \ldots$$
$$= -\frac{6\sqrt{1+(a+bx)^2}}{b} + \frac{6(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\ldots}{}$$

Mathematica [A] time = 0.0288498, size = 70, normalized size = 0.9

$$\frac{-6\sqrt{(a+bx)^2+1} + (a+bx)\sinh^{-1}(a+bx)^3 - 3\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2 + 6(a+bx)\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + (a + b*x)^2] + 6*(a + b*x)*ArcSinh[a + b*x] - 3*Sqrt[1 + (a +
b*x)^2]*ArcSinh[a + b*x]^2 + (a + b*x)*ArcSinh[a + b*x]^3)/b
```

Maple [A] time = 0.028, size = 67, normalized size = 0.9

$$\frac{1}{b}\left((\text{Arcsinh}\,(bx+a))^3\,(bx+a) - 3\,(\text{Arcsinh}\,(bx+a))^2\sqrt{1+(bx+a)^2} + 6\,(bx+a)\,\text{Arcsinh}\,(bx+a) - 6\sqrt{1+(bx+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x
+a)*arcsinh(b*x+a)-6*(1+(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.38706, size = 346, normalized size = 4.44

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3\,\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+6\,(b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 3*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 6*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [A] time = 0.727157, size = 109, normalized size = 1.4

$$\begin{cases}\frac{a\operatorname{asinh}^{3}(a+bx)}{b}+\frac{6a\operatorname{asinh}(a+bx)}{b}+x\operatorname{asinh}^{3}(a+bx)+6x\operatorname{asinh}(a+bx)-\frac{3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^{2}(a+bx)}{b}-\frac{6\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{f} \\ x\operatorname{asinh}^{3}(a) & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**3/b + 6*a*asinh(a + b*x)/b + x*asinh(a + b*x)*
*3 + 6*x*asinh(a + b*x) - 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/b - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a
)**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3, x)
```

3.78 $\int \frac{\sinh^{-1}(a+bx)^3}{x}\,dx$

Optimal. Leaf size=275

$$3\sinh^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+3\sinh^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-6\sinh^{-1}(a+bx)\mathrm{PolyLo}$$

```
[Out] -ArcSinh[a + b*x]^4/4 + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1
+ a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 +
 a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^
2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])
] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4, E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rubi [A] time = 0.398384, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5799, 5561, 2190, 2531, 6609, 2282, 6589}

$$3\sinh^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+3\sinh^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-6\sinh^{-1}(a+bx)\mathrm{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -ArcSinh[a + b*x]^4/4 + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1
+ a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 +
 a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^
2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])
] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4, E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \frac{\text{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\right.}{b}$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}}}{a+}\right.$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}}}{a+}\right.$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}}}{a+}\right.$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}}}{a+}\right.$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}}}{a+}\right.$$

Mathematica [A] time = 0.0328524, size = 346, normalized size = 1.26

$$3\sinh^{-1}(a+bx)^2\text{PolyLog}\left(2, -\frac{e^{\sinh^{-1}(a+bx)}}{b\left(-\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)}\right) + 3\sinh^{-1}(a+bx)^2\text{PolyLog}\left(2, -\frac{e^{\sinh^{-1}(a+bx)}}{b\left(\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)}\right) - 6\sinh^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -ArcSinh[a + b*x]^4/4 + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x]/((-(a
/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x]/
((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 3*ArcSinh[a + b*x]^2*PolyLog[2, -(E^ArcSi
nh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 3*ArcSinh[a + b*x]^2*PolyLog
[2, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 6*ArcSinh[a + b
*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] - 6*Ar
cSinh[a + b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*
b))] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4,
E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \frac{(\text{Arcsinh}\,(bx+a))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\left(bx+a\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}^3\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\left(bx+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

3.79 $\int \frac{\sinh^{-1}(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=268

$$-\frac{6b\sinh^{-1}(a+bx)\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\sinh^{-1}(a+bx)\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}+\frac{6b\mathrm{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+b}}{a-\sqrt{a^2+1}}\right.}{\sqrt{a^2+1}}$$

```
[Out] -(ArcSinh[a + b*x]^3/x) - (3*b*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x
]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (3*b*ArcSinh[a + b*x]^2*Log[1 - E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (6*b*ArcSinh[a + b*x]
*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (6*b*A
rcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1
+ a^2] + (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 +
a^2] - (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^
2]
```

Rubi [A] time = 0.580218, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5865, 5801, 5831, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{6b\sinh^{-1}(a+bx)\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\sinh^{-1}(a+bx)\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}+\frac{6b\mathrm{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+b}}{a-\sqrt{a^2+1}}\right.}{\sqrt{a^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^3/x) - (3*b*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x
]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (3*b*ArcSinh[a + b*x]^2*Log[1 - E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (6*b*ArcSinh[a + b*x]
*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (6*b*A
rcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1
+ a^2] + (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 +
a^2] - (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^
2]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
```

```
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 3\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 3\,\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 6\,\text{Subst}\left(\int \frac{e^x x^2}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + \frac{6\,\text{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{\sqrt{1+a^2}} - \frac{6\,\text{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}+\frac{2\sqrt{1+}}{b}}\right.}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{s}}{}\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{s}}{}\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{s}}{}\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{s}}{}\right.}{\sqrt{1+a^2}}$$

Mathematica [A] time = 0.112068, size = 259, normalized size = 0.97

$$-\frac{6bx\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) - 6bx\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right) - 6bx\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a}}{a-\sqrt{a^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -((Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 - 3*b*x*ArcSinh[a + b*x]^2*Log[(a + Sqr
t[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + 3*b*x*ArcSinh[a + b
*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] +
 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]
- 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
 - 6*b*x*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b*x*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \frac{(\text{Arcsinh}(bx+a))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x^2, x)
```

3.80 $\int \frac{\sinh^{-1}(a+bx)^3}{x^3}\,dx$

Optimal. Leaf size=514

$$\frac{3ab^2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{3ab^2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{3/2}}+\frac{3b^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}}}{a-\sqrt{}}\right)}{a^2+1}$$

```
[Out] (-3*b^2*ArcSinh[a + b*x]^2)/(2*(1 + a^2)) - (3*b*Sqrt[1 + (a + b*x)^2]*ArcS
inh[a + b*x]^2)/(2*(1 + a^2)*x) - ArcSinh[a + b*x]^3/(2*x^2) + (3*b^2*ArcSi
nh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3
*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/
(2*(1 + a^2)^(3/2)) + (3*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)) + (3*b^2*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3*a*b^2*ArcSinh[a + b*
x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (3
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b
^2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1
+ a^2)^(3/2) - (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])
/(1 + a^2)^(3/2) + (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(1 + a^2)^(3/2)
```

Rubi [A] time = 0.883413, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {5865, 5801, 5831, 3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{3ab^2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{3ab^2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{3/2}}+\frac{3b^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}}}{a-\sqrt{}}\right)}{a^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*b^2*ArcSinh[a + b*x]^2)/(2*(1 + a^2)) - (3*b*Sqrt[1 + (a + b*x)^2]*ArcS
inh[a + b*x]^2)/(2*(1 + a^2)*x) - ArcSinh[a + b*x]^3/(2*x^2) + (3*b^2*ArcSi
nh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3
*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/
(2*(1 + a^2)^(3/2)) + (3*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)) + (3*b^2*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3*a*b^2*ArcSinh[a + b*
x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (3
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b
^2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1
+ a^2)^(3/2) - (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])
/(1 + a^2)^(3/2) + (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(1 + a^2)^(3/2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(\right.}{1+a^2}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{(3b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2\sinh^{-1}(a+}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2\sinh^{-1}(a+}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2\sinh^{-1}(a+}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2\sinh^{-1}(a+}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2\sinh^{-1}(a+}{}$$

Mathematica [A] time = 0.235749, size = 524, normalized size = 1.02

$$\frac{6b^2x^2\left(\sqrt{a^2+1}+a\sinh^{-1}(a+bx)\right)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+6b^2x^2\left(\sqrt{a^2+1}-a\sinh^{-1}(a+bx)\right)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+}}{\sqrt{a^2+1}+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]^2 - 3*Sqrt[1 + a^2]*b*x*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2 - Sqrt[1 + a^2]*ArcSinh[a + b*
x]^3 - a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSi
nh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2]
)] - 3*a*b^2*x^2*ArcSinh[a + b*x]^2*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a +
b*x])/(a + Sqrt[1 + a^2])] + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]*Log[(
-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] + 3*a*b^2*x^
2*ArcSinh[a + b*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sq
rt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] + a*ArcSinh[a + b*x])*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] - a*ArcSi
nh[a + b*x])*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])] - 6*a*b^2*x
^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*a*b^2*x^2*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)*x^2)
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int \frac{(\text{Arcsinh}(bx+a))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x^3,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}(bx+a)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}^3(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x^3, x)
```

3.81 $\int \frac{x^2}{\sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=60

$$\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}-\frac{a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}$$

```
[Out] -CoshIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*CoshIntegral[ArcSinh[a + b*x
]])/b^3 + CoshIntegral[3*ArcSinh[a + b*x]]/(4*b^3) - (a*SinhIntegral[2*ArcS
inh[a + b*x]])/b^3
```

Rubi [A] time = 0.527069, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5805, 6741, 12, 6742, 3301, 5448, 3298}

$$\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}-\frac{a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x],x]
```

```
[Out] -CoshIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*CoshIntegral[ArcSinh[a + b*x
]])/b^3 + CoshIntegral[3*ArcSinh[a + b*x]]/(4*b^3) - (a*SinhIntegral[2*ArcS
inh[a + b*x]])/b^3
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{b^2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2\cosh(x)}{x}-\frac{2a\cosh(x)\sinh(x)}{x}+\frac{\cosh(x)\sinh^2(x)}{x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx\right)}{b^3}$$

$$= \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} + \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - (2a)\,\text{Subst}\Big(\int$$

$$= \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{4b^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{sinh}\right.}{4b^3}$$

$$= -\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3} + \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} + \frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3} - \frac{a\text{Shi}\left(2\sinh^{-1}(a\right.}{b^3}$$

Mathematica [A] time = 0.135993, size = 44, normalized size = 0.73

$$\frac{\left(4a^2-1\right)\text{Chi}\left(\sinh^{-1}(a+bx)\right)+\text{Chi}\left(3\sinh^{-1}(a+bx)\right)-4a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x],x]
```

```
[Out] ((-1 + 4*a^2)*CoshIntegral[ArcSinh[a + b*x]] + CoshIntegral[3*ArcSinh[a + b
*x]] - 4*a*SinhIntegral[2*ArcSinh[a + b*x]])/(4*b^3)
```

Maple [A] time = 0.04, size = 49, normalized size = 0.8

$$\frac{1}{b^3}\left(-a\mathrm{Shi}\left(2\,\mathrm{Arcsinh}\left(bx+a\right)\right)-\frac{\mathrm{Chi}\left(\mathrm{Arcsinh}\left(bx+a\right)\right)}{4}+\frac{\mathrm{Chi}\left(3\,\mathrm{Arcsinh}\left(bx+a\right)\right)}{4}+a^2\mathrm{Chi}\left(\mathrm{Arcsinh}\left(bx+a\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a),x)
```

```
[Out] 1/b^3*(-a*Shi(2*arcsinh(b*x+a))-1/4*Chi(arcsinh(b*x+a))+1/4*Chi(3*arcsinh(b
*x+a))+a^2*Chi(arcsinh(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\mathrm{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\mathrm{arsinh}\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\mathrm{asinh}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a),x)
```

```
[Out] Integral(x**2/asinh(a + b*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

3.82 $\int \frac{x}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=30

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Rubi [A] time = 0.211343, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5865, 5805, 6741, 12, 6742, 3301, 5448, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{bx}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a\cosh(x)}{x}+\frac{\cosh(x)\sinh(x)}{x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= -\frac{a\,\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= -\frac{a\,\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b^2}\\
&= -\frac{a\,\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.050618, size = 30, normalized size = 1.

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\,\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Maple [A] time = 0.03, size = 27, normalized size = 0.9

$$\frac{1}{b^2}\left(\frac{\mathrm{Shi}\left(2\,\mathrm{Arcsinh}\left(bx+a\right)\right)}{2}-a\mathrm{Chi}\left(\mathrm{Arcsinh}\left(bx+a\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*Shi(2*arcsinh(b*x+a))-a*Chi(arcsinh(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\mathrm{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\mathrm{arsinh}\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\mathrm{asinh}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a),x)
```

```
[Out] Integral(x/asinh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\mathrm{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a), x)
```

3.83 $\int \frac{1}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

Rubi [A] time = 0.0233665, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5863, 5657, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0069485, size = 11, normalized size = 1.

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

Maple [A] time = 0.023, size = 12, normalized size = 1.1

$$\frac{\mathrm{Chi}\left(\mathrm{Arcsinh}\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a),x)
```

```
[Out] Chi(arcsinh(b*x+a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\mathrm{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/arcsinh(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\mathrm{arsinh}\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(1/arcsinh(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\mathrm{asinh}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a),x)
```

```
[Out] Integral(1/asinh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(1/arcsinh(b*x + a), x)
```

3.84 $\int \frac{1}{x \sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x \sinh^{-1}(a+bx)}, x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a + b*x]), x]
```

Rubi [A] time = 0.0440705, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(a+bx)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(a+bx)} dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)} dx, x, a+bx\right)}{b}$$

Mathematica [A] time = 0.190473, size = 0, normalized size = 0.

$$\int \frac{1}{x \sinh^{-1}(a+bx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a + b*x]), x]
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{x\text{Arcsinh}\,(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a),x)
```

```
[Out] int(1/x/arcsinh(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(bx + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \operatorname{arsinh}(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(b*x + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{asinh}(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a),x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(bx + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

3.85 $\int \frac{x^2}{\sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=154

$$\frac{a^2\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{a^2\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}-\frac{2a\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(a+bx\right.}{4b^3}$$

```
[Out] -((a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x])) + (2*a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]) - ((a + b*x)^2*Sqrt[1 + (a + b*x)
^2])/(b^3*ArcSinh[a + b*x]) - (2*a*CoshIntegral[2*ArcSinh[a + b*x]])/b^3 -
SinhIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*SinhIntegral[ArcSinh[a + b*x]
])/b^3 + (3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Rubi [A] time = 0.216496, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5865, 5803, 5655, 5779, 3298, 5665, 3301}

$$\frac{a^2\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{a^2\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}-\frac{2a\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(a+bx\right.}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x])) + (2*a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]) - ((a + b*x)^2*Sqrt[1 + (a + b*x)
^2])/(b^3*ArcSinh[a + b*x]) - (2*a*CoshIntegral[2*ArcSinh[a + b*x]])/b^3 -
SinhIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*SinhIntegral[ArcSinh[a + b*x]
])/b^3 + (3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sinh^{-1}(x)^2} - \frac{2ax}{b^2\sinh^{-1}(x)^2} + \frac{x^2}{b^2\sinh^{-1}(x)^2}\right) dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\,\text{Subst}\left(\int \frac{}{\text{sinh}}\right.}{} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int\left(-\frac{\text{si}}{}\right.\right.}{} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{2a\text{Chi}\left(2\sinh\right.}{b^3} \\
&= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{2a\text{Chi}\left(2\sinh\right.}{b^3}
\end{aligned}
$$

Mathematica [A] time = 0.43585, size = 83, normalized size = 0.54

$$
\frac{-\frac{4b^2x^2\sqrt{a^2+2abx+b^2x^2+1}}{\sinh^{-1}(a+bx)} + \left(4a^2-1\right)\text{Shi}\left(\sinh^{-1}(a+bx)\right) - 8a\text{Chi}\left(2\sinh^{-1}(a+bx)\right) + 3\text{Shi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] ((-4*b^2*x^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/ArcSinh[a + b*x] - 8*a*Cosh
Integral[2*ArcSinh[a + b*x]] + (-1 + 4*a^2)*SinhIntegral[ArcSinh[a + b*x]]
+ 3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Maple [A] time = 0.051, size = 146, normalized size = 1.

$$\frac{1}{b^3}\left(-\frac{a\left(2\,\mathrm{Chi}\left(2\,\mathrm{Arcsinh}\left(bx+a\right)\right)\mathrm{Arcsinh}\left(bx+a\right)-\sinh\left(2\,\mathrm{Arcsinh}\left(bx+a\right)\right)\right)}{\mathrm{Arcsinh}\left(bx+a\right)}+\frac{1}{4\,\mathrm{Arcsinh}\left(bx+a\right)}\sqrt{1+\left(bx+a\right)^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(-a*(2*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/a
rcsinh(b*x+a)+1/4/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/4*Shi(arcsinh(b*x+a)
)-1/4/arcsinh(b*x+a)*cosh(3*arcsinh(b*x+a))+3/4*Shi(3*arcsinh(b*x+a))+a^2*(
Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3x^5+3\,ab^2x^4+\left(3\,a^2b+b\right)x^3+\left(a^3+a\right)x^2+\left(b^2x^4+2\,abx^3+\left(a^2+1\right)x^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}+\int\frac{3\,b^5x^6+14\,a\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*
a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a
*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b
*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((3*b^5*x^6 + 14*a*
b^4*x^5 + 2*(13*a^2*b^3 + 3*b^3)*x^4 + 8*(3*a^3*b^2 + 2*a*b^2)*x^3 + (11*a^
4*b + 14*a^2*b + 3*b)*x^2 + (3*b^3*x^4 + 8*a*b^2*x^3 + (7*a^2*b + b)*x^2 +
2*(a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^5 + 2*a^3 + a)*x + (6*b
^4*x^5 + 22*a*b^3*x^4 + (30*a^2*b^2 + 7*b^2)*x^3 + (18*a^3*b + 13*a*b)*x^2
+ 2*(2*a^4 + 3*a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^5*x^4 + 4
*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2
*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*
x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\mathrm{arsinh}\left(bx+a\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2/asinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a)^2, x)
```

3.86 $\int \frac{x}{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=84

$$\frac{\operatorname{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\operatorname{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{a\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(b^2*ArcSinh[a + b*x]) - ((a + b*x)*Sqrt[1 + (a +
 b*x)^2])/(b^2*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b^2 - (
a*SinhIntegral[ArcSinh[a + b*x]])/b^2
```

Rubi [A] time = 0.131585, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5865, 5803, 5655, 5779, 3298, 5665, 3301}

$$\frac{\operatorname{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\operatorname{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{a\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(b^2*ArcSinh[a + b*x]) - ((a + b*x)*Sqrt[1 + (a +
 b*x)^2])/(b^2*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b^2 - (
a*SinhIntegral[ArcSinh[a + b*x]])/b^2
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sinh^{-1}(x)^2}+\frac{x}{b\sinh^{-1}(x)^2}\right)dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{S}}{}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,d\right.}{b}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\text{Shi}\left(\sinh^{-1}(a+b\right.}{b^2}$$

Mathematica [A] time = 0.162088, size = 62, normalized size = 0.74

$$-\frac{-\sinh^{-1}(a+bx)\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+a\sinh^{-1}(a+bx)\text{Shi}\left(\sinh^{-1}(a+bx)\right)+bx\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((b*x*Sqrt[1 + (a + b*x)^2] - ArcSinh[a + b*x]*CoshIntegral[2*ArcSinh[a +
b*x]] + a*ArcSinh[a + b*x]*SinhIntegral[ArcSinh[a + b*x]])/(b^2*ArcSinh[a +
 b*x]))
```

Maple [A] time = 0.036, size = 73, normalized size = 0.9

$$\frac{1}{b^2}\left(-\frac{\sinh(2\,\text{Arcsinh}(bx+a))}{2\,\text{Arcsinh}(bx+a)} + \text{Chi}(2\,\text{Arcsinh}(bx+a)) - \frac{a}{\text{Arcsinh}(bx+a)}\left(\text{Shi}(\text{Arcsinh}(bx+a))\,\text{Arcsinh}(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(-1/2/arcsinh(b*x+a)*sinh(2*arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a))-a*(
Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3x^4 + 3\,ab^2x^3 + \left(3\,a^2b + b\right)x^2 + \left(a^3 + a\right)x + \left(b^2x^3 + 2\,abx^2 + \left(a^2 + 1\right)x\right)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{\left(b^3x^2 + 2\,ab^2x + a^2b + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(b^2x + ab\right) + b\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)} + \int \frac{2\,b^5x^5 + 9\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*
b*x^2 + (a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2
*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((2*b^5*x^5 + 9*a*b^4*x
^4 + a^5 + 4*(4*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2 + 5*a*b^2)*x^2 +
(2*b^3*x^3 + 5*a*b^2*x^2 + 4*a^2*b*x + a^3 + a)*(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*(3*a^4*b + 4*a^2*b + b)*x + (4*b^4*x^4 + 14*a*b^3*x^3 + 2*a^4 + 2*(9
*a^2*b^2 + 2*b^2)*x^2 + 3*a^2 + (10*a^3*b + 7*a*b)*x + 1)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) + a)/((b^5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*
b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2
*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\text{arsinh}\left(bx + a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}^2\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x/asinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^2, x)
```

3.87 $\int \frac{1}{\sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=38

$$\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b\sinh^{-1}(a+bx)}$$

[Out] -(Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x])) + SinhIntegral[ArcSinh[a + b*x]]/b

---

Rubi [A] time = 0.07259, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5655, 5779, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a + b*x]^(-2),x]

[Out] -(Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x])) + SinhIntegral[ArcSinh[a + b*x]]/b

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\textbf{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Mathematica [A] time = 0.0219833, size = 35, normalized size = 0.92

$$\frac{\textbf{Shi}\left(\sinh^{-1}(a+bx)\right) - \frac{\sqrt{(a+bx)^2+1}}{\sinh^{-1}(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-2),x]
```

```
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]) + SinhIntegral[ArcSinh[a + b*x]]
)/b
```

Maple [A] time = 0.027, size = 34, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{1}{\text{Arcsinh}\,(bx+a)}\sqrt{1+(bx+a)^2} + \text{Shi}\,(\text{Arcsinh}\,(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+Shi(arcsinh(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} + a}{\left(b^3x^2 + 2\,ab^2x + a^2b + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(b^2x + ab\right) + b\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)} + \int \frac{}{\left(b^4x^4 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
 + integrate((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*
```

```
x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 - 1) + 2*a^2 + 4*(a^3*b +
 a*b)*x + (2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2
+ b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2) + 2*a^
2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arsinh}(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^(-2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**2,x)
```

```
[Out] Integral(asinh(a + b*x)**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^(-2), x)
```

3.88 $\int \frac{1}{x\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\sinh^{-1}(a+bx)^2},x\right)$$

[Out] Unintegrable[1/(x*ArcSinh[a + b*x]^2), x]

Rubi [A] time = 0.0386165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(a+bx)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*ArcSinh[a + b*x]^2),x]

[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^2), x], x, a + b*x]/b

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

Mathematica [A] time = 2.04346, size = 0, normalized size = 0.

$$\int \frac{1}{x\sinh^{-1}(a+bx)^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*ArcSinh[a + b*x]^2),x]

[Out] Integrate[1/(x*ArcSinh[a + b*x]^2), x]

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\text{Arcsinh}\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arcsinh(b*x+a)^2,x)

[Out] int(1/x/arcsinh(b*x+a)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} + a}{\left(b^3x^3 + 2\,ab^2x^2 + \left(a^2b + b\right)x + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(b^2x^2 + abx\right)\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)} - \int \frac{}{\left(b^5x^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*(b^2*x^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))) - integrate((a*b^4*x^4 + 4*a^2*b^3*x^3 + a^5 + 2*a^3 + 2*(3*a^3*
b^2 + a*b^2)*x^2 + (a*b^2*x^2 + a^3 + 2*(a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 4*(a^4*b + a^2*b)*x + (2*a*b^3*x^3 + 2*a^4 + 2*(3*a^2*b^2 +
 b^2)*x^2 + 3*a^2 + (6*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1) + a)/((b^5*x^6 + 4*a*b^4*x^5 + 2*(3*a^2*b^3 + b^3)*x^4 + 4*(a^3*b^2 + a
*b^2)*x^3 + (a^4*b + 2*a^2*b + b)*x^2 + (b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(b^4*x^5 + 3*a*b^3*x^4 + (3*a^2*b^2 + b^
2)*x^3 + (a^3*b + a*b)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a
+ sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\,\text{arsinh}\left(bx + a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(b*x + a)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{asinh}^2\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\text{arsinh}\left(bx + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)^2), x)
```

3.89 $\int \frac{x^2}{\sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=257

$$\frac{a^2 \text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^3}-\frac{a^2(a+bx)}{2b^3\sinh^{-1}(a+bx)}-\frac{a^2\sqrt{(a+bx)^2+1}}{2b^3\sinh^{-1}(a+bx)^2}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{8b^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{8b^3}$$

```
[Out] -(a^2*Sqrt[1 + (a + b*x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + (a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) - ((a + b*x)^2*Sqrt[1 + (a + b*
x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + a/(b^3*ArcSinh[a + b*x]) - (a + b*x)/(b
^3*ArcSinh[a + b*x]) - (a^2*(a + b*x))/(2*b^3*ArcSinh[a + b*x]) + (2*a*(a +
 b*x)^2)/(b^3*ArcSinh[a + b*x]) - (3*(a + b*x)^3)/(2*b^3*ArcSinh[a + b*x])
- CoshIntegral[ArcSinh[a + b*x]]/(8*b^3) + (a^2*CoshIntegral[ArcSinh[a + b*
x]])/(2*b^3) + (9*CoshIntegral[3*ArcSinh[a + b*x]])/(8*b^3) - (2*a*SinhInte
gral[2*ArcSinh[a + b*x]])/b^3
```

Rubi [A] time = 0.497086, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5865, 5803, 5655, 5774, 5657, 3301, 5667, 5669, 5448, 12, 3298, 5675}

$$\frac{a^2 \text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^3}-\frac{a^2(a+bx)}{2b^3\sinh^{-1}(a+bx)}-\frac{a^2\sqrt{(a+bx)^2+1}}{2b^3\sinh^{-1}(a+bx)^2}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{8b^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x]^3,x]
```


```
[Out] -(a^2*Sqrt[1 + (a + b*x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + (a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) - ((a + b*x)^2*Sqrt[1 + (a + b*
x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + a/(b^3*ArcSinh[a + b*x]) - (a + b*x)/(b
^3*ArcSinh[a + b*x]) - (a^2*(a + b*x))/(2*b^3*ArcSinh[a + b*x]) + (2*a*(a +
 b*x)^2)/(b^3*ArcSinh[a + b*x]) - (3*(a + b*x)^3)/(2*b^3*ArcSinh[a + b*x])
- CoshIntegral[ArcSinh[a + b*x]]/(8*b^3) + (a^2*CoshIntegral[ArcSinh[a + b*
x]])/(2*b^3) + (9*CoshIntegral[3*ArcSinh[a + b*x]])/(8*b^3) - (2*a*SinhInte
gral[2*ArcSinh[a + b*x]])/b^3
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sinh^{-1}(x)^3} - \frac{2ax}{b^2\sinh^{-1}(x)^3} + \frac{x^2}{b^2\sinh^{-1}(x)^3}\right)dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x\right.}}{b^3}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}\,s}\right.}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a}{b^3\sinh^{-1}(a+bx}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a}{b^3\sinh^{-1}(a+bx}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a}{b^3\sinh^{-1}(a+bx}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a}{b^3\sinh^{-1}(a+bx}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a}{b^3\sinh^{-1}(a+bx}$$

Mathematica [A] time = 0.391158, size = 110, normalized size = 0.43

$$\frac{-\frac{4bx\left(bx\sqrt{a^2+2abx+b^2x^2+1}+\left(2a^2+5abx+3b^2x^2+2\right)\sinh^{-1}(a+bx)\right)}{\sinh^{-1}(a+bx)^2} + \left(4a^2-1\right)\text{Chi}\left(\sinh^{-1}(a+bx)\right) + 9\text{Chi}\left(3\sinh^{-1}(a+bx)\right) - 16a\text{Sh}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] ((-4*b*x*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (2 + 2*a^2 + 5*a*b*x + 3*
b^2*x^2)*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2 + (-1 + 4*a^2)*CoshIntegral[
ArcSinh[a + b*x]] + 9*CoshIntegral[3*ArcSinh[a + b*x]] - 16*a*SinhIntegral[
2*ArcSinh[a + b*x]])/(8*b^3)
```

Maple [A] time = 0.053, size = 215, normalized size = 0.8

$$\frac{1}{b^3}\left(-\frac{a\left(4\,\text{Shi}\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\left(\text{Arcsinh}\left(bx+a\right)\right)^2 - 2\,\cosh\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\text{Arcsinh}\left(bx+a\right) - \sinh\left(2\,\text{Arcsin}\right.\right.}{2\,\left(\text{Arcsinh}\left(bx+a\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(-1/2*a*(4*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*cosh(2*arcsinh(b*
x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)^2+1/8/arcsinh(b
*x+a)^2*(1+(b*x+a)^2)^(1/2)+1/8/arcsinh(b*x+a)*(b*x+a)-1/8*Chi(arcsinh(b*x+
a))-1/8/arcsinh(b*x+a)^2*cosh(3*arcsinh(b*x+a))-3/8/arcsinh(b*x+a)*sinh(3*a
rcsinh(b*x+a))+9/8*Chi(3*arcsinh(b*x+a))+1/2*a^2*(Chi(arcsinh(b*x+a))*arcsi
nh(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^9 + 7*a*b^7*x^8 + 3*(7*a^2*b^6 + b^6)*x^7 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^6 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^5 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^4 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^3 + (a^7*b
+ 3*a^5*b + 3*a^3*b + a*b)*x^2 + (b^5*x^6 + 4*a*b^4*x^5 + (6*a^2*b^3 + b^3)
*x^4 + 2*(2*a^3*b^2 + a*b^2)*x^3 + (a^4*b + a^2*b)*x^2)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2) + (3*b^6*x^7 + 15*a*b^5*x^6 + 5*(6*a^2*b^4 + b^4)*x^5 + 15
*(2*a^3*b^3 + a*b^3)*x^4 + (15*a^4*b^2 + 15*a^2*b^2 + 2*b^2)*x^3 + (3*a^5*b
 + 5*a^3*b + 2*a*b)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (3*b^8*x^9 + 23*a*
b^7*x^8 + (77*a^2*b^6 + 9*b^6)*x^7 + 3*(49*a^3*b^5 + 17*a*b^5)*x^6 + (175*a
^4*b^4 + 120*a^2*b^4 + 9*b^4)*x^5 + (133*a^5*b^3 + 150*a^3*b^3 + 33*a*b^3)*
x^4 + 3*(21*a^6*b^2 + 35*a^4*b^2 + 15*a^2*b^2 + b^2)*x^3 + (17*a^7*b + 39*a
^5*b + 27*a^3*b + 5*a*b)*x^2 + (3*b^5*x^6 + 14*a*b^4*x^5 + 2*(13*a^2*b^3 +
2*b^3)*x^4 + 12*(2*a^3*b^2 + a*b^2)*x^3 + (11*a^4*b + 12*a^2*b + b)*x^2 + 2
*(a^5 + 2*a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (9*b^6*x^7 + 51
*a*b^5*x^6 + (120*a^2*b^4 + 17*b^4)*x^5 + 5*(30*a^3*b^3 + 13*a*b^3)*x^4 + (
105*a^4*b^2 + 93*a^2*b^2 + 10*b^2)*x^3 + (39*a^5*b + 59*a^3*b + 20*a*b)*x^2
 + 2*(3*a^6 + 7*a^4 + 5*a^2 + 1)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^8
+ 3*a^6 + 3*a^4 + a^2)*x + (9*b^7*x^8 + 60*a*b^6*x^7 + (171*a^2*b^5 + 22*b^
5)*x^6 + 2*(135*a^3*b^4 + 52*a*b^4)*x^5 + (255*a^4*b^3 + 196*a^2*b^3 + 18*b
^3)*x^4 + 2*(72*a^5*b^2 + 92*a^3*b^2 + 25*a*b^2)*x^3 + (45*a^6*b + 86*a^4*b
 + 46*a^2*b + 5*b)*x^2 + 2*(3*a^7 + 8*a^5 + 7*a^3 + 2*a)*x)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^7*x^8 + 18*a*b^6*x^7 + (45*a^2*b^5 + 7*b^5)*x^6 + 4*(15*a^3*b^4 + 7*a*b^4)
*x^5 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^4 + 2*(9*a^5*b^2 + 14*a^3*b^2 +
5*a*b^2)*x^3 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(5*a^2*b^6
+ b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b^4 + 6*a^2
*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 + a^2*b^2 +
 (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2
+ 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a*b^6*x^4 +
 a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3*b^4 + 3*a
*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integrate(1/2*(9
*b^10*x^10 + 82*a*b^9*x^9 + 2*a^10 + 2*(167*a^2*b^8 + 18*b^8)*x^8 + 8*a^8 +
 32*(25*a^3*b^7 + 8*a*b^7)*x^7 + 2*(623*a^4*b^6 + 394*a^2*b^6 + 27*b^6)*x^6
 + 12*a^6 + 4*(329*a^5*b^5 + 342*a^3*b^5 + 69*a*b^5)*x^5 + 4*(238*a^6*b^4 +
 365*a^4*b^4 + 144*a^2*b^4 + 9*b^4)*x^4 + 8*a^4 + 16*(29*a^7*b^3 + 61*a^5*b
^3 + 39*a^3*b^3 + 7*a*b^3)*x^3 + (9*b^6*x^6 + 46*a*b^5*x^5 + 2*a^6 + 4*(24*
a^2*b^4 + b^4)*x^4 + 4*a^4 + 8*(13*a^3*b^3 + 2*a*b^3)*x^3 + (61*a^4*b^2 + 2
4*a^2*b^2 - b^2)*x^2 + 2*a^2 + 2*(9*a^5*b + 8*a^3*b - a*b)*x)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + (145*a^8*b^2 + 396*a^6*b^2 + 366*a^4*b^2 + 124*a^2*b^2
```

```
 + 9*b^2)*x^2 + (36*b^7*x^7 + 220*a*b^6*x^6 + 8*a^7 + 8*(71*a^2*b^5 + 6*b^5
)*x^5 + 20*a^5 + 16*(50*a^3*b^4 + 13*a*b^4)*x^4 + (660*a^4*b^3 + 356*a^2*b^
3 + 13*b^3)*x^3 + 16*a^3 + (316*a^5*b^2 + 300*a^3*b^2 + 39*a*b^2)*x^2 + 2*(
40*a^6*b + 62*a^4*b + 21*a^2*b - b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^
(3/2) + (54*b^8*x^8 + 384*a*b^7*x^7 + 12*a^8 + 6*(197*a^2*b^6 + 20*b^6)*x^6
 + 36*a^6 + 12*(171*a^3*b^5 + 52*a*b^5)*x^5 + (2190*a^4*b^4 + 1332*a^2*b^4
+ 83*b^4)*x^4 + 38*a^4 + 4*(366*a^5*b^3 + 372*a^3*b^3 + 71*a*b^3)*x^3 + (59
4*a^6*b^2 + 912*a^4*b^2 + 357*a^2*b^2 + 19*b^2)*x^2 + 16*a^2 + 2*(66*a^7*b
+ 144*a^5*b + 97*a^3*b + 19*a*b)*x + 2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*a
^2 + 2*(13*a^9*b + 44*a^7*b + 54*a^5*b + 28*a^3*b + 5*a*b)*x + (36*b^9*x^9
+ 292*a*b^8*x^8 + 8*a^9 + 4*(261*a^2*b^7 + 28*b^7)*x^7 + 28*a^7 + 4*(539*a^
3*b^6 + 172*a*b^6)*x^6 + (2828*a^4*b^5 + 1788*a^2*b^5 + 123*b^5)*x^5 + 36*a
^5 + (2436*a^5*b^4 + 2540*a^3*b^4 + 519*a*b^4)*x^4 + (1372*a^6*b^3 + 2120*a
^4*b^3 + 855*a^2*b^3 + 57*b^3)*x^3 + 20*a^3 + (484*a^7*b^2 + 1032*a^5*b^2 +
 681*a^3*b^2 + 133*a*b^2)*x^2 + 2*(48*a^8*b + 134*a^6*b + 129*a^4*b + 48*a^
2*b + 5*b)*x + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^8 + 8*a*b^9
*x^7 + a^8*b^2 + 4*a^6*b^2 + 4*(7*a^2*b^8 + b^8)*x^6 + 6*a^4*b^2 + 8*(7*a^3
*b^7 + 3*a*b^7)*x^5 + 2*(35*a^4*b^6 + 30*a^2*b^6 + 3*b^6)*x^4 + 4*a^2*b^2 +
 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^3 + (b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*
b^4*x^2 + 4*a^3*b^3*x + a^4*b^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6
*b^4 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^2 + 4*(b^7*x^5 + 5*a*b^6*x^4 + a^5*b
^2 + a^3*b^2 + (10*a^2*b^5 + b^5)*x^3 + (10*a^3*b^4 + 3*a*b^4)*x^2 + (5*a^4
*b^3 + 3*a^2*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^6 + 6*a
*b^7*x^5 + a^6*b^2 + 2*a^4*b^2 + (15*a^2*b^6 + 2*b^6)*x^4 + a^2*b^2 + 4*(5*
a^3*b^5 + 2*a*b^5)*x^3 + (15*a^4*b^4 + 12*a^2*b^4 + b^4)*x^2 + 2*(3*a^5*b^3
 + 4*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + b^2 + 8*(a^7*b^3 +
 3*a^5*b^3 + 3*a^3*b^3 + a*b^3)*x + 4*(b^9*x^7 + 7*a*b^8*x^6 + a^7*b^2 + 3*
a^5*b^2 + 3*(7*a^2*b^7 + b^7)*x^5 + 3*a^3*b^2 + 5*(7*a^3*b^6 + 3*a*b^6)*x^4
 + (35*a^4*b^5 + 30*a^2*b^5 + 3*b^5)*x^3 + a*b^2 + 3*(7*a^5*b^4 + 10*a^3*b^
4 + 3*a*b^4)*x^2 + (7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3)*x)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\operatorname{arsinh}(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a)**3,x)
```

```
[Out] Integral(x**2/asinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arsinh}(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="giac")

[Out] integrate(x^2/arcsinh(b*x + a)^3, x)
```

3.90 $\int \frac{x}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{(a+bx)^2}{b^2\sinh^{-1}(a+bx)}+\frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{2b^2\sinh^{-1}(a+bx)^2}-$$

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 +
(a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - 1/(2*b^2*ArcSinh[a + b*x]) + (a*
(a + b*x))/(2*b^2*ArcSinh[a + b*x]) - (a + b*x)^2/(b^2*ArcSinh[a + b*x]) -
(a*CoshIntegral[ArcSinh[a + b*x]])/(2*b^2) + SinhIntegral[2*ArcSinh[a + b*x
]]/b^2
```

Rubi [A] time = 0.249732, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.2, Rules used = {5865, 5803, 5655, 5774, 5657, 3301, 5667, 5669, 5448, 12, 3298, 5675}

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{(a+bx)^2}{b^2\sinh^{-1}(a+bx)}+\frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{2b^2\sinh^{-1}(a+bx)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 +
(a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - 1/(2*b^2*ArcSinh[a + b*x]) + (a*
(a + b*x))/(2*b^2*ArcSinh[a + b*x]) - (a + b*x)^2/(b^2*ArcSinh[a + b*x]) -
(a*CoshIntegral[ArcSinh[a + b*x]])/(2*b^2) + SinhIntegral[2*ArcSinh[a + b*x
]]/b^2
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
```

```
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sinh^{-1}(x)^3}+\frac{x}{b\sinh^{-1}(x)^3}\right)dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{2b^2} + \text{Subs}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)} - \frac{}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)} - \frac{}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)} - \frac{}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)} - \frac{}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)} - \frac{}{b^2}$$

Mathematica [A] time = 0.10173, size = 117, normalized size = 0.8

$$-\frac{bx\sqrt{a^2+2abx+b^2x^2+1}+a^2\sinh^{-1}(a+bx)+2b^2x^2\sinh^{-1}(a+bx)+a\sinh^{-1}(a+bx)^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)-2\sinh}{2b^2\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] -(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + ArcSinh[a + b*x] + a^2*ArcSinh[a
+ b*x] + 3*a*b*x*ArcSinh[a + b*x] + 2*b^2*x^2*ArcSinh[a + b*x] + a*ArcSinh[
a + b*x]^2*CoshIntegral[ArcSinh[a + b*x]] - 2*ArcSinh[a + b*x]^2*SinhIntegr
al[2*ArcSinh[a + b*x]])/(2*b^2*ArcSinh[a + b*x]^2)
```

Maple [A] time = 0.037, size = 107, normalized size = 0.7

$$\frac{1}{b^2}\left(-\frac{\sinh(2\,\text{Arcsinh}(bx+a))}{4\,(\text{Arcsinh}(bx+a))^2} - \frac{\cosh(2\,\text{Arcsinh}(bx+a))}{2\,\text{Arcsinh}(bx+a)} + \text{Shi}(2\,\text{Arcsinh}(bx+a)) - \frac{a}{2\,(\text{Arcsinh}(bx+a))^2}\left(\text{Chi}(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(-1/4/arcsinh(b*x+a)^2*sinh(2*arcsinh(b*x+a))-1/2/arcsinh(b*x+a)*cosh
(2*arcsinh(b*x+a))+Shi(2*arcsinh(b*x+a))-1/2*a*(Chi(arcsinh(b*x+a))*arcsinh
```

```
(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (2*b^8*x^8 + 15*a*b^7*x^
7 + a^8 + (49*a^2*b^6 + 6*b^6)*x^6 + 3*a^6 + (91*a^3*b^5 + 33*a*b^5)*x^5 +
3*(35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 3*a^4 + (77*a^5*b^3 + 90*a^3*b^3
+ 21*a*b^3)*x^3 + (35*a^6*b^2 + 60*a^4*b^2 + 27*a^2*b^2 + 2*b^2)*x^2 + (2*b
^5*x^5 + 9*a*b^4*x^4 + a^5 + 2*(8*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2
 + 3*a*b^2)*x^2 + 6*(a^4*b + a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + (6*b^6*x^6 + 33*a*b^5*x^5 + 3*a^6 + 5*(15*a^2*b^4 + 2*b^4)*x^4 + 7*a^
4 + (90*a^3*b^3 + 37*a*b^3)*x^3 + (60*a^4*b^2 + 51*a^2*b^2 + 5*b^2)*x^2 + 5
*a^2 + (21*a^5*b + 31*a^3*b + 10*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ a^2 + 3*(3*a^7*b + 7*a^5*b + 5*a^3*b + a*b)*x + (6*b^7*x^7 + 39*a*b^6*x^6
 + 3*a^7 + 2*(54*a^2*b^5 + 7*b^5)*x^5 + 8*a^5 + (165*a^3*b^4 + 64*a*b^4)*x^
4 + (150*a^4*b^3 + 116*a^2*b^3 + 11*b^3)*x^3 + 7*a^3 + (81*a^5*b^2 + 104*a^
3*b^2 + 29*a*b^2)*x^2 + (24*a^6*b + 46*a^4*b + 25*a^2*b + 3*b)*x + 2*a)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)) + (3*b^7*x^7 + 18*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^
4 + 7*a*b^4)*x^4 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 1
4*a^3*b^2 + 5*a*b^2)*x^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(
5*a^2*b^6 + b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b
^4 + 6*a^2*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 +
 a^2*b^2 + (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*
b*x + a^2 + 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a
*b^6*x^4 + a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3
*b^4 + 3*a*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integr
ate(1/2*(4*b^9*x^9 + 35*a*b^8*x^8 + 3*a^9 + 8*(17*a^2*b^7 + 2*b^7)*x^7 + 12
*a^7 + 4*(77*a^3*b^6 + 27*a*b^6)*x^6 + 8*(56*a^4*b^5 + 39*a^2*b^5 + 3*b^5)*
x^5 + 18*a^5 + 2*(217*a^5*b^4 + 250*a^3*b^4 + 57*a*b^4)*x^4 + 8*(35*a^6*b^3
 + 60*a^4*b^3 + 27*a^2*b^3 + 2*b^3)*x^3 + (4*b^5*x^5 + 19*a*b^4*x^4 + 36*a^
2*b^3*x^3 + 34*a^3*b^2*x^2 + 16*a^4*b*x + 3*a^5 - 3*a)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1)^2 + 12*a^3 + 4*(29*a^7*b^2 + 69*a^5*b^2 + 51*a^3*b^2 + 11*a*b^2)*
x^2 + (16*b^6*x^6 + 92*a*b^5*x^5 + 12*a^6 + 4*(55*a^2*b^4 + 4*b^4)*x^4 + 12
*a^4 + 20*(14*a^3*b^3 + 3*a*b^3)*x^3 + 4*(50*a^4*b^2 + 21*a^2*b^2)*x^2 - 3*
a^2 + (76*a^5*b + 52*a^3*b - 3*a*b)*x - 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + 3*(8*b^7*x^7 + 54*a*b^6*x^6 + 6*a^7 + 4*(39*a^2*b^5 + 4*b^5)*x^5 + 12
*a^5 + 2*(125*a^3*b^4 + 38*a*b^4)*x^4 + 8*(30*a^4*b^3 + 18*a^2*b^3 + b^3)*x
^3 + 7*a^3 + (138*a^5*b^2 + 136*a^3*b^2 + 23*a*b^2)*x^2 + 2*(22*a^6*b + 32*
a^4*b + 11*a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(7*a^8*b + 22*a^
6*b + 24*a^4*b + 10*a^2*b + b)*x + (16*b^8*x^8 + 124*a*b^7*x^7 + 12*a^8 + 1
```

```
2*(35*a^2*b^6 + 4*b^6)*x^6 + 36*a^6 + 4*(203*a^3*b^5 + 69*a*b^5)*x^5 + 4*(2
45*a^4*b^4 + 165*a^2*b^4 + 12*b^4)*x^4 + 39*a^4 + 3*(252*a^5*b^3 + 280*a^3*
b^3 + 61*a*b^3)*x^3 + (364*a^6*b^2 + 600*a^4*b^2 + 261*a^2*b^2 + 19*b^2)*x^
2 + 18*a^2 + (100*a^7*b + 228*a^5*b + 165*a^3*b + 37*a*b)*x + 3)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1) + 3*a)/((b^9*x^8 + 8*a*b^8*x^7 + a^8*b + 4*a^6*b +
4*(7*a^2*b^7 + b^7)*x^6 + 8*(7*a^3*b^6 + 3*a*b^6)*x^5 + 6*a^4*b + 2*(35*a^4
*b^5 + 30*a^2*b^5 + 3*b^5)*x^4 + 8*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^3 +
 (b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 4*a^2*b + 4*(7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3
)*x^2 + 4*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + a^3*b + (10*a^2*b^4 + b^4)*x^3 +
 (10*a^3*b^3 + 3*a*b^3)*x^2 + (5*a^4*b^2 + 3*a^2*b^2)*x)*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2) + 6*(b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 2*a^4*b + (15*a^2*b^
5 + 2*b^5)*x^4 + 4*(5*a^3*b^4 + 2*a*b^4)*x^3 + a^2*b + (15*a^4*b^3 + 12*a^2
*b^3 + b^3)*x^2 + 2*(3*a^5*b^2 + 4*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1) + 8*(a^7*b^2 + 3*a^5*b^2 + 3*a^3*b^2 + a*b^2)*x + 4*(b^8*x^7 + 7*
a*b^7*x^6 + a^7*b + 3*a^5*b + 3*(7*a^2*b^6 + b^6)*x^5 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^4 + 3*a^3*b + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^3 + 3*(7*a^5*b^3 +
 10*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^
2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\text{arsinh}\,(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\text{asinh}^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a)**3,x)
```

```
[Out] Integral(x/asinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\text{arsinh}\,(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^3, x)
```

3.91 $\int \frac{1}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=63

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}}{2b\sinh^{-1}(a+bx)^2}$$

```
[Out] -Sqrt[1 + (a + b*x)^2]/(2*b*ArcSinh[a + b*x]^2) - (a + b*x)/(2*b*ArcSinh[a
+ b*x]) + CoshIntegral[ArcSinh[a + b*x]]/(2*b)
```

Rubi [A] time = 0.0786598, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5863, 5655, 5774, 5657, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-3),x]
```

```
[Out] -Sqrt[1 + (a + b*x)^2]/(2*b*ArcSinh[a + b*x]^2) - (a + b*x)/(2*b*ArcSinh[a
+ b*x]) + CoshIntegral[ArcSinh[a + b*x]]/(2*b)
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sinh^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{2b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} - \frac{a+bx}{2b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{2b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} - \frac{a+bx}{2b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} - \frac{a+bx}{2b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.0699588, size = 53, normalized size = 0.84

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right) - \frac{a+bx}{\sinh^{-1}(a+bx)} - \frac{\sqrt{(a+bx)^2+1}}{\sinh^{-1}(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-3),x]
```

```
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]^2) - (a + b*x)/ArcSinh[a + b*x] +
 CoshIntegral[ArcSinh[a + b*x]])/(2*b)
```

Maple [A] time = 0.024, size = 51, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{1}{2\left(\text{Arcsinh}\left(bx+a\right)\right)^2}\sqrt{1+\left(bx+a\right)^2} - \frac{bx+a}{2\,\text{Arcsinh}\left(bx+a\right)} + \frac{\text{Chi}\left(\text{Arcsinh}\left(bx+a\right)\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/2/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-1/2/arcsinh(b*x+a)*(b*x+a)+1
/2*Chi(arcsinh(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^7*x^
7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*
a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 +
 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3
*b*x + a^4 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^5 + 5*a*b^4*
x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*
a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9
*a^2*b + b)*x + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + 3*(15*a^2*b^4 + 2*b^4)*
x^4 + 6*a^4 + 12*(5*a^3*b^3 + 2*a*b^3)*x^3 + (45*a^4*b^2 + 36*a^2*b^2 + 4*b
^2)*x^2 + 4*a^2 + 2*(9*a^5*b + 12*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1) + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/((
b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^
3*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4
*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(
2*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^
2 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 +
b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2
)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2) + integrate(1/2*(b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^
2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30
*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 +
(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4 + 3)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (4*
b^5*x^5 + 20*a*b^4*x^4 + 4*a^5 + 4*(10*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4*(10*a
^3*b^2 + 3*a*b^2)*x^2 + (20*a^4*b + 12*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + 3*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6 + 2*(15*a^2*b^4
 + 2*b^4)*x^4 + 4*a^4 + 8*(5*a^3*b^3 + 2*a*b^3)*x^3 + (30*a^4*b^2 + 24*a^2*
b^2 + b^2)*x^2 + a^2 + 2*(6*a^5*b + 8*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*
x + a^2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (4*b^7*x^7 +
 28*a*b^6*x^6 + 4*a^7 + 12*(7*a^2*b^5 + b^5)*x^5 + 12*a^5 + 20*(7*a^3*b^4 +
 3*a*b^4)*x^4 + (140*a^4*b^3 + 120*a^2*b^3 + 9*b^3)*x^3 + 9*a^3 + 3*(28*a^5
*b^2 + 40*a^3*b^2 + 9*a*b^2)*x^2 + (28*a^6*b + 60*a^4*b + 27*a^2*b + b)*x +
 a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^8*x^8 + 8*a*b^7*x^7 + a^8 +
4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b
^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)
*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 +
4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2
 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*a^2 + 8*(a^7*b
 + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b
^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*
b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^
6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)
*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\operatorname{arsinh}\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^(-3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{asinh}^{3}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**3,x)
```

```
[Out] Integral(asinh(a + b*x)**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{arsinh}\left(bx+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^(-3), x)
```

3.92 $\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{x\sinh^{-1}(a+bx)^3},x\right)$$

```
[Out] Unintegrable[1/(x*ArcSinh[a + b*x]^3), x]
```

Rubi [A] time = 0.0395817, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]^3),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^3}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 2.02481, size = 0, normalized size = 0.

$$\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^3),x]
```

```
[Out] Integrate[1/(x*ArcSinh[a + b*x]^3), x]
```

Maple [A] time = 0.101, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\text{Arcsinh}\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a)^3,x)
```

```
[Out] int(1/x/arcsinh(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x - (a*b^7*x^7 + 7*a^2*b^6*x
^6 + a^8 + 3*a^6 + 3*(7*a^3*b^5 + a*b^5)*x^5 + 5*(7*a^4*b^4 + 3*a^2*b^4)*x^
4 + 3*a^4 + (35*a^5*b^3 + 30*a^3*b^3 + 3*a*b^3)*x^3 + 3*(7*a^6*b^2 + 10*a^4
*b^2 + 3*a^2*b^2)*x^2 + (a*b^4*x^4 + a^5 + 2*(2*a^2*b^3 + b^3)*x^3 + 2*a^3
+ 6*(a^3*b^2 + a*b^2)*x^2 + 2*(2*a^4*b + 3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + (3*a*b^5*x^5 + 3*a^6 + (15*a^2*b^4 + 4*b^4)*x^4 + 7
*a^4 + (30*a^3*b^3 + 19*a*b^3)*x^3 + (30*a^4*b^2 + 33*a^2*b^2 + 5*b^2)*x^2
+ 5*a^2 + 5*(3*a^5*b + 5*a^3*b + 2*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + a^2 + (7*a^7*b + 15*a^5*b + 9*a^3*b + a*b)*x + (3*a*b^6*x^6 + 3*a^7 + 2
*(9*a^2*b^5 + b^5)*x^5 + 8*a^5 + (45*a^3*b^4 + 16*a*b^4)*x^4 + (60*a^4*b^3
+ 44*a^2*b^3 + 3*b^3)*x^3 + 7*a^3 + (45*a^5*b^2 + 56*a^3*b^2 + 13*a*b^2)*x^
2 + (18*a^6*b + 34*a^4*b + 17*a^2*b + b)*x + 2*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^7*x^7 + 1
8*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^4 + 7*a*b^4)*x^4 + (45
*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 14*a^3*b^2 + 5*a*b^2)*x
^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/((b^8*x^8 + 6*a*b^7*x^7 + 3*(5*a^2*b^6 + b^6)*x^6 + 4*(5*a^3*b^5 + 3*a*b^
5)*x^5 + 3*(5*a^4*b^4 + 6*a^2*b^4 + b^4)*x^4 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b
^3)*x^3 + (a^6*b^2 + 3*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + (b^5*x^5 + 3*a*b^4*
x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*
(b^6*x^6 + 4*a*b^5*x^5 + (6*a^2*b^4 + b^4)*x^4 + 2*(2*a^3*b^3 + a*b^3)*x^3
+ (a^4*b^2 + a^2*b^2)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 3*(b^7*x^7 + 5*a
*b^6*x^6 + 2*(5*a^2*b^5 + b^5)*x^5 + 2*(5*a^3*b^4 + 3*a*b^4)*x^4 + (5*a^4*b
^3 + 6*a^2*b^3 + b^3)*x^3 + (a^5*b^2 + 2*a^3*b^2 + a*b^2)*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*(a*b^9*x^9 + 10*a^2*b^8*x^8 + 2*a^10 + 8*a^8 + 4*(11*a^3*b^
7 + a*b^7)*x^7 + 16*(7*a^4*b^6 + 2*a^2*b^6)*x^6 + 12*a^6 + 2*(91*a^5*b^5 +
54*a^3*b^5 + 3*a*b^5)*x^5 + 4*(49*a^6*b^4 + 50*a^4*b^4 + 9*a^2*b^4)*x^4 + 8
*a^4 + 4*(35*a^7*b^3 + 55*a^5*b^3 + 21*a^3*b^3 + a*b^3)*x^3 + (a*b^5*x^5 +
2*a^6 + 2*(3*a^2*b^4 + 2*b^4)*x^4 + 4*a^4 + 2*(7*a^3*b^3 + 8*a*b^3)*x^3 + 8
*(2*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + 2*a^2 + (9*a^5*b + 16*a^3*b + 7*a*b)*x
)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 16*(4*a^8*b^2 + 9*a^6*b^2 + 6*a^4*b^2 +
 a^2*b^2)*x^2 + (4*a*b^6*x^6 + 8*a^7 + 4*(7*a^2*b^5 + 3*b^5)*x^5 + 20*a^5 +
 16*(5*a^3*b^4 + 4*a*b^4)*x^4 + 2*(60*a^4*b^3 + 70*a^2*b^3 + 11*b^3)*x^3 +
16*a^3 + (100*a^5*b^2 + 156*a^3*b^2 + 57*a*b^2)*x^2 + (44*a^6*b + 88*a^4*b
+ 51*a^2*b + 7*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*a*b^7*x
^7 + 12*a^8 + 12*(4*a^2*b^6 + b^6)*x^6 + 36*a^6 + 6*(27*a^3*b^5 + 14*a*b^5)
*x^5 + 4*(75*a^4*b^4 + 63*a^2*b^4 + 5*b^4)*x^4 + 38*a^4 + (330*a^5*b^3 + 40
8*a^3*b^3 + 95*a*b^3)*x^3 + 2*(108*a^6*b^2 + 186*a^4*b^2 + 84*a^2*b^2 + 5*b
^2)*x^2 + 16*a^2 + (78*a^7*b + 180*a^5*b + 131*a^3*b + 29*a*b)*x + 2)*(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 2*a^2 + (17*a^9*b + 52*a^7*b + 54*a^5*b + 20*a^3
*b + a*b)*x + (4*a*b^8*x^8 + 8*a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 28*a^7 + 20*
(7*a^3*b^6 + 2*a*b^6)*x^6 + 2*(154*a^4*b^5 + 84*a^2*b^5 + 3*b^5)*x^5 + 36*a
```

```
^5 + (420*a^5*b^4 + 380*a^3*b^4 + 51*a*b^4)*x^4 + (364*a^6*b^3 + 500*a^4*b^
3 + 153*a^2*b^3 + 3*b^3)*x^3 + 20*a^3 + (196*a^7*b^2 + 384*a^5*b^2 + 213*a^
3*b^2 + 25*a*b^2)*x^2 + (60*a^8*b + 160*a^6*b + 141*a^4*b + 42*a^2*b + b)*x
 + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^11 + 8*a*b^9*x^10 + 4*(
7*a^2*b^8 + b^8)*x^9 + 8*(7*a^3*b^7 + 3*a*b^7)*x^8 + 2*(35*a^4*b^6 + 30*a^2
*b^6 + 3*b^6)*x^7 + 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^6 + 4*(7*a^6*b^4
 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^5 + 8*(a^7*b^3 + 3*a^5*b^3 + 3*a^3*b^3 +
 a*b^3)*x^4 + (a^8*b^2 + 4*a^6*b^2 + 6*a^4*b^2 + 4*a^2*b^2 + b^2)*x^3 + (b^
6*x^7 + 4*a*b^5*x^6 + 6*a^2*b^4*x^5 + 4*a^3*b^3*x^4 + a^4*b^2*x^3)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^2 + 4*(b^7*x^8 + 5*a*b^6*x^7 + (10*a^2*b^5 + b^5)*x^6
 + (10*a^3*b^4 + 3*a*b^4)*x^5 + (5*a^4*b^3 + 3*a^2*b^3)*x^4 + (a^5*b^2 + a^
3*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^9 + 6*a*b^7*x^8
+ (15*a^2*b^6 + 2*b^6)*x^7 + 4*(5*a^3*b^5 + 2*a*b^5)*x^6 + (15*a^4*b^4 + 12
*a^2*b^4 + b^4)*x^5 + 2*(3*a^5*b^3 + 4*a^3*b^3 + a*b^3)*x^4 + (a^6*b^2 + 2*
a^4*b^2 + a^2*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(b^9*x^10 + 7*a*b
^8*x^9 + 3*(7*a^2*b^7 + b^7)*x^8 + 5*(7*a^3*b^6 + 3*a*b^6)*x^7 + (35*a^4*b^
5 + 30*a^2*b^5 + 3*b^5)*x^6 + 3*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^5 + (7
*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3)*x^4 + (a^7*b^2 + 3*a^5*b^2 + 3*a^3
*b^2 + a*b^2)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \operatorname{arsinh}(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(b*x + a)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a)**3,x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arsinh}(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)^3), x)
```

3.93 $\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(x^m \left(a+b\sinh^{-1}(c+dx)\right)^n,x\right)$$

```
[Out] Unintegrable[x^m*(a + b*ArcSinh[c + d*x])^n, x]
```

Rubi [A] time = 0.0534858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Defer[Subst][Defer[Int][(-(c/d) + x/d)^m*(a + b*ArcSinh[x])^n, x], x, c + d
*x]/d
```

Rubi steps

$$\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx=\frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)^m \left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 0.418615, size = 0, normalized size = 0.

$$\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n, x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int x^m \left(a+b\text{Arcsinh}\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx+c)+a)^n x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{arsinh}(dx+c)+a)^n x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^m (a + b \operatorname{asinh}(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(d*x+c))**n,x)
```

```
[Out] Integral(x**m*(a + b*asinh(c + d*x))**n, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Timed out
```

3.94 $\int x^2 \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=545

$$\frac{c^2 e^{-\frac{a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^3}-\frac{c^2 e^{a/b} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(\frac{a+b}{\ }\right.}{}$$

```
[Out] (3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c +
d*x]))/b])/(8*d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (2^(-2 -
 n)*c*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))
/b])/(d^3*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - ((a + b*ArcSinh[
c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(8*d^3*E^(a/b)*(-(
(a + b*ArcSinh[c + d*x])/b))^n) + (c^2*(a + b*ArcSinh[c + d*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^3*E^(a/b)*(-((a + b*ArcSinh[c + d*
x])/b))^n) + (E^(a/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSin
h[c + d*x])/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)^n) - (c^2*E^(a/b)*(a +
b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^3*((a
+ b*ArcSinh[c + d*x])/b)^n) - (2^(-2 - n)*c*E^((2*a)/b)*(a + b*ArcSinh[c +
d*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^3*((a + b*ArcSinh[
c + d*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[
1 + n, (3*(a + b*ArcSinh[c + d*x]))/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)
^n)
```

Rubi [A] time = 1.15162, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5865, 5805, 6741, 12, 6742, 3307, 2181, 5448, 3308}

$$\frac{c^2 e^{-\frac{a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^3}-\frac{c^2 e^{a/b} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(\frac{a+b}{\ }\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```


```
[Out] (3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c +
d*x]))/b])/(8*d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (2^(-2 -
 n)*c*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))
/b])/(d^3*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - ((a + b*ArcSinh[
c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(8*d^3*E^(a/b)*(-(
(a + b*ArcSinh[c + d*x])/b))^n) + (c^2*(a + b*ArcSinh[c + d*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^3*E^(a/b)*(-((a + b*ArcSinh[c + d*
x])/b))^n) + (E^(a/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSin
h[c + d*x])/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)^n) - (c^2*E^(a/b)*(a +
b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^3*((a
+ b*ArcSinh[c + d*x])/b)^n) - (2^(-2 - n)*c*E^((2*a)/b)*(a + b*ArcSinh[c +
d*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^3*((a + b*ArcSinh[
c + d*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[
1 + n, (3*(a + b*ArcSinh[c + d*x]))/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)
^n)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int x^2\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)^2\left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\frac{(a+bx)^n\cosh(x)(c-\sinh(x))^2}{d^2}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)(c-\sinh(x))^2\,dx,x,\sinh^{-1}(c+dx)\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int\left(c^2(a+bx)^n\cosh(x)-2c(a+bx)^n\cosh(x)\sinh(x)+(a+bx)^n\cosh(x)\sinh\right.\right.}{d^3}$$

$$= \frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^2(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^3} - \frac{(2c)\,\text{Subst}\left(\int(a+bx)^n\,\text{c}\right.}{}$$

$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)dx,x,\sinh^{-1}(c+dx)\right)}{d^3} - \frac{(2c)}{}$$

$$= \frac{c^2e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^3} - \frac{c^2e^{a/b}\,(}{}$$

$$= \frac{c^2e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^3} - \frac{c^2e^{a/b}\,(}{}$$

$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8d^3} -$$

Mathematica [A] time = 0.966835, size = 345, normalized size = 0.63

$$\frac{2^{-n-3}3^{-n-1}e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(\left(4c^2-1\right)2^n3^{n+1}e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\text{Gamma}\left(n+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*(-(2^n*3^(1 + n)*(-1 + 4*
c^2)*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSi
nh[c + d*x]]) + 2^n*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c + d*x]))/b] - 2*3^(1 + n)*c*E^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[
1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*3^(1 + n)*(-1 + 4*c^2)*E^((2*
a)/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)
] - E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*(2*3^(1 + n)*c*Gamma[1 +
n, (2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*E^(a/b)*Gamma[1 + n, (3*(a + b*Arc
Sinh[c + d*x]))/b])))/(d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^
n)
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int x^2\left(a+b\text{Arcsinh}\,(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx+c)+a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{arsinh}(dx+c)+a)^n x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a+b \operatorname{asinh}(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**n,x)
```

```
[Out] Integral(x**2*(a + b*asinh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx+c)+a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

3.95 $\int x \left(a+b\sinh^{-1}(c+dx)\right)^n dx$

Optimal. Leaf size=267

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)}{}$$

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c +
d*x]))/b])/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (c*(a + b*
ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^2*E^(
a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) + (c*E^(a/b)*(a + b*ArcSinh[c + d*x
])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^2*((a + b*ArcSinh[c + d
*x])/b)^n) + (2^(-3 - n)*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n
, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^2*((a + b*ArcSinh[c + d*x])/b)^n)
```

Rubi [A] time = 0.483293, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5865, 5805, 6741, 12, 6742, 3307, 2181, 5448, 3308}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c +
d*x]))/b])/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (c*(a + b*
ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^2*E^(
a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) + (c*E^(a/b)*(a + b*ArcSinh[c + d*x
])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^2*((a + b*ArcSinh[c + d
*x])/b)^n) + (2^(-3 - n)*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n
, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^2*((a + b*ArcSinh[c + d*x])/b)^n)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$
$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^n\cosh(x)(-c+\sinh(x))}{d}dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)(-c+\sinh(x))\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}$$
$$=\frac{\text{Subst}\left(\int\left(-c(a+bx)^n\cosh(x)+(a+bx)^n\cosh(x)\sinh(x)\right)dx,x,\sinh^{-1}(c+dx)\right)}{d^2}$$
$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}-\frac{c\,\text{Subst}\left(\int(a+bx)^n\cosh(\right.}{d^2}$$
$$=\frac{\text{Subst}\left(\int\frac{1}{2}(a+bx)^n\sinh(2x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}-\frac{c\,\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx,x,\text{si}\right.}{2d^2}$$
$$=-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^2}+\frac{ce^{a/b}\left(a\right.}{}$$
$$=-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^2}+\frac{ce^{a/b}\left(a\right.}{}$$
$$=\frac{2^{-3-n}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\cdots$$

Mathematica [A] time = 0.179955, size = 228, normalized size = 0.85

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-c2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*(2^(2 + n)*c*E^((3*a)/b)*(-((a + b*A
rcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSinh[c + d*x]] + (a/b + ArcSin
h[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] - 2^(2 + n)*c*E
^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b
)] + E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, (2*(a + b*A
rcSinh[c + d*x]))/b]))/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))
^n)
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int x\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx+c)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{arsinh}(dx+c)+a)^n x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{asinh}(c + dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**n,x)
```

```
[Out] Integral(x*(a + b*asinh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx+c)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

3.96 $\int \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=128

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}-\frac{e^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\frac{a+b\sinh}{b}\right.}{}$$

```
[Out] ((a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2
*d*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (E^(a/b)*(a + b*ArcSinh[c +
 d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d*((a + b*ArcSinh[c +
 d*x])/b)^n)
```

Rubi [A] time = 0.123552, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5657, 3307, 2181}

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}-\frac{e^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\frac{a+b\sinh}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^n,x]
```


```
[Out] ((a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2
*d*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (E^(a/b)*(a + b*ArcSinh[c +
 d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d*((a + b*ArcSinh[c +
 d*x])/b)^n)
```


Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^n\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int x^n\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx, x, a+b\,\text{s}\right.}{2bd}$$
$$= \frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d} - \frac{e^{a/b}\left(a+\right.}{}$$

Mathematica [A] time = 0.113041, size = 109, normalized size = 0.85

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^{-n}\text{Gam}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*(-((E^((2*a)/b)*Gamma[1 + n, a/b + ArcSinh[c +
d*x]])/(a/b + ArcSinh[c + d*x])^n) + Gamma[1 + n, -((a + b*ArcSinh[c + d*x]
)/b)]/(-((a + b*ArcSinh[c + d*x])/b))^n))/(2*d*E^(a/b))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\,\text{arsinh}\,(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\,\text{asinh}\,(c+dx))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**n,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}\,(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

3.97 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c + d*x])^n/x, x]
```

Rubi [A] time = 0.0595191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^n/x,x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^n/(-(c/d) + x/d), x], x, c + d*x
]/d
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^n}{-\frac{c}{d}+\frac{x}{d}}\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 0.170924, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n/x,x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^n/x, x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(a+b\text{Arcsinh}\,(dx+c))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^n/x,x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \operatorname{arsinh}(dx+c)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b \operatorname{asinh}(c+dx))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**n/x,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**n/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

3.98 $\int x^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=496

$$\frac{\sqrt{\pi}\sqrt{b}c^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}-\frac{\sqrt{\pi}\sqrt{b}c^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}+\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}-\frac{\sqrt{\pi}\sqrt{b}e}{}$$

```
[Out] (c^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^3 + ((c + d*x)^3*Sqrt[a + b*
ArcSinh[c + d*x]])/(3*d^3) - (c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh
[c + d*x]])/(2*d^3) - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d^3) + (Sqrt[b]*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(4*d^3) + (Sqrt[b]*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*d^3) + (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3)
 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^3*E^
(a/b)) - (Sqrt[b]*c^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/
(4*d^3*E^(a/b)) + (Sqrt[b]*c*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(8*d^3*E^((2*a)/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3*E^((3*a)/b))
```

Rubi [A] time = 1.8469, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5805, 6741, 6742, 5325, 5298, 2205, 2204, 5324, 5299, 5372, 5300}

$$\frac{\sqrt{\pi}\sqrt{b}c^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}-\frac{\sqrt{\pi}\sqrt{b}c^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}+\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}-\frac{\sqrt{\pi}\sqrt{b}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] (c^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^3 + ((c + d*x)^3*Sqrt[a + b*
ArcSinh[c + d*x]])/(3*d^3) - (c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh
[c + d*x]])/(2*d^3) - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d^3) + (Sqrt[b]*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(4*d^3) + (Sqrt[b]*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*d^3) + (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3)
 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^3*E^
(a/b)) - (Sqrt[b]*c^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/
(4*d^3*E^(a/b)) + (Sqrt[b]*c*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(8*d^3*E^((2*a)/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5300

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\int x^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2 dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2 dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2 dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int\left(c^2x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+cx^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\right.\right.\right.}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}+\frac{(2c)\,\text{Su}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\sqrt{a+b\sinh}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\sqrt{a+b\sinh}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\sqrt{a+b\sinh}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\sqrt{a+b\sinh}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\sqrt{a+b\sinh}}{}$$

Mathematica [A] time = 1.80739, size = 656, normalized size = 1.32

$$\frac{9\sqrt{\pi}\sqrt{b}\left(4c^2-1\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+36\sqrt{\pi}\sqrt{b}c^2\sinh\left(\frac{a}{b}\right)\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-36\sqrt{\pi}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-36*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] + 144*c^2*(c + d*x)*Sqrt[a + b*
ArcSinh[c + d*x]] - 72*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*
x]] + Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]] + 9*Sqrt[b]*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]] - 36*Sqrt[b]*c^2*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - 9*Sqrt[b]*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 36*Sqrt[b]*c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 9*Sqrt[b]*(-1 + 4*c^2
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]
) - 9*Sqrt[b]*c*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*Sinh[(2*a)/b] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[b]*Sqrt[3*Pi]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] + Sqrt[b]*
Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/
b] + 12*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[3*ArcSinh[c + d*x]])/(144*d^3)
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int x^2 \sqrt{a + b\mathrm{Arcsinh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arsinh}\,(dx + c) + a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

3.99 $\int x\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{\sqrt{\pi}\sqrt{b}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}+\frac{\sqrt{\pi}\sqrt{b}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\cdots}}{\cdots}$$

```
[Out] -((c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^2) + (Sqrt[a + b*ArcSinh[c +
 d*x]]*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2) - (Sqrt[b]*E^((2*a)/b)*Sqrt[
Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d^2) + (Sqrt
[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2*E^(a/b))
- (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(16*d^2*E^((2*a)/b))
```

Rubi [A] time = 0.699855, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5805, 6741, 6742, 5325, 5298, 2205, 2204, 5324, 5299}

$$-\frac{\sqrt{\pi}\sqrt{b}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}+\frac{\sqrt{\pi}\sqrt{b}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -((c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^2) + (Sqrt[a + b*ArcSinh[c +
 d*x]]*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2) - (Sqrt[b]*E^((2*a)/b)*Sqrt[
Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d^2) + (Sqrt
[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2*E^(a/b))
- (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(16*d^2*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int x\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int\sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= -\frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$
$$= -\frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$
$$= -\frac{2\,\text{Subst}\left(\int\left(cx^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$
$$= -\frac{\text{Subst}\left(\int x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}\right.\right.}{}$$
$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c+dx)\right)}{4d^2} - \frac{\text{Su}}{}$$
$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c+dx)\right)}{4d^2} - \frac{\text{Su}}{}$$
$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c+dx)\right)}{4d^2} - \frac{\sqrt{}}{}$$

Mathematica [A] time = 1.76584, size = 251, normalized size = 0.97

$$\frac{-16ce^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)-\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (8*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - (16*c*Sqrt[a + b
*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c + d*x]])/Sqrt
[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)]/Sqrt[
-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b]) -
Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cos
h[(2*a)/b] + Sinh[(2*a)/b]))/(32*d^2)
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int x\sqrt{a+b\text{Arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}(dx+c)+a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a+b \operatorname{asinh}(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}(dx+c)+a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

3.100 $\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rubi [A] time = 0.24912, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} - \frac{b\,\text{Subst}\left(\int \ldots\right.}{}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2d} - \frac{\text{Sub}\ldots}{}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a\ldots}}{}\right.}{4d}$$

Mathematica [A] time = 0.0737752, size = 111, normalized size = 0.97

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}} - \frac{e^{\frac{2a}{b}}\,\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \sqrt{a + b\text{Arcsinh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\text{asinh}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

3.101 $\int x\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=326

$$-\frac{3\sqrt{\pi}b^{3/2}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d^2}-\frac{3\sqrt{\pi}b^{3/2}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}+\ldots$$

```
[Out] (3*b*c*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d^2) - (c*(c
+ d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/d^2 + ((a + b*ArcSinh[c + d*x])^(3/2
)*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (3*b^(3/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2) - (3*b^(3/2)*E^((2*a)/b)*Sqrt[P
i/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d^2) - (3*b^(
3/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2*E^(a/b))
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(64*d^2*E^((2*a)/b)) - (3*b*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[2*ArcSinh
[c + d*x]])/(16*d^2)
```

Rubi [A] time = 0.964722, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5805, 6741, 6742, 5325, 5324, 5299, 2205, 2204, 5298}

$$-\frac{3\sqrt{\pi}b^{3/2}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d^2}-\frac{3\sqrt{\pi}b^{3/2}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (3*b*c*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d^2) - (c*(c
+ d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/d^2 + ((a + b*ArcSinh[c + d*x])^(3/2
)*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (3*b^(3/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2) - (3*b^(3/2)*E^((2*a)/b)*Sqrt[P
i/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d^2) - (3*b^(
3/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2*E^(a/b))
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(64*d^2*E^((2*a)/b)) - (3*b*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[2*ArcSinh
[c + d*x]])/(16*d^2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^{3/2}dx,x,c+dx\right)}{d}$$
$$=\frac{\text{Subst}\left(\int(a+bx)^{3/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$=-\frac{2\,\text{Subst}\left(\int x^4\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$
$$=-\frac{2\,\text{Subst}\left(\int x^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$
$$=-\frac{2\,\text{Subst}\left(\int\left(cx^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+d}\right.}{bd^2}$$
$$=-\frac{\text{Subst}\left(\int x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}-\frac{(2c)\,\text{Subst}\left(\int x^4\,\text{co}\right.}{}$$
$$=-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\cosh\left(2\sinh^{-1}(c\right.}{4d^2}$$
$$=\frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+$$
$$=\frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+$$
$$=\frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+$$

Mathematica [A] time = 5.19937, size = 582, normalized size = 1.79

$$-64ace^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)-16\sqrt{b}c\left(\sqrt{\pi}(3b-2a)\left(\sinh\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] ((-64*a*c*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + Arc
Sinh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh
[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) - 16*Sqrt[b]*c
*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c +
 d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 4*a*(8*Sqrt[a + b*
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b])
- Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(C
osh[(2*a)/b] + Sinh[(2*a)/b])) + Sqrt[b]*((4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b])
+ (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
```

```
]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*
(4*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] - 3*Sinh[2*ArcSinh[c + d*x]]))
)/(128*d^2)
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int x\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)

[Out] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(3/2),x)

[Out] Integral(x*(a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

3.102 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+$$

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rubi [A] time = 0.248604, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{3b}{}$$

Mathematica [A] time = 0.141323, size = 272, normalized size = 1.81

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\left(\sqrt{\pi}(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\cosh\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{asinh}(c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.103 $\int x \left(a + b \sinh^{-1}(c + dx)\right)^{5/2} dx$

Optimal. Leaf size=389

$$-\frac{15\sqrt{\pi}b^{5/2}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d^2}+\frac{15\sqrt{\pi}b^{5/2}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-1$$

```
[Out] (-15*b^2*c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d^2) + (5*b*c*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d^2) - (c*(c + d*x)*(a +
b*ArcSinh[c + d*x])^(5/2))/d^2 + (15*b^2*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[
2*ArcSinh[c + d*x]])/(64*d^2) + ((a + b*ArcSinh[c + d*x])^(5/2)*Cosh[2*ArcS
inh[c + d*x]])/(4*d^2) - (15*b^(5/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcS
inh[c + d*x]]/Sqrt[b]])/(16*d^2) - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d^2) + (15*b^(5/2)*c*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^2*E^(a/b)) - (15*
b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2
56*d^2*E^((2*a)/b)) - (5*b*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[2*ArcSinh[c
+ d*x]])/(16*d^2)
```

Rubi [A] time = 1.12634, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5805, 6741, 6742, 5325, 5324, 5298, 2205, 2204, 5299}

$$-\frac{15\sqrt{\pi}b^{5/2}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d^2}+\frac{15\sqrt{\pi}b^{5/2}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-1$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (-15*b^2*c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d^2) + (5*b*c*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d^2) - (c*(c + d*x)*(a +
b*ArcSinh[c + d*x])^(5/2))/d^2 + (15*b^2*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[
2*ArcSinh[c + d*x]])/(64*d^2) + ((a + b*ArcSinh[c + d*x])^(5/2)*Cosh[2*ArcS
inh[c + d*x]])/(4*d^2) - (15*b^(5/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcS
inh[c + d*x]]/Sqrt[b]])/(16*d^2) - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d^2) + (15*b^(5/2)*c*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^2*E^(a/b)) - (15*
b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2
56*d^2*E^((2*a)/b)) - (5*b*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[2*ArcSinh[c
+ d*x]])/(16*d^2)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
```

```
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^{5/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{2\,\text{Subst}\left(\int x^6\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$=-\frac{2\,\text{Subst}\left(\int x^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$=-\frac{2\,\text{Subst}\left(\int\left(cx^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$=-\frac{\text{Subst}\left(\int x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}-\frac{(2c)\,\text{Subst}\left(\int x^6\cosh\right.}{}$$

$$=-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d^2}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\cosh\left(2\sinh^{-1}(c+d\right.}{4d^2}$$

$$=\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d^2}+$$

$$=-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2}$$

$$=-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2}$$

$$=-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2}$$

Mathematica [B] time = 9.2564, size = 939, normalized size = 2.41

$$480c\sqrt{\pi}\cosh\left(\frac{a}{b}\right)\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)b^{5/2}-15\sqrt{2\pi}\cosh\left(\frac{2a}{b}\right)\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)b^{5/2}-480c\sqrt{\pi}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(\right.}{\sqrt{b}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-1920*b^2*c^2*Sqrt[a + b*ArcSinh[c + d*x]] - 1920*b^2*c*d*x*Sqrt[a + b*Arc
Sinh[c + d*x]] + 1280*a*b*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*Sqrt[a + b*Ar
cSinh[c + d*x]] - 1024*a*b*c^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]
] - 1024*a*b*c*d*x*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]] + 1280*b^2
*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c
+ d*x]] - 512*b^2*c^2*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] - 512
*b^2*c*d*x*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] + 128*a^2*Sqrt[a
 + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 120*b^2*Sqrt[a + b*ArcSin
h[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 256*a*b*ArcSinh[c + d*x]*Sqrt[a + b*
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 128*b^2*ArcSinh[c + d*x]^2*Sqr
```

```
t[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - 128*a^2*Sqrt[b]*c*Sqrt
[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 480*b^(5/2)*c*S
qrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 15*b^(5/2)*S
qrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
] + (256*a^2*b*c*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]] + (256*a^2*b*c*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)])/(E^(a/b)*Sqr
t[a + b*ArcSinh[c + d*x]]) + 128*a^2*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 480*b^(5/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 32*Sqrt[b]*(4*a^2 - 15*b^2)*c*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 15*b
^(5/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh
[(2*a)/b] - 15*b^(5/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 160*a*b*Sqrt[a + b*ArcSinh[c +
 d*x]]*Sinh[2*ArcSinh[c + d*x]] - 160*b^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSi
nh[c + d*x]]*Sinh[2*ArcSinh[c + d*x]])/(512*d^2)
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int x\left(a+b\mathrm{Arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{arsinh}\left(dx+c\right)+a\right)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

3.104 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-5b$$

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rubi [A] time = 0.388462, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-5b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d}-\frac{(5b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2d}$$

$$=-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d}+\frac{(15}{}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c}{}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c}{}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c}{}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c}{}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c}{}$$

Mathematica [B] time = 1.43188, size = 458, normalized size = 2.56

$$8a^2e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)+\sqrt{b}\left(\sqrt{\pi}\left(4a^2-12ab+15b^2\right)(\ldots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.105 $\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=411

$$\frac{\sqrt{\pi}c^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}+\frac{\sqrt{\pi}c^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}+\frac{\sqrt{\frac{\pi}{2}}ce^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\ldots\right)}{8\sqrt{\ldots}}$$

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^
3) + (c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sq
rt[b]*d^3) + (c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^3) + (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d^3) - (Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^3*E^(a/b)) + (c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d^3*E^(a/b)) - (c*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^
3*E^((2*a)/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sq
rt[b]])/(8*Sqrt[b]*d^3*E^((3*a)/b))
```

Rubi [A] time = 0.856515, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5865, 5805, 6741, 6742, 5299, 2205, 2204, 5298, 5618}

$$\frac{\sqrt{\pi}c^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}+\frac{\sqrt{\pi}c^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}+\frac{\sqrt{\frac{\pi}{2}}ce^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3}-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\ldots\right)}{8\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^
3) + (c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sq
rt[b]*d^3) + (c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^3) + (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d^3) - (Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^3*E^(a/b)) + (c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d^3*E^(a/b)) - (c*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^
3*E^((2*a)/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sq
rt[b]])/(8*Sqrt[b]*d^3*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{c}{d}+\frac{x}{d}\right)^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(c^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+c\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)dx,\right.}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3} + \frac{(2c)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{1}{4}\cosh\left(\frac{3a}{b}-\frac{3x^2}{b}\right)-\frac{1}{4}\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3} + \frac{c\,\text{S}}{}$$

$$= \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a-}}{}\right.}{2\sqrt{b}d}$$

$$= \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a-}}{}\right.}{2\sqrt{b}d}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.}{2\sqrt{b}d^3}$$

Mathematica [A] time = 1.04065, size = 471, normalized size = 1.15

$$\frac{\sqrt{\pi}\left(3\left(4c^2-1\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-12c^2\sinh\left(\frac{a}{b}\right)\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+12c^2\cosh\left(\frac{a}{b}\right)\text{E}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[3]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]] - 3*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 12*c^2
*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 6*Sqrt[2]*c*Cosh[(2
*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + Sqrt[3]*Cosh[
(3*a)/b]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 3*Erfi[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 12*c^2*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 3*(-1 + 4*c^2)*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 6*Sqrt[2]*c*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + 6*Sqrt[2]*c*Erf[(Sqrt[2]*Sqr
```

```
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[
3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] - Sqrt
[3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b]))/(2
4*Sqrt[b]*d^3)
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a + b\mathrm{Arcsinh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b\,\mathrm{arsinh}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.106 $\int \frac{x}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}-\frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}$$

```
[Out] -(c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*
d^2) - (E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(4*Sqrt[b]*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sq
rt[b]])/(2*Sqrt[b]*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(4*Sqrt[b]*d^2*E^((2*a)/b))
```

Rubi [A] time = 0.391827, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5865, 5805, 6741, 6742, 5299, 2205, 2204, 5298}

$$-\frac{\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}-\frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] -(c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*
d^2) - (E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(4*Sqrt[b]*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sq
rt[b]])/(2*Sqrt[b]*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(4*Sqrt[b]*d^2*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{2\operatorname{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$= -\frac{2\operatorname{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$= -\frac{2\operatorname{Subst}\left(\int \left(c\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$= -\frac{\operatorname{Subst}\left(\int \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\operatorname{Subst}\left(\int \cosh\left(\frac{a}{b}-\right.\right.}{}$$

$$= -\frac{\operatorname{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd^2} + \frac{\operatorname{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{si}\right.}{2bd^2}$$

$$= -\frac{ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2} - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b}\right.}{}}{2\sqrt{b}d^2}$$

Mathematica [A] time = 0.959589, size = 217, normalized size = 1.06

$$\frac{\frac{e^{-\frac{a}{b}}\left(4ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-4c\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{\sqrt{2\pi}\left(\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+}}{}\right.\right.}{}}{8d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((4*c*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c +
 d*x]] - 4*c*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSin
h[c + d*x])/b)])/(E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]]) - (Sqrt[2*Pi]*(Erfi
[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*
a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 + Sinh[(2*a)/b])))/Sqrt[b])/(8*d^2)
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+b\text{Arcsinh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b\,\text{arsinh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{b \operatorname{arsinh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.107 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rubi [A] time = 0.128954, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5863, 5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(\right.}{2bd}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-}}\right.}{bd}$$
$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Mathematica [A] time = 0.107971, size = 111, normalized size = 1.21

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)}{2d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\text{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b \operatorname{asinh}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.108 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}-\frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{Erfi}\left(\sqrt{2}\cdots\right.}{b^{3/2}\cdots}$$

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (c*E^(a/
b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d^2) + (E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(b
^(3/2)*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3
/2)*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(b^(3/2)*d^2*E^((2*a)/b))
```

Rubi [A] time = 0.542365, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5803, 5655, 5779, 3308, 2180, 2204, 2205, 5665, 3307}

$$\frac{\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}-\frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{Erfi}\left(\sqrt{2}\cdots\right.}{b^{3/2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (c*E^(a/
b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d^2) + (E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(b
^(3/2)*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3
/2)*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(b^(3/2)*d^2*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{3/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{3/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \right.}{bd^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sin\right.}{bd^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \right.}{b^2 d}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c}}{\sqrt{b}}\right.}{b^{3/2}d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+}}{\sqrt{b}}\right.}{b^{3/2}d^2}$$

Mathematica [A] time = 2.55441, size = 301, normalized size = 1.12

$$\frac{\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-\frac{2\sqrt{b}}{}}{2b^{3/2}d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -((c*(-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*
x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^Arc
Sinh[c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSi
nh[c + d*x])/b)]))/(b*d^2*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh
[c + d*x]])) + (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*Sin
h[2*ArcSinh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]])/(2*b^(3/2)*d^2)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int x\left(a+b\mathrm{Arcsinh}\left(dx+c\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\,\mathrm{arsinh}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\,\mathrm{arsinh}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.109 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rubi [A] time = 0.238905, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\int \frac{e}{\sqrt{a}}\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{b^2d} + \frac{2\,\text{S}}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{b^{3/2}d}$$

Mathematica [A] time = 0.0632539, size = 155, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e^{\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{G}\right.}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(dx + c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

3.110 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=365

$$-\frac{2\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}+\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\text{Erfi}}{}$$

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) -
4/(3*b^2*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (4*c*(c + d*x))/(3*b^2*d^2*Sqr
t[a + b*ArcSinh[c + d*x]]) - (8*(c + d*x)^2)/(3*b^2*d^2*Sqrt[a + b*ArcSinh[
c + d*x]]) - (2*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(3*b^(5/2)*d^2) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2) - (2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d^2*E^(a/b)) + (2*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2*E^((2*a)/b))
```

Rubi [A] time = 0.882786, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {5865, 5803, 5655, 5774, 5657, 3307, 2180, 2205, 2204, 5667, 5669, 5448, 12, 3308, 5675}

$$-\frac{2\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}+\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\text{Erfi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) -
4/(3*b^2*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (4*c*(c + d*x))/(3*b^2*d^2*Sqr
t[a + b*ArcSinh[c + d*x]]) - (8*(c + d*x)^2)/(3*b^2*d^2*Sqrt[a + b*ArcSinh[
c + d*x]]) - (2*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(3*b^(5/2)*d^2) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2) - (2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d^2*E^(a/b)) + (2*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{5/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{5/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}\ldots}\right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\sinh\ldots}}$$

Mathematica [A] time = 2.6056, size = 375, normalized size = 1.03

$$\frac{\sqrt{b}ce^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(2be^{\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+2e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\text{Gar}\ldots\right.}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((Sqrt[b]*c*(E^(a/b)*(-2*a + b + 2*a*E^(2*ArcSinh[c + d*x]) + b*E^(2*ArcSin
h[c + d*x]) + 2*b*(-1 + E^(2*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 2*E^((2
*a)/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d
*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] + 2*b*E^ArcSinh[c + d*x]*(-((a + b*
ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/(E^
((a + b*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2)) + 2*Sqrt[2*Pi]
*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh
[(2*a)/b]) - 2*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (Sqrt[b]*(4*(a + b*ArcSinh[c + d*x])*C
osh[2*ArcSinh[c + d*x]] + b*Sinh[2*ArcSinh[c + d*x]]))/(a + b*ArcSinh[c + d
*x])^(3/2))/(3*b^(5/2)*d^2)
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int x\left(a+b\mathrm{Arcsinh}\left(dx+c\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\,\mathrm{arsinh}\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.111 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}}{3bd\left(a+b\sinh^{-1}(c+dx\right.}$$

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rubi [A] time = 0.275583, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5657, 3307, 2180, 2205, 2204}

$$\frac{2\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}}{3bd\left(a+b\sinh^{-1}(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(}\right.}{3b^2d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,d\right.}{3}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx\right.}{3}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x\right.}{3}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{3b^{5/2}d}$$

Mathematica [A] time = 0.553154, size = 207, normalized size = 1.31

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-2be^{\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-2e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

3.112 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=445

$$\frac{4\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}-\frac{4\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\text{Erfi}(\ldots)}{15\ldots}$$

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) -
4/(15*b^2*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*c*(c + d*x))/(15*b^2*d^2
*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*(c + d*x)^2)/(15*b^2*d^2*(a + b*ArcSi
nh[c + d*x])^(3/2)) + (8*c*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a + b*Ar
cSinh[c + d*x]]) - (32*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a
+ b*ArcSinh[c + d*x]]) + (4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(15*b^(7/2)*d^2) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2) - (4*c*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d^2*E^(a/b)) + (8*Sqrt[
2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2
*E^((2*a)/b))
```

Rubi [A] time = 1.04675, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5865, 5803, 5655, 5774, 5779, 3308, 2180, 2204, 2205, 5667, 5665, 3307, 5675}

$$\frac{4\sqrt{\pi}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}-\frac{4\sqrt{\pi}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\text{Erfi}(\ldots)}{15\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) -
4/(15*b^2*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*c*(c + d*x))/(15*b^2*d^2
*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*(c + d*x)^2)/(15*b^2*d^2*(a + b*ArcSi
nh[c + d*x])^(3/2)) + (8*c*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a + b*Ar
cSinh[c + d*x]]) - (32*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a
+ b*ArcSinh[c + d*x]]) + (4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(15*b^(7/2)*d^2) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2) - (4*c*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d^2*E^(a/b)) + (8*Sqrt[
2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2
*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{7/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{7/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}}\right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4}{15b^2d^2\left(a+b\sinh\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4}{15b^2d^2\left(a+b\sinh\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4}{15b^2d^2\left(a+b\sinh\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4}{15b^2d^2\left(a+b\sinh\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4}{15b^2d^2\left(a+b\sinh\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4}{15b^2d^2\left(a+b\sinh\right.}$$

Mathematica [A] time = 2.50408, size = 429, normalized size = 0.96

$$\frac{\frac{\sqrt{b}\left(-\sinh\left(2\sinh^{-1}(c+dx)\right)\left(16\left(a+b\sinh^{-1}(c+dx)\right)^2+3b^2\right)-4b\cosh\left(2\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)\right)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + 8\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{Erf}}{15b^{7/2}d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -(c*(-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]
) + b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x
] + 8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma
[1/2, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSi
```

```
nh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d
*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b)))/(30*b^
3*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + 8*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Si
nh[(2*a)/b]) + (Sqrt[b]*(-4*b*(a + b*ArcSinh[c + d*x])*Cosh[2*ArcSinh[c + d
*x]] - (3*b^2 + 16*(a + b*ArcSinh[c + d*x])^2)*Sinh[2*ArcSinh[c + d*x]]))/(
a + b*ArcSinh[c + d*x])^(5/2))/(15*b^(7/2)*d^2)
```

Maple [F] time = 0.156, size = 0, normalized size = 0.

$$\int x\,(a+b\mathrm{Arcsinh}\,(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b \operatorname{arsinh}\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.113 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{(c+dx)^2}}{15b^3d\sqrt{a+b\sinh}}$$

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rubi [A] time = 0.41112, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{(c+dx)^2}}{15b^3d\sqrt{a+b\sinh}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```


```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```


Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{5bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{4\,\text{Subst}\left(\int \frac{1}{(a+b\sinh}\right.}{15}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c+d}}{15b^3d\sqrt{a+b\sinh^{-}}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c+d}}{15b^3d\sqrt{a+b\sinh^{-}}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c+d}}{15b^3d\sqrt{a+b\sinh^{-}}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c+d}}{15b^3d\sqrt{a+b\sinh^{-}}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c+d}}{15b^3d\sqrt{a+b\sinh^{-}}}$$

Mathematica [A] time = 0.112945, size = 238, normalized size = 1.22

$$8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right) - 4e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(2b\left(-\frac{a}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\,(dx+c)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

3.114 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=91

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b(e(c+dx))^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2])/
(d*e^2*(1 + m)*(2 + m))
```

Rubi [A] time = 0.0728582, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5865, 5661, 364}

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b(e(c+dx))^{m+2}{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2])/
(d*e^2*(1 + m)*(2 + m))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{1+x^2}} dx, x, c+dx\right)}{de(1+m)}$$
$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b(e(c+dx))^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\right.}{de^2(1+m)(2+m)}$$

Mathematica [A] time = 0.0437118, size = 79, normalized size = 0.87

$$-\frac{(c+dx)(e(c+dx))^m \left(b(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-(c+dx)^2\right)-(m+2)\left(a+b\sinh^{-1}(c+dx)\right)\right)}{d(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(e*(c + d*x))^m*(-((2 + m)*(a + b*ArcSinh[c + d*x])) + b*(c +
d*x)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2]))/(d*(1 + m
)*(2 + m)))
```

Maple [F] time = 1.579, size = 0, normalized size = 0.

$$\int (dex+ce)^m (a+b\text{Arcsinh}(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\,\text{arsinh}(dx+c)+a)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c+dx))^m (a+b\operatorname{asinh}(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(dx+c)+a)(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

3.115 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=100

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d}-\frac{be^4\left((c+dx)^2+1\right)^{5/2}}{25d}+\frac{2be^4\left((c+dx)^2+1\right)^{3/2}}{15d}-\frac{be^4\sqrt{(c+dx)^2+1}}{5d}$$

```
[Out] -(b*e^4*Sqrt[1 + (c + d*x)^2])/(5*d) + (2*b*e^4*(1 + (c + d*x)^2)^(3/2))/(1
5*d) - (b*e^4*(1 + (c + d*x)^2)^(5/2))/(25*d) + (e^4*(c + d*x)^5*(a + b*Arc
Sinh[c + d*x]))/(5*d)
```

Rubi [A] time = 0.0789751, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 266, 43}

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d}-\frac{be^4\left((c+dx)^2+1\right)^{5/2}}{25d}+\frac{2be^4\left((c+dx)^2+1\right)^{3/2}}{15d}-\frac{be^4\sqrt{(c+dx)^2+1}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(b*e^4*Sqrt[1 + (c + d*x)^2])/(5*d) + (2*b*e^4*(1 + (c + d*x)^2)^(3/2))/(1
5*d) - (b*e^4*(1 + (c + d*x)^2)^(5/2))/(25*d) + (e^4*(c + d*x)^5*(a + b*Arc
Sinh[c + d*x]))/(5*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}} dx, x, c+dx\right)}{5d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}} dx, x, (c+dx)^2\right)}{10d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}} - 2\sqrt{1+x} + (1+x\right.\right.}{10d}$$
$$= -\frac{be^4\sqrt{1+(c+dx)^2}}{5d} + \frac{2be^4\left(1+(c+dx)^2\right)^{3/2}}{15d} - \frac{be^4\left(1+(c+dx)^2\right)^{5/2}}{25d} + \frac{e^4(c+}{}$$

Mathematica [A] time = 0.0977003, size = 71, normalized size = 0.71

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right) - \frac{1}{75}b\sqrt{(c+dx)^2+1}\left(-10(c+dx)^2+3\left((c+dx)^2+1\right)^2+5\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*(-(b*Sqrt[1 + (c + d*x)^2]*(5 - 10*(c + d*x)^2 + 3*(1 + (c + d*x)^2)^2
))/75 + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5))/d
```

Maple [A] time = 0.01, size = 93, normalized size = 0.9

$$\frac{1}{d}\left(\frac{(dx+c)^5 e^4 a}{5} + e^4 b\left(\frac{(dx+c)^5 \operatorname{Arcsinh}(dx+c)}{5} - \frac{(dx+c)^4}{25}\sqrt{1+(dx+c)^2} + \frac{4\,(dx+c)^2}{75}\sqrt{1+(dx+c)^2} - \frac{8}{75}\sqrt{1+(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a+e^4*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4
*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^
(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43287, size = 608, normalized size = 6.08

$$15\,ad^5e^4x^5 + 75\,acd^4e^4x^4 + 150\,ac^2d^3e^4x^3 + 150\,ac^3d^2e^4x^2 + 75\,ac^4de^4x + 15\left(bd^5e^4x^5 + 5\,bcd^4e^4x^4 + 10\,bc^2d^3e^4x^3 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*a
*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x^4
 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5*e^
4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (3*b*d^4*e^4*x^4 + 12
*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 - 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 - 2*b*c)*d*e^
4*x + (3*b*c^4 - 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 4.66256, size = 527, normalized size = 5.27

$$\begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4\operatorname{asinh}(c+dx)}{5d} + bc^4e^4x\operatorname{asinh}(c+dx) - \frac{bc^4e^4\sqrt{c^2+2cdx+d^2x^2+1}}{25d} \\ c^4e^4x\left(a+b\operatorname{asinh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*asinh(c + d*x)/(5*d)
 + b*c**4*e**4*x*asinh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)/(25*d) + 2*b*c**3*d*e**4*x**2*asinh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 2*b*c**2*d**2*e**4*x**3*asinh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(75*d) + b*c*d**3*e**4*x**4*asin
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 + b*d**4*e**4*x**5*as
inh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(
c)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.116 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=105

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}(c+dx)^3}{16d}+\frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)}{32d}-\frac{3be^3\sinh^{-1}(c+dx)}{32d}$$

```
[Out] (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(32*d) - (b*e^3*(c + d*x)^3*Sqrt[
1 + (c + d*x)^2])/(16*d) - (3*b*e^3*ArcSinh[c + d*x])/(32*d) + (e^3*(c + d*
x)^4*(a + b*ArcSinh[c + d*x]))/(4*d)
```

Rubi [A] time = 0.0699616, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 321, 215}

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}(c+dx)^3}{16d}+\frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)}{32d}-\frac{3be^3\sinh^{-1}(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(32*d) - (b*e^3*(c + d*x)^3*Sqrt[
1 + (c + d*x)^2])/(16*d) - (3*b*e^3*ArcSinh[c + d*x])/(32*d) + (e^3*(c + d*
x)^4*(a + b*ArcSinh[c + d*x]))/(4*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (ce+dex)^3\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}dx,x,c+dx\right)}{4d}$$
$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d} + \frac{\left(3be^3\right)}{}$$
$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+dx)^4\left(a\right.}{}$$
$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} - \frac{3be^3\sinh^{-1}(c}{32d}$$

Mathematica [A] time = 0.0730688, size = 83, normalized size = 0.79

$$\frac{e^3\left(8(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)-2b\sqrt{(c+dx)^2+1}(c+dx)^3+3b\sqrt{(c+dx)^2+1}(c+dx)-3b\sinh^{-1}(c+dx)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (c + d
*x)^2] - 3*b*ArcSinh[c + d*x] + 8*(c + d*x)^4*(a + b*ArcSinh[c + d*x])))/(3
2*d)
```

Maple [A] time = 0.005, size = 86, normalized size = 0.8

$$\frac{1}{d}\left(\frac{(dx+c)^4e^3a}{4}+e^3b\left(\frac{(dx+c)^4\,\text{Arcsinh}\,(dx+c)}{4}-\frac{(dx+c)^3}{16}\sqrt{1+(dx+c)^2}+\frac{3\,dx+3\,c}{32}\sqrt{1+(dx+c)^2}-\frac{3\,\text{Arcsin}}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a+e^3*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3
*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34303, size = 495, normalized size = 4.71

$$\frac{8\,ad^4e^3x^4 + 32\,acd^3e^3x^3 + 48\,ac^2d^2e^3x^2 + 32\,ac^3de^3x + \left(8\,bd^4e^3x^4 + 32\,bcd^3e^3x^3 + 48\,bc^2d^2e^3x^2 + 32\,bc^3de^3x + \left(8\,bc^4 - 3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c^
3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2 +
32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 - b)*d*e^
3*x + (2*b*c^3 - 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 2.32864, size = 394, normalized size = 3.75

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{asinh}(c+dx)}{4d} + bc^3e^3x\operatorname{asinh}(c+dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2+1}}{16d} + \frac{3bc^2de^3x^2\operatorname{asinh}(c+d}{2} \\ c^3e^3x\,(a + b\operatorname{asinh}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*asinh(c + d*x)/(4*d) + b*c**3*e**3*x*asinh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*asinh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/16 + b*c*d**2*e**3*x**3*asinh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(32*d) + b*d**3*e**3*x**4*asinh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/32 - 3*b*e**3*asinh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*asinh(c)), True))
```

Giac [B] time = 2.72075, size = 807, normalized size = 7.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(24*a*d^3*x^4 + 96*a*c*d^2*x^3 + 144*a*c^2*d*x^2 - 96*(d*(c*log(abs(-c
*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)))/(d*abs(d)) + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)))*b*c^3 + 72*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 -
1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)))/(
```

```
d^2*abs(d)))*d)*b*c^2*d + 16*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11
*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))*abs(d)))/(d^3*abs(d)))*d)*b*c*d^2 + (24*x^4*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)/d^7)*x - 5*(10*c^3*d^2
 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(abs(-c*d - (x*abs(d) - sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)))/(d^4*abs(d)))*d)*b*d^3 + 96*a*c^3*x)
*e^3
```

3.117 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=76

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d}-\frac{be^2\left((c+dx)^2+1\right)^{3/2}}{9d}+\frac{be^2\sqrt{(c+dx)^2+1}}{3d}$$

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2])/(3*d) - (b*e^2*(1 + (c + d*x)^2)^(3/2))/(9*d)
 + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(3*d)
```

Rubi [A] time = 0.0656067, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 266, 43}

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d}-\frac{be^2\left((c+dx)^2+1\right)^{3/2}}{9d}+\frac{be^2\sqrt{(c+dx)^2+1}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2])/(3*d) - (b*e^2*(1 + (c + d*x)^2)^(3/2))/(9*d)
 + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(3*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2 \,\text{Subst}\left(\int x^2 \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}} dx, x, c+dx\right)}{3d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}} dx, x, (c+dx)^2\right)}{6d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right) dx,\right.}{6d}$$
$$= \frac{be^2\sqrt{1+(c+dx)^2}}{3d} - \frac{be^2\left(1+(c+dx)^2\right)^{3/2}}{9d} + \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+\right.}{3d}$$

Mathematica [A] time = 0.0440766, size = 64, normalized size = 0.84

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)-\frac{1}{9}b\left(c^2+2cdx+d^2x^2-2\right)\sqrt{(c+dx)^2+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-(b*(-2 + c^2 + 2*c*d*x + d^2*x^2)*Sqrt[1 + (c + d*x)^2])/9 + ((c + d
*x)^3*(a + b*ArcSinh[c + d*x]))/3))/d
```

Maple [A] time = 0.006, size = 73, normalized size = 1.

$$\frac{1}{d}\left(\frac{(dx+c)^3 e^2a}{3}+e^2b\left(\frac{(dx+c)^3 \operatorname{Arcsinh}(dx+c)}{3}-\frac{(dx+c)^2}{9}\sqrt{1+(dx+c)^2}+\frac{2}{9}\sqrt{1+(dx+c)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a+e^2*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*
(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39488, size = 365, normalized size = 4.8

$$\frac{3\,ad^3e^2x^3 + 9\,acd^2e^2x^2 + 9\,ac^2de^2x + 3\left(bd^3e^2x^3 + 3\,bcd^2e^2x^2 + 3\,bc^2de^2x + bc^3e^2\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*x
^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 - 2*b
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 1.21131, size = 258, normalized size = 3.39

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{asinh}(c+dx)}{3d} + bc^2e^2x\operatorname{asinh}(c+dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + bcde^2x^2\operatorname{asinh}(c+dx) - \frac{2bc}{} \\ c^2e^2x\left(a + b\operatorname{asinh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*asinh(c + d*x)/(3*d) + b*c**2*e**2*x*asinh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + b*c*d*e**2*x**2*asinh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + b*d**2*e**2*x**3*asinh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*asinh(c)), True))
```

Giac [B] time = 2.2866, size = 548, normalized size = 7.21

$$\frac{1}{18}\left(6\,ad^2x^3 + 18\,acdx^2 - 18\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/18*(6*a*d^2*x^3 + 18*a*c*d*x^2 - 18*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^
2 + 9*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(abs(-c*d - (x*a
bs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)))/(d^2*abs(d)))*d)*b*c*d
+ (6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c
^3 - 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs
(d)))/(d^3*abs(d)))*d)*b*d^2 + 18*a*c^2*x)*e^2
```

3.118 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=68

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{(c+dx)^2+1}(c+dx)}{4d}+\frac{be\sinh^{-1}(c+dx)}{4d}$$

```
[Out] -(b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(4*d) + (b*e*ArcSinh[c + d*x])/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(2*d)
```

Rubi [A] time = 0.0387322, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5865, 12, 5661, 321, 215}

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{(c+dx)^2+1}(c+dx)}{4d}+\frac{be\sinh^{-1}(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(4*d) + (b*e*ArcSinh[c + d*x])/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}dx,x,c+dx\right)}{2d}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + \frac{(be)\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{be\sinh^{-1}(c+dx)}{4d} + \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+\right.}{2d}$$

Mathematica [A] time = 0.0619275, size = 57, normalized size = 0.84

$$\frac{e\left(2(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)-b\sqrt{(c+dx)^2+1}(c+dx)+b\sinh^{-1}(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e*(-(b*(c + d*x)*Sqrt[1 + (c + d*x)^2]) + b*ArcSinh[c + d*x] + 2*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])))/(4*d)
```

Maple [A] time = 0.004, size = 62, normalized size = 0.9

$$\frac{1}{d}\left(\frac{(dx+c)^2 ea}{2}+eb\left(\frac{\text{Arcsinh}(dx+c)(dx+c)^2}{2}-\frac{dx+c}{4}\sqrt{1+(dx+c)^2}+\frac{\text{Arcsinh}(dx+c)}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a+e*b*(1/2*arcsinh(d*x+c)*(d*x+c)^2-1/4*(d*x+c)*(1+(d*
x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40071, size = 257, normalized size = 3.78

$$\frac{2\,ad^2ex^2 + 4\,acdex + \left(2\,bd^2ex^2 + 4\,bcdex + \left(2\,bc^2 + b\right)e\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - (bdex + bce)\sqrt{d^2x^2 +}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
+ b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d*e*x + b*c*e
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 0.437205, size = 148, normalized size = 2.18

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{asinh}(c+dx)}{2d} + bcex\operatorname{asinh}(c+dx) - \frac{bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{bdex^2\operatorname{asinh}(c+dx)}{2} - \frac{bex\sqrt{c^2+2cdx+d^2x^2+1}}{4} + \frac{be\operatorname{asin}}{} \\ cex\left(a + b\operatorname{asinh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*asinh(c + d*x)/(2*d) + b*c*e*x
*asinh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(4*d) + b*d*e*
x**2*asinh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + b*e*
asinh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c)), True))
```

Giac [B] time = 1.96356, size = 331, normalized size = 4.87

$$\frac{1}{4}\left(2\,adx^2 - 4\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{d^2x^{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x^2 - 4*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) -
x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c + (2*x^2*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*abs(d)))/(d^2*abs(d)))*d)*b*d + 4*a*c*x)*e
```

3.119 $\int \left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=39

$$ax-\frac{b\sqrt{(c+dx)^2+1}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}$$

```
[Out] a*x - (b*Sqrt[1 + (c + d*x)^2])/d + (b*(c + d*x)*ArcSinh[c + d*x])/d
```

Rubi [A] time = 0.0227585, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5863, 5653, 261}

$$ax-\frac{b\sqrt{(c+dx)^2+1}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + (c + d*x)^2])/d + (b*(c + d*x)*ArcSinh[c + d*x])/d
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sinh^{-1}(c+dx)\right)\,dx &= ax+b\int \sinh^{-1}(c+dx)\,dx\\
&= ax+\frac{b\,\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{d}\\
&= ax-\frac{b\sqrt{1+(c+dx)^2}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0281951, size = 50, normalized size = 1.28

$$ax-\frac{b\left(\sqrt{c^2+2cdx+d^2x^2+1}-c\sinh^{-1}(c+dx)\right)}{d}+bx\sinh^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcSinh[c + d*x] - (b*(Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] - c*ArcS
inh[c + d*x]))/d
```

Maple [A] time = 0.002, size = 36, normalized size = 0.9

$$ax+\frac{b}{d}\left((dx+c)\operatorname{Arcsinh}(dx+c)-\sqrt{1+(dx+c)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(d*x+c),x)
```

```
[Out] a*x+b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

Maxima [A] time = 1.11765, size = 47, normalized size = 1.21

$$ax+\frac{\left((dx+c)\operatorname{arsinh}(dx+c)-\sqrt{(dx+c)^{2}+1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b/d
```

Fricas [A] time = 2.42392, size = 154, normalized size = 3.95

$$\frac{adx+(bdx+bc)\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}\right)-\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b)/d
```

Sympy [A] time = 0.183411, size = 51, normalized size = 1.31

$$ax+b\left(\begin{cases}\frac{c\operatorname{asinh}(c+dx)}{d}+x\operatorname{asinh}(c+dx)-\frac{\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{d} & \text{for}\ d\neq 0\\ x\operatorname{asinh}(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*asinh(c + d*x)/d + x*asinh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*asinh(c), True))
```

Giac [B] time = 1.39628, size = 134, normalized size = 3.44

$$-\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)\right)b+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d
*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt((d
*x + c)^2 + 1)))*b + a*x
```

3.120 $\int \frac{a+b\sinh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=81

$$-\frac{b\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)}{2de}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}$$

```
[Out] (a + b*ArcSinh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcSinh[c + d*x])*Log[1 - E^
(-2*ArcSinh[c + d*x])])/(d*e) - (b*PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(2*
d*e)
```

Rubi [A] time = 0.111794, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5659, 3716, 2190, 2279, 2391}

$$\frac{b\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcSinh[c + d*x])*Log[1 - E
^(2*ArcSinh[c + d*x])])/(d*e) + (b*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*d
*e)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{ex}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x}\,dx,x,c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{b\,\text{Subst}\left(\int \log\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{b\,\text{Subst}\left(\int \frac{\log(}{}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{b\text{Li}_2\left(e^{2\sinh^{-1}(c\right.}}{2de}$$

Mathematica [A] time = 0.0239479, size = 70, normalized size = 0.86

$$\frac{b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)-\left(a+b\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)-2b\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\right)}{2bde}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])*(a + b*ArcSinh[c + d*x] - 2*b*Log[1 - E^(2*ArcS
inh[c + d*x])])) + b^2*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*b*d*e)
```

Maple [A] time = 0.033, size = 159, normalized size = 2.

$$\frac{a\ln(dx+c)}{de} - \frac{b\left(\text{Arcsinh}(dx+c)\right)^2}{2\,de} + \frac{b\text{Arcsinh}(dx+c)}{de}\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right) + \frac{b}{de}\text{polylog}\left(2,-dx-c-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] 1/d*a/e*ln(d*x+c)-1/2/d*b/e*arcsinh(d*x+c)^2+1/d*b/e*arcsinh(d*x+c)*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+1/d*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*
b/e*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*b/e*polylog(2,d*x+c+
(1+(d*x+c)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsinh}(dx + c) + a}{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c+dx}\, dx + \int \frac{b \operatorname{asinh}(c+dx)}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(dx + c) + a}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.121 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

```
[Out] -((a + b*ArcSinh[c + d*x])/(d*e^2*(c + d*x))) - (b*ArcTanh[Sqrt[1 + (c + d*
x)^2]])/(d*e^2)
```

Rubi [A] time = 0.0540328, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5661, 266, 63, 207}

$$-\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])/(d*e^2*(c + d*x))) - (b*ArcTanh[Sqrt[1 + (c + d*
x)^2]])/(d*e^2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^2x^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{2de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c+dx)^2}\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} - \frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}
\end{aligned}
$$

Mathematica [A] time = 0.0320227, size = 44, normalized size = 0.9

$$
\frac{-\frac{a+b\sinh^{-1}(c+dx)}{c+dx} - b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])/(c + d*x)) - b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/
(d*e^2)
```

Maple [A] time = 0.005, size = 54, normalized size = 1.1

$$
\frac{1}{d}\left(-\frac{a}{e^2\,(dx+c)} + \frac{b}{e^2}\left(-\frac{\text{Arcsinh}\,(dx+c)}{dx+c} - \text{Artanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2
)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73795, size = 410, normalized size = 8.37

$$\frac{bdx\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-ac-\left(bcdx+bc^2\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}+1\right)+(bdx+bc)\,\mathrm{lo}}{cd^2e^2x+c^2de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - a*c - (b*c*d*x +
b*c^2)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + (b*d*x + b*c
)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (b*c*d*x + b*c^2)*log
(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1))/(c*d^2*e^2*x + c^2*d*e^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*asinh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [B] time = 1.35275, size = 177, normalized size = 3.61

$$-b\left(\frac{e^{(-1)}\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)}{(dxe+ce)d}+\frac{de^{(-2)}\log\left(\sqrt{\frac{e^2}{(dxe+ce)^2}+1}+\frac{\sqrt{d^2}e}{(dxe+ce)d}\right)}{|d|^2\operatorname{sgn}\left(\frac{1}{dxe+ce}\right)\operatorname{sgn}(d)}\right)-\frac{ae^{(-1)}}{(dxe+ce)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] -b*(e^(-1)*log(d*x + c + sqrt((d*x + c)^2 + 1))/((d*x*e + c*e)*d) + d*e^(-2
)*log(sqrt(e^2/(d*x*e + c*e)^2 + 1) + sqrt(d^2)*e/((d*x*e + c*e)*d))/(abs(d
)^2*sgn(1/(d*x*e + c*e))*sgn(d))) - a*e^(-1)/((d*x*e + c*e)*d)
```

3.122 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2de^3(c+dx)}$$

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(
2*d*e^3*(c + d*x)^2)
```

Rubi [A] time = 0.0509626, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5865, 12, 5661, 264}

$$-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2de^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(
2*d*e^3*(c + d*x)^2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2de^3}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)} - \frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Mathematica [A] time = 0.049776, size = 47, normalized size = 0.8

$$-\frac{a+b(c+dx)\sqrt{(c+dx)^2+1}+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -(a + b*(c + d*x)*Sqrt[1 + (c + d*x)^2] + b*ArcSinh[c + d*x])/(2*d*e^3*(c +
 d*x)^2)
```

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{1}{d}\left(-\frac{a}{2\,e^3\,(dx+c)^2}+\frac{b}{e^3}\left(-\frac{\text{Arcsinh}\,(dx+c)}{2\,(dx+c)^2}-\frac{1}{2\,dx+2\,c}\sqrt{1+(dx+c)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*
(1+(d*x+c)^2)^(1/2)))
```

Maxima [B] time = 1.13941, size = 158, normalized size = 2.68

$$-\frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}d}{d^3e^3x+cd^2e^3}+\frac{\text{arsinh}\,(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)-\frac{a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsi
nh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x^2
 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [B] time = 2.63617, size = 257, normalized size = 4.36

$$\frac{ad^2x^2 + 2\,acdx - bc^2\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - \left(bc^2dx + bc^3\right)\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{2\left(c^2d^3e^3x^2 + 2\,c^3d^2e^3x + c^4de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) - (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*
e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\operatorname{arsinh}\left(dx + c\right) + a}{\left(dex + ce\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

3.123 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^4}\,dx$

Optimal. Leaf size=84

$$-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\sqrt{(c+dx)^2+1}}{6de^4(c+dx)^2}+\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4}$$

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(6*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])
/(3*d*e^4*(c + d*x)^3) + (b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(6*d*e^4)
```

Rubi [A] time = 0.0709212, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5661, 266, 51, 63, 207}

$$-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\sqrt{(c+dx)^2+1}}{6de^4(c+dx)^2}+\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(6*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])
/(3*d*e^4*(c + d*x)^3) + (b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(6*d*e^4)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^4}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^4x^4}\,dx,x,c+dx\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^4}\,dx,x,c+dx\right)}{de^4}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de^4}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{6de^4}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{12de^4}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+(c+dx)^2}\right)}{6de^4}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}
\end{aligned}
$$

Mathematica [A] time = 0.0873561, size = 74, normalized size = 0.88

$$
-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)+b(c+dx)\left(\sqrt{(c+dx)^2+1}-(c+dx)^2\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)\right)}{6de^4(c+dx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -(2*(a + b*ArcSinh[c + d*x]) + b*(c + d*x)*(Sqrt[1 + (c + d*x)^2] - (c + d*
x)^2*ArcTanh[Sqrt[1 + (c + d*x)^2]]))/(6*d*e^4*(c + d*x)^3)
```

Maple [A] time = 0.006, size = 74, normalized size = 0.9

$$
\frac{1}{d}\left(-\frac{a}{3\,e^4\,(dx+c)^3}+\frac{b}{e^4}\left(-\frac{\operatorname{Arcsinh}(dx+c)}{3\,(dx+c)^3}-\frac{1}{6\,(dx+c)^2}\sqrt{1+(dx+c)^2}+\frac{1}{6}\operatorname{Artanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x)
```

```
[Out] 1/d*(-1/3*a/e^4/(d*x+c)^3+b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^
2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}b\left(\frac{2\left(d^2x^2+2\,cdx+c^2+\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)\right)}{d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4}-\frac{i\left(\log\left(\frac{i\left(d^2x+cd\right)}{d}+1\right)-\log\left(-\frac{i\left(d^2x+cd\right)}{d}+1\right)\right)}{de^4}-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(2*(d^2*x^2 + 2*c*d*x + c^2 + log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)))/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) -
 I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d^2*x + c*d)/d + 1))/(d*e^4) - 6*i
ntegrate(1/3/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d
^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^
4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x + (d^5*e^4*x^5 + 5
*c*d^4*e^4*x^4 + c^5*e^4 + c^3*e^4 + (10*c^2*d^3*e^4 + d^3*e^4)*x^3 + (10*c
^3*d^2*e^4 + 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 + 3*c^2*d*e^4)*x)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*
d^2*e^4*x + c^3*d*e^4)
```

Fricas [B] time = 2.82804, size = 760, normalized size = 9.05

$$-\frac{2\,ac^3-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\,bc^5dx+bc^6\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*
c^5*d*x + b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) - 2*
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)) + (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x +
 b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) + (b*c^3*d*x
+ b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^3*d^4*e^4*x^3 + 3*c^4*d^3*e^
4*x^2 + 3*c^5*d^2*e^4*x + c^6*d*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b\,\mathrm{asinh}\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4), x) + Integral(b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(dx + c) + a}{(dex + ce)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

3.124 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^5}\,dx$

Optimal. Leaf size=90

$$-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{(c+dx)^2+1}}{6de^5(c+dx)}-\frac{b\sqrt{(c+dx)^2+1}}{12de^5(c+dx)^3}$$

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[1 + (c + d*x)^2
])/(6*d*e^5*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(4*d*e^5*(c + d*x)^4)
```

Rubi [A] time = 0.0667661, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 271, 264}

$$-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{(c+dx)^2+1}}{6de^5(c+dx)}-\frac{b\sqrt{(c+dx)^2+1}}{12de^5(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[1 + (c + d*x)^2
])/(6*d*e^5*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(4*d*e^5*(c + d*x)^4)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^5}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^5x^5}\,dx, x, c+dx\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^5}\,dx, x, c+dx\right)}{de^5}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx, x, c+dx\right)}{4de^5}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}-\frac{b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{6de^5}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}+\frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}
\end{aligned}
$$

Mathematica [A] time = 0.0567509, size = 61, normalized size = 0.68

$$
-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)+b(c+dx)\sqrt{(c+dx)^2+1}\left(1-2(c+dx)^2\right)}{12de^5(c+dx)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -(b*(c + d*x)*(1 - 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] + 3*(a + b*ArcSinh[
c + d*x]))/(12*d*e^5*(c + d*x)^4)
```

Maple [A] time = 0.004, size = 80, normalized size = 0.9

$$
\frac{1}{d}\left(-\frac{a}{4\,e^5\,(dx+c)^4}+\frac{b}{e^5}\left(-\frac{\operatorname{Arcsinh}(dx+c)}{4\,(dx+c)^4}-\frac{1}{12\,(dx+c)^3}\sqrt{1+(dx+c)^2}+\frac{1}{6\,dx+6\,c}\sqrt{1+(dx+c)^2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arcsinh(d*x+c)-1/12/(d*x+c)
^3*(1+(d*x+c)^2)^(1/2)+1/6/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

Maxima [B] time = 1.23636, size = 348, normalized size = 3.87

$$
\frac{1}{12}\,b\left(\frac{\left(2\,d^4x^4+8\,cd^3x^3+2\,c^4+\left(12\,c^2d^2+d^2\right)x^2+c^2+2\left(4\,c^3d+cd\right)x-1\right)d}{\left(d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5\right)\sqrt{d^2x^2+2\,cdx+c^2+1}}-\frac{3\,\operatorname{arsinh}(dx+c)}{d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 + d^2)*x^2 + c^2 + 2
*(4*c^3*d + c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x
 + c^3*d^2*e^5)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*arcsinh(d*x + c)/(d^
5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e
^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2
*e^5*x + c^4*d*e^5)
```

Fricas [B] time = 2.81177, size = 448, normalized size = 4.98

$$\frac{3\,ad^4x^4 + 12\,acd^3x^3 + 18\,ac^2d^2x^2 + 12\,ac^3dx - 3\,bc^4\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) + \left(2\,bc^4d^3x^3 + 6\,bc^5d^2x^2 + 2\,b\right.}{12\,\left(c^4d^5e^5x^4 + 4\,c^5d^4e^5x^3 + 6\,c^6d^3e^5x^2 + 4\,c^7d^2e^5x + c^8de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (2*b*c^4*d^3*x^3 + 6
*b*c^5*d^2*x^2 + 2*b*c^7 - b*c^5 + (6*b*c^6 - b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*x^2
+ 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx + \int \frac{b\operatorname{asinh}(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*asinh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(dx + c) + a}{(dex + ce)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^5, x)
```

3.125 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^6}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{(c+dx)^2+1}}{40de^6(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{20de^6(c+dx)^4}-\frac{3b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{40de^6}$$

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt[1 + (c + d*x)
^2])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])/(5*d*e^6*(c + d*x)^5
) - (3*b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(40*d*e^6)
```

Rubi [A] time = 0.0862735, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5661, 266, 51, 63, 207}

$$-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{(c+dx)^2+1}}{40de^6(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{20de^6(c+dx)^4}-\frac{3b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{40de^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt[1 + (c + d*x)
^2])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])/(5*d*e^6*(c + d*x)^5
) - (3*b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(40*d*e^6)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^6}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^6x^6}\,dx,x,c+dx\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^6}\,dx,x,c+dx\right)}{de^6}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{x^5\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de^6}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{10de^6}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}-\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{40de^6}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,\right.}{80de^6}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\right.}{40de^6}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}-\frac{3b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right.}{40de^6}
\end{aligned}$$

Mathematica [C] time = 0.0343711, size = 64, normalized size = 0.56

$$\frac{-\frac{1}{5}b\sqrt{(c+dx)^2+1}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},(c+dx)^2+1\right)-\frac{a+b\sinh^{-1}(c+dx)}{5(c+dx)^5}}{de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] (-(a + b*ArcSinh[c + d*x])/(5*(c + d*x)^5) - (b*Sqrt[1 + (c + d*x)^2]*Hyper
geometric2F1[1/2, 3, 3/2, 1 + (c + d*x)^2])/5)/(d*e^6)
```

Maple [A] time = 0.006, size = 94, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a}{5\,e^{6}\left(dx+c\right)^{5}}+\frac{b}{e^{6}}\left(-\frac{\operatorname{Arcsinh}\left(dx+c\right)}{5\,\left(dx+c\right)^{5}}-\frac{1}{20\,\left(dx+c\right)^{4}}\sqrt{1+\left(dx+c\right)^{2}}+\frac{3}{40\,\left(dx+c\right)^{2}}\sqrt{1+\left(dx+c\right)^{2}}-\frac{3}{40}\operatorname{Artan}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x)
```

```
[Out] 1/d*(-1/5*a/e^6/(d*x+c)^5+b/e^6*(-1/5/(d*x+c)^5*arcsinh(d*x+c)-1/20/(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)+3/40/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/40*arctanh(1/(1
+(d*x+c)^2)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{30}\,b\left(\frac{2\left(3\,d^{4}x^{4}+12\,cd^{3}x^{3}+3\,c^{4}+\left(18\,c^{2}d^{2}-d^{2}\right)x^{2}-c^{2}+2\left(6\,c^{3}d-cd\right)x-3\,\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}\right)\right)}{d^{6}e^{6}x^{5}+5\,cd^{5}e^{6}x^{4}+10\,c^{2}d^{4}e^{6}x^{3}+10\,c^{3}d^{3}e^{6}x^{2}+5\,c^{4}d^{2}e^{6}x+c^{5}de^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] 1/30*b*(2*(3*d^4*x^4 + 12*c*d^3*x^3 + 3*c^4 + (18*c^2*d^2 - d^2)*x^2 - c^2
+ 2*(6*c^3*d - c*d)*x - 3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))
/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 +
 5*c^4*d^2*e^6*x + c^5*d*e^6) - 3*I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d
^2*x + c*d)/d + 1))/(d*e^6) + 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*e^6*x
^7 + c^8*e^6 + c^6*e^6 + (28*c^2*d^6*e^6 + d^6*e^6)*x^6 + 2*(28*c^3*d^5*e^6
 + 3*c*d^5*e^6)*x^5 + 5*(14*c^4*d^4*e^6 + 3*c^2*d^4*e^6)*x^4 + 4*(14*c^5*d^
3*e^6 + 5*c^3*d^3*e^6)*x^3 + (28*c^6*d^2*e^6 + 15*c^4*d^2*e^6)*x^2 + 2*(4*c
^7*d*e^6 + 3*c^5*d*e^6)*x + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e^6 + c^5*
e^6 + (21*c^2*d^5*e^6 + d^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 + c*d^4*e^6)*x^4 +
5*(7*c^4*d^3*e^6 + 2*c^2*d^3*e^6)*x^3 + (21*c^5*d^2*e^6 + 10*c^3*d^2*e^6)*x
^2 + (7*c^6*d*e^6 + 5*c^4*d*e^6)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))
 - 1/5*a/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e
^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^6)
```

Fricas [B] time = 3.35301, size = 1116, normalized size = 9.7

$$-\frac{8\,ac^{5}-8\left(bd^{5}x^{5}+5\,bcd^{4}x^{4}+10\,bc^{2}d^{3}x^{3}+10\,bc^{3}d^{2}x^{2}+5\,bc^{4}dx\right)\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}\right)+3\left(bc^{5}d^{5}x^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3
*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
```

```
3*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 +
5*b*c^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)
 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b
*c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(b*
c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 + 5*b*c
^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) - (3
*b*c^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 - 2*b*c^6 + (9*b*c^7 - 2*b*c^5)*
d*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^5*d^6*e^6*x^5 + 5*c^6*d^5*e^6*x^
4 + 10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x + c^10*d*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx + \int \frac{b\operatorname{asinh}(c+dx)}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*asinh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(dx+c)+a}{(dex+ce)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

3.126 $\int (ce + dex)^m \left(a + b \sinh^{-1}(c + dx)\right)^2 dx$

Optimal. Leaf size=187

$$\frac{2b^2(e(c+dx))^{m+3}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},-(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)} - \frac{2b(e(c+dx))^{m+2}\text{Hyper}}{}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, -(c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)) + (2*b^2*(e*(c + d*x))
^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 -(c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rubi [A] time = 0.206553, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5865, 5661, 5762}

$$\frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)} - \frac{2b(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)(a}{de^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, -(c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)) + (2*b^2*(e*(c + d*x))
^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 -(c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}} dx \ldots\right.}{de(1+m)}$$
$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{2b(e(c+dx))^{2+m}\left(a+b\sinh^{-1}(c+ \ldots\right.}{de^2(1+m \ldots}$$

Mathematica [A] time = 0.129712, size = 155, normalized size = 0.83

$$\frac{(c+dx)(e(c+dx))^m \left(\frac{2b^2(c+dx)^2 \text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2}\right\},\left\{\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2}\right\},-(c+dx)^2\right)}{(m+2)(m+3)} - \frac{2b(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-(c+d \ldots\right.}{m+2} \ldots\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcSinh[c + d*x])^2 - (2*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*
x)^2])/(2 + m) + (2*b^2*(c + d*x)^2*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 +
m/2}, {2 + m/2, 5/2 + m/2}, -(c + d*x)^2])/((2 + m)*(3 + m))))/(d*(1 + m))
```

Maple [F] time = 1.458, size = 0, normalized size = 0.

$$\int (dex+ce)^m (a+b\text{Arcsinh}\,(dx+c))^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{arsinh}(dx+c)^2 + 2\,ab \operatorname{arsinh}(dx+c) + a^2\right)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*(d*e*x + c
*e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c+dx))^m (a+b\operatorname{asinh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(dx+c)+a)^2 (dex+ce)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

3.127 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=197

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{25d}+\frac{8be^4\sqrt{(c+dx)^2+1}(c+dx)^2}{75d}$$

```
[Out] (16*b^2*e^4*x)/75 - (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(75*
d) + (8*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(
75*d) - (2*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])
)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^2)/(5*d)
```

Rubi [A] time = 0.306813, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5758, 5717, 8, 30}

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{25d}+\frac{8be^4\sqrt{(c+dx)^2+1}(c+dx)^2}{75d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```


```
[Out] (16*b^2*e^4*x)/75 - (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(75*
d) + (8*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(
75*d) - (2*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])
)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^2)/(5*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^4x^4 \left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4 \text{Subst}\left(\int x^4 \left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)^2}{5d} - \frac{\left(2be^4\right) \text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}} dx, x\right.}{5d}$$

$$= -\frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{25d} + \frac{e^4(c+dx)^5\left(a+b\,\text{s}\right.}{5d}$$

$$= \frac{2b^2e^4(c+dx)^5}{125d} + \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{75d} - 2b$$

$$= -\frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{75d}$$

$$= \frac{16}{75}b^2e^4x - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}\left(a+b\,\text{s}\right.}{75d}$$

Mathematica [A] time = 0.248307, size = 192, normalized size = 0.97

$$e^4\left(9\left(25a^2+2b^2\right)(c+dx)^5+30ab\sqrt{(c+dx)^2+1}\left(-3(c+dx)^4+4(c+dx)^2-8\right)+30b\sinh^{-1}(c+dx)\left(15a(c+dx)^5-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*(240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[1 + (c + d*x)^2]*(-8 + 4*(c + d*x)^2 - 3*(c + d*x)^4) + 30
*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[1 + (c + d*x)^2] + 4*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2] - 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 2
```

```
25*b^2*(c + d*x)^5*ArcSinh[c + d*x]^2))/(1125*d)
```

Maple [A] time = 0.039, size = 282, normalized size = 1.4

$$\frac{1}{d}\left(\frac{(dx+c)^5 e^4 a^2}{5}+e^4 b^2\left(\frac{(dx+c)^3 (\mathrm{Arcsinh}(dx+c))^2\left(1+(dx+c)^2\right)}{5}-\frac{(\mathrm{Arcsinh}(dx+c))^2 (dx+c)\left(1+(dx+c)^2\right)}{5}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^2+e^4*b^2*(1/5*(d*x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+
c)^2)-1/5*arcsinh(d*x+c)^2*(d*x+c)*(1+(d*x+c)^2)+1/5*arcsinh(d*x+c)^2*(d*x+
c)-2/25*arcsinh(d*x+c)*(d*x+c)^2*(1+(d*x+c)^2)^(3/2)+14/75*arcsinh(d*x+c)*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)-16/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2/125
*(1+(d*x+c)^2)^2*(d*x+c)+298/1125*d*x+298/1125*c-76/1125*(1+(d*x+c)^2)*(d*x
+c))+2*e^4*a*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(
1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69383, size = 1331, normalized size = 6.76

$$\frac{9\left(25\,a^2+2\,b^2\right)d^5e^4x^5+45\left(25\,a^2+2\,b^2\right)cd^4e^4x^4+10\left(9\left(25\,a^2+2\,b^2\right)c^2-4\,b^2\right)d^3e^4x^3+30\left(3\left(25\,a^2+2\,b^2\right)c^3-4\,b^2c\right)d^{\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
+ 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2)*
c^3 - 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^
2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^3*e^
4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 30*(15*a*b*d^5*e^4*x^5 + 75*a*b
*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 + 75*a*b
*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e^4*x^3 +
 2*(9*b^2*c^2 - 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*e^4*x + (3*b
^2*c^4 - 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 30*(3*a*b*d^4*e^4*x^4 + 12*a*b*
c*d^3*e^4*x^3 + 2*(9*a*b*c^2 - 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^3 - 2*a*b*c)
```

```
*d*e^4*x + (3*a*b*c^4 - 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))/d
```

Sympy [A] time = 11.1037, size = 1268, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
*4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
sinh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*asinh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*asinh(c
+ d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*asinh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/25 + 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*asinh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/75 + 2*a*b*d**4*e**4*x**5*asinh(c + d*x)/5 - 2*a*b*d
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)/(75*d) + b**2*c**5*e**4*asinh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*asinh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*asinh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*asinh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*c**2
*e**4*x/75 + 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(75*d) + b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)/25 - 8*b**2*c*d*e**4*x**2/75 + 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + b**2*d**4*e**4*x**5*asinh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*d**2*e**4*x**3/225 + 8*b**
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^2, x)
```

3.128 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=172

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{8d}+\frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)\left(a\right.}{16d}$$

```
[Out] (-3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) + (3*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(16*d) - (b*e^3*(c
 + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(8*d) - (3*e^3*(a
 + b*ArcSinh[c + d*x])^2)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x]
)^2)/(4*d)
```

Rubi [A] time = 0.257649, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{8d}+\frac{3be^3\sqrt{(c+dx)^2+1}(c+dx)\left(a\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (-3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) + (3*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(16*d) - (b*e^3*(c
 + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(8*d) - (3*e^3*(a
 + b*ArcSinh[c + d*x])^2)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x]
)^2)/(4*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}} dx, x, \ldots\right.}{2d}$$

$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{8d} + \frac{e^3(c+dx)^4\left(a+b\,\text{si}\ldots\right.}{4d}$$

$$= \frac{b^2e^3(c+dx)^4}{32d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{16d} - \frac{be^3(\ldots}{\ldots}$$

$$= -\frac{3b^2e^3(c+dx)^2}{32d} + \frac{b^2e^3(c+dx)^4}{32d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh\ldots\right.}{16d}$$

Mathematica [A] time = 0.189873, size = 170, normalized size = 0.99

$$\frac{e^3\left(\left(8a^2+b^2\right)(c+dx)^4+2ab\left(3-2(c+dx)^2\right)\sqrt{(c+dx)^2+1}(c+dx)+2b(c+dx)\sinh^{-1}(c+dx)\left(8a(c+dx)^3-2b\sqrt{\ldots}\right.\right.}{32\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^3*(-3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*(c + d*x)*(3 -
 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] - 6*a*b*ArcSinh[c + d*x] + 2*b*(c + d
*x)*(8*a*(c + d*x)^3 + 3*b*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*x]
^2))/(32*d)
```

Maple [A] time = 0.043, size = 229, normalized size = 1.3

$$\frac{1}{d}\left(\frac{(dx+c)^4e^3a^2}{4}+e^3b^2\left(\frac{(dx+c)^2(\text{Arcsinh}(dx+c))^2\left(1+(dx+c)^2\right)}{4}-\frac{(\text{Arcsinh}(dx+c))^2\left(1+(dx+c)^2\right)}{4}-\frac{\text{Arcsi}\ldots}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a^2+e^3*b^2*(1/4*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+
c)^2)-1/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/8*arcsinh(d*x+c)*(d*x+c)*(1+(d*x
+c)^2)^(3/2)+5/16*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)+5/32*arcsinh(d
*x+c)^2+1/32*(d*x+c)^2*(1+(d*x+c)^2)-1/8*(d*x+c)^2-1/8)+2*e^3*a*b*(1/4*(d*x
+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*
x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78894, size = 1034, normalized size = 6.01

$$\frac{\left(8\,a^2+b^2\right)d^4e^3x^4+4\left(8\,a^2+b^2\right)cd^3e^3x^3+3\left(2\left(8\,a^2+b^2\right)c^2-b^2\right)d^2e^3x^2+2\left(2\left(8\,a^2+b^2\right)c^3-3\,b^2c\right)de^3x+\left(8\,b^2d^4e^3x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*a
^2 + b^2)*c^2 - b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*e^3*
x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 + 32
*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48*a*b*
c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b^2*d^3
*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 - b^2)*d*e^3*x + (2*b^2*c^3 -
 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x^2 + 3*(2
*a*b*c^2 - a*b)*d*e^3*x + (2*a*b*c^3 - 3*a*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))/d
```

Sympy [A] time = 6.29332, size = 916, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*asinh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*asinh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 2*a*b*c*d**2*e**3*x**3*asinh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + a*b*d**3*e**3*x**4*asinh(c +
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 - 3*a*b*e**3*asinh(c + d*x)/
(16*d) + b**2*c**4*e**3*asinh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*asinh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)*asinh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/8 - 3*b**2*c*e**3*x/16 + 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*asinh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 3*b**2*d*e**3*x**2/32 + 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/16 - 3*b**2*e**3*a
sinh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**2, True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^2, x)
```

3.129 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=136

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{(c+dx)^2+1}(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{4be^2\sqrt{(c+dx)^2+1}\left(a+b\sin\right.}{9d}$$

```
[Out] (-4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) + (4*b*e^2*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) - (2*b*e^2*(c + d*x)^2*Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh
[c + d*x])^2)/(3*d)
```

Rubi [A] time = 0.204961, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5758, 5717, 8, 30}

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{(c+dx)^2+1}(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{4be^2\sqrt{(c+dx)^2+1}\left(a+b\sin\right.}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```


```
[Out] (-4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) + (4*b*e^2*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) - (2*b*e^2*(c + d*x)^2*Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh
[c + d*x])^2)/(3*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{\left(2be^2\right)\text{Subst}\left(\int\frac{x^3\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\,dx,x\right.}{3d}$$
$$=-\frac{2be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{e^2(c+dx)^3\left(a+b\,\mathrm{s}\right.}{3d}$$
$$=\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{2be^2(c+dx}{}$$
$$=-\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-$$

Mathematica [A] time = 0.166324, size = 147, normalized size = 1.08

$$\frac{e^2\left(\left(9a^2+2b^2\right)(c+dx)^3+6ab\left(2-(c+dx)^2\right)\sqrt{(c+dx)^2+1}+6b\sinh^{-1}(c+dx)\left(3a(c+dx)^3-b\sqrt{(c+dx)^2+1}(c+\right.\right.}{27d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*(-12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*(2 - (c + d*x
)^2)*Sqrt[1 + (c + d*x)^2] + 6*b*(3*a*(c + d*x)^3 + 2*b*Sqrt[1 + (c + d*x)^
2] - b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 9*b^2*(c + d*x
)^3*ArcSinh[c + d*x]^2))/(27*d)
```

Maple [A] time = 0.033, size = 192, normalized size = 1.4

$$\frac{1}{d}\left(\frac{(dx+c)^3e^2a^2}{3}+e^2b^2\left(\frac{(\text{Arcsinh}\,(dx+c))^2(dx+c)\left(1+(dx+c)^2\right)}{3}-\frac{(\text{Arcsinh}\,(dx+c))^2(dx+c)}{3}-\frac{2\,\text{Arcsinh}\,(dx\right.\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^2+e^2*b^2*(1/3*arcsinh(d*x+c)^2*(d*x+c)*(1+(d*x+c)
^2)-1/3*arcsinh(d*x+c)^2*(d*x+c)-2/9*arcsinh(d*x+c)*(d*x+c)^2*(1+(d*x+c)^2)
^(1/2)+4/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2/27*(1+(d*x+c)^2)*(d*x+c)-14
/27*d*x-14/27*c)+2*e^2*a*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.89639, size = 767, normalized size = 5.64

$$\frac{\left(9\,a^2+2\,b^2\right)d^3e^2x^3+3\left(9\,a^2+2\,b^2\right)cd^2e^2x^2+3\left(\left(9\,a^2+2\,b^2\right)c^2-4\,b^2\right)de^2x+9\left(b^2d^3e^2x^3+3\,b^2cd^2e^2x^2+3\,b^2c^2de^2x+b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((9
*a^2 + 2*b^2)*c^2 - 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2*x
^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*d*e^
2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 - 2*b^2
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 - 2*a*b)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 3.18346, size = 610, normalized size = 4.49

$$\begin{cases} a^2c^2e^2x + a^2cde^2x^2 + \frac{a^2d^2e^2x^3}{3} + \frac{2abc^3e^2\operatorname{asinh}(c+dx)}{3d} + 2abc^2e^2x\operatorname{asinh}(c+dx) - \frac{2abc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + 2abcde^2x^2\operatorname{asinh}(c \cdots \\ c^2e^2x\left(a+b\operatorname{asinh}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
2*a*b*c**3*e**2*asinh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*asinh(c + d*x) - 2
*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a*b*c*d*e**2*
```

```
x**2*asinh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9
 + 2*a*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/9 + 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
(9*d) + b**2*c**3*e**2*asinh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*asinh(c +
 d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*asinh(c + d*x)**2 +
2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/9 + b**2*d**2*e**2*x**3*asinh(c + d*x)**2/3 + 2*b**2*d**2
*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/9 - 4*b**2*e**2*x/9 + 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**2,
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^2, x)
```

3.130 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=103

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{(c+dx)^2+1}(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{b^2e(c$$

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x]))/(2*d) + (e*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.144862, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{(c+dx)^2+1}(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{b^2e(c$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x]))/(2*d) + (e*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])^2)/(2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}} dx, x, c+\right.}{d}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + \frac{e(c+dx)^2\left(a+b\sinh^{-}\right.}{2d}$$
$$= \frac{b^2e(c+dx)^2}{4d} - \frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + \frac{e\left(a+b\,\mathrm{s}\right.}{}$$

Mathematica [A] time = 0.177152, size = 120, normalized size = 1.17

$$\frac{e\left(\left(2a^2+b^2\right)(c+dx)^2-2ab\sqrt{(c+dx)^2+1}(c+dx)+2b(c+dx)\sinh^{-1}(c+dx)\left(2a(c+dx)-b\sqrt{(c+dx)^2+1}\right)+2ab\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*((2*a^2 + b^2)*(c + d*x)^2 - 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] + 2*a
*b*ArcSinh[c + d*x] + 2*b*(c + d*x)*(2*a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2
])*ArcSinh[c + d*x] + b^2*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^2))/(4*d)
```

Maple [A] time = 0.027, size = 135, normalized size = 1.3

$$\frac{1}{d}\left(\frac{(dx+c)^2 ea^2}{2} + eb^2\left(\frac{(\text{Arcsinh}\,(dx+c))^2\left(1+(dx+c)^2\right)}{2} - \frac{\text{Arcsinh}\,(dx+c)\,(dx+c)}{2}\sqrt{1+(dx+c)^2} - \frac{(\text{Arcsinh}\,(d}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a^2+e*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcs
inh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1
/4)+2*e*a*b*(1/2*arcsinh(d*x+c)*(d*x+c)^2-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1
/4*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96909, size = 533, normalized size = 5.17

$$\frac{\left(2\,a^2+b^2\right)d^2ex^2+2\left(2\,a^2+b^2\right)cdex+\left(2\,b^2d^2ex^2+4\,b^2cdex+\left(2\,b^2c^2+b^2\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2 +
 4*b^2*c*d*e*x + (2*b^2*c^2 + b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 + a*b)*e -
(b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))/d
```

Sympy [A] time = 1.09358, size = 335, normalized size = 3.25

$$\begin{cases} a^2cex+\frac{a^2dex^2}{2}+\frac{abc^2e\operatorname{asinh}(c+dx)}{d}+2abcex\operatorname{asinh}(c+dx)-\frac{abce\sqrt{c^2+2cdx+d^2x^2+1}}{2d}+abdex^2\operatorname{asinh}(c+dx)-\frac{abex\sqrt{c^2+2cdx+d^2x^2}}{2} \\ cex\left(a+b\operatorname{asinh}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*asinh(c + d*x)/d + 2*a
*b*c*e*x*asinh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d
) + a*b*d*e*x**2*asinh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/2 + a*b*e*asinh(c + d*x)/(2*d) + b**2*c**2*e*asinh(c + d*x)**2/(2*d) +
b**2*c*e*x*asinh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(2*d) + b**2*d*e*x**2*asinh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/2 + b**2*e*asinh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))
**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)(b\operatorname{arsinh}(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.131 $\int \left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$-\frac{2b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + ((c + d*
x)*(a + b*ArcSinh[c + d*x])^2)/d
```

Rubi [A] time = 0.0701418, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5653, 5717, 8}

$$-\frac{2b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + ((c + d*
x)*(a + b*ArcSinh[c + d*x])^2)/d
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}} dx, x, c+dx\right)}{d}$$
$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{\left(2b^2\right)}{}$$
$$= 2b^2x - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}$$

Mathematica [A] time = 0.0956592, size = 87, normalized size = 1.53

$$\frac{\left(a^2+2b^2\right)(c+dx) - 2ab\sqrt{(c+dx)^2+1} + 2b\sinh^{-1}(c+dx)\left(ac+adx+b\left(-\sqrt{(c+dx)^2+1}\right)\right) + b^2(c+dx)\sinh^{-1}(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2] + 2*b*(a*c + a*d*x -
 b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(c + d*x)*ArcSinh[c + d*x]
^2)/d
```

Maple [A] time = 0.028, size = 90, normalized size = 1.6

$$\frac{1}{d}\left((dx+c)a^2 + b^2\left((\text{Arcsinh}(dx+c))^2(dx+c) - 2\,\text{Arcsinh}(dx+c)\sqrt{1+(dx+c)^2} + 2dx + 2c\right) + 2ab\left((dx+c)\,\text{A}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*((d*x+c)*a^2+b^2*(arcsinh(d*x+c)^2*(d*x+c)-2*arcsinh(d*x+c)*(1+(d*x+c)^
2)^(1/2)+2*d*x+2*c)+2*a*b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66711, size = 333, normalized size = 5.84

$$\frac{\left(a^2+2b^2\right)dx + \left(b^2dx+b^2c\right)\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)^2 - 2\sqrt{d^2x^2+2cdx+c^2+1}\,ab + 2\left(abdx+abc-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)))/d
```

Sympy [A] time = 0.401433, size = 143, normalized size = 2.51

$$\begin{cases} a^{2}x + \frac{2abc\operatorname{asinh}(c+dx)}{d} + 2abx\operatorname{asinh}(c+dx) - \frac{2ab\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{d} + \frac{b^{2}c\operatorname{asinh}^{2}(c+dx)}{d} + b^{2}x\operatorname{asinh}^{2}(c+dx) + 2b^{2}x - \frac{2b^{2}\sqrt{c^{2}+2 \\ x\left(a+b\operatorname{asinh}(c)\right)^{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*asinh(c + d*x)/d + 2*a*b*x*asinh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + b**2*c*asinh(c + d*x)**2/d + b*
*2*x*asinh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(dx+c)+a)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2, x)
```

3.132 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=116

$$-\frac{b\text{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}(c+dx)}\right)}{2de}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\cdots$$

```
[Out] (a + b*ArcSinh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcSinh[c + d*x])^2*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (b*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^
(-2*ArcSinh[c + d*x])])/(d*e) - (b^2*PolyLog[3, E^(-2*ArcSinh[c + d*x])])/(
2*d*e)
```

Rubi [A] time = 0.197995, antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5659, 3716, 2190, 2531, 2282, 6589}

$$\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcSinh[c + d*x])^2*Log[1 -
 E^(2*ArcSinh[c + d*x])])/(d*e) + (b*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^
(2*ArcSinh[c + d*x])])/(d*e) - (b^2*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(2*
d*e)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{ex}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x}\,dx,x,c+dx\right)}{de}\\
&= \frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}-\frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}-\frac{(2b)\,\text{Subst}}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}+\frac{b\left(a+b\sin\right.}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}+\frac{b\left(a+b\sin\right.}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}+\frac{b\left(a+b\sin\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.0455482, size = 100, normalized size = 0.86

$$\frac{6b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)-3b^3\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)-2\left(a+b\sinh^{-1}(c+dx)\right)^2\left(a+b\,\mathrm{s}\right.}{6bde}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2*(a + b*ArcSinh[c + d*x] - 3*b*Log[1 - E^(2*A
rcSinh[c + d*x])]) + 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSinh
[c + d*x])] - 3*b^3*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(6*b*d*e)
```

Maple [B] time = 0.031, size = 404, normalized size = 3.5

$$\frac{a^2\ln(dx+c)}{de} - \frac{b^2(\mathrm{Arcsinh}(dx+c))^3}{3de} + \frac{b^2(\mathrm{Arcsinh}(dx+c))^2}{de}\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right) + 2\frac{b^2\mathrm{Arcsinh}(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^2/e*ln(d*x+c)-1/3/d*b^2/e*arcsinh(d*x+c)^3+1/d*b^2/e*arcsinh(d*x+c)^2
*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2/d*b^2/e*arcsinh(d*x+c)*polylog(2,-d*x-c-
(1+(d*x+c)^2)^(1/2))-2/d*b^2/e*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*b^
2/e*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2/d*b^2/e*arcsinh(d*x+
c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2/d*b^2/e*polylog(3,d*x+c+(1+(d*x+c
)^2)^(1/2))-1/d*a*b/e*arcsinh(d*x+c)^2+2/d*a*b/e*arcsinh(d*x+c)*ln(1+d*x+c+
(1+(d*x+c)^2)^(1/2))+2/d*a*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2/d*a*
b/e*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2/d*a*b/e*polylog(2,d*x+
c+(1+(d*x+c)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{arsinh}(dx+c)^2 + 2\,ab\,\mathrm{arsinh}(dx+c) + a^2}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d*e*x + c
*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*asinh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^2}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

3.133 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{2b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{de^2}+\frac{2b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{4b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de}$$

```
[Out] -((a + b*ArcSinh[c + d*x])^2/(d*e^2*(c + d*x))) - (4*b*(a + b*ArcSinh[c + d
*x])*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (2*b^2*PolyLog[2, -E^ArcSinh[c
+ d*x]])/(d*e^2) + (2*b^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rubi [A] time = 0.175678, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5760, 4182, 2279, 2391}

$$-\frac{2b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{de^2}+\frac{2b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{4b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^2/(d*e^2*(c + d*x))) - (4*b*(a + b*ArcSinh[c + d
*x])*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (2*b^2*PolyLog[2, -E^ArcSinh[c
+ d*x]])/(d*e^2) + (2*b^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{\left(2b^2\right)\text{Sub}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{\left(2b^2\right)\text{Sub}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{2b^2\text{Li}_2\left(-\right.}{}$$

Mathematica [A] time = 0.590954, size = 154, normalized size = 1.54

$$\frac{b^2\left(2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right)-2\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)+\sinh^{-1}(c+dx)\left(-\frac{\sinh^{-1}(c+dx)}{c+dx}+2\log\left(1-e^{-\sinh^{-1}(c+dx}\right.\right.\right.}{de^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) + a*b*((-2*ArcSinh[c + d*x])/(c + d*x) + 2*Log[(2*Sinh[Ar
cSinh[c + d*x]/2]^2)/(c + d*x)]) + b^2*(ArcSinh[c + d*x]*(-(ArcSinh[c + d*x
]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] - 2*Log[1 + E^(-ArcSinh[c +
 d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 2*PolyLog[2, E^(-ArcSinh[
```

```
c + d*x])]))/(d*e^2)
```

Maple [A] time = 0.046, size = 229, normalized size = 2.3

$$-\frac{a^2}{de^2(dx+c)}-\frac{b^2(\operatorname{Arcsinh}(dx+c))^2}{de^2(dx+c)}-2\frac{b^2\operatorname{Arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{de^2}-2\frac{b^2\operatorname{polylog}\left(2,-d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2*arcsinh(d*x+c)^2/(d*x+c)-2/d*b^2/e^2*arcsi
nh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*b^2*polylog(2,-d*x-c-(1+(d*x+c)
^2)^(1/2))/d/e^2+2/d*b^2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))
+2*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-2/d*a*b/e^2/(d*x+c)*arcsi
nh(d*x+c)-2/d*a*b/e^2*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(dx+c)^2+2\,ab\operatorname{arsinh}(dx+c)+a^2}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^2*e^2*x
^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^2+2cdx+d^2x^2}\,dx+\int\frac{b^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{2ab\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*asinh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*asinh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

3.134 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=85

$$-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}$$

```
[Out] -((b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(d*e^3*(c + d*x))) - (
a + b*ArcSinh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c + d*x])/(d*e^3
)
```

Rubi [A] time = 0.136605, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 12, 5661, 5723, 29}

$$-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(d*e^3*(c + d*x))) - (
a + b*ArcSinh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c + d*x])/(d*e^3
)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^3x^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^3}\,dx, x, c+dx\right)}{de^3}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^3}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x}\,dx, \ldots\right.}{de^3}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3}$$

Mathematica [A] time = 0.238107, size = 120, normalized size = 1.41

$$-\frac{a\left(a+2b(c+dx)\sqrt{c^2+2cdx+d^2x^2+1}\right)+2b\sinh^{-1}(c+dx)\left(a+b(c+dx)\sqrt{c^2+2cdx+d^2x^2+1}\right)-2b^2(c+dx)^2\log(\ldots}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -(a*(a + 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) + 2*b*(a + b*(c +
 d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])*ArcSinh[c + d*x] + b^2*ArcSinh[c +
 d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x])/(2*d*e^3*(c + d*x)^2)
```

Maple [B] time = 0.062, size = 180, normalized size = 2.1

$$-\frac{a^2}{2\,de^3\,(dx+c)^2} - \frac{b^2\text{Arcsinh}\,(dx+c)}{de^3} - \frac{b^2\text{Arcsinh}\,(dx+c)}{de^3\,(dx+c)}\sqrt{1+(dx+c)^2} - \frac{b^2\,(\text{Arcsinh}\,(dx+c))^2}{2\,de^3\,(dx+c)^2} + \frac{b^2}{de^3}\ln\left(\left(dx+c\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^2/e^3/(d*x+c)^2-1/d*b^2/e^3*arcsinh(d*x+c)-1/d*b^2/e^3*arcsinh(d*x
+c)/(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/2/d*b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+1/
d*b^2/e^3*ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1)-1/d*a*b/e^3/(d*x+c)^2*arcsinh
(d*x+c)-1/d*a*b/e^3/(d*x+c)*(1+(d*x+c)^2)^(1/2)
```

Maxima [B] time = 1.29577, size = 311, normalized size = 3.66

$$-\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}d\,\operatorname{arsinh}(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log(dx+c)}{de^3}\right)b^2-ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}d}{d^3e^3x+cd^2e^3}+\frac{\operatorname{arsinh}(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^2*e
^3) - log(d*x + c)/(d*e^3))*b^2 - a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/
(d^3*e^3*x + c*d^2*e^3) + arcsinh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c
^2*d*e^3)) - 1/2*b^2*arcsinh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*
d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

Fricas [B] time = 3.04284, size = 714, normalized size = 8.4

$$-\frac{2\,abc^2d^2x^2+4\,abc^3dx+2\,abc^4+b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2+a^2c^2-2\left(abd^2x^2+2\,abcdx-\left(b^2c^2d\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 + b^2*c^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + a^2*c^2 - 2*(a*b*d^2*x^2 + 2*a*b*c*
d*x - (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
 + b^2*c^4)*log(d*x + c) - 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
 - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^
3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ab\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

3.135 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=169

$$\frac{b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}\right.}{3de^4(c\ }$$

```
[Out] -b^2/(3*d*e^4*(c + d*x)) - (b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])^2/(3*d*e^4*(c + d*x)^3)
 + (2*b*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]])/(3*d*e^4) + (
b^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(3*d*e^4) - (b^2*PolyLog[2, E^ArcSinh[
c + d*x]])/(3*d*e^4)
```

Rubi [A] time = 0.246695, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5747, 5760, 4182, 2279, 2391, 30}

$$\frac{b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}\right.}{3de^4(c\ }$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -b^2/(3*d*e^4*(c + d*x)) - (b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])^2/(3*d*e^4*(c + d*x)^3)
 + (2*b*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]])/(3*d*e^4) + (
b^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(3*d*e^4) - (b^2*PolyLog[2, E^ArcSinh[
c + d*x]])/(3*d*e^4)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^3\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de^4}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} - \frac{b\,\text{Subst}\left(\int \frac{a+b}{x}\right.}{}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} -$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} +$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} +$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} +$$

Mathematica [A] time = 1.67858, size = 212, normalized size = 1.25

$$-\frac{b^2\left(4(c+dx)^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right) - 4(c+dx)^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right) + 4(c+dx)^2 + 4\sinh^{-1}(c+dx)^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -(4*a^2 + a*b*(8*ArcSinh[c + d*x] + 2*Sinh[2*ArcSinh[c + d*x]] + Log[Tanh[A
rcSinh[c + d*x]/2]]*(-3*(c + d*x) + Sinh[3*ArcSinh[c + d*x]])) + b^2*(4*(c
+ d*x)^2 + 4*ArcSinh[c + d*x]^2 + 4*(c + d*x)^3*PolyLog[2, -E^(-ArcSinh[c +
 d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + ArcSinh[c + d*x
]*(2*Sinh[2*ArcSinh[c + d*x]] + (Log[1 - E^(-ArcSinh[c + d*x])] - Log[1 + E
^(-ArcSinh[c + d*x])])*(-3*(c + d*x) + Sinh[3*ArcSinh[c + d*x]]))))/(12*d*e
^4*(c + d*x)^3)
```

Maple [A] time = 0.08, size = 310, normalized size = 1.8

$$-\frac{a^2}{3\,de^4\,(dx+c)^3} - \frac{b^2\text{Arcsinh}\,(dx+c)}{3\,de^4\,(dx+c)^2}\sqrt{1+(dx+c)^2} - \frac{b^2\,(\text{Arcsinh}\,(dx+c))^2}{3\,de^4\,(dx+c)^3} - \frac{b^2}{3\,de^4\,(dx+c)} + \frac{b^2\text{Arcsinh}\,(dx+c}{3\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^2/e^4/(d*x+c)^3-1/3/d*b^2/e^4/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*arcsin
h(d*x+c)-1/3/d*b^2/e^4/(d*x+c)^3*arcsinh(d*x+c)^2-1/3*b^2/d/e^4/(d*x+c)+1/3
/d*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*b^2*polylog(2
,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/3/d*b^2/e^4*arcsinh(d*x+c)*ln(1-d*x-c-
(1+(d*x+c)^2)^(1/2))-1/3*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/3
/d*a*b/e^4/(d*x+c)^3*arcsinh(d*x+c)-1/3/d*a*b/e^4/(d*x+c)^2*(1+(d*x+c)^2)^(
1/2)+1/3/d*a*b/e^4*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{a^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}+\int\frac{}{3\left(d^7e^4x^7+7\,cd^6e^4x^6+c^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 + b
^2*d^3)*x^3 + 3*(c^3 + c)*a*b + (c^3 + c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^2)
*x^2 + (3*(3*c^2*d + d)*a*b + (3*c^2*d + d)*b^2)*x + (b^2*c^2 + 3*(c^2 + 1)
*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*c*d + b^2*c*d)*x)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^7
*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5*e^4)
*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4
)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d*e^4)
*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*e^4 +
 d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 + 2*c^
2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\operatorname{arsinh}\left(dx+c\right)^2+2\,ab\operatorname{arsinh}\left(dx+c\right)+a^2}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^4*e^4*x
^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{b^2\operatorname{asinh}^2\left(c+dx\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{2ab\operatorname{asinh}\left(c+dx\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*asinh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

3.136 $\int (ce + dex)^m \left(a + b\sinh^{-1}(c + dx)\right)^3 \, dx$

Optimal. Leaf size=86

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de(m+1)}-\frac{3b\text{Unintegrable}\left(\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{e(m+1)}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Uni
ntegrable[((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/Sqrt[1 + (c +
d*x)^2], x])/(e*(1 + m))
```

Rubi [A] time = 0.173323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de(1+m)} - \frac{(3b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}} \, d\right.}{de(1+m)}$$

Mathematica [A] time = 3.50164, size = 0, normalized size = 0.

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A] time = 1.48, size = 0, normalized size = 0.

$$\int (dex+ce)^m \left(a+b\text{Arcsinh}\,(dx+c)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2b \operatorname{arsinh}(dx+c) + a^3\right)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (e(c+dx))^m (a+b\operatorname{asinh}(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(dx+c)+a)^3(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

3.137 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=326

$$\frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d}-\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c-\right.}{5d}$$

```
[Out] (16*a*b^2*e^4*x)/25 - (298*b^3*e^4*Sqrt[1 + (c + d*x)^2])/(375*d) + (76*b^3
*e^4*(1 + (c + d*x)^2)^(3/2))/(1125*d) - (6*b^3*e^4*(1 + (c + d*x)^2)^(5/2)
)/(625*d) + (16*b^3*e^4*(c + d*x)*ArcSinh[c + d*x])/(25*d) - (8*b^2*e^4*(c
+ d*x)^3*(a + b*ArcSinh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*A
rcSinh[c + d*x]))/(125*d) - (8*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c
 + d*x])^2)/(25*d) + (4*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcS
inh[c + d*x])^2)/(25*d) - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b
*ArcSinh[c + d*x])^2)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^3)
/(5*d)
```

Rubi [A] time = 0.473741, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5758, 5717, 5653, 261, 266, 43}

$$\frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d}-\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c-\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```


```
[Out] (16*a*b^2*e^4*x)/25 - (298*b^3*e^4*Sqrt[1 + (c + d*x)^2])/(375*d) + (76*b^3
*e^4*(1 + (c + d*x)^2)^(3/2))/(1125*d) - (6*b^3*e^4*(1 + (c + d*x)^2)^(5/2)
)/(625*d) + (16*b^3*e^4*(c + d*x)*ArcSinh[c + d*x])/(25*d) - (8*b^2*e^4*(c
+ d*x)^3*(a + b*ArcSinh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*A
rcSinh[c + d*x]))/(125*d) - (8*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c
 + d*x])^2)/(25*d) + (4*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcS
inh[c + d*x])^2)/(25*d) - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b
*ArcSinh[c + d*x])^2)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^3)
/(5*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^3}{5d} - \frac{(3be^4)\,\text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}} dx, x, c\right.}{5d}$$
$$= -\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{25d} + \frac{e^4(c+dx)^5\left(a+b\sin\right.}{5d}$$
$$= \frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d} + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\,\text{si}\right.}{25d}$$
$$= -\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx\right.}{125d}$$
$$= \frac{16}{25}ab^2e^4x - \frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{6b^2e^4(c+dx)^5\left(a+b\sin\right.}{125d}$$
$$= \frac{16}{25}ab^2e^4x - \frac{6b^3e^4\sqrt{1+(c+dx)^2}}{125d} + \frac{4b^3e^4\left(1+(c+dx)^2\right)^{3/2}}{125d} - \frac{6b^3e^4\left(1+(c+\right.}{625d}$$
$$= \frac{16}{25}ab^2e^4x - \frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d} + \frac{76b^3e^4\left(1+(c+dx)^2\right)^{3/2}}{1125d} - \frac{6b^3e^4\left(1+(\right.}{62}$$

Mathematica [A] time = 0.465942, size = 355, normalized size = 1.09

$$\frac{e^4\left(3a\left(25a^2+6b^2\right)(c+dx)^5+\frac{1}{15}b\sqrt{(c+dx)^2+1}\left(-27\left(25a^2+2b^2\right)(c+dx)^4+4\left(225a^2+68b^2\right)(c+dx)^2-8\left(225a^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) - 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[1 + (c + d*x)^2]*(-8*(225*a^2 + 518*b^2) + 4*(225*a^2 +
68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 - b*(-240*b^2*(c
 + d*x) + 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*(c + d*x)^5 + 2
40*a*b*Sqrt[1 + (c + d*x)^2] - 120*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] +
90*a*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 15*b^2*(-15*a*
(c + d*x)^5 + 8*b*Sqrt[1 + (c + d*x)^2] - 4*b*(c + d*x)^2*Sqrt[1 + (c + d*x
)^2] + 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 75*b^3*(
c + d*x)^5*ArcSinh[c + d*x]^3))/(375*d)
```

Maple [A] time = 0.05, size = 548, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^3+e^4*b^3*(1/5*(d*x+c)^3*arcsinh(d*x+c)^3*(1+(d*x+
c)^2)-1/5*arcsinh(d*x+c)^3*(d*x+c)*(1+(d*x+c)^2)+1/5*arcsinh(d*x+c)^3*(d*x+
c)-3/25*arcsinh(d*x+c)^2*(d*x+c)^2*(1+(d*x+c)^2)^(3/2)+7/25*arcsinh(d*x+c)^
2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/25*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+6
/125*arcsinh(d*x+c)*(d*x+c)*(1+(d*x+c)^2)^2+298/375*(d*x+c)*arcsinh(d*x+c)-
76/375*(d*x+c)*(1+(d*x+c)^2)*arcsinh(d*x+c)-6/625*(d*x+c)^2*(1+(d*x+c)^2)^(
3/2)+326/5625*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-4144/5625*(1+(d*x+c)^2)^(1/2))+
3*e^4*a*b^2*(1/5*(d*x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/5*arcsinh(d*x+c
)^2*(d*x+c)*(1+(d*x+c)^2)+1/5*arcsinh(d*x+c)^2*(d*x+c)-2/25*arcsinh(d*x+c)*
(d*x+c)^2*(1+(d*x+c)^2)^(3/2)+14/75*arcsinh(d*x+c)*(d*x+c)^2*(1+(d*x+c)^2)^
(1/2)-16/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2/125*(1+(d*x+c)^2)^2*(d*x+c
)+298/1125*d*x+298/1125*c-76/1125*(1+(d*x+c)^2)*(d*x+c))+3*e^4*a^2*b*(1/5*(
d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(
1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.16982, size = 2325, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e^
4*x^4 - 150*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 - 450*(4*a*b^2
*c - (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 - 225*(8*a*b^2*c^2 - (25*a^3 + 6*a
*b^2)*c^4 - 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4*x^4
 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x + b^
3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 225*(15*a*b
^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3 + 150*a
*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3*b^3*d^4
*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 - 2*b^3)*d^2*e^4*x^2 + 4*(3*
b^3*c^3 - 2*b^3*c)*d*e^4*x + (3*b^3*c^4 - 4*b^3*c^2 + 8*b^3)*e^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^
2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*e^4*
x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 - 30*(4*b^3*c - 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 - 15*(8*b^3*c^2 - 3*(25*a^2*b + 2*b^3)*c
^4 - 16*b^3)*d*e^4*x - (40*b^3*c^3 - 9*(25*a^2*b + 2*b^3)*c^5 - 240*b^3*c)*
e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*(9*a*b^2*c^2 - 2
*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 - 2*a*b^2*c)*d*e^4*x + (3*a*b^2*c^4 -
4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (27*(25*a^2*b + 2*b^3)*d^4*e^4*x^4 +
 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 - 2*(450*a^2*b + 136*b^3 - 81*(25*a^2
*b + 2*b^3)*c^2)*d^2*e^4*x^2 + 4*(27*(25*a^2*b + 2*b^3)*c^3 - 2*(225*a^2*b
+ 68*b^3)*c)*d*e^4*x + (27*(25*a^2*b + 2*b^3)*c^4 + 1800*a^2*b + 4144*b^3 -
```

```
4*(225*a^2*b + 68*b^3)*c^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 25.807, size = 2518, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
*4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*asinh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*asinh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*asinh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*asinh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*asinh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25
+ 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 3*a**2*b*d**4
*e**4*x**5*asinh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/25 + 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a*b*
*2*c**5*e**4*asinh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*asinh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)*asinh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*asinh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*asi
nh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c**2*
e**4*x/25 + 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c*d*e**4*x**2/25 + 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*asinh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*d*
*2*e**4*x**3/75 + 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + b**3*c**5*e**4*asinh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*asinh(c + d*x)/(125*d) + b**3*c**4*e**4*x*asinh(
c + d*x)**3 + 6*b**3*c**4*e**4*x*asinh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*asi
nh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*asinh(c + d*x)/25 - 12*b**3*c**3*
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**3*e**4*asi
nh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*asinh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*asinh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**2*e**4*x*asinh(c + d*
x)/25 + 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x
)**2/(25*d) + 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*asinh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*asi
nh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)*asinh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*c*d*e**4*x**2*asinh(c + d*x)/25 + 8*b**3*c*e**4*
```

```
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*c*e**4*asinh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*asinh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*asinh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)*asinh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*d**2*e**4*x**3*asinh(c + d*x)/75 + 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*e**4*x*asin
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^3, x)
```

3.138 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^3 dx$

Optimal. Leaf size=279

$$\frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d}-\frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d}-\frac{3b}{}$$

```
[Out] (45*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(256*d) - (3*b^3*e^3*(c + d*x)
^3*Sqrt[1 + (c + d*x)^2])/(128*d) - (45*b^3*e^3*ArcSinh[c + d*x])/(256*d) -
 (9*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(32*d) + (3*b^2*e^3*(c +
d*x)^4*(a + b*ArcSinh[c + d*x]))/(32*d) + (9*b*e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(32*d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^3)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^3)/(4*d)
```

Rubi [A] time = 0.385716, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5758, 5675, 321, 215}

$$\frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d}-\frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```


```
[Out] (45*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(256*d) - (3*b^3*e^3*(c + d*x)
^3*Sqrt[1 + (c + d*x)^2])/(128*d) - (45*b^3*e^3*ArcSinh[c + d*x])/(256*d) -
 (9*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(32*d) + (3*b^2*e^3*(c +
d*x)^4*(a + b*ArcSinh[c + d*x]))/(32*d) + (9*b*e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(32*d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^3)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^3)/(4*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,\right.}{4d}$$

$$= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d} + \frac{e^3(c+dx)^4\left(a+b\right.}{4}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d} + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\right.}{32d}$$

$$= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{32d} +$$

$$= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{9b^2e^3(c+}{}$$

$$= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{45b^3e^3\,\text{si}}{2}$$

Mathematica [A] time = 0.383546, size = 303, normalized size = 1.09

$$\frac{e^3\left(8a\left(8a^2+3b^2\right)(c+dx)^4+3b\sqrt{(c+dx)^2+1}(c+dx)\left(3\left(8a^2+5b^2\right)-2\left(8a^2+b^2\right)(c+dx)^2\right)-24b(c+dx)\sinh^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^3*(-72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*(c + d*
x)*Sqrt[1 + (c + d*x)^2]*(3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2)*(c + d*x)^2)
- 9*b*(8*a^2 + 5*b^2)*ArcSinh[c + d*x] - 24*b*(c + d*x)*(3*b^2*(c + d*x) -
8*a^2*(c + d*x)^3 - b^2*(c + d*x)^3 - 6*a*b*Sqrt[1 + (c + d*x)^2] + 4*a*b*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 24*b^2*(-3*a + 8*a*(c
+ d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (
c + d*x)^2])*ArcSinh[c + d*x]^2 + 8*b^3*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*
x]^3))/(256*d)
```

Maple [A] time = 0.041, size = 433, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a^3+e^3*b^3*(1/4*(d*x+c)^2*arcsinh(d*x+c)^3*(1+(d*x+
c)^2)-1/4*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/16*arcsinh(d*x+c)^2*(d*x+c)*(1+(
d*x+c)^2)^(3/2)+15/32*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)+5/32*arc
sinh(d*x+c)^3+3/32*arcsinh(d*x+c)*(d*x+c)^2*(1+(d*x+c)^2)-3/128*(1+(d*x+c)^
2)^(3/2)*(d*x+c)+51/256*(d*x+c)*(1+(d*x+c)^2)^(1/2)+51/256*arcsinh(d*x+c)-3
/8*(1+(d*x+c)^2)*arcsinh(d*x+c))+3*e^3*a*b^2*(1/4*(d*x+c)^2*arcsinh(d*x+c)^
2*(1+(d*x+c)^2)-1/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/8*arcsinh(d*x+c)*(d*x+
c)*(1+(d*x+c)^2)^(3/2)+5/16*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)+5/32
*arcsinh(d*x+c)^2+1/32*(d*x+c)^2*(1+(d*x+c)^2)-1/8*(d*x+c)^2-1/8)+3*e^3*a^2
*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d
*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.05081, size = 1773, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^3
 - 24*(3*a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^2*c - 2*(
```

```
8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*x^3
 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^3)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 24*(8*a*b^2*d^4*e^3*x^4
 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d*e^3*x
 + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^3*x^2 +
 3*(2*b^3*c^2 - b^3)*d*e^3*x + (2*b^3*c^3 - 3*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(8
*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 - 24*(b^3 -
 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 - 16*(3*b^3*c - 2*(8*a^2*b + b^3)*c^3)*
d*e^3*x - (24*b^3*c^2 - 8*(8*a^2*b + b^3)*c^4 + 24*a^2*b + 15*b^3)*e^3 - 16
*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2 - a*b^2)*d*e
^3*x + (2*a*b^2*c^3 - 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(2*(8*a^2*b + b^3)*d^3*e
^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 - 3*(8*a^2*b + 5*b^3 - 2*(8*a^2*b
+ b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b + 5*b^3)*c)*e^3)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 12.6469, size = 1828, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*asinh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*asinh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*asinh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*asinh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/16 + 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*asinh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/32 - 9*a**2*b*e**3*asinh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*asinh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*c*e**3*x/16 + 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*asinh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*d
*e**3*x**2/32 + 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)/16 - 9*a*b**2*e**3*asinh(c + d*x)**2/(32*d) + b**3*c**4*e**3*asinh
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*asinh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*asinh(c + d*x)**3 + 3*b**3*c**3*e**3*x*asinh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*asinh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*asinh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c**2*
e**3*asinh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*asinh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
```

```
 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c*e**3*x*asinh(c + d*x)/16 + 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(32*d) + 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(256*d) + b**3*d**3*e**3*
x**4*asinh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*asinh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*d*
e**3*x**2*asinh(c + d*x)/32 + 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**2/32 + 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/256 - 3*b**3*e**3*asinh(c + d*x)**3/(32*d) - 45*b**3*e**3*asinh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^3, x)
```

3.139 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=227

$$\frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{4}{3}ab^2e^2x+\frac{2be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\sqrt{(c+d}}{}$$

```
[Out] (-4*a*b^2*e^2*x)/3 + (14*b^3*e^2*Sqrt[1 + (c + d*x)^2])/(9*d) - (2*b^3*e^2*
(1 + (c + d*x)^2)^(3/2))/(27*d) - (4*b^3*e^2*(c + d*x)*ArcSinh[c + d*x])/(3
*d) + (2*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(9*d) + (2*b*e^2*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) - (b*e^2*(c + d*x)^2*S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(
a + b*ArcSinh[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.299909, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5758, 5717, 5653, 261, 266, 43}

$$\frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{4}{3}ab^2e^2x+\frac{2be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\sqrt{(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```


```
[Out] (-4*a*b^2*e^2*x)/3 + (14*b^3*e^2*Sqrt[1 + (c + d*x)^2])/(9*d) - (2*b^3*e^2*
(1 + (c + d*x)^2)^(3/2))/(27*d) - (4*b^3*e^2*(c + d*x)*ArcSinh[c + d*x])/(3
*d) + (2*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(9*d) + (2*b*e^2*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) - (b*e^2*(c + d*x)^2*S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(
a + b*ArcSinh[c + d*x])^3)/(3*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^3}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,x\right.}{d}$$
$$= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d} + \frac{e^2(c+dx)^3\left(a+b\,\text{s}\right.}{3d}$$
$$= \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{2be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(\right.}{3d}$$
$$= -\frac{4}{3}ab^2e^2x + \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{2be^2\sqrt{1+(c+dx)^2}\left(\right.}{3}$$
$$= -\frac{4}{3}ab^2e^2x - \frac{4b^3e^2(c+dx)\sinh^{-1}(c+dx)}{3d} + \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c\right.}{9d}$$
$$= -\frac{4}{3}ab^2e^2x + \frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d} - \frac{2b^3e^2\left(1+(c+dx)^2\right)^{3/2}}{27d} - \frac{4b^3e^2(c+}{}$$

Mathematica [A] time = 0.311478, size = 258, normalized size = 1.14

$$\frac{e^2\left(a\left(3a^2+2b^2\right)(c+dx)^3+\frac{1}{3}b\sqrt{(c+dx)^2+1}\left(-\left(9a^2+2b^2\right)(c+dx)^2+18a^2+40b^2\right)-b\sinh^{-1}(c+dx)\left(-9a^2(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*(-12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[1 + (c
+ d*x)^2]*(18*a^2 + 40*b^2 - (9*a^2 + 2*b^2)*(c + d*x)^2))/3 - b*(12*b^2*(c
 + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 - 12*a*b*Sqrt[1 + (c + d*x)
^2] + 6*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 3*b^2*(-3
*a*(c + d*x)^3 - 2*b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*
x)^2])*ArcSinh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcSinh[c + d*x]^3))/(9*d)
```

Maple [A] time = 0.037, size = 360, normalized size = 1.6

$$\frac{1}{d}\left(\frac{(dx+c)^3e^2a^3}{3} + e^2b^3\left(\frac{(\text{Arcsinh}\,(dx+c))^3(dx+c)\left(1+(dx+c)^2\right)}{3} - \frac{(\text{Arcsinh}\,(dx+c))^3(dx+c)}{3} - \frac{(\text{Arcsinh}\,(dx}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^3+e^2*b^3*(1/3*arcsinh(d*x+c)^3*(d*x+c)*(1+(d*x+c)
^2)-1/3*arcsinh(d*x+c)^3*(d*x+c)-1/3*arcsinh(d*x+c)^2*(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)+2/3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(d*x+c)*(1+(d*x+c)^2)
```

```
*arcsinh(d*x+c)-14/9*(d*x+c)*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2)^(1
/2)+40/27*(1+(d*x+c)^2)^(1/2))+3*e^2*a*b^2*(1/3*arcsinh(d*x+c)^2*(d*x+c)*(1
+(d*x+c)^2)-1/3*arcsinh(d*x+c)^2*(d*x+c)-2/9*arcsinh(d*x+c)*(d*x+c)^2*(1+(d
*x+c)^2)^(1/2)+4/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2/27*(1+(d*x+c)^2)*(d
*x+c)-14/27*d*x-14/27*c)+3*e^2*a^2*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87802, size = 1285, normalized size = 5.66

$$\frac{3\left(3\,a^3+2\,ab^2\right)d^3e^2x^3+9\left(3\,a^3+2\,ab^2\right)cd^2e^2x^2-9\left(4\,ab^2-\left(3\,a^3+2\,ab^2\right)c^2\right)de^2x+9\left(b^3d^3e^2x^3+3\,b^3cd^2e^2x^2+3\,b^3c^2de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2 -
 9*(4*a*b^2 - (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3*c
*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^2 +
9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x
+ (b^3*c^2 - 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x^3 + 3*
(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 - 3*(4*b^3 - (9*a^2*b + 2*b^3)*c^2)*d*e^2*x
 - (12*b^3*c - (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 + 2*a*b^2*
c*d*e^2*x + (a*b^2*c^2 - 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - ((9*a^2*b + 2*b^3)*d^2*e^
2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x - (18*a^2*b + 40*b^3 - (9*a^2*b + 2*b
^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 7.19988, size = 1173, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*asinh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*asinh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(3*d) + 3*a**2*b*c*d*e
```

```
**2*x**2*asinh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/3 + a**2*b*d**2*e**2*x**3*asinh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/3 + 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(3*d) + a*b**2*c**3*e**2*asinh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*asinh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
asinh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*asinh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 - 4*a*b**2*e**2*x/3 + 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + b**3*c**3*e**
2*asinh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*asinh(c + d*x)**3 + 2*b**3*c**2*e**2*x*asinh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**3*c*d*e**2*x*
*2*asinh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 4*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 4*b**3*c*e**2*asinh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*asi
nh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27
 - 4*b**3*e**2*x*asinh(c + d*x)/3 + 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)**2/(3*d) + 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^3, x)
```

3.140 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=161

$$\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+\right.}{2d}$$

```
[Out] (-3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(8*d) + (3*b^3*e*ArcSinh[c + d*x
])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(4*d) - (3*b*e*(c
 + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(a + b
*ArcSinh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^3)/(2
*d)
```

Rubi [A] time = 0.212607, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5661, 5758, 5675, 321, 215}

$$\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(8*d) + (3*b^3*e*ArcSinh[c + d*x
])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(4*d) - (3*b*e*(c
 + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(a + b
*ArcSinh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^3)/(2
*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^3}{2d}-\frac{(3be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,x,c\right.}{2d}$$
$$=-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\sin\right.}{2d}$$
$$=\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sin\right.}{4d}$$
$$=-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(}{}$$
$$=-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d}+\frac{3b^3e\sinh^{-1}(c+dx)}{8d}+\frac{3b^2e(c+dx)^2\left(a+b\right.}{4d}$$

Mathematica [A] time = 0.214428, size = 200, normalized size = 1.24

$$\frac{e\left(2a\left(2a^2+3b^2\right)(c+dx)^2-3b\left(2a^2+b^2\right)(c+dx)\sqrt{(c+dx)^2+1}+3b\left(2a^2+b^2\right)\sinh^{-1}(c+dx)-6b(c+dx)\sinh^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*(c + d*x)*Sqrt[1 +
(c + d*x)^2] + 3*b*(2*a^2 + b^2)*ArcSinh[c + d*x] - 6*b*(c + d*x)*(-2*a^2*(
c + d*x) - b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] +
```

```
6*b^2*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c + d*x)^2])*ArcSinh[c +
 d*x]^2 + 2*b^3*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^3))/(8*d)
```

Maple [A] time = 0.032, size = 243, normalized size = 1.5

$$\frac{1}{d}\left(\frac{(dx+c)^2 ea^3}{2}+eb^3\left(\frac{(\operatorname{Arcsinh}(dx+c))^3\left(1+(dx+c)^2\right)}{2}-\frac{3\,(\operatorname{Arcsinh}(dx+c))^2\,(dx+c)}{4}\sqrt{1+(dx+c)^2}-\frac{(\operatorname{Arcsinh}}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a^3+e*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/4*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*(1+(d*x+c
)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*x+c))+3*e
*a*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcsinh(d*x+c)*(1+(d*x+c)^2)
^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+3*e*a^2*b*(1/2*arcsi
nh(d*x+c)*(d*x+c)^2-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.77998, size = 891, normalized size = 5.53

$$\frac{2\left(2\,a^3+3\,ab^2\right)d^2ex^2+4\left(2\,a^3+3\,ab^2\right)cdex+2\left(2\,b^3d^2ex^2+4\,b^3cdex+\left(2\,b^3c^2+b^3\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^3
*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 + b^3)*e)*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*a*b
^2*c^2 + a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(2*(2*a^2*b + b^3)
*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x + (2*a^2*b + b^3 + 2*(2*a^2*b + b^3)
*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*((2*a^2*b + b^3)*d*e*x +
 (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 2.80014, size = 685, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*asinh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*asinh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/(4*d) + 3*a**2*b*d*e*x**2*asinh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 3*a**2*b*e*asinh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*asinh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*asinh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*asinh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/2 + 3*a*
b**2*e*asinh(c + d*x)**2/(4*d) + b**3*c**2*e*asinh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*asinh(c + d*x)/(4*d) + b**3*c*e*x*asinh(c + d*x)**3 + 3*b**3*c*e*
x*asinh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) +
 b**3*d*e*x**2*asinh(c + d*x)**3/2 + 3*b**3*d*e*x**2*asinh(c + d*x)/4 - 3*b
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + b**3*e*asinh(c + d*x)**3/(4*d)
+ 3*b**3*e*asinh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**3, Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.141 $\int \left(a + b\sinh^{-1}(c + dx)\right)^3 dx$

Optimal. Leaf size=100

$$6ab^2x - \frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} - \frac{6b^3\sqrt{(c+dx)^2+1}}{d} + \frac{6b^3(c+dx)}{}$$

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[1 + (c + d*x)^2])/d + (6*b^3*(c + d*x)*ArcSinh[c +
d*x])/d - (3*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/d + ((c +
d*x)*(a + b*ArcSinh[c + d*x])^3)/d
```

Rubi [A] time = 0.108284, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5653, 5717, 261}

$$6ab^2x - \frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} - \frac{6b^3\sqrt{(c+dx)^2+1}}{d} + \frac{6b^3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[1 + (c + d*x)^2])/d + (6*b^3*(c + d*x)*ArcSinh[c +
d*x])/d - (3*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/d + ((c +
d*x)*(a + b*ArcSinh[c + d*x])^3)/d
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} - \frac{(3b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} + \frac{(6b^2$$
$$= 6ab^2x - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}$$
$$= 6ab^2x + \frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c$$
$$= 6ab^2x - \frac{6b^3\sqrt{1+(c+dx)^2}}{d} + \frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\,\text{s}\right.}{d}$$

Mathematica [A] time = 0.15662, size = 147, normalized size = 1.47

$$\frac{a\left(a^2+6b^2\right)(c+dx)-3b\left(a^2+2b^2\right)\sqrt{(c+dx)^2+1}-3b\sinh^{-1}(c+dx)\left(a^2(-(c+dx))+2ab\sqrt{(c+dx)^2+1}-2b^2(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[1 + (c + d*x)^2] - 3*b*
(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[
c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x
]^2 + b^3*(c + d*x)*ArcSinh[c + d*x]^3)/d
```

Maple [A] time = 0.029, size = 160, normalized size = 1.6

$$\frac{1}{d}\left(a^3(dx+c)+b^3\left((\text{Arcsinh}(dx+c))^3(dx+c)-3(\text{Arcsinh}(dx+c))^2\sqrt{1+(dx+c)^2}+6(dx+c)\text{Arcsinh}(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+b^3*(arcsinh(d*x+c)^3*(d*x+c)-3*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+3*a*b^2*(arcsinh
(d*x+c)^2*(d*x+c)-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+3*a^2*b*(
(d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73792, size = 548, normalized size = 5.48

$$\left(b^3dx + b^3c\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^3 + \left(a^3 + 6\,ab^2\right)dx + 3\left(ab^2dx + ab^2c - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}b^3\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 3*(2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*(a^2*b + 2*b^3))/d
```

Sympy [A] time = 1.06046, size = 282, normalized size = 2.82

$$\begin{cases} a^3x + \frac{3a^2bc\operatorname{asinh}(c+dx)}{d} + 3a^2bx\operatorname{asinh}(c+dx) - \frac{3a^2b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{3ab^2c\operatorname{asinh}^2(c+dx)}{d} + 3ab^2x\operatorname{asinh}^2(c+dx) + 6ab^2x - \\ x\left(a + b\operatorname{asinh}(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*asinh(c + d*x)/d + 3*a**2*b*x*asinh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 3*a*b**2*c*asinh(c + d
*x)**2/d + 3*a*b**2*x*asinh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + b**3*c*asinh(c + d*x)**3/d + 6
*b**3*c*asinh(c + d*x)/d + b**3*x*asinh(c + d*x)**3 + 6*b**3*x*asinh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*(a + b*asinh(c))**
3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{arsinh}(dx + c) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3, x)
```

3.142 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{ce+dex}\,dx$

Optimal. Leaf size=155

$$-\frac{3b^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{2de}-\frac{3b\text{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de}-\frac{3b}{}$$

```
[Out] (a + b*ArcSinh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcSinh[c + d*x])^3*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])*PolyL
og[3, E^(-2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^3*PolyLog[4, E^(-2*ArcSinh[c
 + d*x])])/(4*d*e)
```

Rubi [A] time = 0.223581, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{2de}+\frac{3b\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de}+\frac{3b^3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcSinh[c + d*x])^3*Log[1 -
 E^(2*ArcSinh[c + d*x])])/(d*e) + (3*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2
, E^(2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])*PolyLo
g[3, E^(2*ArcSinh[c + d*x])])/(2*d*e) + (3*b^3*PolyLog[4, E^(2*ArcSinh[c +
d*x])])/(4*d*e)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x}\,dx,x,c+dx\right)}{de}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{(3b)\,\text{Su}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b\left(a+\right.}{}$$

Mathematica [A] time = 0.0467424, size = 128, normalized size = 0.83

$$\frac{-6b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)+6b\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2+3b^3\text{P}}{4de}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])^4/b) + 4*(a + b*ArcSinh[c + d*x])^3*Log[1 - E^(
2*ArcSinh[c + d*x])] + 6*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[3, E^(2*ArcSinh[c +
d*x])] + 3*b^3*PolyLog[4, E^(2*ArcSinh[c + d*x])])/(4*d*e)
```

Maple [B] time = 0.053, size = 736, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^3/e*ln(d*x+c)-1/4/d*b^3/e*arcsinh(d*x+c)^4+1/d*b^3/e*arcsinh(d*x+c)^3
*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3/d*b^3/e*arcsinh(d*x+c)^2*polylog(2,-d*x-
c-(1+(d*x+c)^2)^(1/2))-6/d*b^3/e*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)
^2)^(1/2))+6/d*b^3/e*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*b^3/e*arcsin
```

```
h(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*b^3/e*arcsinh(d*x+c)^2*polyl
og(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*b^3/e*arcsinh(d*x+c)*polylog(3,d*x+c+(1
+(d*x+c)^2)^(1/2))+6/d*b^3/e*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-1/d*a*b^2
/e*arcsinh(d*x+c)^3+3/d*a*b^2/e*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(
1/2))+6/d*a*b^2/e*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-6/d*
a*b^2/e*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*a*b^2/e*arcsinh(d*x+c)^2*
ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e*arcsinh(d*x+c)*polylog(2,d*x+c+
(1+(d*x+c)^2)^(1/2))-6/d*a*b^2/e*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-3/2/d
*a^2*b/e*arcsinh(d*x+c)^2+3/d*a^2*b/e*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^
2)^(1/2))+3/d*a^2*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*a^2*b/e*arc
sinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*a^2*b/e*polylog(2,d*x+c+(1+
(d*x+c)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2b \operatorname{arsinh}(dx+c) + a^3}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*asinh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*asinh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
sinh(c + d*x)/(c + d*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")

[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

3.143 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{6b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^3\text{Poly}}{}$$

```
[Out] -((a + b*ArcSinh[c + d*x])^3/(d*e^2*(c + d*x))) - (6*b*(a + b*ArcSinh[c + d
*x])^2*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^3*PolyLog[3, -E^ArcSinh[c
 + d*x]])/(d*e^2) - (6*b^3*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rubi [A] time = 0.255148, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5661, 5760, 4182, 2531, 2282, 6589}

$$-\frac{6b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^3\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^3/(d*e^2*(c + d*x))) - (6*b*(a + b*ArcSinh[c + d
*x])^2*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^3*PolyLog[3, -E^ArcSinh[c
 + d*x]])/(d*e^2) - (6*b^3*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
```

```
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^2x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^2}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\operatorname{csch}(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{(6b^2)\,\text{Su}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{6b^2\left(a+\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{6b^2\left(a+\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{6b^2\left(a+\right.}{}$$

Mathematica [A] time = 0.779462, size = 315, normalized size = 1.9

$$3ab^2\left(2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right) - 2\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right) + \sinh^{-1}(c+dx)\left(-\frac{\sinh^{-1}(c+dx)}{c+dx} + 2\log\left(1-e^{-\sinh^{-1}(c+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^3/(c + d*x)) - (3*a^2*b*ArcSinh[c + d*x])/(c + d*x) + 3*a^2*b*Log[c +
d*x] - 3*a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]] + 3*a*b^2*(ArcSin
h[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])
] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])
] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + b^3*(-(ArcSinh[c + d*x]^3/(c + d
*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c +
d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-
ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] +
 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]
))/(d*e^2)
```

Maple [B] time = 0.046, size = 481, normalized size = 2.9

$$-\frac{a^3}{de^2(dx+c)} - \frac{b^3(\text{Arcsinh}(dx+c))^3}{de^2(dx+c)} - 3\,\frac{b^3(\text{Arcsinh}(dx+c))^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{de^2} - 6\,\frac{b^3\text{Arcsinh}(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^3/e^2/(d*x+c)-1/d*b^3/e^2*arcsinh(d*x+c)^3/(d*x+c)-3/d*b^3/e^2*arcsi
nh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*b^3/e^2*arcsinh(d*x+c)*poly
log(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+6*b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2
))/d/e^2+3/d*b^3/e^2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*b
^3/e^2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6*b^3*polylog(3,
d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-3/d*a*b^2/e^2/(d*x+c)*arcsinh(d*x+c)^2-6/d
*a*b^2/e^2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a*b^2/e^2*pol
ylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-
(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-3/d
*a^2*b/e^2/(d*x+c)*arcsinh(d*x+c)-3/d*a^2*b/e^2*arctanh(1/(1+(d*x+c)^2)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2b \operatorname{arsinh}(dx+c) + a^3}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3 \operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2 \operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b \operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*asinh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*asinh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

3.144 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{3b^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)}{2de^3}+\frac{3b^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh\right.}{2de^3(c+dx)}$$

```
[Out] (3*b*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^3) - (3*b*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^3/(
2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcSinh[c + d*x])*Log[1 - E^(-2*ArcSin
h[c + d*x])])/(d*e^3) - (3*b^3*PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(2*d*e^
3)
```

Rubi [A] time = 0.260505, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5723, 5659, 3716, 2190, 2279, 2391}

$$\frac{3b^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)}{2de^3}+\frac{3b^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(\right.}{2de^3(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-3*b*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^3) - (3*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^3/
(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcSinh[c + d*x])*Log[1 - E^(2*ArcSin
h[c + d*x])])/(d*e^3) + (3*b^3*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*d*e^3
)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
```

```
c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2de^3}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{\left(3b^2\right)\text{Subst}}{}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{\left(3b^2\right)\text{Subst}}{}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sin\right.}{2de^3(}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sin\right.}{2de^3(}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sin\right.}{2de^3(}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sin\right.}{2de^3(}$$

Mathematica [A] time = 0.7836, size = 229, normalized size = 1.46

$$-\frac{3b^3(c+dx)^2\text{PolyLog}\left(2, e^{-2\sinh^{-1}(c+dx)}\right) + a\left(a\left(a+3b(c+dx)\sqrt{c^2+2cdx+d^2x^2+1}\right) - 6b^2(c+dx)^2\log(c+dx)\right) +}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -(3*b^2*(a + b*(c + d*x)*(-c - d*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]))*Ar
cSinh[c + d*x]^2 + b^3*ArcSinh[c + d*x]^3 + 3*b*ArcSinh[c + d*x]*(a*(a + 2*
b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) - 2*b^2*(c + d*x)^2*Log[1 -
E^(-2*ArcSinh[c + d*x])]) + a*(a*(a + 3*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x
+ d^2*x^2]) - 6*b^2*(c + d*x)^2*Log[c + d*x]) + 3*b^3*(c + d*x)^2*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(2*d*e^3*(c + d*x)^2)
```

Maple [B] time = 0.089, size = 409, normalized size = 2.6

$$-\frac{a^3}{2\,de^3\,(dx+c)^2} - \frac{3\,b^3\,(\text{Arcsinh}\,(dx+c))^2}{2\,de^3\,(dx+c)}\sqrt{1+(dx+c)^2} - \frac{3\,b^3\,(\text{Arcsinh}\,(dx+c))^2}{2\,de^3} - \frac{b^3\,(\text{Arcsinh}\,(dx+c))^3}{2\,de^3\,(dx+c)^2} + 3\,\frac{b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^3/e^3/(d*x+c)^2-3/2/d*b^3/e^3*arcsinh(d*x+c)^2/(d*x+c)*(1+(d*x+c)^
2)^(1/2)-3/2/d*b^3/e^3*arcsinh(d*x+c)^2-1/2/d*b^3/e^3*arcsinh(d*x+c)^3/(d*x
+c)^2+3/d*b^3/e^3*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^
3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^3*arcsinh(d*x+c)*ln(1-d*x
-c-(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-3/
d*a*b^2/e^3*arcsinh(d*x+c)-3/d*a*b^2/e^3*arcsinh(d*x+c)/(d*x+c)*(1+(d*x+c)^
2)^(1/2)-3/2/d*a*b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+3/d*a*b^2/e^3*ln((d*x+c
+(1+(d*x+c)^2)^(1/2))^2-1)-3/2/d*a^2*b/e^3/(d*x+c)^2*arcsinh(d*x+c)-3/2/d*a
^2*b/e^3/(d*x+c)*(1+(d*x+c)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2b \operatorname{arsinh}(dx+c) + a^3}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3 \operatorname{asinh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2 \operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b \operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^3}{(dex + ce)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

3.145 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=261

$$\frac{b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^3\text{PolyLog}}{}$$

```
[Out] -((b^2*(a + b*ArcSinh[c + d*x]))/(d*e^4*(c + d*x))) - (b*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c +
d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcSinh[c + d*x])^2*ArcTanh[E^Arc
Sinh[c + d*x]])/(d*e^4) - (b^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4) + (b
^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) - (b^2
*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (b^3*Po
lyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (b^3*PolyLog[3, E^ArcSinh[c + d*x]
])/(d*e^4)
```

Rubi [A] time = 0.410011, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5865, 12, 5661, 5747, 5760, 4182, 2531, 2282, 6589, 266, 63, 207}

$$\frac{b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```



```
[Out] -((b^2*(a + b*ArcSinh[c + d*x]))/(d*e^4*(c + d*x))) - (b*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c +
d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcSinh[c + d*x])^2*ArcTanh[E^Arc
Sinh[c + d*x]])/(d*e^4) - (b^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4) + (b
^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) - (b^2
*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (b^3*Po
lyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (b^3*PolyLog[3, E^ArcSinh[c + d*x]
])/(d*e^4)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} - \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}\right.}{x\sqrt{}}\right.}{}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c\right.}{3de^4(c+d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c\right.}{3de^4(c+d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c\right.}{3de^4(c+d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c\right.}{3de^4(c+d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c\right.}{3de^4(c+d}$$

Mathematica [B] time = 7.53295, size = 694, normalized size = 2.66

$$\frac{ab^2\left(-8\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right) - \frac{2\left(-4(c+dx)^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)+4\sinh^{-1}(c+dx)^2+2\sinh^{-1}(c+dx)\sinh\left(2\sinh^{-1}(c+dx)\right)-3(c+dx)\,\text{si}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -a^3/(3*d*e^4*(c + d*x)^3) - (a^2*b*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*d
*e^4*(c + d*x)^2) - (a^2*b*ArcSinh[c + d*x])/(d*e^4*(c + d*x)^3) - (a^2*b*L
og[c + d*x])/(2*d*e^4) + (a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]])
/(2*d*e^4) + (a*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x])] - (2*(-2 + 4*Arc
Sinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d*x)*ArcSinh[c + d*x]
*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c + d*x]*Log[1 + E^(-
```

```
ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + 2*Ar
cSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d*x]*Log[1 - E^(-ArcS
inh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d*x]*Log[1 + E^(-ArcS
inh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))/(8*d*e^4) + (b^3*(-
24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcSinh[c + d*x]^3*Coth[Ar
cSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh[c + d*x]/2]^2 - (c +
d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 - 24*ArcSinh[c + d*x]^2*
Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*Log[1 + E^(-ArcSinh[
c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*ArcSinh[c + d*x]*PolyLog
[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c
+ d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[3, E^(-ArcSin
h[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c + d*x]/2]^2 - (16*ArcSin
h[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x]*
Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[ArcSinh[c + d*x]/2]))/
(48*d*e^4)
```

Maple [B] time = 0.094, size = 651, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^3/e^4/(d*x+c)^3-1/2/d*b^3/e^4/(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)-1/3/d*b^3/e^4/(d*x+c)^3*arcsinh(d*x+c)^3-1/d*b^3/e^4/(d*x+c)*arc
sinh(d*x+c)+1/2/d*b^3/e^4*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+
1/d*b^3/e^4*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-b^3*polylo
g(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/2/d*b^3/e^4*arcsinh(d*x+c)^2*ln(1-d
*x-c-(1+(d*x+c)^2)^(1/2))-1/d*b^3/e^4*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*
x+c)^2)^(1/2))+b^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/d*b^3/e^4*a
rctanh(d*x+c+(1+(d*x+c)^2)^(1/2))-1/d*a*b^2/e^4/(d*x+c)^2*arcsinh(d*x+c)*(1
+(d*x+c)^2)^(1/2)-1/d*a*b^2/e^4/(d*x+c)^3*arcsinh(d*x+c)^2-1/d*a*b^2/e^4/(d
*x+c)+1/d*a*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/d*a*b^
2/e^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/d*a*b^2/e^4*arcsinh(d*x+c)*ln
(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/d*a*b^2/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(
1/2))-1/d*a^2*b/e^4/(d*x+c)^3*arcsinh(d*x+c)-1/2/d*a^2*b/e^4/(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+1/2/d*a^2*b/e^4*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 + c)*a*b^2
 + (c^3 + c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c*d
^2)*x^2 + (3*(3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 + 3*(c^2
 + 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*(3*a*b^2*c*d + b^3*c*d)*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
```

```
 + 1))^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d + d)*a^2*b*x + (
c^3 + c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 + 1)*a^2*b)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
)/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d
^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*
d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4
*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4
+ 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\operatorname{arsinh}(dx+c)^3 + 3\,ab^2\operatorname{arsinh}(dx+c)^2 + 3\,a^2b\operatorname{arsinh}(dx+c) + a^3}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4
*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3a^2b\operatorname{asinh}(c+d}{c^4+4c^3dx+6c^2d^2x^2+4cd}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*asinh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*asinh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

3.146 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=86

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de(m+1)}-\frac{4b\text{Unintegrable}\left(\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{e(m+1)}$$

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Uni
ntegrable[((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^3)/Sqrt[1 + (c +
d*x)^2], x])/(e*(1 + m))
```

Rubi [A] time = 0.178817, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de(1+m)}-\frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\sinh^{-1}(x)\right.}{\sqrt{1+x^2}}\right.}{de(1+m)}$$

Mathematica [A] time = 1.80833, size = 0, normalized size = 0.

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A] time = 1.5, size = 0, normalized size = 0.

$$\int (dex+ce)^m (a+b\text{Arcsinh}\,(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4\right)(dex+ce)^m\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(dx+c)+a)^4 (dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

3.147 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=349

$$-\frac{3b^3e^3(c+dx)^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{45b^3e^3(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{64d}+\frac{3b^2e^3}{}$$

```
[Out] (-45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) + (45*b
^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(64*d) - (
3*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(32*d
) - (45*b^2*e^3*(a + b*ArcSinh[c + d*x])^2)/(128*d) - (9*b^2*e^3*(c + d*x)^
2*(a + b*ArcSinh[c + d*x])^2)/(16*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcSin
h[c + d*x])^2)/(16*d) + (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x])^3)/(8*d) - (b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*A
rcSinh[c + d*x])^3)/(4*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^4)/(32*d) + (e^
3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^4)/(4*d)
```

Rubi [A] time = 0.67252, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$-\frac{3b^3e^3(c+dx)^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{45b^3e^3(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{64d}+\frac{3b^2e^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```


```
[Out] (-45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) + (45*b
^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(64*d) - (
3*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(32*d
) - (45*b^2*e^3*(a + b*ArcSinh[c + d*x])^2)/(128*d) - (9*b^2*e^3*(c + d*x)^
2*(a + b*ArcSinh[c + d*x])^2)/(16*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcSin
h[c + d*x])^2)/(16*d) + (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x])^3)/(8*d) - (b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*A
rcSinh[c + d*x])^3)/(4*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^4)/(32*d) + (e^
3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^4)/(4*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^4}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}} dx, x, c+ \ldots\right.}{d}$$
$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d} + \frac{e^3(c+dx)^4\left(a+b\sinh \ldots\right.}{4d}$$
$$= \frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\,\text{si}\ldots\right.}{8d}$$
$$= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{32d} - \frac{9b^2e^3(c+dx)^2\left(a+b\ldots\right.}{16d}$$
$$= \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{64d} - \frac{3b^3e\ldots}{}$$
$$= -\frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sin\ldots\right.}{64d}$$

Mathematica [A] time = 0.592948, size = 475, normalized size = 1.36

$$\frac{e^3\left(\left(24a^2b^2+32a^4+3b^4\right)(c+dx)^4-9b^2\left(8a^2+5b^2\right)(c+dx)^2+2ab\sqrt{(c+dx)^2+1}(c+dx)\left(-2\left(8a^2+3b^2\right)(c+dx)^2+\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*(-9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
 + d*x)^4 + 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(24*a^2 + 45*b^2 - 2*(8*a
^2 + 3*b^2)*(c + d*x)^2) - 6*a*b*(8*a^2 + 15*b^2)*ArcSinh[c + d*x] + 2*b*(c
 + d*x)*(-72*a*b^2*(c + d*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 +
72*a^2*b*Sqrt[1 + (c + d*x)^2] + 45*b^3*Sqrt[1 + (c + d*x)^2] - 48*a^2*b*(c
 + d*x)^2*Sqrt[1 + (c + d*x)^2] - 6*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*
ArcSinh[c + d*x] + 3*b^2*(-24*a^2 - 15*b^2 - 24*b^2*(c + d*x)^2 + 64*a^2*(c
 + d*x)^4 + 8*b^2*(c + d*x)^4 + 48*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 32
*a*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 16*b^3*(-3*a +
 8*a*(c + d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sq
rt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + 4*b^4*(-3 + 8*(c + d*x)^4)*ArcSin
h[c + d*x]^4))/(128*d)
```

Maple [B] time = 0.065, size = 683, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a^4+e^3*b^4*(1/4*(d*x+c)^2*arcsinh(d*x+c)^4*(1+(d*x+
c)^2)-1/4*arcsinh(d*x+c)^4*(1+(d*x+c)^2)-1/4*arcsinh(d*x+c)^3*(d*x+c)*(1+(d
*x+c)^2)^(3/2)+5/8*arcsinh(d*x+c)^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)+5/32*arcsin
h(d*x+c)^4+3/16*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-3/32*arcsinh(d*x+c
)*(d*x+c)*(1+(d*x+c)^2)^(3/2)+51/64*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x
+c)+51/128*arcsinh(d*x+c)^2+3/128*(d*x+c)^2*(1+(d*x+c)^2)-3/4*arcsinh(d*x+c
)^2*(1+(d*x+c)^2)-3/8*(d*x+c)^2-3/8)+4*e^3*a*b^3*(1/4*(d*x+c)^2*arcsinh(d*x
+c)^3*(1+(d*x+c)^2)-1/4*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/16*arcsinh(d*x+c)^
2*(d*x+c)*(1+(d*x+c)^2)^(3/2)+15/32*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d
*x+c)+5/32*arcsinh(d*x+c)^3+3/32*arcsinh(d*x+c)*(d*x+c)^2*(1+(d*x+c)^2)-3/1
28*(1+(d*x+c)^2)^(3/2)*(d*x+c)+51/256*(d*x+c)*(1+(d*x+c)^2)^(1/2)+51/256*ar
csinh(d*x+c)-3/8*(1+(d*x+c)^2)*arcsinh(d*x+c))+6*e^3*a^2*b^2*(1/4*(d*x+c)^2
*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/8*arcs
inh(d*x+c)*(d*x+c)*(1+(d*x+c)^2)^(3/2)+5/16*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1
/2)*(d*x+c)+5/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^2*(1+(d*x+c)^2)-1/8*(d*x+c)^
2-1/8)+4*e^3*a^3*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^
2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.34543, size = 2631, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c*d^3*e^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3
*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*
b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 + 48*
b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4 + 32*
a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*x + (8*
a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2 + 3*(2*
b^4*c^2 - b^4)*d*e^3*x + (2*b^4*c^3 - 3*b^4*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(8*(8*a^
2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 - 24*(b^4 - 2
*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3
)*d*e^3*x - (24*b^4*c^2 - 8*(8*a^2*b^2 + b^4)*c^4 + 24*a^2*b^2 + 15*b^4)*e^
3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*b^3*c^2 - a*b^
3)*d*e^3*x + (2*a*b^3*c^3 - 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*(8*(8*a^3*b + 3*
a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3 - 24*(3*a*b^3 - 2
*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^3*c - 2*(8*a^3*b + 3*a*b^
3)*c^3)*d*e^3*x - (72*a*b^3*c^2 - 8*(8*a^3*b + 3*a*b^3)*c^4 + 24*a^3*b + 45
*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6*(8*a^2*b^2 + b^4)*c*d^
2*e^3*x^2 - 3*(8*a^2*b^2 + 5*b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d*e^3*x + (2*(8
*a^2*b^2 + b^4)*c^3 - 3*(8*a^2*b^2 + 5*b^4)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(2*(8*a^3*
b + 3*a*b^3)*d^3*e^3*x^3 + 6*(8*a^3*b + 3*a*b^3)*c*d^2*e^3*x^2 - 3*(8*a^3*b
 + 15*a*b^3 - 2*(8*a^3*b + 3*a*b^3)*c^2)*d*e^3*x + (2*(8*a^3*b + 3*a*b^3)*c
^3 - 3*(8*a^3*b + 15*a*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 28.8438, size = 2876, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*asinh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*asinh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*as
inh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
4 + 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) + a**3*b*d**
3*e**3*x**4*asinh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/4 + 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 - 3
*a**3*b*e**3*asinh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*asinh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2 + 9*a**2*b**2*c
**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/4 - 9*a**2*b**2*c*e**3*x/8 + 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(8*d) + 3*a**2*
```

```
b**2*d**3*e**3*x**4*asinh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*
x)/4 - 9*a**2*b**2*d*e**3*x**2/16 + 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a**2*b**2*e**3*asinh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*asinh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*asinh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*asinh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*as
inh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*asinh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*asinh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/32 - 9*a*b**3*c**2*e**3*asinh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*asinh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32
 - 9*a*b**3*c*e**3*x*asinh(c + d*x)/4 + 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)**2/(8*d) + 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/(64*d) + a*b**3*d**3*e**3*x**4*asinh(c + d*x)**3 + 3
*a*b**3*d**3*e**3*x**4*asinh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32 - 9*a*b**3*d*e**3*x**2*asinh(c + d*x
)/8 + 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**
2/8 + 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/64 - 3*a*b**3*e
**3*asinh(c + d*x)**3/(8*d) - 45*a*b**3*e**3*asinh(c + d*x)/(64*d) + b**4*c
**4*e**3*asinh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*asinh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*asinh(c + d*x)**4 + 3*b**4*c**3*e**3*x*asinh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*asinh(c + d*x)
**4/2 + 9*b**4*c**2*d*e**3*x**2*asinh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/32 - 9*b**4*c**2*e**3*asinh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*asinh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*asinh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*c*e**3*x*asinh(c + d*x)**2/8 - 45*b**
4*c*e**3*x/64 + 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)**3/(8*d) + 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*asinh(c + d*x)**4/4 + 3*b**4*d**3*e*
*3*x**4*asinh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*d
*e**3*x**2*asinh(c + d*x)**2/16 - 45*b**4*d*e**3*x**2/128 + 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/8 + 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/64 - 3*b**4*e**3*asinh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*asinh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*asinh(c))**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^4, x)
```

3.148 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=281

$$\frac{160b^3e^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}+\frac{4b^2e^2(c+dx)^3}{}$$

```
[Out] (-160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) + (160*b^3*e^2*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^3*e^2*(c + d*x)^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^2*e^2*(c + d*x)
*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (4*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[
c + d*x])^2)/(9*d) + (8*b*e^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
)^3)/(9*d) - (4*b*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
d*x])^3)/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^4)/(3*d)
```

Rubi [A] time = 0.484952, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5661, 5758, 5717, 5653, 8, 30}

$$\frac{160b^3e^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}+\frac{4b^2e^2(c+dx)^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```


```
[Out] (-160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) + (160*b^3*e^2*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^3*e^2*(c + d*x)^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^2*e^2*(c + d*x)
*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (4*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[
c + d*x])^2)/(9*d) + (8*b*e^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
)^3)/(9*d) - (4*b*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
d*x])^3)/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^4)/(3*d)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^4}{3d}-\frac{\left(4be^2\right)\text{Subst}\left(\int\frac{x^3\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}}\,dx,x,c\right.}{3d}$$

$$=-\frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9d}+\frac{e^2(c+dx)^3\left(a+b\sin\right.}{3d}$$

$$=\frac{4b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{9d}+\frac{8be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{9d}$$

$$=-\frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^2e^2(c+dx)\left(a+b\right.}{3d}$$

$$=\frac{8b^4e^2(c+dx)^3}{81d}+\frac{160b^3e^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+d}{}$$

$$=-\frac{160}{27}b^4e^2x+\frac{8b^4e^2(c+dx)^3}{81d}+\frac{160b^3e^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}$$

Mathematica [A] time = 0.441229, size = 412, normalized size = 1.47

$$\frac{e^2\left(\left(36a^2b^2+27a^4+8b^4\right)(c+dx)^3-24b^2\left(9a^2+20b^2\right)(c+dx)+12ab\sqrt{(c+dx)^2+1}\left(-\left(3a^2+2b^2\right)(c+dx)^2+6a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*(-24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
 + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2]*(6*a^2 + 40*b^2 - (3*a^2 + 2*b^2)*
(c + d*x)^2) + 12*b*(-36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3 + 6*a*b^2*(c +
 d*x)^3 + 18*a^2*b*Sqrt[1 + (c + d*x)^2] + 40*b^3*Sqrt[1 + (c + d*x)^2] - 9
*a^2*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] - 2*b^3*(c + d*x)^2*Sqrt[1 + (c +
d*x)^2])*ArcSinh[c + d*x] + 18*b^2*(-12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 +
 2*b^2*(c + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2] - 6*a*b*(c + d*x)^2*Sqrt[
1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 - 36*b^3*(-3*a*(c + d*x)^3 - 2*b*Sqrt[
1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3
+ 27*b^4*(c + d*x)^3*ArcSinh[c + d*x]^4))/(81*d)
```

Maple [B] time = 0.04, size = 567, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^4+e^2*b^4*(1/3*(d*x+c)*arcsinh(d*x+c)^4*(1+(d*x+c)
^2)-1/3*arcsinh(d*x+c)^4*(d*x+c)-4/9*arcsinh(d*x+c)^3*(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)+8/9*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4/9*arcsinh(d*x+c)^2*(d*x
+c)*(1+(d*x+c)^2)-28/9*arcsinh(d*x+c)^2*(d*x+c)-8/27*arcsinh(d*x+c)*(d*x+c)
^2*(1+(d*x+c)^2)^(1/2)+160/27*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+8/81*(1+(d
*x+c)^2)*(d*x+c)-488/81*d*x-488/81*c)+4*e^2*a*b^3*(1/3*arcsinh(d*x+c)^3*(d*
x+c)*(1+(d*x+c)^2)-1/3*arcsinh(d*x+c)^3*(d*x+c)-1/3*arcsinh(d*x+c)^2*(d*x+c
)^2*(1+(d*x+c)^2)^(1/2)+2/3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(d*x+c
)*(1+(d*x+c)^2)*arcsinh(d*x+c)-14/9*(d*x+c)*arcsinh(d*x+c)-2/27*(d*x+c)^2*(
1+(d*x+c)^2)^(1/2)+40/27*(1+(d*x+c)^2)^(1/2))+6*e^2*a^2*b^2*(1/3*arcsinh(d*
x+c)^2*(d*x+c)*(1+(d*x+c)^2)-1/3*arcsinh(d*x+c)^2*(d*x+c)-2/9*arcsinh(d*x+c
)*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2/27
*(1+(d*x+c)^2)*(d*x+c)-14/27*d*x-14/27*c)+4*e^2*a^3*b*(1/3*(d*x+c)^3*arcsin
h(d*x+c)-1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99754, size = 1894, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
8*b^4)*c*d^2*e^2*x^2 - 3*(72*a^2*b^2 + 160*b^4 - (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2*d*
e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 3
6*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x + 3*a*
b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 - 2*b^4)*e^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d
^2*e^2*x^2 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x - (12*b^4*c - (9*a
^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2*x + (a*b^
3*c^2 - 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^3*e^2*x^3 +
9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 - 9*(4*a*b^3 - (3*a^3*b + 2*a*b^3)*c^2)
*d*e^2*x - 3*(12*a*b^3*c - (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9*a^2*b^2 + 2*b
^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x - (18*a^2*b^2 + 40*b^4 -
(9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 12*((3*a^3*b + 2*a*b^3)*d^2*e^2*x^
2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x - (6*a^3*b + 40*a*b^3 - (3*a^3*b + 2*a*
b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 13.9048, size = 1889, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
4*a**3*b*c**3*e**2*asinh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*asinh(c + d*
x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 4*a**3
*b*c*d*e**2*x**2*asinh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/9 + 4*a**3*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 4*a**3*b*d*e**
2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 8*a**3*b*e**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a**2*b**2*c**3*e**2*asinh(c + d*x)**2/d +
 6*a**2*b**2*c**2*e**2*x*asinh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*
a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*
d) + 6*a**2*b**2*c*d*e**2*x**2*asinh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**
2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*asinh(c + d*x)**2 + 4*a**2*b**2*d**2*e**
2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/3 - 8*a**2*b**2*e**2*x/3 + 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*asinh(c + d*x)*
*3/(3*d) + 8*a*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*a
sinh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*asinh(c + d*x)/3 - 4*a*b**3*c**2*e*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 8*a*b**3*
c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + 4*a*b**3*c*d*e**2*x
**2*asinh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 8*a*b**3*
```

```
c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 16*a*b*
*3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 16*a*b**3*c*e**2*asin
h(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 8*a*b**3*d
**2*e**2*x**3*asinh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/27 - 16*a*b**3*e**2*x*asinh(c + d*x)/3 + 8*a*b**3*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) + 160*a*b**3*
e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**4*c**3*e**2*asinh(c +
 d*x)**4/(3*d) + 4*b**4*c**3*e**2*asinh(c + d*x)**2/(9*d) + b**4*c**2*e**2*
x*asinh(c + d*x)**4 + 4*b**4*c**2*e**2*x*asinh(c + d*x)**2/3 + 8*b**4*c**2*
e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asin
h(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*asinh(c + d*x)**4 + 4*b**4*c*d*e**2*
x**2*asinh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*c*e**2*asinh(c +
 d*x)**2/(3*d) + b**4*d**2*e**2*x**3*asinh(c + d*x)**4/3 + 4*b**4*d**2*e**2
*x**3*asinh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 8*b**4*d*e**2*x**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*e**2*x*as
inh(c + d*x)**2/3 - 160*b**4*e**2*x/27 + 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)*asinh(c + d*x)**3/(9*d) + 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asin
h(c))**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^4, x)
```

3.149 $\int (ce + dex)\left(a + b\sinh^{-1}(c + dx)\right)^4 dx$

Optimal. Leaf size=195

$$-\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}+\frac{3b^2e\left(a+b\sinh^{-1}(c+dx\right)}{4d}$$

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x]))/(2*d) + (3*b^2*e*(a + b*ArcSinh[c + d*x])^2)/(4*d) +
(3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/(2*d) - (b*e*(c + d*x)*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/d + (e*(a + b*ArcSinh[c + d*
x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^4)/(2*d)
```

Rubi [A] time = 0.320859, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$-\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}+\frac{3b^2e\left(a+b\sinh^{-1}(c+dx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x]))/(2*d) + (3*b^2*e*(a + b*ArcSinh[c + d*x])^2)/(4*d) +
(3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/(2*d) - (b*e*(c + d*x)*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/d + (e*(a + b*ArcSinh[c + d*
x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^4)/(2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^4}{2d}-\frac{(2be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}}\,dx,x,c\right.}{d}$$
$$=-\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}+\frac{e(c+dx)^2\left(a+b\sinh\right.}{2d}$$
$$=\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sin\right.}{d}$$
$$=-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\right.}{2d}$$
$$=\frac{3b^4e(c+dx)^2}{4d}-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e\left(\right.}{}$$

Mathematica [A] time = 0.310301, size = 300, normalized size = 1.54

$$\frac{e\left(\left(6a^2b^2+2a^4+3b^4\right)(c+dx)^2-2ab\left(2a^2+3b^2\right)(c+dx)\sqrt{(c+dx)^2+1}+3b^2\sinh^{-1}(c+dx)^2\left(4a^2(c+dx)^2+2a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*(c + d*
x)*Sqrt[1 + (c + d*x)^2] + 2*a*b*(2*a^2 + 3*b^2)*ArcSinh[c + d*x] - 2*b*(c
+ d*x)*(-4*a^3*(c + d*x) - 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[1 + (c + d*x)^2
] + 3*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 3*b^2*(2*a^2 + b^2 + 4*
a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]
)*ArcSinh[c + d*x]^2 + 4*b^3*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c
 + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^4
))/(4*d)
```

Maple [B] time = 0.069, size = 371, normalized size = 1.9

$$\frac{1}{d}\left(\frac{(dx+c)^2 ea^4}{2}+eb^4\left(\frac{(\text{Arcsinh}\,(dx+c))^4\left(1+(dx+c)^2\right)}{2}-(\text{Arcsinh}\,(dx+c))^3(dx+c)\sqrt{1+(dx+c)^2}-\frac{(\text{Arcsinh}\,(d}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a^4+e*b^4*(1/2*arcsinh(d*x+c)^4*(1+(d*x+c)^2)-arcsinh(
d*x+c)^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^4+3/2*arcsinh(d*x+c
)^2*(1+(d*x+c)^2)-3/2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/4*arcsin
h(d*x+c)^2+3/4*(d*x+c)^2+3/4)+4*e*a*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)
-3/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*
(1+(d*x+c)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*
x+c))+6*e*a^2*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcsinh(d*x+c)*(1
+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+4*e*a^3*b
*(1/2*arcsinh(d*x+c)*(d*x+c)^2-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(
d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96975, size = 1287, normalized size = 6.6

$$\frac{\left(2a^4+6a^2b^2+3b^4\right)d^2ex^2+2\left(2a^4+6a^2b^2+3b^4\right)cdex+\left(2b^4d^2ex^2+4b^4cdex+\left(2b^4c^2+b^4\right)e\right)\log\left(dx+c+\sqrt{d^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)*
c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 + b^4)*e)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b^3*
c*d*e*x + (2*a*b^3*c^2 + a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(2
*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x + (2*a^2*b^2 + b
^4 + 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 +
2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x + (2*a^3
*b + 3*a*b^3 + 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4)*d*e*x +
(2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x + (2*a^3*b +
 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 6.31193, size = 1027, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*asinh(c + d*x)/d
+ 4*a**3*b*c*e*x*asinh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)/d + 2*a**3*b*d*e*x**2*asinh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1) + a**3*b*e*asinh(c + d*x)/d + 3*a**2*b**2*c**2*e*asinh(c
 + d*x)**2/d + 6*a**2*b**2*c*e*x*asinh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*asinh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x) + 3*a**2*b**2*e*asinh
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*asinh(c + d*x)**3/d + 3*a*b**3*c**2*e*
asinh(c + d*x)/d + 4*a*b**3*c*e*x*asinh(c + d*x)**3 + 6*a*b**3*c*e*x*asinh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d) + 2*a*b**3*
d*e*x**2*asinh(c + d*x)**3 + 3*a*b**3*d*e*x**2*asinh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/2 + a*b**3*e*asinh(c + d*x)**3/d + 3*a*b*
*3*e*asinh(c + d*x)/(2*d) + b**4*c**2*e*asinh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*asinh(c + d*x)**2/(2*d) + b**4*c*e*x*asinh(c + d*x)**4 + 3*b**4*c*e*x*
asinh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/(2*d) + b**4*d*e*x**2*asinh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*asinh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/2 + b**4*e*asinh(c + d*x)**4/(4*d) + 3*b**4*e*asinh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.150 $\int \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=115

$$-\frac{24b^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+\right.}{d}$$

```
[Out] 24*b^4*x - (24*b^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + (12*
b^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^2)/d - (4*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcSinh[c + d*x])^4)/d
```

Rubi [A] time = 0.159742, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5653, 5717, 8}

$$-\frac{24b^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + (12*
b^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^2)/d - (4*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcSinh[c + d*x])^4)/d
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4}{d} - \frac{(4b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}} dx, x, c+dx\right)}{d}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4}{d} + \frac{(12b}{}$$

$$= \frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} - \frac{4b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} + \frac{(c}{}$$

$$= -\frac{24b^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} -$$

$$= 24b^4x - \frac{24b^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

Mathematica [A] time = 0.250068, size = 226, normalized size = 1.97

$$\frac{\left(12a^2b^2+a^4+24b^4\right)(c+dx) - 4ab\left(a^2+6b^2\right)\sqrt{(c+dx)^2+1} + 6b^2\sinh^{-1}(c+dx)^2\left(a^2(c+dx) - 2ab\sqrt{(c+dx)^2+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[1 + (c +
d*x)^2] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*b*Sqrt[1 + (c +
 d*x)^2] + 6*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 6*b^2*(a^2*(c +
d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 -
4*b^3*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(
c + d*x)*ArcSinh[c + d*x]^4)/d
```

Maple [B] time = 0.03, size = 245, normalized size = 2.1

$$\frac{1}{d}\left((dx+c)a^4 + b^4\left((\text{Arcsinh}(dx+c))^4(dx+c) - 4(\text{Arcsinh}(dx+c))^3\sqrt{1+(dx+c)^2} + 12(\text{Arcsinh}(dx+c))^2(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*(arcsinh(d*x+c)^4*(d*x+c)-4*arcsinh(d*x+c)^3*(1+(d*x+c
)^2)^(1/2)+12*arcsinh(d*x+c)^2*(d*x+c)-24*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2
)+24*d*x+24*c)+4*a*b^3*(arcsinh(d*x+c)^3*(d*x+c)-3*arcsinh(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+6*a^2*b^2*(arc
sinh(d*x+c)^2*(d*x+c)-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+4*a^3
*b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75134, size = 784, normalized size = 6.82

$$\frac{\left(b^4dx + b^4c\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^4 + 4\left(ab^3dx + ab^3c - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}b^4\right)\log\left(dx + c + \sqrt{d^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A] time = 2.20034, size = 444, normalized size = 3.86

$$\begin{cases} a^4x + \frac{4a^3bc\operatorname{asinh}(c+dx)}{d} + 4a^3bx\operatorname{asinh}(c+dx) - \frac{4a^3b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{6a^2b^2c\operatorname{asinh}^2(c+dx)}{d} + 6a^2b^2x\operatorname{asinh}^2(c+dx) + 12a^2b^2 \\ x\,(a + b\operatorname{asinh}(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*asinh(c + d*x)/d + 4*a**3*b*x*asinh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 6*a**2*b**2*c*asinh(c
+ d*x)**2/d + 6*a**2*b**2*x*asinh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 4*a*b**3*c*asinh
(c + d*x)**3/d + 24*a*b**3*c*asinh(c + d*x)/d + 4*a*b**3*x*asinh(c + d*x)**
3 + 24*a*b**3*x*asinh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d
 + b**4*c*asinh(c + d*x)**4/d + 12*b**4*c*asinh(c + d*x)**2/d + b**4*x*asin
h(c + d*x)**4 + 12*b**4*x*asinh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**4, Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arsinh}(dx + c) + a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4, x)
```

3.151 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{ce+dex}\,dx$

Optimal. Leaf size=186

$$-\frac{3b^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de}-\frac{3b^3\text{PolyLog}\left(4,e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{2bP}{}$$

```
[Out] (a + b*ArcSinh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcSinh[c + d*x])^4*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyL
og[3, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^3*(a + b*ArcSinh[c + d*x])*Pol
yLog[4, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, E^(-2*ArcSinh[c
 + d*x])])/(2*d*e)
```

Rubi [A] time = 0.258579, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de}+\frac{3b^3\text{PolyLog}\left(4,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}+\frac{2bPol}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcSinh[c + d*x])^4*Log[1 -
 E^(2*ArcSinh[c + d*x])])/(d*e) + (2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2
, E^(2*ArcSinh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLo
g[3, E^(2*ArcSinh[c + d*x])])/(d*e) + (3*b^3*(a + b*ArcSinh[c + d*x])*PolyL
og[4, E^(2*ArcSinh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d
*x])])/(2*d*e)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{ex}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x}\,dx,x,c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^4\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^4}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{(4b)\,\text{Subst}}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{2b\left(a+b\,\text{si}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{2b\left(a+b\,\text{si}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{2b\left(a+b\,\text{si}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{2b\left(a+b\,\text{si}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{2b\left(a+b\,\text{si}\right.}{}$$

Mathematica [A] time = 0.0641496, size = 157, normalized size = 0.84

$$\frac{-3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2+3b^3\text{PolyLog}\left(4,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)+2b\text{Poly}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (-(a + b*ArcSinh[c + d*x])^5/(5*b) + (a + b*ArcSinh[c + d*x])^4*Log[1 - E^(
2*ArcSinh[c + d*x])] + 2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[3, E^(2*ArcSinh[c
+ d*x])] + 3*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[4, E^(2*ArcSinh[c + d*x])
] - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d*x])])/2)/(d*e)
```

Maple [B] time = 0.06, size = 1153, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x)
```

```
[Out] 4/d*a^3*b/e*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-1/d*a*b^3/e*arcsinh(d*x+c)
^4+24/d*a*b^3/e*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*a*b^3/e*polylog(
4,d*x+c+(1+(d*x+c)^2)^(1/2))-12/d*a^2*b^2/e*polylog(3,d*x+c+(1+(d*x+c)^2)^(
1/2))-2/d*a^2*b^2/e*arcsinh(d*x+c)^3-12/d*a^2*b^2/e*polylog(3,-d*x-c-(1+(d*
x+c)^2)^(1/2))-2/d*a^3*b/e*arcsinh(d*x+c)^2+4/d*a^3*b/e*polylog(2,-d*x-c-(1
+(d*x+c)^2)^(1/2))-12/d*b^4/e*arcsinh(d*x+c)^2*polylog(3,-d*x-c-(1+(d*x+c)^
2)^(1/2))+24/d*b^4/e*arcsinh(d*x+c)*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+1
/d*b^4/e*arcsinh(d*x+c)^4*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+4/d*b^4/e*arcsinh
(d*x+c)^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-12/d*b^4/e*arcsinh(d*x+c)^2*
polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24/d*b^4/e*arcsinh(d*x+c)*polylog(4,d*
x+c+(1+(d*x+c)^2)^(1/2))+1/d*b^4/e*arcsinh(d*x+c)^4*ln(1+d*x+c+(1+(d*x+c)^2
)^(1/2))+4/d*a*b^3/e*arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+12/d*
a*b^3/e*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^2/
e*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a^2*b^2/e*arcsin
h(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^2/e*arcsinh(d*x+c)*po
lylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*a*b^3/e*arcsinh(d*x+c)*polylog(3,-d
*x-c-(1+(d*x+c)^2)^(1/2))-24/d*a*b^3/e*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d
*x+c)^2)^(1/2))+4/d*a^3*b/e*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+
4/d*a*b^3/e*arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a^2*b^2/e*
arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*a^3*b/e*arcsinh(d*x+c)
*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*b^4/e*arcsinh(d*x+c)^3*polylog(2,-d*x-
c-(1+(d*x+c)^2)^(1/2))-1/5/d*b^4/e*arcsinh(d*x+c)^5-24/d*b^4/e*polylog(5,-d
*x-c-(1+(d*x+c)^2)^(1/2))-24/d*b^4/e*polylog(5,d*x+c+(1+(d*x+c)^2)^(1/2))+1
/d*a^4/e*ln(d*x+c)+12/d*a*b^3/e*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x+c
)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*asinh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*asinh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
2*asinh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*asinh(c + d*x)/(c + d
*x), x))/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

3.152 $\int \frac{\left(a+b \sinh^{-1}(c+dx)\right)^4}{(ce+dex)^2} \, dx$

Optimal. Leaf size=234

$$\frac{24b^3\text{PolyLog}\left(3,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{24b^3\text{PolyLog}\left(3,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{12b^{2}}{}$$

```
[Out] -((a + b*ArcSinh[c + d*x])^4/(d*e^2*(c + d*x))) - (8*b*(a + b*ArcSinh[c + d
*x])^3*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (12*b^2*(a + b*ArcSinh[c + d*
x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (12*b^2*(a + b*ArcSinh[c +
 d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (24*b^3*(a + b*ArcSinh[c
 + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^3*(a + b*ArcSinh[
c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^4*PolyLog[4, -E^A
rcSinh[c + d*x]])/(d*e^2) + (24*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rubi [A] time = 0.317163, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5760, 4182, 2531, 6609, 2282, 6589}

$$\frac{24b^3\text{PolyLog}\left(3,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{24b^3\text{PolyLog}\left(3,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{12b^{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```


```
[Out] -((a + b*ArcSinh[c + d*x])^4/(d*e^2*(c + d*x))) - (8*b*(a + b*ArcSinh[c + d
*x])^3*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (12*b^2*(a + b*ArcSinh[c + d*
x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (12*b^2*(a + b*ArcSinh[c +
 d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (24*b^3*(a + b*ArcSinh[c
 + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^3*(a + b*ArcSinh[
c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^4*PolyLog[4, -E^A
rcSinh[c + d*x]])/(d*e^2) + (24*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^2)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int (a+bx)^3\text{csch}(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{(12b^2)}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12b^2(}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12b^2(}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12b^2(}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12b^2(}{}$$

Mathematica [B] time = 1.76449, size = 501, normalized size = 2.14

$$\frac{12a^2b^2\left(2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right)-2\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)+\sinh^{-1}(c+dx)\left(-\frac{\sinh^{-1}(c+dx)}{c+dx}+2\log\left(1-e^{-\sin}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^4)/(c + d*x) + 4*a^3*b*((-2*ArcSinh[c + d*x])/(c + d*x) + 2*Log[(2*S
inh[ArcSinh[c + d*x]/2]^2)/(c + d*x)]) + 12*a^2*b^2*(ArcSinh[c + d*x]*(-(Ar
cSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] - 2*Log[1 + E^
(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 2*PolyLog[2,
 E^(-ArcSinh[c + d*x])]) + 8*a*b^3*(-(ArcSinh[c + d*x]^3/(c + d*x)) + 3*Arc
Sinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c + d*x]^2*Log[1
 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c +
d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] + 6*PolyLog[3
, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]) + b^4*(Pi^
4 - 2*ArcSinh[c + d*x]^4 - (2*ArcSinh[c + d*x]^4)/(c + d*x) - 8*ArcSinh[c +
 d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] + 8*ArcSinh[c + d*x]^3*Log[1 - E^Arc
Sinh[c + d*x]] + 24*ArcSinh[c + d*x]^2*PolyLog[2, -E^(-ArcSinh[c + d*x])] +
 24*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] + 48*ArcSinh[c + d*x]
*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 48*ArcSinh[c + d*x]*PolyLog[3, E^ArcS
inh[c + d*x]] + 48*PolyLog[4, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[4, E^Arc
Sinh[c + d*x]]))/(2*d*e^2)
```

Maple [B] time = 0.047, size = 820, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^4/e^2/(d*x+c)-1/d*b^4/e^2*arcsinh(d*x+c)^4/(d*x+c)-4/d*b^4/e^2*arcsi
nh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-12/d*b^4/e^2*arcsinh(d*x+c)^2*p
olylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*b^4/e^2*arcsinh(d*x+c)*polylog(3,
-d*x-c-(1+(d*x+c)^2)^(1/2))-24*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/
e^2+4/d*b^4/e^2*arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*b^4/e
^2*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*b^4/e^2*arcsi
nh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24*b^4*polylog(4,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^2-4/d*a*b^3/e^2/(d*x+c)*arcsinh(d*x+c)^3-12/d*a*b^3/e^2
*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*a*b^3/e^2*arcsinh(d*
x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*a*b^3/e^2*polylog(3,-d*x-c-
(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/e^2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^
2)^(1/2))+24/d*a*b^3/e^2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)
)-24/d*a*b^3/e^2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a^2*b^2/e^2/(d*x+
c)*arcsinh(d*x+c)^2-12/d*a^2*b^2/e^2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2
)^(1/2))-12/d*a^2*b^2/e^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^
2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^2/e^2*polyl
og(2,d*x+c+(1+(d*x+c)^2)^(1/2))-4/d*a^3*b/e^2/(d*x+c)*arcsinh(d*x+c)-4/d*a^
3*b/e^2*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^4\,\mathrm{arsinh}\,(dx+c)^4+4\,ab^3\,\mathrm{arsinh}\,(dx+c)^3+6\,a^2b^2\,\mathrm{arsinh}\,(dx+c)^2+4\,a^3b\,\mathrm{arsinh}\,(dx+c)+a^4}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b \operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*asinh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*asinh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*asinh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^2, x)
```

3.153 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx$

Optimal. Leaf size=186

$$-\frac{6b^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b^4\text{PolyLog}\left(3,e^{-2\sinh^{-1}(c+dx)}\right)}{de^3}+\frac{6b^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)}{de^3}$$

```
[Out] (2*b*(a + b*ArcSinh[c + d*x])^3)/(d*e^3) - (2*b*Sqrt[1 + (c + d*x)^2]*(a +
b*ArcSinh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^4/(2*d*
e^3*(c + d*x)^2) + (6*b^2*(a + b*ArcSinh[c + d*x])^2*Log[1 - E^(-2*ArcSinh[
c + d*x])])/(d*e^3) - (6*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(-2*ArcS
inh[c + d*x])])/(d*e^3) - (3*b^4*PolyLog[3, E^(-2*ArcSinh[c + d*x])])/(d*e^
3)
```

Rubi [A] time = 0.32809, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5661, 5723, 5659, 3716, 2190, 2531, 2282, 6589}

$$\frac{6b^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b^4\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)}{de^3}+\frac{6b^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\left(a+\right.}{de^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (-2*b*(a + b*ArcSinh[c + d*x])^3)/(d*e^3) - (2*b*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^4/(2*d
*e^3*(c + d*x)^2) + (6*b^2*(a + b*ArcSinh[c + d*x])^2*Log[1 - E^(2*ArcSinh[
c + d*x])])/(d*e^3) + (6*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSi
nh[c + d*x])])/(d*e^3) - (3*b^4*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
```

```
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(6b^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(6b^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}\right.}{2de^3(c+}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}\right.}{2de^3(c+}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}\right.}{2de^3(c+}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}\right.}{2de^3(c+}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}\right.}{2de^3(c+}$$

Mathematica [C] time = 1.23193, size = 360, normalized size = 1.94

$$8ab^3\left(\sinh^{-1}(c+dx)\left(-\frac{\sinh^{-1}(c+dx)^2}{(c+dx)^2} - \frac{3\sqrt{(c+dx)^2+1}\sinh^{-1}(c+dx)}{c+dx} + 3\sinh^{-1}(c+dx) + 6\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\right) - 3\text{PolyLog}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] ((-2*a^4)/(c + d*x)^2 - (8*a^3*b*Sqrt[1 + (c + d*x)^2])/(c + d*x) - (8*a^3*
b*ArcSinh[c + d*x])/(c + d*x)^2 - (2*b^4*ArcSinh[c + d*x]^4)/(c + d*x)^2 +
24*a^2*b^2*(-((Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x)) - ArcSinh
[c + d*x]^2/(2*(c + d*x)^2) + Log[c + d*x]) + 8*a*b^3*(ArcSinh[c + d*x]*(3*
ArcSinh[c + d*x] - (3*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x) - A
rcSinh[c + d*x]^2/(c + d*x)^2 + 6*Log[1 - E^(-2*ArcSinh[c + d*x])]) - 3*Pol
yLog[2, E^(-2*ArcSinh[c + d*x])]) + b^4*(I*Pi^3 - 8*ArcSinh[c + d*x]^3 - (8
*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x]^3)/(c + d*x) + 24*ArcSinh[c + d*x]^
2*Log[1 - E^(2*ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]*PolyLog[2, E^(2*Arc
Sinh[c + d*x])] - 12*PolyLog[3, E^(2*ArcSinh[c + d*x])]))/(4*d*e^3)
```

Maple [B] time = 0.066, size = 723, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^4/e^3/(d*x+c)^2-2/d*b^4/e^3*arcsinh(d*x+c)^3/(d*x+c)*(1+(d*x+c)^2)
^(1/2)-2/d*b^4/e^3*arcsinh(d*x+c)^3-1/2/d*b^4/e^3*arcsinh(d*x+c)^4/(d*x+c)^
2+6/d*b^4/e^3*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+12/d*b^4/e^3
*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-12/d*b^4/e^3*polylog(
3,-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*b^4/e^3*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d
*x+c)^2)^(1/2))+12/d*b^4/e^3*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(
1/2))-12/d*b^4/e^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a*b^3/e^3*arcsi
nh(d*x+c)^2/(d*x+c)*(1+(d*x+c)^2)^(1/2)-6/d*a*b^3/e^3*arcsinh(d*x+c)^2-2/d*
a*b^3/e^3*arcsinh(d*x+c)^3/(d*x+c)^2+12/d*a*b^3/e^3*arcsinh(d*x+c)*ln(1+d*x
+c+(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/e^3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2)
)+12/d*a*b^3/e^3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/
e^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a^2*b^2/e^3*arcsinh(d*x+c)-6/d
*a^2*b^2/e^3*arcsinh(d*x+c)/(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/d*a^2*b^2/e^3*arc
sinh(d*x+c)^2/(d*x+c)^2+6/d*a^2*b^2/e^3*ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1)
-2/d*a^3*b/e^3/(d*x+c)^2*arcsinh(d*x+c)-2/d*a^3*b/e^3/(d*x+c)*(1+(d*x+c)^2)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4a^3b \operatorname{asin}}{c^3+3c^2dx+3}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*asinh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^4}{(dex + ce)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

3.154 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^4} dx$

Optimal. Leaf size=385

$$-\frac{4b^3\text{PolyLog}\left(3,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{4b^3\text{PolyLog}\left(3,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{2b^2\text{P}}{}$$

```
[Out] (-2*b^2*(a + b*ArcSinh[c + d*x])^2)/(d*e^4*(c + d*x)) - (2*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[
c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcSinh[c + d*x])*ArcTanh
[E^ArcSinh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcSinh[c + d*x])^3*ArcTanh[E^A
rcSinh[c + d*x]])/(3*d*e^4) - (4*b^4*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^
4) + (2*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*
e^4) + (4*b^4*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (2*b^2*(a + b*ArcSi
nh[c + d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^4*PolyLog[4, -
E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^
4)
```

Rubi [A] time = 0.564892, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5865, 12, 5661, 5747, 5760, 4182, 2531, 6609, 2282, 6589, 2279, 2391}

$$-\frac{4b^3\text{PolyLog}\left(3,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{4b^3\text{PolyLog}\left(3,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{2b^2\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```


```
[Out] (-2*b^2*(a + b*ArcSinh[c + d*x])^2)/(d*e^4*(c + d*x)) - (2*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[
c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcSinh[c + d*x])*ArcTanh
[E^ArcSinh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcSinh[c + d*x])^3*ArcTanh[E^A
rcSinh[c + d*x]])/(3*d*e^4) - (4*b^4*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^
4) + (2*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*
e^4) + (4*b^4*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (2*b^2*(a + b*ArcSi
nh[c + d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^4*PolyLog[4, -
E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^
4)
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^3\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de^4}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} - \frac{(2b)\,\text{Subst}\Big(}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sin\right.}{3de^4}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sin\right.}{3de^4}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sin\right.}{3de^4}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sin\right.}{3de^4}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sin\right.}{3de^4}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sin\right.}{3de^4}$$

Mathematica [B] time = 8.92788, size = 1182, normalized size = 3.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] -a^4/(3*d*e^4*(c + d*x)^3) + (a^2*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x])
] - (2*(-2 + 4*ArcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d*x
)*ArcSinh[c + d*x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c +
 d*x]*Log[1 + E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh
[c + d*x])] + 2*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d*x
]*Log[1 - E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d*x
]*Log[1 + E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))/(
4*d*e^4) + (a*b^3*(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcSin
h[c + d*x]^3*Coth[ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh[c
 + d*x]/2]^2 - (c + d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 - 24
*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*
Log[1 + E^(-ArcSinh[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*ArcS
inh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*PolyL
og[2, E^(-ArcSinh[c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48*P
olyLog[3, E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c + d*
x]/2]^2 - (16*ArcSinh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 +
24*ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[Ar
cSinh[c + d*x]/2]))/(12*d*e^4) + (b^4*(-2*Pi^4 + 4*ArcSinh[c + d*x]^4 - 24*
ArcSinh[c + d*x]^2*Coth[ArcSinh[c + d*x]/2] + 2*ArcSinh[c + d*x]^4*Coth[Arc
Sinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^2 - ((c +
d*x)*ArcSinh[c + d*x]^4*Csch[ArcSinh[c + d*x]/2]^4)/2 + 96*ArcSinh[c + d*x]
*Log[1 - E^(-ArcSinh[c + d*x])] - 96*ArcSinh[c + d*x]*Log[1 + E^(-ArcSinh[c
 + d*x])] + 16*ArcSinh[c + d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] - 16*ArcSi
nh[c + d*x]^3*Log[1 - E^ArcSinh[c + d*x]] - 48*(-2 + ArcSinh[c + d*x]^2)*Po
lyLog[2, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[2, E^(-ArcSinh[c + d*x])] - 4
8*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] - 96*ArcSinh[c + d*x]*P
olyLog[3, -E^(-ArcSinh[c + d*x])] + 96*ArcSinh[c + d*x]*PolyLog[3, E^ArcSin
h[c + d*x]] - 96*PolyLog[4, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[4, E^ArcSi
nh[c + d*x]] - 4*ArcSinh[c + d*x]^3*Sech[ArcSinh[c + d*x]/2]^2 - (8*ArcSinh
[c + d*x]^4*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x]^2
*Tanh[ArcSinh[c + d*x]/2] - 2*ArcSinh[c + d*x]^4*Tanh[ArcSinh[c + d*x]/2]))
/(24*d*e^4) + (4*a^3*b*((ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2])/12 - Cs
ch[ArcSinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2]*Cs
ch[ArcSinh[c + d*x]/2]^2)/24 - Log[Tanh[ArcSinh[c + d*x]/2]]/6 - Sech[ArcSi
nh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2])/12 - (Arc
Sinh[c + d*x]*Sech[ArcSinh[c + d*x]/2]^2*Tanh[ArcSinh[c + d*x]/2])/24))/(d*
e^4)
```

Maple [B] time = 0.126, size = 1202, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

```
[Out] -4/3/d*a*b^3/e^4/(d*x+c)^3*arcsinh(d*x+c)^3-4/d*a*b^3/e^4/(d*x+c)*arcsinh(d
*x+c)+2/d*a*b^3/e^4*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*a*
b^3/e^4*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/d*a*b^3/e^4*
arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-4/3/d*a^3*b/e^4/(d*x+c)^3*
arcsinh(d*x+c)-2/3/d*a^3*b/e^4/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-4/d*a*b^3/e^4*
arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4/(d*x+c)
^3*arcsinh(d*x+c)^2+2/d*a^2*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)
^(1/2))-2/d*a^2*b^2/e^4*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-2/3/
d*b^4/e^4/(d*x+c)^2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-4/d*a*b^3/e^4*poly
log(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+4/d*a*b^3/e^4*polylog(3,d*x+c+(1+(d*x+c)^
2)^(1/2))-8/d*a*b^3/e^4*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))+2/d*a^2*b^2/e^4*
```

```
polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4*polylog(2,d*x+c+(1+(d
*x+c)^2)^(1/2))+2/3/d*a^3*b/e^4*arctanh(1/(1+(d*x+c)^2)^(1/2))-1/3/d*b^4/e^
4/(d*x+c)^3*arcsinh(d*x+c)^4-2/d*b^4/e^4/(d*x+c)*arcsinh(d*x+c)^2+4/d*b^4/e
^4*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-4/d*b^4/e^4*arcsinh(
d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4/(d*x+c)+2/3/d*b^4/e^
4*arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2/d*b^4/e^4*arcsinh(d*x+
c)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-4/d*b^4/e^4*arcsinh(d*x+c)*polyl
og(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/3/d*b^4/e^4*arcsinh(d*x+c)^3*ln(1-d*x-c-
(1+(d*x+c)^2)^(1/2))-2/d*b^4/e^4*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c
)^2)^(1/2))-4*b^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog
(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1
/2))/d/e^4-4*b^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+4/d*b^4/e^4*arc
sinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4/(d*x+c)^2*arcsi
nh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/d*a*b^3/e^4/(d*x+c)^2*arcsinh(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)-1/3/d*a^4/e^4/(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 + c)
*a*b^3 + (c^3 + c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 + b
^4*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 +
3*(c^2 + 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(3*a*b^3*c*d + b^4*c*d)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a
^2*b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 +
1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (3*c^2*d +
d)*a^3*b*x + (c^3 + c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (c^2 + 1)*a
^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^
2*d^5*e^4 + d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3
*e^4 + 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*
d*e^4 + 5*c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4
 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(
5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+\ldots}{c^4+4c^3dx+6c^2d^2x^2+4c\ldots}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*asinh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*asinh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*asinh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

3.155 $\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(e(c+dx))^m}{a+b\sinh^{-1}(c+dx)},x\right)$$

[Out] Unintegrable[(e*(c + d*x))^m/(a + b*ArcSinh[c + d*x]), x]

Rubi [A] time = 0.0603417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]

[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcSinh[x]), x], x, c + d*x]/d

Rubi steps

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 1.11493, size = 0, normalized size = 0.

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]

[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]), x]

Maple [A] time = 0.622, size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^m}{a+b\text{Arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)

[Out] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^m}{b\,\mathrm{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dex + ce)^m}{b\,\mathrm{arsinh}\,(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\,(c + dx))^m}{a + b\,\mathrm{asinh}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m/(a + b*asinh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^m}{b\,\mathrm{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

3.156 $\int \frac{(ce+dex)^4}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16bd}-\frac{e^4 \ldots}{}$$

```
[Out] (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b*d) - (3*e^4*C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) + (e^4*
Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) - (e^4
*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b*d) + (3*e^4*Sinh[
(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) - (e^4*Sinh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d)
```

Rubi [A] time = 0.468893, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1} \ldots\right)}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]])/(8*b*d) - (3*e^4*Cosh[
(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(16*b*d) + (e^4*Cosh[(
5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcSinh[c + d*x]])/(16*b*d) - (e^4*Sinh[a/
b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(8*b*d) + (3*e^4*Sinh[(3*a)/b]*Sin
hIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(16*b*d) - (e^4*Sinh[(5*a)/b]*Sinh
Integral[(5*a)/b + 5*ArcSinh[c + d*x]])/(16*b*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{16d}+\frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8d}-\frac{(3}{}$$
$$= \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8d}-\frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}\right.}{a+b}\right.}{16d}$$
$$= \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{16bd}+\frac{e^4\cos}{}$$

Mathematica [A] time = 0.302412, size = 151, normalized size = 0.71

$$\frac{e^4\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*(2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b
```

```
+ ArcSinh[c + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3
*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*Sin
hIntegral[5*(a/b + ArcSinh[c + d*x])]))/(16*b*d)
```

Maple [A] time = 0.164, size = 194, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{e^4}{32\,b}e^{5\,\frac{a}{b}}\mathrm{Ei}\left(1,5\,\mathrm{Arcsinh}\,(dx+c)+5\,\frac{a}{b}\right)+\frac{3\,e^4}{32\,b}e^{3\,\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\,(dx+c)+3\,\frac{a}{b}\right)-\frac{e^4}{16\,b}e^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(-1/32*e^4/b*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a
/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/16*e^4/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/
b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32*e^4/b*exp(-3*a/b)*Ei
(1,-3*arcsinh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5
*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4}{b\,\mathrm{arsinh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b*arcsinh(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

3.157 $\int \frac{(ce+dex)^3}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8bd}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\ldots$$

```
[Out] (e^3*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/(4*b*d) -
(e^3*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(4*a)/b])/(8*b*d) -
(e^3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d) +
(e^3*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(8*b*d)
```

Rubi [A] time = 0.340267, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{8bd}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}\ldots\right)}{4bd}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b])/(4*b*d) - (e
^3*CoshIntegral[(4*a)/b + 4*ArcSinh[c + d*x]]*Sinh[(4*a)/b])/(8*b*d) - (e^3
*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(4*b*d) + (e^3*C
osh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(8*b*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}$$
$$= -\frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} + \frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{\ldots}\right.}{8\ldots}$$
$$= \frac{e^3\textbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{4bd} - \frac{e^3\textbf{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)\sinh\left(\frac{4a}{b}\right)}{8bd} - \frac{e^3\,\text{co}\ldots}{\ldots}$$

Mathematica [A] time = 0.212484, size = 109, normalized size = 0.75

$$\frac{e^3\left(2\sinh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\textbf{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-2\cosh\left(\frac{2a}{b}\right)\textbf{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c\ldots\right.\right.\right.}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - CoshIntegr
al[4*(a/b + ArcSinh[c + d*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral
[2*(a/b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c + d*x])]))/(8*b*d)
```

Maple [A] time = 0.079, size = 134, normalized size = 0.9

$$\frac{1}{d}\left(\frac{e^3}{16\,b}\mathrm{e}^{4\,\frac{a}{b}}\mathrm{Ei}\left(1,4\,\mathrm{Arcsinh}\,(dx+c)+4\,\frac{a}{b}\right)-\frac{e^3}{8\,b}\mathrm{e}^{2\,\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{Arcsinh}\,(dx+c)+2\,\frac{a}{b}\right)+\frac{e^3}{8\,b}\mathrm{e}^{-2\,\frac{a}{b}}\mathrm{Ei}\left(1,-2\,\mathrm{Arcsinh}\,(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}{b\,\mathrm{arsinh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcsi
nh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int\frac{c^3}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int\frac{d^3x^3}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int\frac{3cd^2x^2}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int\frac{3c^2dx}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**3*(Integral(c**3/(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/(a + b*
asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*asinh(c + d*x)), x) + I
ntegral(3*c**2*d*x/(a + b*asinh(c + d*x)), x))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^3}{b \operatorname{arsinh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

3.158 $\int \frac{(ce+dex)^2}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2\sinh}{}$$

```
[Out] -(e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b*d) + (e^2*Co
sh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d) + (e^2*Si
nh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b*d) - (e^2*Sinh[(3*a)
/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d)
```

Rubi [A] time = 0.292862, antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(e^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]])/(4*b*d) + (e^2*Cosh[(
3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(4*b*d) + (e^2*Sinh[a/b
]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(4*b*d) - (e^2*Sinh[(3*a)/b]*SinhIn
tegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(4*b*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ce+dex)^2}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}\\
&= -\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}+\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}\\
&= -\frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}\right.}{a+b}\right.}{4d}\\
&= -\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\sin}{}
\end{aligned}$$

Mathematica [A] time = 0.178464, size = 102, normalized size = 0.72

$$\frac{e^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*Cos
hIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSin
h[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b
*d)
```

Maple [A] time = 0.071, size = 130, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{e^2}{8b}\mathrm{e}^{3\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\,(dx+c)+3\,\frac{a}{b}\right)+\frac{e^2}{8b}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(dx+c)+\frac{a}{b}\right)+\frac{e^2}{8b}\mathrm{e}^{-\frac{a}{b}}\mathrm{Ei}\left(1,-\mathrm{Arcsinh}\,(dx+c)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(-1/8*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/8*e^2/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/8*
e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b\,\mathrm{arsinh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arcsinh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{d^2x^2}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{2cdx}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**2*(Integral(c**2/(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
asinh(c + d*x)), x) + Integral(2*c*d*x/(a + b*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

3.159 $\int \frac{ce+dex}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2bd}$$

```
[Out] -(e*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b*d) + (
e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b*d)
```

Rubi [A] time = 0.161335, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(e*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b])/(2*b*d) + (e*
Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(2*b*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{ce+dex}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{ex}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{e\,\mathrm{Subst}\left(\int \frac{x}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{e\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}\\
&= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d} - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+\right.}{a+bx}\right.}{2d}\\
&= -\frac{e\,\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.0866393, size = 61, normalized size = 0.88

$$-\frac{e\left(\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)-\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(e*(CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/
b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]))/(2*b*d)
```

Maple [A] time = 0.036, size = 66, normalized size = 1.

$$\frac{1}{d}\left(\frac{e}{4\,b}\mathrm{e}^{2\,\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{Arcsinh}\,(dx+c)+2\,\frac{a}{b}\right)-\frac{e}{4\,b}\mathrm{e}^{-2\,\frac{a}{b}}\mathrm{Ei}\left(1,-2\,\mathrm{Arcsinh}\,(dx+c)-2\,\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(d*x+c)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{b \operatorname{arsinh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{dex + ce}{b \operatorname{arsinh}(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a + b \operatorname{asinh}(c + dx)} \, dx + \int \frac{dx}{a + b \operatorname{asinh}(c + dx)} \, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
```

```
[Out] e*(Integral(c/(a + b*asinh(c + d*x)), x) + Integral(d*x/(a + b*asinh(c + d*
x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{b \operatorname{arsinh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

3.160 $\int \frac{1}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) - (Sinh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d)
```

Rubi [A] time = 0.0889073, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5863, 5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) - (Sinh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d)
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x}{bd}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

Mathematica [A] time = 0.0215911, size = 49, normalized size = 0.84

$$\frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c + d*x]])/(b*d)
```

Maple [A] time = 0.027, size = 60, normalized size = 1.

$$\frac{1}{d}\left(-\frac{1}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{Arcsinh}\,(dx+c)+\frac{a}{b}\right)-\frac{1}{2b}e^{-\frac{a}{b}}\text{Ei}\left(1,-\text{Arcsinh}\,(dx+c)-\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(-1/2/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh
(d*x+c)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\,\text{arsinh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\,\text{asinh}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

3.161 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)},x\right)}{e}$$

[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])), x]/e

---

Rubi [A] time = 0.066273, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)}dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)}dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.814729, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])), x]

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(a+b\text{Arcsinh}(dx+c))}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{adex+ace+(bdex+bce)\operatorname{arsinh}(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arcsinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ac+adx+bc\operatorname{asinh}(c+dx)+bdx\operatorname{asinh}(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral(1/(a*c + a*d*x + b*c*asinh(c + d*x) + b*d*x*asinh(c + d*x)), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

3.162 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=256

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}+\cdots$$

```
[Out] -((e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^4*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(8*b^2*d) + (9*e^
4*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(16*b^2*d) -
(5*e^4*CoshIntegral[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/(16*b^2*
d) + (e^4*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b^2*d) - (
9*e^4*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b^2*d
) + (5*e^4*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*
b^2*d)
```

Rubi [A] time = 0.409048, antiderivative size = 252, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8b^2d}+\frac{9e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{16b^2d}-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\sinh^{-1}\cdots\right)}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^4*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b])/(8*b^2*d) + (9*e^4*Co
shIntegral[(3*a)/b + 3*ArcSinh[c + d*x]]*Sinh[(3*a)/b])/(16*b^2*d) - (5*e^4
*CoshIntegral[(5*a)/b + 5*ArcSinh[c + d*x]]*Sinh[(5*a)/b])/(16*b^2*d) + (e^
4*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(8*b^2*d) - (9*e^4*Cosh[(
3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(16*b^2*d) + (5*e^4*Cos
h[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c + d*x]])/(16*b^2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
```

```
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^4\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)} - \frac{9\sinh(3x)}{16(a+bx)} + \frac{5\sinh(5x)}{16(a+bx)}\right)dx,x,\sinh^{-1}(\right.}{bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8bd} + \frac{\left(5e^4\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8bd} -$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^4\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2d} + \frac{9e^4\textbf{Chi}\left(\frac{3a}{b}+3\sin\right.}{}$$

Mathematica [A] time = 1.08561, size = 281, normalized size = 1.1

$$\frac{e^4\left(16\left(3\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-3\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) +
16*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - CoshIntegral[3*(a/b
```

```
+ ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh
[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) - 5*(1
0*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b +
ArcSinh[c + d*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]
*Sinh[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcSinh[c + d*x])])))/(16*b^2*d)
```

Maple [B] time = 0.207, size = 602, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/32*(16*(d*x+c)^5-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-12*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c-(1+(d*x+c)^2)^(1/2))*e^4/b/(a+b*arcsi
nh(d*x+c))+5/32*e^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-3/32*(4*(d*
x+c)^3-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+3*d*x+3*c-(1+(d*x+c)^2)^(1/2))*e^4/b
/(a+b*arcsinh(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+
1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4/b/(a+b*arcsinh(d*x+c))+1/16*e^4/b^2*e
xp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/16*e^4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+
b*arcsinh(d*x+c))-1/16*e^4/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32*e^4
/b*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/
2))/(a+b*arcsinh(d*x+c))+9/32*e^4/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*
a/b)-1/32*e^4/b*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)
+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsi
nh(d*x+c))-5/32*e^4/b^2*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5
*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^
3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d
*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*
e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 +
 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^
2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b
*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((5*d^8*e^4
*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 + 10*c^6*e^4 + 5*c^4*e^4 + 10*(14*c^2*d
^6*e^4 + d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 + 3*c*d^5*e^4)*x^5 + 5*(70*c^4*d
^4*e^4 + 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^3*e^4 + 10*c^3*d^3*e^
4 + c*d^3*e^4)*x^3 + 10*(14*c^6*d^2*e^4 + 15*c^4*d^2*e^4 + 3*c^2*d^2*e^4)*x
^2 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^5 + 5*c^6*e^4 + 3*c^4*e^4 + 3*(25*c^2*
d^4*e^4 + d^4*e^4)*x^4 + 4*(25*c^3*d^3*e^4 + 3*c*d^3*e^4)*x^3 + 3*(25*c^4*d
```

```
^2*e^4 + 6*c^2*d^2*e^4)*x^2 + 6*(5*c^5*d*e^4 + 2*c^3*d*e^4)*x)*(d^2*x^2 + 2
*c*d*x + c^2 + 1) + 20*(2*c^7*d*e^4 + 3*c^5*d*e^4 + c^3*d*e^4)*x + (10*d^7*
e^4*x^7 + 70*c*d^6*e^4*x^6 + 10*c^7*e^4 + 13*c^5*e^4 + 4*c^3*e^4 + (210*c^2
*d^5*e^4 + 13*d^5*e^4)*x^5 + 5*(70*c^3*d^4*e^4 + 13*c*d^4*e^4)*x^4 + 2*(175
*c^4*d^3*e^4 + 65*c^2*d^3*e^4 + 2*d^3*e^4)*x^3 + 2*(105*c^5*d^2*e^4 + 65*c^
3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (70*c^6*d*e^4 + 65*c^4*d*e^4 + 12*c^2*d*e^4)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b +
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b
^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*
b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2
*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^2\,\text{arsinh}\,(dx+c)^2 + 2\,ab\,\text{arsinh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^2, x)
```

3.163 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=188

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-$$

```
[Out] -((e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^2*d)
 + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(2*b^2*
d) + (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^
2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(2*
b^2*d)
```

Rubi [A] time = 0.310706, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{2b^2d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh\right.}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(2*b^2*d) +
 (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(2*b^2*d) +
 (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(2*b^2*d) -
 (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(2*b^2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{2(a+bx)}\right)dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd} + \frac{e^3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^2d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{C}}{}$$

Mathematica [A] time = 0.860798, size = 193, normalized size = 1.03

$$-\frac{e^3\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -(e^3*((2*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + C
osh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Cosh[(4*a)/b]*CoshI
ntegral[4*(a/b + ArcSinh[c + d*x])] - 3*Log[a + b*ArcSinh[c + d*x]] - 4*Sin
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])] + 3*(Log[a + b*ArcSinh[
c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]) + Sinh[
```

```
(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(2*b^2*d)
```

Maple [B] time = 0.105, size = 388, normalized size = 2.1

$$\frac{1}{d}\left(\frac{e^3}{(16a+16b\mathrm{Arcsinh}\,(dx+c))\,b}\left(8\,(dx+c)^4-8\,(dx+c)^3\sqrt{1+(dx+c)^2}+8\,(dx+c)^2-4\,(dx+c)\sqrt{1+(dx+c)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/16*(8*(d*x+c)^4-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^2-4*(d*x+c
)*(1+(d*x+c)^2)^(1/2)+1)*e^3/(a+b*arcsinh(d*x+c))/b-1/4*e^3/b^2*exp(4*a/b)*
Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)
+1)*e^3/(a+b*arcsinh(d*x+c))/b+1/4*e^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)
+2*a/b)+1/8*e^3/b*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsin
h(d*x+c))+1/4*e^3/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/16*e^3/b*
(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+
c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/4*e^3/b^2*exp(-4*a/b)*Ei(1,-4*arcsinh
(d*x+c)-4*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^4
*e^3)*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*d^
2*e^3)*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^3*x
^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e^3 +
 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 +
2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2
*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((4*d^7*e^3*x^7
 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 + 8*c^5*e^3 + 4*c^3*e^3 + 4*(21*c^2*d^5*e^3
 + 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 + 2*c*d^4*e^3)*x^4 + 4*(35*c^4*d^3*e^
3 + 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 4*(21*c^5*d^2*e^3 + 20*c^3*d^2*e^3 + 3*
c*d^2*e^3)*x^2 + 2*(2*d^5*e^3*x^5 + 10*c*d^4*e^3*x^4 + 2*c^5*e^3 + c^3*e^3
+ (20*c^2*d^3*e^3 + d^3*e^3)*x^3 + (20*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (10
*c^4*d*e^3 + 3*c^2*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(7*c^6*d*e^3
 + 10*c^4*d*e^3 + 3*c^2*d*e^3)*x + (8*d^6*e^3*x^6 + 48*c*d^5*e^3*x^5 + 8*c^
6*e^3 + 10*c^4*e^3 + 3*c^2*e^3 + 10*(12*c^2*d^4*e^3 + d^4*e^3)*x^4 + 40*(4*
c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(40*c^4*d^2*e^3 + 20*c^2*d^2*e^3 + d^2*e^3
)*x^2 + 2*(24*c^5*d*e^3 + 20*c^3*d*e^3 + 3*c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2
 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d
*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)
*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
+ 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*s
```

```
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c
)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^2\,\text{arsinh}\,(dx+c)^2 + 2\,ab\,\text{arsinh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*arc
sinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int\frac{c^3}{a^2 + 2ab\,\text{asinh}\,(c+dx) + b^2\,\text{asinh}^2\,(c+dx)}\,dx + \int\frac{d^3x^3}{a^2 + 2ab\,\text{asinh}\,(c+dx) + b^2\,\text{asinh}^2\,(c+dx)}\,dx + \int\frac{}{a^2 + 2ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2
*asinh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^2, x)
```

3.164 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=184

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4b^2d}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4b^2d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cos}{}$$

```
[Out] -((e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) +
 (e^2*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(4*b^2*d) - (3*e^
2*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(4*b^2*d) - (
e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b^2*d) + (3*e^2*
Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b^2*d)
```

Rubi [A] time = 0.28505, antiderivative size = 180, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4b^2d}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{4b^2d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) +
 (e^2*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b])/(4*b^2*d) - (3*e^2*Co
shIntegral[(3*a)/b + 3*ArcSinh[c + d*x]]*Sinh[(3*a)/b])/(4*b^2*d) - (e^2*Co
sh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(4*b^2*d) + (3*e^2*Cosh[(3*a)
/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(4*b^2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2}dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^2}dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^2}dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4(a+bx)}+\frac{3\sinh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}dx, x, \sinh^{-1}(c+dx)\right)}{4bd} + \frac{\left(3e^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}dx, x, \sinh^{-1}(c+dx)\right)}{4bd} +$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2d} - \frac{3e^2\text{Chi}\left(\frac{3a}{b}+3\sin\right.}{}$$

Mathematica [A] time = 0.733205, size = 138, normalized size = 0.75

$$\frac{e^2\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Arc
Sinh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*
x]] + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b^2*d)
```

Maple [A] time = 0.093, size = 342, normalized size = 1.9

$$\frac{1}{d}\left(\frac{e^2}{8\,b\,(a+b\mathrm{Arcsinh}\,(dx+c))}\left(4\,(dx+c)^3-4\,(dx+c)^2\,\sqrt{1+(dx+c)^2}+3\,dx+3\,c-\sqrt{1+(dx+c)^2}\right)+\frac{3\,e^2}{8\,b^2}\mathrm{e}^{3\,\frac{a}{b}}\mathrm{Ei}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/8*(4*(d*x+c)^3-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+3*d*x+3*c-(1+(d*x+c)^
2)^(1/2))*e^2/b/(a+b*arcsinh(d*x+c))+3/8*e^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(
d*x+c)+3*a/b)-1/8*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2/b/(a+b*arcsinh(d*x+c))-1
/8*e^2/b^2*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/8*e^2/b*(d*x+c+(1+(d*x+c)^2)
^(1/2))/(a+b*arcsinh(d*x+c))+1/8*e^2/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b
)-1/8*e^2/b*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+
c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-3/8*e^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d
*x+c)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 + c^3*e^2 + (10*c^2*d^3*e^2 + d^3
*e^2)*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*x^2 + (5*c^4*d*e^2 + 3*c^2*d*e^2
)*x + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d^2*e^2 +
 d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*
c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a
*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((3*d^6*e^2*x^6 + 18*
c*d^5*e^2*x^5 + 3*c^6*e^2 + 6*c^4*e^2 + 3*(15*c^2*d^4*e^2 + 2*d^4*e^2)*x^4
+ 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 + 2*c*d^3*e^2)*x^3 + 3*(15*c^4*d^2*e^2 + 12
*c^2*d^2*e^2 + d^2*e^2)*x^2 + (3*d^4*e^2*x^4 + 12*c*d^3*e^2*x^3 + 3*c^4*e^2
 + c^2*e^2 + (18*c^2*d^2*e^2 + d^2*e^2)*x^2 + 2*(6*c^3*d*e^2 + c*d*e^2)*x)*
(d^2*x^2 + 2*c*d*x + c^2 + 1) + 6*(3*c^5*d*e^2 + 4*c^3*d*e^2 + c*d*e^2)*x +
 (6*d^5*e^2*x^5 + 30*c*d^4*e^2*x^4 + 6*c^5*e^2 + 7*c^3*e^2 + (60*c^2*d^3*e^
2 + 7*d^3*e^2)*x^3 + 2*c*e^2 + 3*(20*c^3*d^2*e^2 + 7*c*d^2*e^2)*x^2 + (30*c
^4*d*e^2 + 21*c^2*d*e^2 + 2*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a
*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d
)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^
2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2
+ d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*
d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b^2\,\mathrm{arsinh}\,(dx+c)^2+2\,ab\,\mathrm{arsinh}\,(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arcsinh(d*x + c)^2 + 2*
a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a^2 + 2ab \operatorname{asinh}(c+dx) + b^2 \operatorname{asinh}^2(c+dx)}\,dx + \int \frac{d^2x^2}{a^2 + 2ab \operatorname{asinh}(c+dx) + b^2 \operatorname{asinh}^2(c+dx)}\,dx + \int \frac{}{a^2 + 2ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d
*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^2, x)
```

3.165 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sqrt{(c+dx)^2+1}(c+dx)}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

```
[Out] -((e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) + (e*
Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(b^2*d) - (e*Si
nh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(b^2*d)
```

Rubi [A] time = 0.147562, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sqrt{(c+dx)^2+1}(c+dx)}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) + (e*
Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(b^2*d) - (e*Sinh
[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(b^2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d} - \frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}}{}$$

Mathematica [A] time = 0.295022, size = 97, normalized size = 0.94

$$\frac{e\left(-\frac{b\sqrt{c^2+2cdx+d^2x^2+1}(c+dx)}{a+b\sinh^{-1}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(a + b*ArcSinh[c + d*
x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

Maple [A] time = 0.049, size = 160, normalized size = 1.6

$$\frac{1}{d}\left(\frac{e}{\left(4a+4b\text{Arcsinh}\,(dx+c)\right)b}\left(2\,(dx+c)^2-2\,(dx+c)\sqrt{1+(dx+c)^2}+1\right)-\frac{e}{2b^2}e^{2\frac{a}{b}}\text{Ei}\left(1,2\,\text{Arcsinh}\,(dx+c)+2\,\frac{a}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e/(a+b*arcsinh(d*x+c
))/b-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4*e/b*(2*(d*x+c)^2
+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2*e/b^2*exp(-2*a/b
)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e + c^2*e + (6*c^2*d^2*e + d^2*e)*x^2 + 2
*(2*c^3*d*e + c*d*e)*x + (d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e + c*e + (3*c^2*
d*e + d*e)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2
*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^
2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^5*e + 4*c^3*e +
 4*(5*c^2*d^3*e + d^3*e)*x^3 + 4*(5*c^3*d^2*e + 3*c*d^2*e)*x^2 + 2*(d^3*e*x
^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2
*c*e + 2*(5*c^4*d*e + 6*c^2*d*e + d*e)*x + (4*d^4*e*x^4 + 16*c*d^3*e*x^3 +
4*c^4*e + 4*c^2*e + 4*(6*c^2*d^2*e + d^2*e)*x^2 + 8*(2*c^3*d*e + c*d*e)*x +
 e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b +
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b
^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*
b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2
*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dex+ce}{b^2\operatorname{arsinh}(dx+c)^2+2\,ab\operatorname{arsinh}(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int\frac{c}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx+\int\frac{dx}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e*(Integral(c/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^2, x)
```

3.166 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

```
[Out] -(Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x]))) - (CoshIntegral[(a
+ b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(b^2*d) + (Cosh[a/b]*SinhIntegral[(a +
b*ArcSinh[c + d*x])/b])/(b^2*d)
```

Rubi [A] time = 0.174356, antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5655, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{b^2d}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x]))) - (CoshIntegral[a/b
 + ArcSinh[c + d*x]]*Sinh[a/b])/(b^2*d) + (Cosh[a/b]*SinhIntegral[a/b + Arc
Sinh[c + d*x]])/(b^2*d)
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} - \frac{\sinh}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{b^2d} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sin\right.}{b^2d}$$

Mathematica [A] time = 0.271571, size = 179, normalized size = 1.97

$$\frac{-\sinh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(c+dx)\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(c+dx)\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] (-(b*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) - a*c*Log[a + b*ArcSinh[c + d*x]] -
 b*c*ArcSinh[c + d*x]*Log[a + b*ArcSinh[c + d*x]] + a*c*Log[d*(a + b*ArcSin
h[c + d*x])] + b*c*ArcSinh[c + d*x]*Log[d*(a + b*ArcSinh[c + d*x])] - (a +
b*ArcSinh[c + d*x])*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] + (a + b
*ArcSinh[c + d*x])*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(b^2*d*(
a + b*ArcSinh[c + d*x]))
```

Maple [A] time = 0.038, size = 128, normalized size = 1.4

$$\frac{1}{d}\left(\frac{1}{2b\left(a+b\text{Arcsinh}\left(dx+c\right)\right)}\left(-\sqrt{1+\left(dx+c\right)^2}+dx+c\right)+\frac{1}{2b^2}e^{\frac{a}{b}}\text{Ei}\left(1,\text{Arcsinh}\left(dx+c\right)+\frac{a}{b}\right)-\frac{1}{2b\left(a+b\text{Arcsinh}\left(d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2*(-(1+(d*x+c)^2)^(1/2)+d*x+c)/b/(a+b*arcsinh(d*x+c))+1/2/b^2*exp(a/
b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(
d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x
^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*
b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((d^4*x^4
+ 4*c*d^3*x^3 + c^4 + 2*(3*c^2*d^2 + d^2)*x^2 + (d^2*x^2 + 2*c*d*x + c^2 +
1)*(d^2*x^2 + 2*c*d*x + c^2 - 1) + 2*c^2 + 4*(c^3*d + c*d)*x + (2*d^3*x^3 +
 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d + d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1) + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c
^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*
b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
 + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 \operatorname{arsinh}(dx+c)^2 + 2\,ab \operatorname{arsinh}(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{arsinh}\left(dx + c\right) + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-2), x)
```

3.167 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])^2), x]/e
```

Rubi [A] time = 0.0636682, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^2), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.40631, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2), x]
```

Maple [A] time = 0.125, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\left(a+b\text{Arcsinh}\,(dx+c)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*e*x^2 + (3*c^2*d^2*e + d^2*e)*
a*b*x + (c^3*d*e + c*d*e)*a*b + (b^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*c^2
*d^2*e + d^2*e)*b^2*x + (c^3*d*e + c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*c*d^
2*e*x + b^2*c^2*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^3*e*x^2 + 2*a*b*c*d^2*e*x + a*b*c^2*
d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - integrate((2*(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(d*x + c) + (2*d^2*x^2 + 4*c*d*x + 2*c^2 + 1)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))/(a*b*d^6*e*x^6 + 6*a*b*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*
e)*a*b*x^4 + 4*(5*c^3*d^3*e + 2*c*d^3*e)*a*b*x^3 + (15*c^4*d^2*e + 12*c^2*d
^2*e + d^2*e)*a*b*x^2 + 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*a*b*x + (c^6*e +
2*c^4*e + c^2*e)*a*b + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 + 6*a*b*c^2*d^2*e
*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^
6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*e)*b^2*x^4 + 4*(5*c^3*d
^3*e + 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*b^2*x^2 +
 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*b^2*x + (c^6*e + 2*c^4*e + c^2*e)*b^2 +
(b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 + 4*b^2*c^3*d*e*x
+ b^2*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^5*e*x^5 + 5*b^2*c*d^4
*e*x^4 + (10*c^2*d^3*e + d^3*e)*b^2*x^3 + (10*c^3*d^2*e + 3*c*d^2*e)*b^2*x^
2 + (5*c^4*d*e + 3*c^2*d*e)*b^2*x + (c^5*e + c^3*e)*b^2)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*
d^5*e*x^5 + 5*a*b*c*d^4*e*x^4 + (10*c^2*d^3*e + d^3*e)*a*b*x^3 + (10*c^3*d^
2*e + 3*c*d^2*e)*a*b*x^2 + (5*c^4*d*e + 3*c^2*d*e)*a*b*x + (c^5*e + c^3*e)*
a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2dex + a^2ce + \left(b^2dex + b^2ce\right)\text{arsinh}\left(dx + c\right)^2 + 2\left(abdex + abce\right)\text{arsinh}\left(dx + c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2
+ 2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{asinh}(c+dx)+2abdx\operatorname{asinh}(c+dx)+b^2c\operatorname{asinh}^2(c+dx)+b^2dx\operatorname{asinh}^2(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*asinh(c + d*x) + 2*a*b*d*x*asinh(c
+ d*x) + b**2*c*asinh(c + d*x)**2 + b**2*d*x*asinh(c + d*x)**2), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2), x)
```

3.168 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=320

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{16b^3d}-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}-\frac{e^4}{\ldots}$$

```
[Out] -(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcSinh[c + d*x])) - (5*e^4*(c + d*x)^
5)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[(a + b*
ArcSinh[c + d*x])/b])/(16*b^3*d) - (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*CoshIntegral
[(5*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) - (e^4*Sinh[a/b]*SinhIntegral[
(a + b*ArcSinh[c + d*x])/b])/(16*b^3*d) + (27*e^4*Sinh[(3*a)/b]*SinhIntegra
l[(3*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) - (25*e^4*Sinh[(5*a)/b]*SinhI
ntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d)
```

Rubi [A] time = 0.891697, antiderivative size = 316, normalized size of antiderivative = 0.99, number of steps used = 26, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{16b^3d}-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{32b^3d}+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}\ldots\right)}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcSinh[c + d*x])) - (5*e^4*(c + d*x)^
5)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[a/b + A
rcSinh[c + d*x]])/(16*b^3*d) - (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c + d*x]])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*a)
/b + 5*ArcSinh[c + d*x]])/(32*b^3*d) - (e^4*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c + d*x]])/(16*b^3*d) + (27*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b +
3*ArcSinh[c + d*x]])/(32*b^3*d) - (25*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*a)/
b + 5*ArcSinh[c + d*x]])/(32*b^3*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{(2e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{bd} + \frac{(5e^4}{}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}$$

Mathematica [A] time = 1.23823, size = 316, normalized size = 0.99

$$\frac{e^4\left(-\frac{16b^2\sqrt{(c+dx)^2+1}(c+dx)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 48\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + \sinh\left(\frac{a}{b}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
2 + (16*b*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x]) + 48*(
-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c +
d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 25*(2*Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[
3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c
 + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c + d*x])])))/(32*b^3*d)
```

Maple [B] time = 0.224, size = 896, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/64*(16*(d*x+c)^5-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-12*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c-(1+(d*x+c)^2)^(1/2))*e^4*(5*b*arcsin
h(d*x+c)+5*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-25/64*e
^4/b^3*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/64*(4*(d*x+c)^3-4*(d*x+c)^
2*(1+(d*x+c)^2)^(1/2)+3*d*x+3*c-(1+(d*x+c)^2)^(1/2))*e^4*(3*b*arcsinh(d*x+c
)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+27/64*e^4/b^3*
exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/32*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e
^4*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^
2)-1/32*e^4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/32*e^4/b*(d*x+c+(1+(d*x
+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/32*e^4/b^2*(d*x+c+(1+(d*x+c)^2)^(1/2
))/(a+b*arcsinh(d*x+c))-1/32*e^4/b^3*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/
64*e^4/b*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^
2)^(1/2))/(a+b*arcsinh(d*x+c))^2+9/64*e^4/b^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64*e
^4/b^3*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/64*e^4/b*(16*(d*x+c)^5+2
0*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-5/64*e^4/b^2*(16*(
d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)
^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-25/64*e^4/
b^3*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*
arcsinh(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{(b \operatorname{arsinh}(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^3, x)
```

3.169 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=247

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}+\frac{e^3}{}$$

```
[Out] -(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (2*e^3*(c + d*x
)^4)/(b^2*d*(a + b*ArcSinh[c + d*x])) + (e^3*CoshIntegral[(2*(a + b*ArcSinh
[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4*(a + b*ArcSi
nh[c + d*x]))/b]*Sinh[(4*a)/b])/(b^3*d) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(
2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*SinhIntegral
[(4*(a + b*ArcSinh[c + d*x]))/b])/(b^3*d)
```

Rubi [A] time = 0.681912, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{b^3d}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (2*e^3*(c + d*x
)^4)/(b^2*d*(a + b*ArcSinh[c + d*x])) + (e^3*CoshIntegral[(2*a)/b + 2*ArcSi
nh[c + d*x]]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4*a)/b + 4*ArcSi
nh[c + d*x]]*Sinh[(4*a)/b])/(b^3*d) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)
/b + 2*ArcSinh[c + d*x]])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)
/b + 4*ArcSinh[c + d*x]])/(b^3*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\left(3e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd} + \cdots$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+\cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+\cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+\cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+\cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+\cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+\cdots\right.}$$

Mathematica [A] time = 0.660613, size = 179, normalized size = 0.72

$$\frac{e^3\left(-\frac{b^2\sqrt{(c+dx)^2+1}(c+dx)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + \sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - 2\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - \cosh\left(\cdots\right.\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^3*(-((b^2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2)
 + (b*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x]) + CoshInte
gral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegral[4*(a/b + Ar
cSinh[c + d*x])]*Sinh[(4*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(
2*b^3*d)
```

Maple [B] time = 0.127, size = 579, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/32*(8*(d*x+c)^4-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^2-4*(d*x+
c)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(4*b*arcsinh(d*x+c)+4*a-b)/b^2/(b^2*arcsinh(d
*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e^3/b^3*exp(4*a/b)*Ei(1,4*arcsinh(d*x
+c)+4*a/b)+1/16*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(2*b*arcs
inh(d*x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4*e
^3/b^3*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/16*e^3/b*(2*(d*x+c)^2+1+2*
(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/8*e^3/b^2*(2*(d*x+c)^
2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/4*e^3/b^3*exp(-2*
a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/32*e^3/b*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d
*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d
*x+c))^2-1/8*e^3/b^2*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/
2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/2*e^3/b^3*exp(-4
*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*arc
sinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{a^3 + 3a^2b\,\text{asinh}\,(c+dx) + 3ab^2\,\text{asinh}^2\,(c+dx) + b^3\,\text{asinh}^3\,(c+dx)}\,dx + \int \frac{d^3x^3}{a^3 + 3a^2b\,\text{asinh}\,(c+dx) + 3ab^2\,\text{asin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
```

```
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{(b \operatorname{arsinh}(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^3, x)
```

3.170 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^3d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8b^3d}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^3d}-\frac{9e^2\sinh}{}$$

```
[Out] -(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (e^2*(c + d*x))/(b^2*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^3)/(
2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcS
inh[c + d*x])/b])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*A
rcSinh[c + d*x]))/b])/(8*b^3*d) + (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c + d*x])/b])/(8*b^3*d) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*Arc
Sinh[c + d*x]))/b])/(8*b^3*d)
```

Rubi [A] time = 0.611828, antiderivative size = 305, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301, 5657}

$$-\frac{9e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8b^3d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{8b^3d}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (e^2*(c + d*x))/(b^2*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^3)/(
2*b^2*d*(a + b*ArcSinh[c + d*x])) - (9*e^2*Cosh[a/b]*CoshIntegral[a/b + Arc
Sinh[c + d*x]])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*A
rcSinh[c + d*x]])/(8*b^3*d) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c
+ d*x])/b])/(b^3*d) + (9*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]
)/(8*b^3*d) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c + d*x
]])/(8*b^3*d) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(b
^3*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{e^2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{bd} + \frac{\left(3e^2\right)\text{Su}}{}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right.}$$

Mathematica [A] time = 0.729361, size = 216, normalized size = 0.88

$$e^2\left(-\frac{4b^2\sqrt{(c+dx)^2+1}(c+dx)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 9\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + \sinh\left(\frac{a}{b}\right)\text{S}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*((-4*b^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2
 + (4*b*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x]) + 8*Cosh[a
/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 8*Sinh[a/b]*SinhIntegral[a/b + A
rcSinh[c + d*x]] + 9*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + C
osh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhInteg
ral[a/b + ArcSinh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c
 + d*x])])))/(8*b^3*d)
```

Maple [B] time = 0.114, size = 507, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/16*(4*(d*x+c)^3-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+3*d*x+3*c-(1+(d*x+c
)^2)^(1/2))*e^2*(3*b*arcsinh(d*x+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*
arcsinh(d*x+c)+a^2)-9/16*e^2/b^3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/
16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh
(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arcsinh(d*x+
c)+a/b)+1/16*e^2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/16*
e^2/b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/16*e^2/b^3*exp(-
a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/16*e^2/b*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^
2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16*e^2/
b^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1
/2))/(a+b*arcsinh(d*x+c))-9/16*e^2/b^3*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3
*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arcsinh(d*x + c)^3 + 3*
a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a^3 + 3a^2b\,\text{asinh}\,(c+dx) + 3ab^2\,\text{asinh}^2\,(c+dx) + b^3\,\text{asinh}^3\,(c+dx)}\,dx + \int \frac{d^2}{a^3 + 3a^2b\,\text{asinh}\,(c+dx) + 3ab^2\,\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)*
*2 + b**3*asinh(c + d*x)**3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^3, x)
```

3.171 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\quad}{2b^2d\left(a+b\right.}$$

```
[Out] -(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2) - e
/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d*(a + b*ArcSinh
[c + d*x])) - (e*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b]
)/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/
(b^3*d)
```

Rubi [A] time = 0.331282, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301, 5675}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\quad}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2) - e
/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d*(a + b*ArcSinh
[c + d*x])) - (e*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b])/
(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(b^3
*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd} + \frac{e\,\text{Sub}}{}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+\right.}$$

Mathematica [A] time = 0.31807, size = 120, normalized size = 0.77

$$\frac{e\left(-\frac{b^2(c+dx)\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^2} - 2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + \frac{b\left(-2(c+\right.}{a+b\sinh^{-}}\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e*(-((b^2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2) + (
b*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 2*CoshIntegral[2*(a/b +
ArcSinh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cSinh[c + d*x])]))/(2*b^3*d)
```

Maple [A] time = 0.061, size = 239, normalized size = 1.5

$$\frac{1}{d}\left(-\frac{e\left(2\,b\text{Arcsinh}\left(dx+c\right)+2\,a-b\right)}{8\,b^2\left(b^2\left(\text{Arcsinh}\left(dx+c\right)\right)^2+2\,ab\text{Arcsinh}\left(dx+c\right)+a^2\right)}\left(2\,\left(dx+c\right)^2-2\,\left(dx+c\right)\sqrt{1+\left(dx+c\right)^2}+1\right)+\frac{e}{2\,b^3}\text{e}^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/8*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e*(2*b*arcsinh(d*x+
c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e/b^3*exp
(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/8*e/b*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d
*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4*e/b^2*(2*(d*x+c)^2+1+2*(d*x+c)*(
1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2*e/b^3*exp(-2*a/b)*Ei(1,-2*arcs
inh(d*x+c)-2*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dex + ce}{b^3 \operatorname{arsinh}(dx + c)^3 + 3\,ab^2 \operatorname{arsinh}(dx + c)^2 + 3\,a^2 b \operatorname{arsinh}(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2
 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a^3 + 3a^2 b \operatorname{asinh}(c + dx) + 3ab^2 \operatorname{asinh}^2(c + dx) + b^3 \operatorname{asinh}^3(c + dx)}\,dx + \int \frac{dx}{a^3 + 3a^2 b \operatorname{asinh}(c + dx) + 3ab^2 \operatorname{asinh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2
+ b**3*asinh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*asinh(c + d*x
) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^3, x)
```

3.172 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\sqrt{(c+dx)^2+1}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

```
[Out] -Sqrt[1 + (c + d*x)^2]/(2*b*d*(a + b*ArcSinh[c + d*x])^2) - (c + d*x)/(2*b^
2*d*(a + b*ArcSinh[c + d*x])) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c +
d*x])/b])/(2*b^3*d) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/
(2*b^3*d)
```

Rubi [A] time = 0.177495, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5863, 5655, 5774, 5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\sqrt{(c+dx)^2+1}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -Sqrt[1 + (c + d*x)^2]/(2*b*d*(a + b*ArcSinh[c + d*x])^2) - (c + d*x)/(2*b^
2*d*(a + b*ArcSinh[c + d*x])) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c +
d*x])/b])/(2*b^3*d) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/
(2*b^3*d)
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b\sinh^{-1}(x)}\right.}{2b^2d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx\right.}{2}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \cos\right.}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a+b\sinh}{b}\right.}{2b^3d}$$

Mathematica [A] time = 0.275074, size = 100, normalized size = 0.8

$$-\frac{\frac{b^2\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^2} - \cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \frac{b(c+dx)}{a+b\sinh^{-1}(c+dx)}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -((b^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2 + (b*(c + d*x))/(a
 + b*ArcSinh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] + S
inh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(2*b^3*d)
```

Maple [A] time = 0.053, size = 190, normalized size = 1.5

$$\frac{1}{d}\left(-\frac{b\mathrm{Arcsinh}\,(dx+c)+a-b}{4\,b^2\left(b^2\,(\mathrm{Arcsinh}\,(dx+c))^2+2\,ab\mathrm{Arcsinh}\,(dx+c)+a^2\right)}\left(-\sqrt{1+(dx+c)^2}+dx+c\right)-\frac{1}{4\,b^3}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/4*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcs
inh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)
+a/b)-1/4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*(d*x
+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arcsin
h(d*x+c)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 + d^5
)*a + (7*c^2*d^5 + d^5)*b)*x^5 + 5*((7*c^3*d^4 + 3*c*d^4)*a + (7*c^3*d^4 +
3*c*d^4)*b)*x^4 + ((35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 + 30*c
^2*d^3 + 3*d^3)*b)*x^3 + 3*((7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d
^2 + 10*c^3*d^2 + 3*c*d^2)*b)*x^2 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c
*d^3)*x^3 + (6*a*c^2*d^2 + (6*c^2*d^2 + d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 +
c^2)*b + 2*(2*a*c^3*d + (2*c^3*d + c*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d
^3 + d^3)*a + 5*(6*c^2*d^3 + d^3)*b)*x^3 + 3*((10*c^3*d^2 + 3*c*d^2)*a + 5*
(2*c^3*d^2 + c*d^2)*b)*x^2 + 3*(c^5 + c^3)*a + (3*c^5 + 5*c^3 + 2*c)*b + (3
*(5*c^4*d + 3*c^2*d)*a + (15*c^4*d + 15*c^2*d + 2*d)*b)*x)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*a + (c^7 + 3*c^5 + 3*c^3 + c)*b +
 ((7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)
*b)*x + (b*d^7*x^7 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b*x^5 + 5*(7*c^3*d
^4 + 3*c*d^4)*b*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b*x^3 + 3*(7*c^5*d^
2 + 10*c^3*d^2 + 3*c*d^2)*b*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*b*x +
(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + (c^4 - 1)*b)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b*d^5*x^5 + 5*b*c*d^4*x^4 + (10*c^2
*d^3 + d^3)*b*x^3 + (10*c^3*d^2 + 3*c*d^2)*b*x^2 + (5*c^4*d + 3*c^2*d)*b*x
+ (c^5 + c^3)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*
b + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + 3*(15*c^2*d^4 + 2*d^4)*b*x^4 + 12*(5*c^
3*d^3 + 2*c*d^3)*b*x^3 + (45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*b*x^2 + 2*(9*c^5
*d + 12*c^3*d + 4*c*d)*b*x + (3*c^6 + 6*c^4 + 4*c^2 + 1)*b)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(
a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^5)*x^5 + (3*(15*c^2*d^4 + 2*d^4)*a
 + (45*c^2*d^4 + 7*d^4)*b)*x^4 + 4*(3*(5*c^3*d^3 + 2*c*d^3)*a + (15*c^3*d^3
 + 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*a + (45*c^4*d^2 + 4
```

```
2*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 + 6*c^4 + 4*c^2 + 1)*a + (3*c^6 + 7*c^4
+ 5*c^2 + 1)*b + 2*((9*c^5*d + 12*c^3*d + 4*c*d)*a + (9*c^5*d + 14*c^3*d +
5*c*d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*b^
2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a^2
*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*d^
7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*c*
d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3 +
3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1
)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 + 2*
(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 +
 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*(5
*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*d
+ 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 +
 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2 +
 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)
*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 +
 d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^4*
d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2*d
^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^4
+ 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)*a
*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*d^
6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3 +
3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c^3
*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*(5
*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^4*
d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(d^8*x^8 + 8*c*d^7*x^7 + c^8 +
4*(7*c^2*d^6 + d^6)*x^6 + 4*c^6 + 8*(7*c^3*d^5 + 3*c*d^5)*x^5 + 2*(35*c^4*d
^4 + 30*c^2*d^4 + 3*d^4)*x^4 + 6*c^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)
*x^3 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + 3)*(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*x^
2 + (4*d^5*x^5 + 20*c*d^4*x^4 + 4*c^5 + 4*(10*c^2*d^3 + d^3)*x^3 + 4*c^3 +
4*(10*c^3*d^2 + 3*c*d^2)*x^2 + (20*c^4*d + 12*c^2*d + 3*d)*x + 3*c)*(d^2*x^
2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(2*d^6*x^6 + 12*c*d^5*x^5 + 2*c^6 + 2*(15*
c^2*d^4 + 2*d^4)*x^4 + 4*c^4 + 8*(5*c^3*d^3 + 2*c*d^3)*x^3 + (30*c^4*d^2 +
24*c^2*d^2 + d^2)*x^2 + c^2 + 2*(6*c^5*d + 8*c^3*d + c*d)*x - 1)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 4*c^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*x + (4*d^
7*x^7 + 28*c*d^6*x^6 + 4*c^7 + 12*(7*c^2*d^5 + d^5)*x^5 + 12*c^5 + 20*(7*c^
3*d^4 + 3*c*d^4)*x^4 + (140*c^4*d^3 + 120*c^2*d^3 + 9*d^3)*x^3 + 9*c^3 + 3*
(28*c^5*d^2 + 40*c^3*d^2 + 9*c*d^2)*x^2 + (28*c^6*d + 60*c^4*d + 27*c^2*d +
 d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)/(a*b^2*d^8*x^8 + 8*a*b^2*
c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x
^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*
d^3 + 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b
^2*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4
 + 4*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*
x^2 + 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b
^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2
 + 3*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d
^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 +
(15*c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^
4*d^2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x +
 (c^6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 +
```

```
8*b^3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3
*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*
d^3 + 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x
^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c
^2 + 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^
3*d*x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c
*d^4*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5
*c^4*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3
/2) + 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(
5*c^3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*
(3*c^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*
x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b
^3*x^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d
 + 9*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*
d^7*x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4
+ 3*c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c
^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d +
d)*a*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3\,\text{arsinh}\,(dx+c)^3+3\,ab^2\,\text{arsinh}\,(dx+c)^2+3\,a^2b\,\text{arsinh}\,(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*a
rcsinh(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+b\,\text{asinh}\,(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3), x)
```

3.173 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])^3), x]/e
```

Rubi [A] time = 0.0619979, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^3), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{de} \end{aligned}$$

Mathematica [A] time = 1.01987, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3), x]
```

Maple [A] time = 0.142, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\left(a+b\text{Arcsinh}\,(dx+c)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3dex + a^3ce + \left(b^3dex + b^3ce\right)\text{arsinh}\left(dx + c\right)^3 + 3\left(ab^2dex + ab^2ce\right)\text{arsinh}\left(dx + c\right)^2 + 3\left(a^2bdex + a^2bce\right)\text{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3
+ 3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c
*e)*arcsinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)(b\,\text{arsinh}\,(dx + c) + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3), x)
```

3.174 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=410

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{48b^4d}+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}-\frac{125e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{96b^4d}+$$

```
[Out] -(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (5*e^4*(c + d
*x)^5)/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^4*(c + d*x)^2*Sqrt[1 + (
c + d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (25*e^4*(c + d*x)^4*Sqrt[1
+ (c + d*x)^2])/(6*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^4*CoshIntegral[(a +
 b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(48*b^4*d) + (27*e^4*CoshIntegral[(3*(a
+ b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(32*b^4*d) - (125*e^4*CoshIntegral
[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/(96*b^4*d) + (e^4*Cosh[a/b]
*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(48*b^4*d) - (27*e^4*Cosh[(3*a)/
b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(32*b^4*d) + (125*e^4*Cosh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(96*b^4*d)
```

Rubi [A] time = 0.877848, antiderivative size = 406, normalized size of antiderivative = 0.99, number of steps used = 24, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{48b^4d}+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{32b^4d}-\frac{125e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh\right.}{96b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (5*e^4*(c + d
*x)^5)/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^4*(c + d*x)^2*Sqrt[1 + (
c + d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (25*e^4*(c + d*x)^4*Sqrt[1
+ (c + d*x)^2])/(6*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^4*CoshIntegral[a/b
+ ArcSinh[c + d*x]]*Sinh[a/b])/(48*b^4*d) + (27*e^4*CoshIntegral[(3*a)/b +
3*ArcSinh[c + d*x]]*Sinh[(3*a)/b])/(32*b^4*d) - (125*e^4*CoshIntegral[(5*a)
/b + 5*ArcSinh[c + d*x]]*Sinh[(5*a)/b])/(96*b^4*d) + (e^4*Cosh[a/b]*SinhInt
egral[a/b + ArcSinh[c + d*x]])/(48*b^4*d) - (27*e^4*Cosh[(3*a)/b]*SinhInteg
ral[(3*a)/b + 3*ArcSinh[c + d*x]])/(32*b^4*d) + (125*e^4*Cosh[(5*a)/b]*Sinh
Integral[(5*a)/b + 5*ArcSinh[c + d*x]])/(96*b^4*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\left(4e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{3bd} + \frac{\left(5e^4\right.}{}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

Mathematica [A] time = 1.91378, size = 410, normalized size = 1.

$$-\frac{e^4\left(\frac{32b^3\sqrt{(c+dx)^2+1}(c+dx)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{16b^2\left(-5(c+dx)^5-4(c+dx)^3\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 384\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^4*((32*b^3*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
3 - (16*b^2*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x])^2 +
(16*b*Sqrt[1 + (c + d*x)^2]*(12*(c + d*x)^2 + 25*(c + d*x)^4))/(a + b*ArcSi
nh[c + d*x]) + 384*(CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - Cosh[a
/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]) + 544*(-3*CoshIntegral[a/b + ArcS
inh[c + d*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*
a)/b] + 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] - Cosh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 125*(10*CoshIntegral[a/b + ArcSin
h[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*
a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]*Sinh[(5*a)/b] - 10*Cosh[a/
b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh[(3*a)/b]*SinhIntegral[3*(a
/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c + d
*x])])))/(96*b^4*d)
```

Maple [B] time = 0.258, size = 1244, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/192*(16*(d*x+c)^5-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-12*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c-(1+(d*x+c)^2)^(1/2))*e^4*(25*b^2*arc
sinh(d*x+c)^2+50*a*b*arcsinh(d*x+c)-5*arcsinh(d*x+c)*b^2+25*a^2-5*a*b+2*b^2
)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsinh(d*x+c)^2*a*b^2+3*a^2*b*arcsinh(d*x+c)
+a^3)+125/192*e^4/b^4*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-1/64*(4*(d*x+
c)^3-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+3*d*x+3*c-(1+(d*x+c)^2)^(1/2))*e^4*(9*
b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*arcsinh(d*x+c)*b^2+9*a^2-3*a*b
+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsinh(d*x+c)^2*a*b^2+3*a^2*b*arcsinh(
d*x+c)+a^3)-27/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/96*(-(1
+(d*x+c)^2)^(1/2)+d*x+c)*e^4*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)-arc
sinh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsinh(d*x+c)^2
*a*b^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/96*e^4/b^4*exp(a/b)*Ei(1,arcsinh(d*x+c
)+a/b)-1/48*e^4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/96*e
^4/b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/96*e^4/b^3*(d*x
+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/96*e^4/b^4*exp(-a/b)*Ei(1,-a
rcsinh(d*x+c)-a/b)+1/32*e^4/b*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)
^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+3/64*e^4/b^2*(4*(d*x+
c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*ar
csinh(d*x+c))^2+9/64*e^4/b^3*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64*e^4/b^4*exp(-3*a/b
)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/96*e^4/b*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d
*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(
d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-5/192*e^4/b^2*(16*(d*x+c)^5+20*(d*x
+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)
^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-25/192*e^4/b^3*(16*(d*x+
c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(
1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-125/192*e^4/b^
4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^4\,\text{arsinh}\,(dx+c)^4 + 4\,ab^3\,\text{arsinh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arsinh}\,(dx+c)^2 + 4\,a^3b\,\text{arsinh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^
2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^4, x)
```

3.175 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=340

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-$$

```
[Out] -(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^3*(c + d*x
)^4)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^3*(c + d*x)*Sqrt[1 + (c + d*
x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (8*e^3*(c + d*x)^3*Sqrt[1 + (c +
d*x)^2])/(3*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^3*Cosh[(2*a)/b]*CoshIntegr
al[(2*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshIn
tegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) + (e^3*Sinh[(2*a)/b]*Sinh
Integral[(2*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*
SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d)
```

Rubi [A] time = 0.695904, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301}

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{3b^4d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sin\right.}{3b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^3*(c + d*x
)^4)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^3*(c + d*x)*Sqrt[1 + (c + d*
x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (8*e^3*(c + d*x)^3*Sqrt[1 + (c +
d*x)^2])/(3*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^3*Cosh[(2*a)/b]*CoshIntegr
al[(2*a)/b + 2*ArcSinh[c + d*x]])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshInte
gral[(4*a)/b + 4*ArcSinh[c + d*x]])/(3*b^4*d) + (e^3*Sinh[(2*a)/b]*SinhInte
gral[(2*a)/b + 2*ArcSinh[c + d*x]])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*SinhIn
tegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(3*b^4*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{e^3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{bd} + \frac{(4e^3 \cdots}{\cdots}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c \cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c \cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c \cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c \cdots\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c \cdots\right.}$$

Mathematica [A] time = 1.11992, size = 318, normalized size = 0.94

$$e^3\left(-\frac{2b^3\sqrt{(c+dx)^2+1}(c+dx)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{b^2\left(-4(c+dx)^4-3(c+dx)^2\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 30\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{si}\cdots\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*((-2*b^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (b^2*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x])^2 - (2*b
*Sqrt[1 + (c + d*x)^2]*(3*(c + d*x) + 8*(c + d*x)^3))/(a + b*ArcSinh[c + d*
x]) + 6*Log[a + b*ArcSinh[c + d*x]] + 30*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b
 + ArcSinh[c + d*x])] - Log[a + b*ArcSinh[c + d*x]] - Sinh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcSinh[c + d*x])]) + 8*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/
b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c + d*
x])] + 3*Log[a + b*ArcSinh[c + d*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b
+ ArcSinh[c + d*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x]
)])))/(6*b^4*d)
```

Maple [B] time = 0.151, size = 800, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(8*(d*x+c)^4-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^2-4*(d*x+c
)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(8*b^2*arcsinh(d*x+c)^2+16*a*b*arcsinh(d*x+c)-
2*arcsinh(d*x+c)*b^2+8*a^2-2*a*b+b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsinh(d
*x+c)^2*a*b^2+3*a^2*b*arcsinh(d*x+c)+a^3)-2/3*e^3/b^4*exp(4*a/b)*Ei(1,4*arc
sinh(d*x+c)+4*a/b)-1/24*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(
2*b^2*arcsinh(d*x+c)^2+4*a*b*arcsinh(d*x+c)-arcsinh(d*x+c)*b^2+2*a^2-a*b+b^
2)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsinh(d*x+c)^2*a*b^2+3*a^2*b*arcsinh(d*x+c
)+a^3)+1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/24*e^3/b*(2*(d
*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+1/24*e^3/b^
2*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/12
*e^3/b^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))
+1/6*e^3/b^4*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/48*e^3/b*(8*(d*x+c
)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/
2)+1)/(a+b*arcsinh(d*x+c))^3-1/24*e^3/b^2*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c
)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c
))^2-1/6*e^3/b^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4
*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-2/3*e^3/b^4*exp(-4*a/b
)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^4\,\text{arsinh}\,(dx+c)^4 + 4\,ab^3\,\text{arsinh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arsinh}\,(dx+c)^2 + 4\,a^3b\,\text{arsinh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*arc
sinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2
 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^4, x)
```

3.176 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=331

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{24b^4d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8b^4d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{24b^4d}+\frac{9e^2\cosh\left(\right.}{}$$

```
[Out] -(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*(c + d*x)^3)
/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*Sqrt[1 + (c + d*x)^2])/(3*b^3*
d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(2*
b^3*d*(a + b*ArcSinh[c + d*x])) + (e^2*CoshIntegral[(a + b*ArcSinh[c + d*x]
)/b]*Sinh[a/b])/(24*b^4*d) - (9*e^2*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]
))/b]*Sinh[(3*a)/b])/(8*b^4*d) - (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh
[c + d*x])/b])/(24*b^4*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*Arc
Sinh[c + d*x]))/b])/(8*b^4*d)
```

Rubi [A] time = 0.674519, antiderivative size = 327, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301, 5655, 5779}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{24b^4d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{8b^4d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*(c + d*x)^3)
/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*Sqrt[1 + (c + d*x)^2])/(3*b^3*
d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(2*
b^3*d*(a + b*ArcSinh[c + d*x])) + (e^2*CoshIntegral[a/b + ArcSinh[c + d*x]]
*Sinh[a/b])/(24*b^4*d) - (9*e^2*CoshIntegral[(3*a)/b + 3*ArcSinh[c + d*x]]*
Sinh[(3*a)/b])/(8*b^4*d) - (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*
x]])/(24*b^4*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c +
 d*x]])/(8*b^4*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{(2e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{3bd} + \frac{e^2\,\text{Su}}{}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+d\right.}$$

Mathematica [A] time = 0.790577, size = 258, normalized size = 0.78

$$e^2\left(-\frac{8b^3(c+dx)^2\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{4b^2\left(-3(c+dx)^3-2(c+dx)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 27\left(3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - \sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (4*b^2*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x])^2 - (4*b
*Sqrt[1 + (c + d*x)^2]*(2 + 9*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 80*C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] + 80*Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c + d*x]] + 27*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/
b] - CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + A
rcSinh[c + d*x])])))/(24*b^4*d)
```

Maple [B] time = 0.14, size = 709, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(4*(d*x+c)^3-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+3*d*x+3*c-(1+(d*x+c)
^2)^(1/2))*e^2*(9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*arcsinh(d*x+
c)*b^2+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsinh(d*x+c)^2*a*b^
2+3*a^2*b*arcsinh(d*x+c)+a^3)+9/16*e^2/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)
+3*a/b)-1/48*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b^2*arcsinh(d*x+c)^2+2*a*b*a
rcsinh(d*x+c)-arcsinh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3
*arcsinh(d*x+c)^2*a*b^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*E
i(1,arcsinh(d*x+c)+a/b)+1/24*e^2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh
(d*x+c))^3+1/48*e^2/b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+
1/48*e^2/b^3*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/48*e^2/b^4*
exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/24*e^2/b*(4*(d*x+c)^3+3*d*x+3*c+4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/16
*e^2/b^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^
2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16*e^2/b^3*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-9/16*e^
2/b^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^4\,\operatorname{arsinh}\left(dx+c\right)^4 + 4\,ab^3\,\operatorname{arsinh}\left(dx+c\right)^3 + 6\,a^2b^2\,\operatorname{arsinh}\left(dx+c\right)^2 + 4\,a^3b\,\operatorname{arsinh}\left(dx+c\right) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arcsinh(d*x + c)^4 + 4*
a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d
*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^4, x)
```

3.177 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=204

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2e\sqrt{(c\ \cdots}}{3b^3d\left(a\ \cdots\right.}$$

```
[Out] -(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3) - e
/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b^2*d*(a + b*Arc
Sinh[c + d*x])^2) - (2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*(a + b*A
rcSinh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d
*x]))/b])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c +
 d*x]))/b])/(3*b^4*d)
```

Rubi [A] time = 0.341499, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301, 5675}

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{3b^4d}-\frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3) - e
/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b^2*d*(a + b*Arc
Sinh[c + d*x])^2) - (2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*(a + b*A
rcSinh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c +
 d*x]])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c +
 d*x]])/(3*b^4*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^4}dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^4}dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^4}dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}dx, x, c+dx\right)}{3bd} + \frac{(2e)\,\text{S}}{}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c\right.}$$

Mathematica [A] time = 0.790984, size = 181, normalized size = 0.89

$$\frac{e\left(-\frac{2b^3(c+dx)\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{b^2\left(-2(c+dx)^2-1\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - 4\left(\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}\right.\right.\right.\right.}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (
b^2*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x])^2 - (4*b*(c + d*x)*Sqrt[
1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2
*(a/b + ArcSinh[c + d*x])] + 4*Log[a + b*ArcSinh[c + d*x]] - 4*(Log[a + b*A
rcSinh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))
)/(6*b^4*d)
```

Maple [A] time = 0.072, size = 333, normalized size = 1.6

$$\frac{1}{d}\left(\frac{e\left(2b^2\left(\text{Arcsinh}\left(dx+c\right)\right)^2+4ab\text{Arcsinh}\left(dx+c\right)-\text{Arcsinh}\left(dx+c\right)b^2+2a^2-ab+b^2\right)}{12b^3\left(b^3\left(\text{Arcsinh}\left(dx+c\right)\right)^3+3\left(\text{Arcsinh}\left(dx+c\right)\right)^2ab^2+3a^2b\text{Arcsinh}\left(dx+c\right)+a^3\right)}\left(2\left(dx+c\right)^2-2\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e*(2*b^2*arcsinh(d*
x+c)^2+4*a*b*arcsinh(d*x+c)-arcsinh(d*x+c)*b^2+2*a^2-a*b+b^2)/b^3/(b^3*arcs
inh(d*x+c)^3+3*arcsinh(d*x+c)^2*a*b^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/3*e/b^4
*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/12*e/b*(2*(d*x+c)^2+1+2*(d*x+c)*
(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12*e/b^2*(2*(d*x+c)^2+1+2*(d*
x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/6*e/b^3*(2*(d*x+c)^2+1+2
*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/3*e/b^4*exp(-2*a/b)*Ei
(1,-2*arcsinh(d*x+c)-2*a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dex + ce}{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3
 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx+c) + a)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^4, x)
```

3.178 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{6b^4d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{\sqrt{(c+dx)^2+}}{6b^3d\left(a+b\sinh^{-1}(\right.}$$

```
[Out] -Sqrt[1 + (c + d*x)^2]/(3*b*d*(a + b*ArcSinh[c + d*x])^3) - (c + d*x)/(6*b^
2*d*(a + b*ArcSinh[c + d*x])^2) - Sqrt[1 + (c + d*x)^2]/(6*b^3*d*(a + b*Arc
Sinh[c + d*x])) - (CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(6*b
^4*d) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(6*b^4*d)
```

Rubi [A] time = 0.267411, antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5863, 5655, 5774, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{6b^4d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{6b^4d}-\frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{\sqrt{(}}{6b^3d\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -Sqrt[1 + (c + d*x)^2]/(3*b*d*(a + b*ArcSinh[c + d*x])^3) - (c + d*x)/(6*b^
2*d*(a + b*ArcSinh[c + d*x])^2) - Sqrt[1 + (c + d*x)^2]/(6*b^3*d*(a + b*Arc
Sinh[c + d*x])) - (CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b])/(6*b^4*d
) + (Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(6*b^4*d)
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\mathrm{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^2}\right.}{6b^2d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c+dx)^2}}{6b^3d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c+dx)^2}}{6b^3d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c+dx)^2}}{6b^3d\left(a+b\sinh^{-1}(c+d\right.}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c+dx)^2}}{6b^3d\left(a+b\sinh^{-1}(c+d\right.}$$

Mathematica [A] time = 0.436369, size = 130, normalized size = 0.81

$$-\frac{\frac{2b^3\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{b^2(c+dx)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + \sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - \cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \frac{b\sqrt{(c+}}{a+b\sinh}}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -((2*b^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (b^2*(c + d*x)
)/(a + b*ArcSinh[c + d*x])^2 + (b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c +
 d*x]) + CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*SinhInt
egral[a/b + ArcSinh[c + d*x]])/(6*b^4*d)
```

Maple [A] time = 0.062, size = 272, normalized size = 1.7

$$\frac{1}{d}\left(\frac{b^2(\mathrm{Arcsinh}\,(dx+c))^2+2\,ab\mathrm{Arcsinh}\,(dx+c)-\mathrm{Arcsinh}\,(dx+c)\,b^2+a^2-ab+2\,b^2}{12\,b^3\left(b^3(\mathrm{Arcsinh}\,(dx+c))^3+3\,(\mathrm{Arcsinh}\,(dx+c))^2\,ab^2+3\,a^2b\mathrm{Arcsinh}\,(dx+c)+a^3\right)}\left(-\sqrt{1+(dx+c)^2}+dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(
d*x+c)-arcsinh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*arcsin
h(d*x+c)^2*a*b^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/12/b^4*exp(a/b)*Ei(1,arcsinh
(d*x+c)+a/b)-1/6/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/
b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/12/b^3*(d*x+c+(1+(
d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arcsinh(d*x+
c)-a/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^4\,\mathrm{arsinh}\,(dx+c)^4+4\,ab^3\,\mathrm{arsinh}\,(dx+c)^3+6\,a^2b^2\,\mathrm{arsinh}\,(dx+c)^2+4\,a^3b\,\mathrm{arsinh}\,(dx+c)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2
*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(dx + c) + a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-4), x)
```

3.179 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])^4), x]/e
```

Rubi [A] time = 0.063398, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^4), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 3.36599, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4), x]
```

Maple [A] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\left(a+b\text{Arcsinh}\,(dx+c)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^4dex + a^4ce + \left(b^4dex + b^4ce\right)\operatorname{arsinh}\left(dx + c\right)^4 + 4\left(ab^3dex + ab^3ce\right)\operatorname{arsinh}\left(dx + c\right)^3 + 6\left(a^2b^2dex + a^2b^2ce\right)\operatorname{ar}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4
+ 4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b
^2*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)(b\operatorname{arsinh}\left(dx + c\right) + a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4), x)
```

3.180 $\int (ce+dex)^4\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}-\cdots$$

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) + (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(64*d) + (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rubi [A] time = 0.875108, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5865, 12, 5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) + (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(64*d) + (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
```

```
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^4\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^4x^4\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,\right.}{10d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{10d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{(ibe^4)\,\text{Subst}\left(\int\left(\frac{5i\sinh(x)}{8\sqrt{a+bx}} - \frac{5i\sinh(3x)}{16\sqrt{a+bx}} +\right.\right.}{10d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{160d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{(be^4)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{320d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-}}\right.}{160d}$$
$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{\sqrt{b}e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d} - \frac{\sqrt{b}}{}$$

Mathematica [A] time = 0.629168, size = 342, normalized size = 0.95

$$e^4e^{-\frac{5a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(-150*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcS
inh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 25*Sqrt[3]*E^((
2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]
))/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*Ar
cSinh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[5]*E^((10*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcSinh[c + d*x]))/
b]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (dex+ce)^4\sqrt{a+b\text{Arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^4\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int c^4\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int d^4x^4\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 4cd^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 6c^2d^2x^2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^4\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.181 $\int (ce+dex)^3 \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{\pi}\sqrt{b}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+$$

```
[Out] (-3*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcSinh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d) + (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) + (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rubi [A] time = 0.665004, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5865, 12, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] (-3*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcSinh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d) + (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) + (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^3x^3\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int x^3\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,\right.}{8d}$$
$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{8d}$$
$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int\left(\frac{3}{8\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cos}{8\sqrt{}}\right.\right.}{8d}$$
$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Su}}{}$$
$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Su}}{}$$
$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{e^3\,\text{Subst}}{}$$
$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{\sqrt{b}e^3e^{\frac{4a}{b}}}{}$$

Mathematica [A] time = 0.281982, size = 223, normalized size = 0.82

$$\frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{3}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right) - 4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Ga}\right.}{128d\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcSinh[c + d*x]))/b] - 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcSinh[c + d*x])/b)]*(-4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcSinh[c +
d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(128*
d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.195, size = 0, normalized size = 0.

$$\int (dex+ce)^3\sqrt{a+b\text{Arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^3\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int c^3\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int d^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 3cd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 3c^2dx\sqrt{a-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(1/2),x)

[Out] e**3*(Integral(c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^3\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)

3.182 $\int (ce + dex)^2 \sqrt{a + b \sinh^{-1}(c + dx)} \, dx$

Optimal. Leaf size=245

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\sqrt{}$$

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d) + (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(48*d) + (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rubi [A] time = 0.650181, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5865, 12, 5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d) + (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(48*d) + (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^2x^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,\right.}{6d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{6d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(ibe^2)\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{a+bx}} - \frac{i\sinh(3x)}{4\sqrt{a+bx}}\right)d\right.}{6d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{24d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} + \frac{(be^2)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{48d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} + \frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-}}\right.}{24d}$$
$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d} + \frac{\sqrt{b}}{}$$

Mathematica [A] time = 0.389586, size = 238, normalized size = 0.97

$$\frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gam}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 9*E^((2*a)/b)*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcSin
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)]
)
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int (dex+ce)^2\sqrt{a+b\text{Arcsinh}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2 \left(\int c^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int d^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 2cdx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*asinh(c + d*x)),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.183 $\int (ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{e\sqrt{}}{}$$

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcSinh[
c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rubi [A] time = 0.429238, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{e\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcSinh[
c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int ex\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+\right.}{4d}$$
$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx\right.}{4d}$$
$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(be)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx,x,s\right.}{4d}$$
$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{e\,\text{Subst}\left(\int e^{\frac{2a}{b}}\right.}{}$$
$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{\sqrt{b}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}}{}$$

Mathematica [A] time = 0.0983304, size = 140, normalized size = 0.85

$$\frac{ee^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{3}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{3}{2}\right.\right.}{8\sqrt{2}d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-
2*(a + b*ArcSinh[c + d*x]))/b] + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (2*(a + b*ArcSinh[c + d*x]))/b]))/(8*Sqrt[2]*d*E^((2*a)/b)
*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (dex+ce)\sqrt{a+b\text{Arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)\sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int c\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int dx\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c*sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x*sqrt(a + b*asin
h(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)\sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.184 $\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rubi [A] time = 0.246222, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```


Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d} - \frac{b\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2d} - \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\,\text{si}\cdots}}{\cdots}\right)}{4d}$$

Mathematica [A] time = 0.0816329, size = 111, normalized size = 0.97

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\mathrm{Arcsinh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\mathrm{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

3.185 $\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[Sqrt[a + b*ArcSinh[c + d*x]]/(c + d*x), x]/e
```

Rubi [A] time = 0.088093, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcSinh[x]]/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\sinh^{-1}(x)}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\sinh^{-1}(x)}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 2.03031, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x), x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\sqrt{a+b\text{Arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+a}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+b\,\operatorname{asinh}\left(c+dx\right)}}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2)/(d*e*x+c*e),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x))/(c + d*x), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\,\operatorname{arsinh}\left(dx+c\right)+a}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.186 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=601

$$\frac{3\sqrt{\pi}b^{3/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{3\sqrt{3\pi}b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{200d}+\cdots$$

```
[Out] (-4*b*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d) + (2*b
*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d)
 - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(5*d) + (3*b^(3/
2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) -
 (b^(3/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(200*d) - (3*b^(3/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d) + (3*b^(3/2)*e^4*E^((5*a)
/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d
) + (3*b^(3/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64
*d*E^(a/b)) - (b^(3/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(200*d*E^((3*a)/b)) - (3*b^(3/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d*E^((3*a)/b)) + (3*b^(3/
2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32
00*d*E^((5*a)/b))
```

Rubi [A] time = 1.66167, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {5865, 12, 5663, 5758, 5717, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$\frac{3\sqrt{\pi}b^{3/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{3\sqrt{3\pi}b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{200d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```


```
[Out] (-4*b*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d) + (2*b
*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d)
 - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(5*d) + (3*b^(3/
2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) -
 (b^(3/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(200*d) - (3*b^(3/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d) + (3*b^(3/2)*e^4*E^((5*a)
/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d
) + (3*b^(3/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64
*d*E^(a/b)) - (b^(3/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(200*d*E^((3*a)/b)) - (3*b^(3/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d*E^((3*a)/b)) + (3*b^(3/
2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32
00*d*E^((5*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{5d} - \frac{\left(3be^4\right)\text{Subst}\left(\int \frac{x^5\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}} dx, x\right.}{10d}$$

$$= -\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{50d} + \frac{e^4(c+dx)^5\left(a+b\sin\right.}{5d}$$

$$= \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} - \frac{3be^4(c+dx)^4\sqrt{1+(c}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$

Mathematica [A] time = 0.467546, size = 343, normalized size = 0.57

$$-\frac{be^4e^{-\frac{5a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(2250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+9\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -(b*e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(2250*E^((6*a)/b)*Sqrt[-((a + b*ArcSin
h[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + 9*Sqrt[5]*Sqrt[a/b + A
rcSinh[c + d*x]]*Gamma[5/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 125*Sqrt[3]*
E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcSinh[c +
d*x]))/b] + 2250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, -((a +
 b*ArcSinh[c + d*x])/b)] - 125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c
+ d*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 9*Sqrt[5]*E^((10*a
)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (5*(a + b*ArcSinh[c + d
*x]))/b]))/(36000*d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int (dex+ce)^4 \left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^4(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.187 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=360

$$-\frac{3\sqrt{\pi}b^{3/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}b^{3/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$

```
[Out] (9*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(64*
d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]
])/(32*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) + (e^3*(c + d*x)^
4*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (3*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[P
i]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d) + (3*b^(3/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(2048*d*E^((4*a)/b)) - (3*b^(3/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(128*d*E^((2*a)/b))
```

Rubi [A] time = 1.05852, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}b^{3/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}b^{3/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```


```
[Out] (9*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(64*
d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]
])/(32*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) + (e^3*(c + d*x)^
4*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (3*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[P
i]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d) + (3*b^(3/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(2048*d*E^((4*a)/b)) - (3*b^(3/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(128*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
```

```
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}} d\right.}{8d}$$

$$= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\left(a+b\right.}{4}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3\sqrt{1+(}{}$$

Mathematica [A] time = 0.286508, size = 225, normalized size = 0.62

$$\frac{be^3e^{-\frac{4a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{5}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+8\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\right.}{512d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5
/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 8*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[5/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*(-8*Sqrt[2]*Gamma[5/2, (2*(a + b*ArcSinh
[c + d*x]))/b] + E^((2*a)/b)*Gamma[5/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/
(512*d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.196, size = 0, normalized size = 0.

$$\int (dex + ce)^3 (a + b\text{Arcsinh}(dx + c))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3 \left(\int ac^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \, dx + \int ad^3 x^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \, dx + \int bc^3 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx) \, dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(a*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**3*x**3
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(3*a*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**
3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c*d**2*
x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c**2*d*x*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.188 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=328

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}-\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+$$

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (b*e^2*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (e^2
*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(3*d) - (3*b^(3/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) + (b^(3/2)*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(96*d) - (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rubi [A] time = 0.882012, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {5865, 12, 5663, 5758, 5717, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}-\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```


```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (b*e^2*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (e^2
*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(3*d) - (3*b^(3/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) + (b^(3/2)*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(96*d) - (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}} dx, x, \right.}{2d}$$

$$= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{e^2(c+dx)^3 \left(a+b\sinh\right.}{3d}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+}}{6d}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+}}{6d}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+}}{6d}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+}}{6d}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+}}{6d}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+}}{6d}$$

Mathematica [A] time = 0.289547, size = 238, normalized size = 0.73

$$-\frac{be^2e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Ga}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -(b*e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh
[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcS
inh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 27*E^((2*a)/b)*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, -((a + b*ArcSinh[c + d*x])/b)] + Sq
rt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (3*(a + b*
ArcSinh[c + d*x]))/b]))/(216*d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^
```

```
2/b^2)])
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (dex+ce)^2 (a+b\mathrm{Arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^2 (b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2 \left(\int ac^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx + \int ad^2x^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx + \int bc^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(a*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*asinh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.189 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}$$

```
[Out] (-3*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d)
 + (e*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh
[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*e*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((2*a)/b))
```

Rubi [A] time = 0.482889, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```


```
[Out] (-3*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d)
 + (e*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh
[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*e*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}\,dx,x\right.}{4d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(c+dx)^2\left(a+b\sin\right.}{2d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh^{-1}(c+\right.}{4d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh^{-1}(c+\right.}{4d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh^{-1}(c+\right.}{4d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh^{-1}(c+\right.}{4d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh^{-1}(c+\right.}{4d}$$
$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh^{-1}(c+\right.}{4d}$$

Mathematica [A] time = 0.104699, size = 142, normalized size = 0.69

$$\frac{bee^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{5}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{5}{2}\right.\right.}{16\sqrt{2}d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2
, (-2*(a + b*ArcSinh[c + d*x]))/b]) + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[5/2, (2*(a + b*ArcSinh[c + d*x]))/b]))/(16*Sqrt[2]*d*E^((2
*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int (dex+ce)\left(a+b\text{Arcsinh}\,(dx+c)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int ac\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int adx\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int bc\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, dx + \int b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(a*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
asinh(c + d*x)), x) + Integral(b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.190 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+ \cdots$$

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rubi [A] time = 0.253528, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+ \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{3/2}dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}dx,x,c+dx\right)}{2d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{\left(3b^2\right)}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b)\,\text{S}}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b)\,\text{S}}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{(3b)\,\text{S}}{}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{3b^{3/2}e}{}$$

Mathematica [A] time = 0.181756, size = 272, normalized size = 1.81

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\left(\sqrt{\pi}(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\text{co}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^{\frac{3}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{asinh}\,(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.191 $\int \frac{(a+b\sinh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{(a+b\sinh^{-1}(c+dx))^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[(a + b*ArcSinh[c + d*x])^(3/2)/(c + d*x), x]/e
```

Rubi [A] time = 0.103211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sinh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^{3/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^{3/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.41524, size = 0, normalized size = 0.

$$\int \frac{(a+b\sinh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{a+b\operatorname{asinh}(c+dx)}}{c+dx}\,dx + \int \frac{b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2)/(d*e*x+c*e),x)
```

```
[Out] (Integral(a*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

3.192 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=701

$$\frac{15\sqrt{\pi}b^{5/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{\sqrt{3\pi}b^{5/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1280d}-\frac{\sqrt{\frac{\pi}{3}}b^{5/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{240d}$$

```
[Out] (2*b^2*e^4*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) - (b^2*e^4*(c + d*
x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(15*d) + (3*b^2*e^4*(c + d*x)^5*Sqrt[a +
 b*ArcSinh[c + d*x]])/(100*d) - (4*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSi
nh[c + d*x])^(3/2))/(15*d) + (2*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^(3/2))/(15*d) - (b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(10*d) + (e^4*(c + d*x)^5*(a + b*ArcSin
h[c + d*x])^(5/2))/(5*d) + (15*b^(5/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) - (b^(5/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d) - (b^(5/2)*e^4*
E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(1280*d) + (3*b^(5/2)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(6400*d) - (15*b^(5/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d*E^(a/b)) + (b^(5/2)*e^4*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d*E^((3*a)/b)) +
 (b^(5/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(1280*d*E^((3*a)/b)) - (3*b^(5/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(6400*d*E^((5*a)/b))
```

Rubi [A] time = 2.20313, antiderivative size = 701, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$\frac{15\sqrt{\pi}b^{5/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{\sqrt{3\pi}b^{5/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1280d}-\frac{\sqrt{\frac{\pi}{3}}b^{5/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (2*b^2*e^4*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) - (b^2*e^4*(c + d*
x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(15*d) + (3*b^2*e^4*(c + d*x)^5*Sqrt[a +
 b*ArcSinh[c + d*x]])/(100*d) - (4*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSi
nh[c + d*x])^(3/2))/(15*d) + (2*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^(3/2))/(15*d) - (b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(10*d) + (e^4*(c + d*x)^5*(a + b*ArcSin
h[c + d*x])^(5/2))/(5*d) + (15*b^(5/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) - (b^(5/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d) - (b^(5/2)*e^4*
E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(1280*d) + (3*b^(5/2)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(6400*d) - (15*b^(5/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d*E^(a/b)) + (b^(5/2)*e^4*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d*E^((3*a)/b)) +
 (b^(5/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(1280*d*E^((3*a)/b)) - (3*b^(5/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(6400*d*E^((5*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}} dx, x\right.}{2d}$$

$$= -\frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{10d} + \frac{e^4(c+dx)^5\left(a+b\,\text{s}\right.}{5d}$$

$$= \frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{100d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\,\text{s}\right.}{15d}$$

$$= -\frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d} + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{100d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}$$

Mathematica [A] time = 0.698151, size = 342, normalized size = 0.49

$$-\frac{e^4e^{-\frac{5a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-33750e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+27\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -(e^4*(a + b*ArcSinh[c + d*x])^(5/2)*(-33750*E^((6*a)/b)*Sqrt[-((a + b*ArcS
inh[c + d*x])/b)]*Gamma[7/2, a/b + ArcSinh[c + d*x]] + 27*Sqrt[5]*Sqrt[a/b
+ ArcSinh[c + d*x]]*Gamma[7/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 625*Sqrt[
3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (-3*(a + b*ArcSinh[c
```

```
 + d*x]))/b] + 33750*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, -(
(a + b*ArcSinh[c + d*x])/b)] + 625*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSin
h[c + d*x])/b)]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 27*Sqrt[5]*E^(
(10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (5*(a + b*ArcSinh[
c + d*x]))/b]))/(540000*d*E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(
3/2))
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int (dex+ce)^4 \left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^4 (b\,\text{arsinh}\left(dx+c\right)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.193 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=455

$$-\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(}{\sqrt{b}}\right.}{16384d}$$

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(2048*d) - (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcSinh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcSinh[c + d*x]])/(256*d) + (15*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*
(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*(c + d*x)^3*Sqrt[1 + (c +
 d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(5/2))/(4*
d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(16384*d) + (15*b^(5/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(512*d) - (15*b^(5/2)*e^3*Sqrt[P
i]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16384*d*E^((4*a)/b)) +
(15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(512*d*E^((2*a)/b))
```

Rubi [A] time = 1.51963, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5865, 12, 5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(}{\sqrt{b}}\right.}{16384d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(2048*d) - (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcSinh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcSinh[c + d*x]])/(256*d) + (15*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*
(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*(c + d*x)^3*Sqrt[1 + (c +
 d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(5/2))/(4*
d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(16384*d) + (15*b^(5/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(512*d) - (15*b^(5/2)*e^3*Sqrt[P
i]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16384*d*E^((4*a)/b)) +
(15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(512*d*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{4d} - \frac{\left(5be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\right.}{8d}$$

$$= -\frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d} + \frac{e^3(c+dx)^4\left(a\right.}{}$$

$$= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a\right.}{64d}$$

$$= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}}}{256d}$$

$$= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}}}{256d}$$

$$= -\frac{45b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx}}{256d}$$

Mathematica [A] time = 0.314397, size = 223, normalized size = 0.49

$$-\frac{e^3e^{-\frac{4a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{7}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-16\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+d\right.}{2048}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -(e^3*(a + b*ArcSinh[c + d*x])^(5/2)*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/
2, (-4*(a + b*ArcSinh[c + d*x]))/b] - 16*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Arc
```

```
Sinh[c + d*x]]*Gamma[7/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sq
rt[-((a + b*ArcSinh[c + d*x])/b)]*(-16*Sqrt[2]*Gamma[7/2, (2*(a + b*ArcSinh
[c + d*x]))/b] + E^((2*a)/b)*Gamma[7/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/
(2048*d*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(3/2))
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int (dex+ce)^3 \left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)

[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^3 (b\,\text{arsinh}\left(dx+c\right)+a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(5/2),x)

[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.194 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=394

$$-\frac{15\sqrt{\pi}b^{5/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{15\sqrt{\pi}b^{5/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}\cdot$$

```
[Out] (-5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
 d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(36*d) + (5*b*e^2*Sqrt[1 + (c + d*x)^
2]*(a + b*ArcSinh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*(c + d*x)^2*Sqrt[1 + (c
 + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (e^2*(c + d*x)^3*(a + b
*ArcSinh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) + (5*b^(5/2)*e^2*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(576*d) + (15*b
^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d*E^(a/
b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rubi [A] time = 1.24447, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}b^{5/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{15\sqrt{\pi}b^{5/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (-5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
 d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(36*d) + (5*b*e^2*Sqrt[1 + (c + d*x)^
2]*(a + b*ArcSinh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*(c + d*x)^2*Sqrt[1 + (c
 + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (e^2*(c + d*x)^3*(a + b
*ArcSinh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) + (5*b^(5/2)*e^2*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(576*d) + (15*b
^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d*E^(a/
b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(576*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
```

```
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$=\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{3d}-\frac{\left(5be^2\right)\text{Subst}\left(\int\frac{x^3\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}dx,\right.}{6d}$$

$$=-\frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{18d}+\frac{e^2(c+dx)^3\left(a+b\right.}{3}$$

$$=\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{36d}+\frac{5be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{9d}$$

$$=-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+d}}{36d}$$

$$=-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+d}}{36d}$$

$$=-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+d}}{36d}$$

$$=-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+d}}{36d}$$

$$=-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+d}}{36d}$$

$$=-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+d}}{36d}$$

Mathematica [A] time = 0.413291, size = 238, normalized size = 0.6

$$-\frac{e^2e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(81e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Ga}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -(e^2*(a + b*ArcSinh[c + d*x])^(5/2)*(81*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[
c + d*x])/b)]*Gamma[7/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcSi
nh[c + d*x]]*Gamma[7/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 81*E^((2*a)/b)*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (3*(a + b*A
rcSinh[c + d*x]))/b]))/(648*d*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2
))^(3/2))
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int (dex + ce)^2 \left(a + b\text{Arcsinh}\left(dx + c\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arsinh}\left(dx + c\right) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.195 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=262

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c-}}{32d}$$

```
[Out] (15*b^2*e*Sqrt[a + b*ArcSinh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqrt
[a + b*ArcSinh[c + d*x]])/(32*d) - (5*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(
a + b*ArcSinh[c + d*x])^(3/2))/(8*d) + (e*(a + b*ArcSinh[c + d*x])^(5/2))/(
4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) - (15*b^(5/2)*e
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rubi [A] time = 0.731318, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c-}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (15*b^2*e*Sqrt[a + b*ArcSinh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqrt
[a + b*ArcSinh[c + d*x]])/(32*d) - (5*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(
a + b*ArcSinh[c + d*x])^(3/2))/(8*d) + (e*(a + b*ArcSinh[c + d*x])^(5/2))/(
4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) - (15*b^(5/2)*e
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d} - \frac{(5be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\,dx,\right.}{4d}$$

$$= -\frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{8d} + \frac{e(c+dx)^2\left(a+b\,\text{s}\right.}{2}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\,\text{s}\right.}{8d}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\,\text{s}\right.}{8d}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\,\text{s}\right.}{8d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - 5b$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - 5b$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - 5b$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - 5b$$

Mathematica [A] time = 0.0757203, size = 126, normalized size = 0.48

$$\frac{ee^{-\frac{2a}{b}}\left(b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{7}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-b^3\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{7}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right.\right.}{32\sqrt{2}d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e*(-(b^3*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh
[c + d*x]))/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (
2*(a + b*ArcSinh[c + d*x]))/b]))/(32*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSi
nh[c + d*x]])
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int (dex+ce)\left(a+b\text{Arcsinh}\,(dx+c)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.196 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-5b\sqrt{\ldots}$$

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rubi [A] time = 0.402411, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-5b\sqrt{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d} - \frac{(5b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}} dx, x, c+dx\right)}{2d}$$

$$= -\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d} +$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} +$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} +$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} +$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} +$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} +$$

Mathematica [B] time = 1.32907, size = 458, normalized size = 2.56

$$8a^2e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)+\sqrt{b}\left(\sqrt{\pi}\left(4a^2-12ab+15b^2\right)\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.197 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[(a + b*ArcSinh[c + d*x])^(5/2)/(c + d*x), x]/e
```

Rubi [A] time = 0.105253, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(5/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{5/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{5/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.07201, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

Maple [A] time = 0.154, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

3.198 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=835

$$\frac{e^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}(c+dx)^5}{5d}+\frac{7b^2e^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}(c+dx)^5}{100d}-\frac{7be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c\right.}{50d}$$

```
[Out] (-1813*b^3*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(1125*d)
 + (119*b^3*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*
x]])/(1125*d) - (21*b^3*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/(1000*d) + (14*b^2*e^4*(c + d*x)*(a + b*ArcSinh[c + d*x])^
(3/2))/(15*d) - (7*b^2*e^4*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(45*
d) + (7*b^2*e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(100*d) - (28*b
*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d) + (14*b*e
^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d)
 - (7*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2
))/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(7/2))/(5*d) + (105*b
^(7/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(256
*d) - (119*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(18000*d) - (21*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*
Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000*d) + (21*b^(7/2
)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(64000*d) + (105*b^(7/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(256*d*E^(a/b)) - (119*b^(7/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(18000*d*E^((3*a)/b)) - (21*b^(7/2)*
e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000
*d*E^((3*a)/b)) + (21*b^(7/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(64000*d*E^((5*a)/b))
```

Rubi [A] time = 3.2885, antiderivative size = 835, normalized size of antiderivative = 1., number of steps used = 77, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$\frac{e^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}(c+dx)^5}{5d}+\frac{7b^2e^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}(c+dx)^5}{100d}-\frac{7be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c\right.}{50d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-1813*b^3*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(1125*d)
 + (119*b^3*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*
x]])/(1125*d) - (21*b^3*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/(1000*d) + (14*b^2*e^4*(c + d*x)*(a + b*ArcSinh[c + d*x])^
(3/2))/(15*d) - (7*b^2*e^4*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(45*
d) + (7*b^2*e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(100*d) - (28*b
*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d) + (14*b*e
^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d)
 - (7*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2
))/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(7/2))/(5*d) + (105*b
^(7/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(256
*d) - (119*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(18000*d) - (21*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*
```

```
Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000*d) + (21*b^(7/2
)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(64000*d) + (105*b^(7/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(256*d*E^(a/b)) - (119*b^(7/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(18000*d*E^((3*a)/b)) - (21*b^(7/2)*
e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000
*d*E^((3*a)/b)) + (21*b^(7/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(64000*d*E^((5*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^4x^4 \left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4 \text{Subst}\left(\int x^4 \left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{5d} - \frac{\left(7be^4\right) \text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}} dx,\right.}{10d}$$

$$= -\frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{50d} + \frac{e^4(c+dx)^5\left(a+b\right.}{5}$$

$$= \frac{7b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{100d} + \frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}\left(a\right.}{75d}$$

$$= -\frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1000d} - \frac{7b^2e^4(c+dx)^3\left(a+\right.}{4}$$

$$= \frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} - \frac{21b^3e^4(c+dx)^4\sqrt{1}}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c}}{1}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c}}{1}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c}}{1}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c}}{1}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c}}{1}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+dx)^2\sqrt{1+(c}}{1}$$

Mathematica [A] time = 0.501047, size = 343, normalized size = 0.41

$$\frac{be^4e^{-\frac{5a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(506250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{9}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+81\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b*e^4*(a + b*ArcSinh[c + d*x])^(5/2)*(506250*E^((6*a)/b)*Sqrt[-((a + b*Arc
Sinh[c + d*x])/b)]*Gamma[9/2, a/b + ArcSinh[c + d*x]] + 81*Sqrt[5]*Sqrt[a/b
 + ArcSinh[c + d*x]]*Gamma[9/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3125*Sqr
t[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (-3*(a + b*ArcSinh
[c + d*x]))/b] + 506250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2,
 -((a + b*ArcSinh[c + d*x])/b)] - 3125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 81*Sqrt[5]
*E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (5*(a + b*ArcS
inh[c + d*x]))/b]))/(8100000*d*E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^
2))^(3/2))
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (dex + ce)^4 (a + b\text{Arcsinh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^4 (b\,\text{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^4 (b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.199 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} \, dx$

Optimal. Leaf size=547

$$-\frac{105\sqrt{\pi}b^{7/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{131072d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{105\sqrt{\pi}b^{7/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sin}}{\sqrt{}}\right)}{131072d}$$

```
[Out] (1575*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(4096*d) - (105*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/(2048*d) - (525*b^2*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(2048
*d) - (105*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (3
5*b^2*e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (21*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(64*d) - (7*
b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(32
*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(7/2))/(32*d) + (e^3*(c + d*x)^4*(a +
 b*ArcSinh[c + d*x])^(7/2))/(4*d) - (105*b^(7/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*E
rf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(131072*d) + (105*b^(7/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(2048*d) + (105*b^(7/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(131072*d*E^((4*a)/b)) - (105*b^(7/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d*E^((2*a)/b))
```

Rubi [A] time = 2.14344, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{105\sqrt{\pi}b^{7/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{131072d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{105\sqrt{\pi}b^{7/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sin}}{\sqrt{}}\right)}{131072d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (1575*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(4096*d) - (105*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/(2048*d) - (525*b^2*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(2048
*d) - (105*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (3
5*b^2*e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (21*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(64*d) - (7*
b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(32
*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(7/2))/(32*d) + (e^3*(c + d*x)^4*(a +
 b*ArcSinh[c + d*x])^(7/2))/(4*d) - (105*b^(7/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*E
rf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(131072*d) + (105*b^(7/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(2048*d) + (105*b^(7/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(131072*d*E^((4*a)/b)) - (105*b^(7/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{4d} - \frac{\left(7be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}\right.}{8d}$$

$$= -\frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{32d} + \frac{e^3(c+dx)^4\left(a\right.}{}$$

$$= \frac{35b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{256d} + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}}{64d}$$

$$= -\frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{105b^2e^3(c+d}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)}{}$$

Mathematica [A] time = 0.313419, size = 225, normalized size = 0.41

$$-\frac{be^3e^{-\frac{4a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{9}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+32\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c\right.}{}$$

819

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -(b*e^3*(a + b*ArcSinh[c + d*x])^(5/2)*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[9/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 32*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b
+ ArcSinh[c + d*x]]*Gamma[9/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/
b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*(-32*Sqrt[2]*Gamma[9/2, (2*(a + b*Ar
cSinh[c + d*x]))/b] + E^((2*a)/b)*Gamma[9/2, (4*(a + b*ArcSinh[c + d*x]))/b
])))/(8192*d*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(3/2))
```

Maple [F] time = 0.196, size = 0, normalized size = 0.

$$\int (dex + ce)^3 (a + b\mathrm{Arcsinh}\,(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^3 (b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^3 (b \operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.200 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=481

$$-\frac{105\sqrt{\pi}b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}-\frac{105\sqrt{\pi}b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}$$

```
[Out] (175*b^3*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(54*d) - (
35*b^3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/
(216*d) - (35*b^2*e^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (3
5*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(108*d) + (7*b*e^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*(c + d
*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(18*d) + (e^2*(
c + d*x)^3*(a + b*ArcSinh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) + (35*b^(7/2)*
e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(3456*d) - (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(128*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rubi [A] time = 1.67977, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{105\sqrt{\pi}b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}-\frac{105\sqrt{\pi}b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (175*b^3*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(54*d) - (
35*b^3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/
(216*d) - (35*b^2*e^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (3
5*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(108*d) + (7*b*e^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*(c + d
*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(18*d) + (e^2*(
c + d*x)^3*(a + b*ArcSinh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) + (35*b^(7/2)*
e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(3456*d) - (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(128*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{3d} - \frac{\left(7be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}\right.}{6d}$$

$$= -\frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{18d} + \frac{e^2(c+dx)^3\left(a\right.}{}$$

$$= \frac{35b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{108d} + \frac{7be^2\sqrt{1+(c+dx)^2}\left(a+b\sin\right.}{9d}$$

$$= -\frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{216d} - \frac{35b^2e^2(c+dx)}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c}}{2}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c}}{2}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c}}{2}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c}}{2}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c}}{2}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c}}{2}$$

Mathematica [A] time = 0.328138, size = 238, normalized size = 0.49

$$\frac{be^2e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-243e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{9}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b*e^2*(a + b*ArcSinh[c + d*x])^(5/2)*(-243*E^((4*a)/b)*Sqrt[-((a + b*ArcSi
nh[c + d*x])/b)]*Gamma[9/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[9/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 243*E^((2*a)/
```

```
b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (3*(a +
 b*ArcSinh[c + d*x]))/b]))/(1944*d*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^
2/b^2))^(3/2))
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int (dex + ce)^2 (a + b\text{Arcsinh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b\,\text{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.201 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=305

$$-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e(c+dx)\sqrt{(c+dx)^2+1}\sqrt{a}}{128d}$$

```
[Out] (-105*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(
128*d) + (35*b^2*e*(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) + (35*b^2*e*(c +
d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (7*b*e*(c + d*x)*Sqrt[1 + (
c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(8*d) + (e*(a + b*ArcSinh[c + d
*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(7/2))/(2*d) -
(105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2*a)/b))
```

Rubi [A] time = 0.795843, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e(c+dx)\sqrt{(c+dx)^2+1}\sqrt{a}}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (-105*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(
128*d) + (35*b^2*e*(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) + (35*b^2*e*(c +
d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (7*b*e*(c + d*x)*Sqrt[1 + (
c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(8*d) + (e*(a + b*ArcSinh[c + d
*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(7/2))/(2*d) -
(105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{7/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{7/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{2d} - \frac{(7be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}dx,x,c\right.}{4d}$$

$$= -\frac{7be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{8d} + \frac{e(c+dx)^2\left(a+b\sinh\right.}{2d}$$

$$= \frac{35b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d} - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\,s\right.}{8d}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e(c+dx)^2\left(a+\right.}{32}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+b\sinh^{-1}(\right.}{64d}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+b\sinh^{-1}(\right.}{64d}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+b\sinh^{-1}(\right.}{64d}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+b\sinh^{-1}(\right.}{64d}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+b\sinh^{-1}(\right.}{64d}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+b\sinh^{-1}(\right.}{64d}$$

Mathematica [A] time = 0.0785757, size = 125, normalized size = 0.41

$$\frac{ee^{-\frac{2a}{b}}\left(b^4\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{9}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+b^4e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{9}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{64\sqrt{2}d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e*(b^4*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-2*(a + b*ArcSinh[c
 + d*x]))/b] + b^4*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (2*(
a + b*ArcSinh[c + d*x]))/b]))/(64*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[
c + d*x]])
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (dex + ce) (a + b\text{Arcsinh} (dx + c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh} (dx + c) + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)(b \operatorname{arsinh} (dx + c) + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.202 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=216

$$\frac{105\sqrt{\pi}b^{7/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}b^{7/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx}}{8d}$$

```
[Out] (-105*b^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d) + (35*b
^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (7*b*Sqrt[1 + (c + d*x
)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c +
d*x])^(7/2))/d + (105*b^(7/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rubi [A] time = 0.418628, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{105\sqrt{\pi}b^{7/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}b^{7/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (-105*b^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d) + (35*b
^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (7*b*Sqrt[1 + (c + d*x
)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c +
d*x])^(7/2))/d + (105*b^(7/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d*E^(a/b))
```


Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{d}-\frac{(7b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2d}$$

$$=-\frac{7b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{d}+\frac{(35$$

$$=\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{7b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d}+$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}$$

<u>Mathematica</u> [B] time = 4.53561, size = 698, normalized size = 3.23

$$16a^3e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\,\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)+6a\sqrt{b}\left(\sqrt{\pi}\left(4a^2-12ab+15b^2\right)\right($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] ((16*a^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[
c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 12*a^2*Sqrt[b
]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c
+ d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 6*a*Sqrt[b]*(4*Sq
rt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSin
h[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b +
15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + S
inh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*ArcS
inh[c + d*x]]*(2*b*(c + d*x)*(-10*a + 35*b*ArcSinh[c + d*x] + 4*b*ArcSinh[c
 + d*x]^3) + Sqrt[1 + (c + d*x)^2]*(-4*a^2 + 4*a*b*ArcSinh[c + d*x] - 7*b^2
*(15 + 4*ArcSinh[c + d*x]^2))) + (8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sq
```

```
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
 (-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(32*d)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(7/2),x)

[Out] int((a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(7/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.203 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable[(a + b*ArcSinh[c + d*x])^(7/2)/(c + d*x), x]/e
```

Rubi [A] time = 0.101734, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(7/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{7/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{7/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.0759, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

Maple [A] time = 0.153, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

3.204 $\int \frac{(ce+dex)^4}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=326

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{E}}{}$$

```
[Out] (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b
]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) - (e^4*Sqrt[3*Pi
]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((3
*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rubi [A] time = 0.636786, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5865, 12, 5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b
]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) - (e^4*Sqrt[3*Pi
]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((3
*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
```

x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}}-\frac{3\cosh(3x)}{16\sqrt{a+bx}}+\frac{\cosh(5x)}{16\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{16d}+\frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{8d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^4\,\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{32d}+$$
$$= \frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{16bd}+\frac{e^4\,\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx,x,\sqrt{a+\right.}{16bd}$$
$$= \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\frac{\sqrt{5}\right.}{32}$$

Mathematica [A] time = 0.359088, size = 320, normalized size = 0.98

$$\frac{e^4e^{-\frac{5a}{b}}\left(-10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}\right.\right.}{b}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(-10*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh
[c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcSinh[c + d*x]))/b] - 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqr
t[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]
+ 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*A
rcSinh[c + d*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Ga
mma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Ar
cSinh[c + d*x]])
```

Maple [F] time = 0.37, size = 0, normalized size = 0.

$$\int (dex+ce)^4\frac{1}{\sqrt{a+b\text{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int \frac{c^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{d^4x^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{4cd^3x^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{6c^2d^2x^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.205 $\int \frac{(ce+dex)^3}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\cdots}{}$$

```
[Out] -(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(
32*Sqrt[b]*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b))
```

Rubi [A] time = 0.455371, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5865, 12, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```


```
[Out] -(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(
32*Sqrt[b]*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{a+bx}}+\frac{\sinh(4x)}{8\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}$$

$$= -\frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \cdots$$

$$= -\frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{8bd} + \frac{e^3\,\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx, x, \sqrt{a+b}\cdots\right)}{8bd}$$

$$= -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{\cdots}}{\cdots}\right)}{32\sqrt{\cdots}}$$

Mathematica [A] time = 0.224919, size = 205, normalized size = 0.94

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)-2\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right.\right.}{32d\sqrt{a+b\sinh^{-1}(c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcSinh[c
+ d*x]]*(-2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b
)*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[a +
b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int (dex+ce)^3\frac{1}{\sqrt{a+b\text{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{3}\left(\int \frac{c^{3}}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx + \int \frac{d^{3}x^{3}}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx + \int \frac{3cd^{2}x^{2}}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx + \int \frac{3c^{2}dx}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^{3}}{\sqrt{b\,\mathrm{arsinh}\,(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.206 $\int \frac{(ce+dex)^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{\pi}e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{3}}e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{3}}e^2e^{-\frac{3a}{b}}\mathrm{E}}{}$$

```
[Out] -(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b
]*d) + (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(8*Sqrt[b]*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]
]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rubi [A] time = 0.455416, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5865, 12, 5669, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{3}}e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{3}}e^2e^{-\frac{3a}{b}}\mathrm{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b
]*d) + (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(8*Sqrt[b]*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]
]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}\\
&= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}\\
&= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= -\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}+\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}\\
&= \frac{e^2\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d}-\frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d}-\frac{e^2 \cdots}{\cdots}\\
&= \frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{4bd}-\frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh \cdots}\right)}{4bd}\\
&= -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}+\frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}-\frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right)}{8\sqrt{b}d}
\end{aligned}
$$

Mathematica [A] time = 0.232459, size = 217, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(c+\right.}{b}\right.\right.}{24d\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]
*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int (dex+ce)^2 \frac{1}{\sqrt{a+b\text{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx+\int \frac{d^2x^2}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx+\int \frac{2cdx}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*asinh(c + d*x)),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{\sqrt{b\operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.207 $\int \frac{ce+dex}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

```
[Out] -(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(4*Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rubi [A] time = 0.24291, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5669, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(4*Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{ce+dex}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}$$

$$= -\frac{e\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd} + \frac{e\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd}$$

$$= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

Mathematica [A] time = 0.0663705, size = 119, normalized size = 1.05

$$\frac{ee^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2}, -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right) + e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2}, \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{4\sqrt{2}d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c + d
*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*Ar
cSinh[c + d*x]))/b]))/(4*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]]
)
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (dex+ce)\frac{1}{\sqrt{a+b\mathrm{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex+ce}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{dx}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x/sqrt(a + b*asin
h(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{\sqrt{b \operatorname{arsinh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.208 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rubi [A] time = 0.12648, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5863, 5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+\right.}{2bd}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c\right.}}{bd}$$

$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Mathematica [A] time = 0.0639863, size = 111, normalized size = 1.21

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)}{2d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\text{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b \operatorname{asinh}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.209 $\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]]), x]/e
```

Rubi [A] time = 0.0979978, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcSinh[x]]), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{ex\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 0.0645254, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]), x]
```

Maple [A] time = 0.171, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\frac{1}{\sqrt{a+b\text{Arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\sqrt{b\operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\sqrt{a+b\operatorname{asinh}(c+dx)}+dx\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(c*sqrt(a + b*asinh(c + d*x)) + d*x*sqrt(a + b*asinh(c + d*x))),
 x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)\sqrt{b\operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

3.210 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{E}}{}$$

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^
(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(16*b^(3/2)*d) - (e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/b)) - (3*e^4*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*
d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rubi [A] time = 0.65091, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5865, 12, 5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```


```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^
(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(16*b^(3/2)*d) - (e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/b)) - (3*e^4*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*
d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\left(2e^4\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} - \frac{9\sinh(3x)}{16\sqrt{a+bx}} + \frac{5\sinh(5x)}{16\sqrt{a+bx}}\right)dx, x, \text{si}}{bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4bd} + \frac{\left(5e^4\right)\text{Subs}}{}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8bd} + \frac{e^4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{4b^2d} + \frac{e^4\,\text{S}}{}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+}}{}\right.}{16b^{3/2}d}$$

Mathematica [A] time = 0.634037, size = 490, normalized size = 1.34

$$\frac{e^4e^{-5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(2e^{\frac{6a}{b}+5\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2}, \frac{a}{b}+\sinh^{-1}(c+dx)\right) + \sqrt{5}e^{5\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-E^((5*a)/b) + 3*E^((5*a)/b + 2*ArcSinh[c + d*x]) - 2*E^((5*a)/b + 4*
ArcSinh[c + d*x]) - 2*E^((5*a)/b + 6*ArcSinh[c + d*x]) + 3*E^((5*a)/b + 8*A
rcSinh[c + d*x]) - E^((5*a)/b + 10*ArcSinh[c + d*x]) + 2*E^((6*a)/b + 5*Arc
Sinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]] + Sqrt[5]*E^(5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Ga
mma[1/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[3]*E^((2*a)/b + 5*ArcSin
h[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSi
nh[c + d*x]))/b] + 2*E^((4*a)/b + 5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh
[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - 3*Sqrt[3]*E^((8*
a)/b + 5*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a +
b*ArcSinh[c + d*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c + d*x]))*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(16*b*d
*E^(5*(a/b + ArcSinh[c + d*x]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int (dex+ce)^4 (a+b\text{Arcsinh}(dx+c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4 \left(\int \frac{c^4}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)} dx + \int \frac{d^4x^4}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asin}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt
(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^4}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.211 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\ \mathrm{I}$$

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
])/(4*b^(3/2)*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d*E^((2*a)/b)
)
```

Rubi [A] time = 0.453131, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5865, 12, 5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\sqrt{\frac{\pi}{2}}e^3e^{-\frac{2a}{b}}\ \mathrm{I}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
])/(4*b^(3/2)*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d*E^((2*a)/b)
)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
```

```
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\left(2e^3\right)\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2\sqrt{a+bx}}+\frac{\cosh(4x)}{2\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+\right.}{bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} + \frac{e^3\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd} - \frac{e^3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{b^2d} - \frac{e^3}{}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.}{2b^{3/2}d}$$

Mathematica [A] time = 0.444133, size = 253, normalized size = 0.97

$$e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] - E^((4*a)/b)*(-(Sqrt[2]*E^((2*a)/b)*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c + d
*x]))/b] - 2*Sinh[2*ArcSinh[c + d*x]] + Sinh[4*ArcSinh[c + d*x]])))/(4*b*d*
E^((4*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int (dex+ce)^{3}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{3}}{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3\left(\int \frac{c^3}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx+\int \frac{d^3x^3}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(3*c*d**2*x*
*2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.212 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$\frac{\sqrt{\pi}e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\mathrm{E}}{}$$

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*b^
(3/2)*d) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(4*b^(3/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rubi [A] time = 0.465021, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5865, 12, 5665, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\mathrm{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```


```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*b^
(3/2)*d) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(4*b^(3/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\left(2e^2\right)\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{3\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+\right.}{bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd} + \frac{\left(3e^2\right)\text{Subs}}{}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4bd} - \frac{e^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2b^2d} - \frac{e^2\,\text{S}}{}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b}}{}\right.}{4b^{3/2}d}$$

Mathematica [A] time = 0.358954, size = 327, normalized size = 1.28

$$e^2e^{-3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-e^{\frac{4a}{b}+3\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}e^{3\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b}{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c + d*x]) + E^((3*a)/b + 4*ArcS
inh[c + d*x]) - E^((3*a)/b + 6*ArcSinh[c + d*x]) - E^((4*a)/b + 3*ArcSinh[c
 + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] +
Sqrt[3]*E^(3*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/
2, (-3*(a + b*ArcSinh[c + d*x]))/b] - E^((2*a)/b + 3*ArcSinh[c + d*x])*Sqrt
[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(4*b*d*E^(3*(a/b + ArcSinh[c + d*x
]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int (dex+ce)^2\,(a+b\text{Arcsinh}\,(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^2\left(\int \frac{c^2}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx+\int \frac{d^2x^2}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(2*c*d*x/(a*
sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.213 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{(c+dx)^2+1}(c+dx)}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) +
 (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rubi [A] time = 0.231014, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{(c+dx)^2+1}(c+dx)}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) +
 (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{bd} + \frac{e\,\text{Subst}\left(\int \frac{e^{2}}{\sqrt{a}}\right.}{}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{b^2d} + \frac{(2}{}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{s}}}{}\right.}{b^{3/2}d}$$

Mathematica [A] time = 0.112226, size = 147, normalized size = 0.99

$$\frac{ee^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right.}{2bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSi
nh[c + d*x]))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1
/2, (2*(a + b*ArcSinh[c + d*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c + d*x]
]))/(2*b*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (dex + ce) (a + b\text{Arcsinh} (dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh} (dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a\sqrt{a + b \operatorname{asinh} (c + dx)} + b\sqrt{a + b \operatorname{asinh} (c + dx)} \operatorname{asinh} (c + dx)} dx + \int \frac{dx}{a\sqrt{a + b \operatorname{asinh} (c + dx)} + b\sqrt{a + b \operatorname{asinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(c/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*
asinh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a
 + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.214 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rubi [A] time = 0.254991, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5863, 5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{b^2d} + \frac{2\,\text{Subs}}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+a}}{\sqrt{b}}\right.}{b^{3/2}d}$$

Mathematica [A] time = 0.114311, size = 155, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e^{\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\text{Gar}\right.}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(dx + c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\text{arsinh}\,(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\,\text{asinh}\,(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

3.215 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2)), x]/e
```

Rubi [A] time = 0.10799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(3/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.0713067, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.176, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ac\sqrt{a+b\operatorname{asinh}(c+dx)}+adx\sqrt{a+b\operatorname{asinh}(c+dx)}+bc\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+bdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/(a*c*sqrt(a + b*asinh(c + d*x)) + a*d*x*sqrt(a + b*asinh(c + d*x
)) + b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b*d*x*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)), x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

3.216 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=437

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{\sqrt{\pi}e^4e^{-}}{}$$

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (20*
e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^4*E^(a/b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d) - (3*e^4*E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8
*b^(5/2)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) - (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^((3*a)/b)) + (5*
e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(24*b^
(5/2)*d*E^((5*a)/b))
```

Rubi [A] time = 1.55474, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{\sqrt{\pi}e^4e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (20*
e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^4*E^(a/b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d) - (3*e^4*E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8
*b^(5/2)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) - (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^((3*a)/b)) + (5*
e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(24*b^
(5/2)*d*E^((5*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right.}{3bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(}}$$

Mathematica [A] time = 3.79014, size = 551, normalized size = 1.26

$$\frac{e^4\left(-4be^{-\frac{a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+e^{-\sinh^{-1}(c+dx)}\left(-4e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\right(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-2*E^ArcSinh[c + d*x]*(2*a + b + 2*b*ArcSinh[c + d*x]) + (4*a - 2*b +
 4*b*ArcSinh[c + d*x] - 4*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c +
 d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSi
nh[c + d*x] + (-(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcSinh[c
+ d*x])) - 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
```

```
5*(a + b*ArcSinh[c + d*x]))/b])/E^((5*a)/b) + (3*E^(3*(a/b + ArcSinh[c + d*
x]))*(6*a + b + 6*b*ArcSinh[c + d*x]) + 18*Sqrt[3]*b*(-((a + b*ArcSinh[c +
d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b])/E^((3*a)/b) -
(4*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/E^(a/b) + (3*(-6*a + b - 6*b*ArcSinh[c + d*x] + 6*Sqrt[3]*E^(3*(
a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*
x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/E^(3*ArcSinh[c + d*x]) + (
10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x])
)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a +
b*ArcSinh[c + d*x]))/b])/E^(5*ArcSinh[c + d*x])))/(48*b^2*d*(a + b*ArcSinh[
c + d*x])^(3/2))
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int (dex+ce)^4 (a+b\mathrm{Arcsinh}(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^4\left(\int \frac{c^4}{a^2\sqrt{a+b\,\mathrm{asinh}(c+dx)}+2ab\sqrt{a+b\,\mathrm{asinh}(c+dx)}\,\mathrm{asinh}(c+dx)+b^2\sqrt{a+b\,\mathrm{asinh}(c+dx)}\,\mathrm{asinh}^2(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2*sqrt(
a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b
**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x
/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.217 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{-}}{}$$

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (16*e^3
*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e^3*E^((4*a)/b)*S
qrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (e^3
*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(3*b^(5/2)*d*E^((4*a)/b)) - (e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rubi [A] time = 1.09579, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (16*e^3
*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e^3*E^((4*a)/b)*S
qrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (e^3
*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(3*b^(5/2)*d*E^((4*a)/b)) - (e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{\left(2e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{bd} + \cdots$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

Mathematica [A] time = 1.74964, size = 390, normalized size = 1.2

$$\frac{e^3e^{-4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-8be^{4\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+4\sqrt{2}be^{\frac{2a}{b}+4\sinh^{-1}(c+dx)}\left(-\frac{a+b\cdots}{}\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-8*b*E^(4*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gam
ma[1/2, (-4*(a + b*ArcSinh[c + d*x]))/b] + 4*Sqrt[2]*b*E^((2*a)/b + 4*ArcSi
nh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*Ar
cSinh[c + d*x]))/b] + (E^((4*a)/b)*(-((-1 + E^(2*ArcSinh[c + d*x]))^2*(b*(-
1 + E^(4*ArcSinh[c + d*x])) + 8*a*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSin
```

```
h[c + d*x])) + 8*b*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x]))*Ar
cSinh[c + d*x])) - 8*Sqrt[2]*E^((2*a)/b + 4*ArcSinh[c + d*x])*Sqrt[a/b + Ar
cSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d
*x]))/b] + 16*E^(4*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(
a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]))/2))/(1
2*b^2*d*E^(4*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int (dex + ce)^3 (a + b\text{Arcsinh}(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^3}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^3 \left(\int \frac{c^3}{a^2\sqrt{a + b \operatorname{asinh}(c + dx)} + 2ab\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx) + b^2\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2(c + dx)} d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
```

```
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^3}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.218 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}}{}$$

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*e^2*
(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) + (e^2*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*
d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d
*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rubi [A] time = 0.959963, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205, 5657}

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*e^2*
(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) + (e^2*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*
d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d
*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
```

```
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c}$$

Mathematica [A] time = 1.44945, size = 389, normalized size = 1.21

$$\frac{e^2e^{-3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-6\sqrt{3}be^{3\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+2be^{\frac{2a}{b}+3\sinh^{-1}(c+dx)}\left(-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a +
b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
```

```
b*ArcSinh[c + d*x]))/b] + 2*b*E^((2*a)/b + 3*ArcSinh[c + d*x])*(-((a + b*Ar
cSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - E^((3
*a)/b)*((-1 + E^(2*ArcSinh[c + d*x]))*(b*(-1 + E^(4*ArcSinh[c + d*x])) + a*
(6 + 4*E^(2*ArcSinh[c + d*x]) + 6*E^(4*ArcSinh[c + d*x])) + 2*b*(3 + 2*E^(2
*ArcSinh[c + d*x]) + 3*E^(4*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSin
h[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/(12*b^2*d*E^(3*(a
/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \left(dex+ce\right)^{2} \left(a+b\mathrm{Arcsinh}\left(dx+c\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)

[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dex+ce\right)^{2}}{\left(b\,\mathrm{arsinh}\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{2}\left(\int \frac{c^{2}}{a^{2}\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}+2ab\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}\,\mathrm{asinh}\left(c+dx\right)+b^{2}\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}\,\mathrm{asinh}^{2}\left(c+dx\right)}\,dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*asinh(c + d*x)) +
2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.219 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{\cdots}{3b^2d\sqrt{a+b\cdots}}$$

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2
)) - (4*e)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (8*e*(c + d*x)^2)/(3*b^
2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e*Sqrt[2*Pi]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/
b))
```

Rubi [A] time = 0.54193, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3308, 2180, 2204, 2205, 5675}

$$-\frac{2\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{\cdots}{3b^2d\sqrt{a+b\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```


```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2
)) - (4*e)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (8*e*(c + d*x)^2)/(3*b^
2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e*Sqrt[2*Pi]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/
b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\mathrm{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{3bd} + \ldots (4$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+}$$

Mathematica [A] time = 0.673967, size = 227, normalized size = 1.09

$$ee^{-2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(e^{\frac{2a}{b}}\left(4\sqrt{2}e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\mathrm{Gamma}\left(\frac{1}{2}, \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\cdot\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e*(-4*Sqrt[2]*b*E^(2*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/
2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*
a*E^(4*ArcSinh[c + d*x]) - b*E^(4*ArcSinh[c + d*x]) - 4*b*(1 + E^(4*ArcSinh
[c + d*x]))*ArcSinh[c + d*x] + 4*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*Arc
Sinh[c + d*x]))/b])))/(6*b^2*d*E^(2*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSin
```

```
h[c + d*x])^(3/2))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int (dex+ce)\,(a+b\mathrm{Arcsinh}\,(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex+ce}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e\left(\int \frac{c}{a^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+b^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2\,(c+dx)}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.220 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+}}{3bd\left(a+b\sinh^{-1}(c-\right.}$$

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rubi [A] time = 0.284528, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5657, 3307, 2180, 2205, 2204}

$$\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+}}{3bd\left(a+b\sinh^{-1}(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh \cdots}}\right.}{3b^2 \cdots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\cdots\right)}{\sqrt{x}}\right.}{\cdots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}\cdots}{\sqrt{x}}\right.}{\cdots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,d\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh\cdots}}{\cdots}\right.}{3b^{5/2}d}$$

Mathematica [A] time = 0.297075, size = 207, normalized size = 1.31

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-2be^{\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-2e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

3.221 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])^(5/2)), x]/e
```

Rubi [A] time = 0.108517, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(5/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 0.0760313, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2c\sqrt{a+b\operatorname{asinh}(c+dx)}+a^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}+2abc\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+2abdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2c\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(a**2*c*sqrt(a + b*asinh(c + d*x)) + a**2*d*x*sqrt(a + b*asinh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*c*sqrt(a + b*asinh(c + d*x))
*asinh(c + d*x)**2 + b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2)
, x)/e
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

3.222 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}-\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}}{}$$

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (
4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e^4*(c +
d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (40*
e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b
^(7/2)*d) + (9*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(20*b^(7/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) - (9*e^
4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(20*b^(7
/2)*d*E^((3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(12*b^(7/2)*d*E^((5*a)/b))
```

Rubi [A] time = 1.46746, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5865, 12, 5667, 5774, 5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}-\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (
4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e^4*(c +
d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (40*
e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b
^(7/2)*d) + (9*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(20*b^(7/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) - (9*e^
4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(20*b^(7
/2)*d*E^((3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(12*b^(7/2)*d*E^((5*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{5bd} + \cdots$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^4(c+dx}{3b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^4(c+dx}{3b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^4(c+dx}{3b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^4(c+dx}{3b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^4(c+dx}{3b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^4(c+dx}{3b^2d\left(a+b\sinh^{-1}(\right.}$$

Mathematica [A] time = 2.58505, size = 701, normalized size = 1.32

$$\frac{e^4\left(e^{-\sinh^{-1}(c+dx)}\left(8e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-8a^2-4\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^4*(-6*b^2*E^ArcSinh[c + d*x] - 3*b^2*E^(5*ArcSinh[c + d*x]) + (-8*a^2 +
4*a*b - 6*b^2 - 4*(4*a - b)*b*ArcSinh[c + d*x] - 8*b^2*ArcSinh[c + d*x]^2 +
 8*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c
 + d*x])^2*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSinh[c + d*x] - (10*(a
+ b*ArcSinh[c + d*x])*(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcS
inh[c + d*x]) + 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-5*(a + b*ArcSinh[c + d*x]))/b]))/E^((5*a)/b) + 9*(b^2*E^(3*ArcSinh[c +
 d*x]) + (2*(a + b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a +
 b + 6*b*ArcSinh[c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3
```

```
/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) - (4*(a + b*
ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*
x]) + 2*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh
[c + d*x])/b)]))/E^(a/b) + (9*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b +
6*b*ArcSinh[c + d*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b +
ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c +
 d*x]))/b]))))/E^(3*ArcSinh[c + d*x]) - (3*b^2 + 10*(a + b*ArcSinh[c + d*x])
*(10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x
]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a
+ b*ArcSinh[c + d*x]))/b]))/E^(5*ArcSinh[c + d*x])))/(240*b^3*d*(a + b*ArcS
inh[c + d*x])^(5/2))
```

Maple [F] time = 0.298, size = 0, normalized size = 0.

$$\int (dex+ce)^4 (a+b\text{Arcsinh}(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b\,\text{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^4}{(b \operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.223 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=420

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2}}{}$$

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16
*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^3*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (128*e
^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (16*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(15*b^(7/2)*d) - (4*e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) - (4*e^3*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)
*d*E^((2*a)/b))
```

Rubi [A] time = 1.09819, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5865, 12, 5667, 5774, 5665, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16
*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^3*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (128*e
^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (16*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(15*b^(7/2)*d) - (4*e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) - (4*e^3*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)
*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{\left(6e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{5bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^3(c+}{15b^2d\left(a+b\sinh\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^3(c+}{15b^2d\left(a+b\sinh\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^3(c+}{15b^2d\left(a+b\sinh\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^3(c+}{15b^2d\left(a+b\sinh\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^3(c+}{15b^2d\left(a+b\sinh\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^3(c+}{15b^2d\left(a+b\sinh\right.}$$

Mathematica [A] time = 2.06066, size = 429, normalized size = 1.02

$$e^3\left(4\left(a+b\sinh^{-1}(c+dx)\right)\left(4\sqrt{2}be^{-\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*(4*(a + b*ArcSinh[c + d*x])*((-4*a)/E^(2*ArcSinh[c + d*x]) + b/E^(2*Ar
cSinh[c + d*x]) - (4*b*ArcSinh[c + d*x])/E^(2*ArcSinh[c + d*x]) + E^(2*ArcS
inh[c + d*x])*(4*a + b + 4*b*ArcSinh[c + d*x]) + (4*Sqrt[2]*b*(-((a + b*Arc
Sinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2
*a)/b) + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[
c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) - 4*(a + b*ArcSinh[c
+ d*x])*((-8*a)/E^(4*ArcSinh[c + d*x]) + (b*(1 - 8*ArcSinh[c + d*x]))/E^(4*
ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x])*(8*a + b + 8*b*ArcSinh[c + d*x])
 + (16*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcSin
```

```
h[c + d*x]))/b])/E^((4*a)/b) + 16*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*
(a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]) + 6*b^
2*Sinh[2*ArcSinh[c + d*x]] - 3*b^2*Sinh[4*ArcSinh[c + d*x]]))/(60*b^3*d*(a
+ b*ArcSinh[c + d*x])^(5/2))
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int (dex+ce)^{3} (a+b\mathrm{Arcsinh}\,(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^{3}}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.224 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Er}}{}$$

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*e
^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^2*Sqrt[1 +
 (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (24*e^2*(c + d*x)^
2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^2*E^(a
/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) - (3
*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(5*b^(7/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]])/(15*b^(7/2)*d*E^(a/b)) + (3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d*E^((3*a)/b))
```

Rubi [A] time = 1.1098, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {5865, 12, 5667, 5774, 5665, 3308, 2180, 2204, 2205, 5655, 5779}

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Er}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*e
^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^2*Sqrt[1 +
 (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (24*e^2*(c + d*x)^
2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^2*E^(a
/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) - (3
*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(5*b^(7/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]])/(15*b^(7/2)*d*E^(a/b)) + (3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d*E^((3*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{5bd} + \cdots$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^2(c+dx\cdots}{5b^2d\left(a+b\sinh^{-1}(\cdots\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^2(c+dx\cdots}{5b^2d\left(a+b\sinh^{-1}(\cdots\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^2(c+dx\cdots}{5b^2d\left(a+b\sinh^{-1}(\cdots\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^2(c+dx\cdots}{5b^2d\left(a+b\sinh^{-1}(\cdots\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^2(c+dx\cdots}{5b^2d\left(a+b\sinh^{-1}(\cdots\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^2(c+dx\cdots}{5b^2d\left(a+b\sinh^{-1}(\cdots\right.}$$

Mathematica [A] time = 1.44251, size = 474, normalized size = 1.16

$$\frac{e^2\left(e^{-\sinh^{-1}(c+dx)}\left(-4e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+4a^2+\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(3*b^2*E^ArcSinh[c + d*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*Arc
Sinh[c + d*x] + 4*b^2*ArcSinh[c + d*x]^2 - 4*E^(a/b + ArcSinh[c + d*x])*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2, a/b + ArcSi
```

```
nh[c + d*x]])/E^ArcSinh[c + d*x] - 3*(b^2*E^(3*ArcSinh[c + d*x]) + (2*(a +
b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a + b + 6*b*ArcSinh[
c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) + (2*(a + b*ArcSinh[c + d*x])
*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a +
 b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/
E^(a/b) - (3*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b + 6*b*ArcSinh[c + d
*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]
*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/E^(
3*ArcSinh[c + d*x])))/(60*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int (dex + ce)^2 (a + b\mathrm{Arcsinh}\,(dx + c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex + ce)^2}{(b\,\mathrm{arsinh}\,(dx + c) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.225 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=252

$$\frac{8\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{32e\sqrt{(}}{15b^3d\sqrt{}}$$

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2
)) - (4*e)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*e*(c + d*x)^2)/(1
5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e*(c + d*x)*Sqrt[1 + (c + d*x
)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (8*e*E^((2*a)/b)*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (8*e
*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/
2)*d*E^((2*a)/b))
```

Rubi [A] time = 0.562214, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5667, 5774, 5665, 3307, 2180, 2204, 2205, 5675}

$$\frac{8\sqrt{2\pi}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{32e\sqrt{(}}{15b^3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```


```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2
)) - (4*e)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*e*(c + d*x)^2)/(1
5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e*(c + d*x)*Sqrt[1 + (c + d*x
)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (8*e*E^((2*a)/b)*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (8*e
*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/
2)*d*E^((2*a)/b))
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
```

```
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{5bd} +$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e(c+}{15b^2d\left(a+b\sin\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e(c+}{15b^2d\left(a+b\sin\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e(c+}{15b^2d\left(a+b\sin\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e(c+}{15b^2d\left(a+b\sin\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e(c+}{15b^2d\left(a+b\sin\right.}$$

Mathematica [A] time = 0.945677, size = 235, normalized size = 0.93

$$-\frac{e\left(\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{-\frac{2a}{b}}\left(8\sqrt{2}b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+2e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(4a\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -(e*((a + b*ArcSinh[c + d*x])*((2*E^(2*(a/b + ArcSinh[c + d*x]))*(4*a + b +
 4*b*ArcSinh[c + d*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*
Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2*a)/b) + (-8*a + 2*b - 8*
b*ArcSinh[c + d*x] + 8*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + Ar
cSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d
*x]))/b])/E^(2*ArcSinh[c + d*x])) + 3*b^2*Sinh[2*ArcSinh[c + d*x]]))/(15*b^
3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (dex+ce)\left(a+b\text{Arcsinh}\,(dx+c)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.226 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{(c+d}}{15b^3d\sqrt{a+b\,\text{si}}}$$

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rubi [A] time = 0.465366, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{(c+d}}{15b^3d\sqrt{a+b\,\text{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{5bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{4\,\text{Subst}\left(\int \frac{}{(a+b}\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c}}{15b^3d\sqrt{a+b\sin}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c}}{15b^3d\sqrt{a+b\sin}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c}}{15b^3d\sqrt{a+b\sin}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c}}{15b^3d\sqrt{a+b\sin}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1+(c}}{15b^3d\sqrt{a+b\sin}}$$

Mathematica [A] time = 0.189301, size = 238, normalized size = 1.22

$$\frac{8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-4e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(2b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

3.227 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=28

$$\frac{\text{Unintegrable}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}},x\right)}{e}$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*ArcSinh[c + d*x])^(7/2)), x]/e
```

Rubi [A] time = 0.117297, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(7/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.0811025, size = 0, normalized size = 0.

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)), x]
```

Maple [A] time = 0.167, size = 0, normalized size = 0.

$$\int \frac{1}{dex+ce}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

3.228 $\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)dx$

Optimal. Leaf size=298

$$-\frac{14be^{7/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{135d\sqrt{(c+dx)^2+1}}+\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de}+28be^2\sqrt{(c}$$

```
[Out] (28*b*e^2*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(405*d) - (4*b*(e*(c +
 d*x))^(7/2)*Sqrt[1 + (c + d*x)^2])/(81*d) - (28*b*e^3*Sqrt[e*(c + d*x)]*Sq
rt[1 + (c + d*x)^2])/(135*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(9/2)*(a + b*
ArcSinh[c + d*x]))/(9*d*e) + (28*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x
)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/
(135*d*Sqrt[1 + (c + d*x)^2]) - (14*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c +
d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2
])/(135*d*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.332207, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 5661, 321, 329, 305, 220, 1196}

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de}+\frac{28be^2\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}}{405d}-\frac{28be^3\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{135d(c+dx+1)}-\frac{14be^{7/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```


```
[Out] (28*b*e^2*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(405*d) - (4*b*(e*(c +
 d*x))^(7/2)*Sqrt[1 + (c + d*x)^2])/(81*d) - (28*b*e^3*Sqrt[e*(c + d*x)]*Sq
rt[1 + (c + d*x)^2])/(135*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(9/2)*(a + b*
ArcSinh[c + d*x]))/(9*d*e) + (28*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x
)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/
(135*d*Sqrt[1 + (c + d*x)^2]) - (14*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c +
d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2
])/(135*d*Sqrt[1 + (c + d*x)^2])
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{1+x^2}}dx, x, c+dx\right)}{9de}$$

$$= -\frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de} + \frac{(1}{}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+}{}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+}{}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+}{}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} - \frac{28be^3\sqrt{}}{}$$

Mathematica [C] time = 0.208894, size = 113, normalized size = 0.38

$$\frac{2(e(c+dx))^{7/2}\left(-14b\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)+45a(c+dx)^3-10b\sqrt{(c+dx)^2+1}(c+dx)^2+14b\sqrt{}\right.}{405d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(45*a*(c + d*x)^3 + 14*b*Sqrt[1 + (c + d*x)^2] - 10*
b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 45*b*(c + d*x)^3*ArcSinh[c + d*x] - 1
4*b*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(405*d*(c + d*x)^2)
```

Maple [C] time = 0.045, size = 238, normalized size = 0.8

$$2\frac{1}{de}\left(1/9\,(dex+ce)^{9/2}a+b\left(1/9\,(dex+ce)^{9/2}\operatorname{Arcsinh}\left(\frac{dex+ce}{e}\right)-2/9\frac{1}{e}\left(1/9\,e^2\,(dex+ce)^{7/2}\sqrt{\frac{(dex+ce)^2}{e^2}+1}-\frac{7\,e^4}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/9*(d*e*x+c*e)^(9/2)*a+b*(1/9*(d*e*x+c*e)^(9/2)*arcsinh(1/e*(d*e*x+
c*e))-2/9/e*(1/9*e^2*(d*e*x+c*e)^(7/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-7/45*e
^4*(d*e*x+c*e)^(3/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)+7/15*I*e^5/(I/e)^(1/2)*(
1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)^2+1)^
(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/
2)*(I/e)^(1/2),I)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ad^3e^3x^3+3\,acd^2e^3x^2+3\,ac^2de^3x+ac^3e^3+\left(bd^3e^3x^3+3\,bcd^2e^3x^2+3\,bc^2de^3x+bc^3e^3\right)\operatorname{arsinh}(dx+c)\right)\sqrt{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^3*e^3*x^3 + 3*a*c*d^2*e^3*x^2 + 3*a*c^2*d*e^3*x + a*c^3*e^3 +
 (b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + b*c^3*e^3)*arcsinh(
```

```
d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{7}{2}}(b \operatorname{arsinh}(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a), x)
```

3.229 $\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)dx$

Optimal. Leaf size=177

$$-\frac{10be^{5/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{147d\sqrt{(c+dx)^2+1}}+\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de}+\frac{20be^2\sqrt{(c}{}$$

```
[Out] (20*b*e^2*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(147*d) - (4*b*(e*(c + d
*x))^(5/2)*Sqrt[1 + (c + d*x)^2])/(49*d) + (2*(e*(c + d*x))^(7/2)*(a + b*Ar
cSinh[c + d*x]))/(7*d*e) - (10*b*e^(5/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x)^
2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(1
47*d*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.159405, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 5661, 321, 329, 220}

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de}+\frac{20be^2\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{147d}-\frac{10be^{5/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\right.\right.}{147d\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (20*b*e^2*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(147*d) - (4*b*(e*(c + d
*x))^(5/2)*Sqrt[1 + (c + d*x)^2])/(49*d) + (2*(e*(c + d*x))^(7/2)*(a + b*Ar
cSinh[c + d*x]))/(7*d*e) - (10*b*e^(5/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x)^
2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(1
47*d*Sqrt[1 + (c + d*x)^2])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}}{\sqrt{1+x^2}}dx, x, c+dx\right)}{7de}$$

$$= -\frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de} + \frac{(1}{}$$

$$= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+d}{}$$

$$= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+d}{}$$

$$= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+d}{}$$

Mathematica [C] time = 0.179269, size = 113, normalized size = 0.64

$$\frac{2(e(c+dx))^{5/2}\left(-10b\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)+21a(c+dx)^3-6b\sqrt{(c+dx)^2+1}(c+dx)^2+10b\sqrt{(c+}\right.}{147d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(21*a*(c + d*x)^3 + 10*b*Sqrt[1 + (c + d*x)^2] - 6*b
*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 21*b*(c + d*x)^3*ArcSinh[c + d*x] - 10
*b*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2]))/(147*d*(c + d*x)^2)
```

Maple [C] time = 0.01, size = 212, normalized size = 1.2

$$2\frac{1}{de}\left(1/7\left(dex+ce\right)^{7/2}a+b\left(1/7\left(dex+ce\right)^{7/2}\text{Arcsinh}\left(\frac{dex+ce}{e}\right)-2/7\frac{1}{e}\left(1/7e^2\left(dex+ce\right)^{5/2}\sqrt{\frac{(dex+ce)^2}{e^2}+1}-\frac{5e^4\sqrt{a}}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arcsinh(1/e*(d*e*x+
c*e))-2/7/e*(1/7*e^2*(d*e*x+c*e)^(5/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-5/21*e
^4*(d*e*x+c*e)^(1/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)+5/21*e^4/(I/e)^(1/2)*(1-
I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)^2+1)^(1
/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ad^2e^2x^2 + 2\,acde^2x + ac^2e^2 + \left(bd^2e^2x^2 + 2\,bcde^2x + bc^2e^2\right)\text{arsinh}\left(dx + c\right)\right)\sqrt{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^2*e^2*x^2 + 2*a*c*d*e^2*x + a*c^2*e^2 + (b*d^2*e^2*x^2 + 2*b*
c*d*e^2*x + b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{5}{2}}(b\,\text{arsinh}\left(dx + c\right) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a), x)
```

3.230 $\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=261

$$\frac{6be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}+\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de}-\frac{12be^{3/2}(c+dx+}{}$$

```
[Out] (-4*b*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(25*d) + (12*b*e*Sqrt[e*(c
 + d*x)]*Sqrt[1 + (c + d*x)^2])/(25*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(5/
2)*(a + b*ArcSinh[c + d*x]))/(5*d*e) - (12*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
+ (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e
]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2]) + (6*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.24261, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 5661, 321, 329, 305, 220, 1196}

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de}+\frac{6be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}-\frac{12be^{3/2}(c+dx+1)\sqrt{\frac{(c+}{(c+}}}{25d\sqrt{(}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(25*d) + (12*b*e*Sqrt[e*(c
 + d*x)]*Sqrt[1 + (c + d*x)^2])/(25*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(5/
2)*(a + b*ArcSinh[c + d*x]))/(5*d*e) - (12*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
+ (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e
]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2]) + (6*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{1+x^2}}dx, x, c+dx\right)}{5de}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de} +$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de} +$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de} +$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{12be\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{25d(1+c+dx)} + \frac{2(e(c}{}$$

Mathematica [C] time = 0.0465343, size = 87, normalized size = 0.33

$$\frac{2(e(c+dx))^{3/2}\left(2b\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)+5ac+5adx-2b\sqrt{(c+dx)^2+1}+5bc\sinh^{-1}(c+dx)\right.}{25d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*a*c + 5*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 5*b*c
*ArcSinh[c + d*x] + 5*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/2, 3
/4, 7/4, -(c + d*x)^2]))/(25*d)
```

Maple [C] time = 0.01, size = 205, normalized size = 0.8

$$2\frac{1}{de}\left(1/5\,(dex+ce)^{5/2}\,a+b\left(1/5\,(dex+ce)^{5/2}\,\mathrm{Arcsinh}\left(\frac{dex+ce}{e}\right)-2/5\,\frac{1}{e}\left(1/5\,e^2\,(dex+ce)^{3/2}\,\sqrt{\frac{(dex+ce)^2}{e^2}}+1-3/5\,ie^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arcsinh(1/e*(d*e*x+
c*e))-2/5/e*(1/5*e^2*(d*e*x+c*e)^(3/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-3/5*I*
e^3/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*
(d*e*x+c*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-Elliptic
E((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(adex+ace+(bdex+bce)\,\mathrm{arsinh}\,(dx+c)\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arcsinh(d*x + c))*sqrt(d*e*x
+ c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\,(c+dx))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}}(b \operatorname{arsinh}(dx + c) + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a), x)
```

3.231 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=142

$$\frac{2b\sqrt{e}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{9d\sqrt{(c+dx)^2+1}}+\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{9d}$$

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(9*d) + (2*(e*(c + d*x))^(3/
2)*(a + b*ArcSinh[c + d*x]))/(3*d*e) + (2*b*Sqrt[e]*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(9*d*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.12639, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 5661, 321, 329, 220}

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{9d}+\frac{2b\sqrt{e}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|\frac{1}{2}\right)}{9d\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(9*d) + (2*(e*(c + d*x))^(3/
2)*(a + b*ArcSinh[c + d*x]))/(3*d*e) + (2*b*Sqrt[e]*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(9*d*Sqrt[1 + (c + d*x)^2])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{1+x^2}}dx,x,c+dx\right)}{3de}$$
$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de} + \frac{(2\ldots}{\ldots}$$
$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de} + \frac{(4\ldots}{\ldots}$$
$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de} + \frac{2b\ldots}{\ldots}$$

Mathematica [C] time = 0.0310172, size = 87, normalized size = 0.61

$$\frac{2\sqrt{e(c+dx)}\left(2b\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)+3ac+3adx-2b\sqrt{(c+dx)^2+1}+3bc\sinh^{-1}(c+dx)+3\ldots}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(3*a*c + 3*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 3*b*c*A
rcSinh[c + d*x] + 3*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/4, 1/2
, 5/4, -(c + d*x)^2]))/(9*d)
```

Maple [C] time = 0.01, size = 179, normalized size = 1.3

$$2\frac{1}{de}\left(1/3\,(dex+ce)^{3/2}a+b\left(1/3\,(dex+ce)^{3/2}\text{Arcsinh}\left(\frac{dex+ce}{e}\right)-2/3\frac{1}{e}\left(1/3\,e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}-1/3\,e^2\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arcsinh(1/e*(d*e*x+
c*e))-2/3/e*(1/3*e^2*(d*e*x+c*e)^(1/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-1/3*e^
```

```
2/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d
*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\,(c+dx)}\,(a+b\,\text{asinh}\,(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))*(d*e*x+c*e)**(1/2),x)

[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

3.232 $\int \frac{a+b\sinh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=223

$$-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{d\sqrt{e}\sqrt{(c+dx)^2+1}}+\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{(c+dx)^2+1}}{de(c+dx}$$

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(d*e*(1 + c + d*x)) + (2*Sqr
t[e*(c + d*x)]*(a + b*ArcSinh[c + d*x]))/(d*e) + (4*b*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2]) - (2*b*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.21161, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 5661, 329, 305, 220, 1196}

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{de(c+dx+1)}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```


```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(d*e*(1 + c + d*x)) + (2*Sqr
t[e*(c + d*x)]*(a + b*ArcSinh[c + d*x]))/(d*e) + (4*b*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2]) - (2*b*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2])
```


Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de} + \frac{(4b)\,\text{Subst}}{}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{de(1+c+dx)} + \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de} + \frac{4b(1+c+dx)\sqrt{\frac{1+(c}{(1+c}}}{d\sqrt{e}}$$

Mathematica [C] time = 0.0303084, size = 61, normalized size = 0.27

$$-\frac{2\sqrt{e(c+dx)}\left(2b(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)-3\left(a+b\sinh^{-1}(c+dx)\right)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-2*Sqrt[e*(c + d*x)]*(-3*(a + b*ArcSinh[c + d*x]) + 2*b*(c + d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(3*d*e)
```

Maple [C] time = 0.011, size = 161, normalized size = 0.7

$$2\frac{1}{de}\left(a\sqrt{dex+ce}+b\left(\sqrt{dex+ce}\operatorname{Arcsinh}\left(\frac{dex+ce}{e}\right)-2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(a*(d*e*x+c*e)^(1/2)+b*((d*e*x+c*e)^(1/2)*arcsinh(1/e*(d*e*x+c*e))-2*
I/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d
*e*x+c*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE(
(d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{arsinh}(dx+c)+a}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{asinh}(c+dx)}{\sqrt{e(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/sqrt(e*(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(dx+c)+a}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

3.233 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{de^{3/2}\sqrt{(c+dx)^2+1}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (2*b*(1 + c + d*x)*
Sqrt[(1 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)
]/Sqrt[e]], 1/2])/(d*e^(3/2)*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.112615, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5865, 5661, 329, 220}

$$\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{(c+dx)^2+1}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (2*b*(1 + c + d*x)*
Sqrt[(1 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)
]/Sqrt[e]], 1/2])/(d*e^(3/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

Mathematica [C] time = 0.0253449, size = 56, normalized size = 0.53

$$-\frac{2\left(-2b(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)+a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] - 2*b*(c + d*x)*Hypergeometric2F1[1/4, 1/2, 5/4
, -(c + d*x)^2]))/(d*e*Sqrt[e*(c + d*x)])
```

Maple [C] time = 0.009, size = 140, normalized size = 1.3

$$2\frac{1}{de}\left(-\frac{a}{\sqrt{dex+ce}}+b\left(-\frac{1}{\sqrt{dex+ce}}\text{Arcsinh}\left(\frac{dex+ce}{e}\right)+2\frac{1}{e}\sqrt{1-\frac{i\,(dex+ce)}{e}}\sqrt{1+\frac{i\,(dex+ce)}{e}}\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arcsinh(1/e*(d*e*x+c*e)
)+2/e/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^
2*(d*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

3.234 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=266

$$\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{3de^{5/2}\sqrt{(c+dx)^2+1}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b\sqrt{(c+dx)^2+1}}{3de^2\sqrt{e(c+dx)}}+\frac{4b\sqrt{(c+dx)}}{3de^3(c}$$

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(3*d*e^2*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[e*(c +
 d*x)]*Sqrt[1 + (c + d*x)^2])/(3*d*e^3*(1 + c + d*x)) - (2*(a + b*ArcSinh[c
 + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2]) + (2*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.237651, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 5661, 325, 329, 305, 220, 1196}

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b\sqrt{(c+dx)^2+1}}{3de^2\sqrt{e(c+dx)}}+\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{3de^3(c+dx+1)}+\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(}}{}\right.\right.}{3de^{5/2}\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(3*d*e^2*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[e*(c +
 d*x)]*Sqrt[1 + (c + d*x)^2])/(3*d*e^3*(1 + c + d*x)) - (2*(a + b*ArcSinh[c
 + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2]) + (2*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{3de^4}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{3de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} + \frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{4b(1+c}{}$$

Mathematica [C] time = 0.0290358, size = 58, normalized size = 0.22

$$-\frac{2\left(2b(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)+a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] + 2*b*(c + d*x)*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c + d*x)^2]))/(3*d*e*(e*(c + d*x))^(3/2))
```

Maple [C] time = 0.011, size = 202, normalized size = 0.8

$$2\frac{1}{de}\left(-1/3\frac{a}{(dex+ce)^{3/2}}+b\left(-1/3\frac{1}{(dex+ce)^{3/2}}\mathrm{Arcsinh}\left(\frac{dex+ce}{e}\right)+2/3\frac{1}{e}\left(-\frac{1}{\sqrt{dex+ce}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}+\frac{i}{e}\sqrt{1-\frac{i(de}{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arcsinh(1/e*(d*e*
x+c*e))+2/3/e*(-(1/e^2*(d*e*x+c*e)^2+1)^(1/2)/(d*e*x+c*e)^(1/2)+I/e/(I/e)^(
1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)
^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*
e)^(1/2)*(I/e)^(1/2),I)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dex+ce}(b\,\mathrm{arsinh}\,(dx+c)+a)}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(dx + c) + a}{(dex + ce)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(5/2), x)
```

3.235 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{15de^{7/2}\sqrt{(c+dx)^2+1}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}-\frac{4b\sqrt{(c+dx)^2+1}}{15de^2(e(c+dx))^{3/2}}$$

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(15*d*e^2*(e*(c + d*x))^(3/2)) - (2*(a + b*Arc
Sinh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) - (2*b*(1 + c + d*x)*Sqrt[(1 +
(c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/(15*d*e^(7/2)*Sqrt[1 + (c + d*x)^2])
```

Rubi [A] time = 0.135654, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 5661, 325, 329, 220}

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}-\frac{4b\sqrt{(c+dx)^2+1}}{15de^2(e(c+dx))^{3/2}}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{15de^{7/2}\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(15*d*e^2*(e*(c + d*x))^(3/2)) - (2*(a + b*Arc
Sinh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) - (2*b*(1 + c + d*x)*Sqrt[(1 +
(c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/(15*d*e^(7/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{15de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx}\right.}{15de^4}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{15de^{7/2}\sqrt{1+(c+dx)^2}}$$

Mathematica [C] time = 0.0356996, size = 61, normalized size = 0.42

$$\frac{-4b(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)-6\left(a+b\sinh^{-1}(c+dx)\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-6*(a + b*ArcSinh[c + d*x]) - 4*b*(c + d*x)*Hypergeometric2F1[-3/4, 1/2, 1
/4, -(c + d*x)^2])/(15*d*e*(e*(c + d*x))^(5/2))
```

Maple [C] time = 0.014, size = 176, normalized size = 1.2

$$2\,\frac{1}{de}\left(-1/5\,\frac{a}{(dex+ce)^{5/2}}+b\left(-1/5\,\frac{1}{(dex+ce)^{5/2}}\text{Arcsinh}\left(\frac{dex+ce}{e}\right)+2/5\,\frac{1}{e}\left(-1/3\,\frac{1}{(dex+ce)^{3/2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}-1/3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arcsinh(1/e*(d*e*
x+c*e))+2/5/e*(-1/3*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)/(d*e*x+c*e)^(3/2)-1/3/e^2
/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*
e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

3.236 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{13/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{13}{4},\frac{13}{4}\right\},\left\{\frac{15}{4},\frac{17}{4}\right\},-(c+dx)^2\right)}{1287de^3}-\frac{8b(e(c+dx))^{11/2}\text{Hypergeometric2F}}{}$$

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c
 + d*x)^2])/(99*d*e^2) + (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1,
 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)^2])/(1287*d*e^3)
```

Rubi [A] time = 0.212018, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3}-\frac{8b(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx\right.}{99de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c
 + d*x)^2])/(99*d*e^2) + (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1,
 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)^2])/(1287*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{9de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\,dx\right.}{9de}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{9de} - \frac{8b(e(c+dx))^{11/2}\left(a+b\sinh^{-1}(c+\right.}{99de}$$

Mathematica [A] time = 0.13285, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{9/2}\left(8b^2(c+dx)^2\text{HypergeometricPFQ}\left(\left\{1,\frac{13}{4},\frac{13}{4}\right\},\left\{\frac{15}{4},\frac{17}{4}\right\},-(c+dx)^2\right) - 52b(c+dx)\text{Hypergeometric2}\right.}{1287de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcSinh[c + d*x])^2 - 52*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c + d*x)^2] + 8*
b^2*(c + d*x)^2*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)
^2]))/(1287*d*e)
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2d^3e^3x^3+3\,a^2cd^2e^3x^2+3\,a^2c^2de^3x+a^2c^3e^3+\left(b^2d^3e^3x^3+3\,b^2cd^2e^3x^2+3\,b^2c^2de^3x+b^2c^3e^3\right)\text{arsinh}\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^3*e^3*x^3 + 3*a^2*c*d^2*e^3*x^2 + 3*a^2*c^2*d*e^3*x + a^2*c
^3*e^3 + (b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*b^2*c^2*d*e^3*x + b^2*c
^3*e^3)*arcsinh(d*x + c)^2 + 2*(a*b*d^3*e^3*x^3 + 3*a*b*c*d^2*e^3*x^2 + 3*a
*b*c^2*d*e^3*x + a*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{7}{2}} (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.237 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{11/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},-(c+dx)^2\right)}{693de^3}-\frac{8b(e(c+dx))^{9/2}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{63d}$$

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c +
 d*x)^2])/(63*d*e^2) + (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 1
1/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2])/(693*d*e^3)
```

Rubi [A] time = 0.221239, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3}-\frac{8b(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{63de^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c +
 d*x)^2])/(63*d*e^2) + (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 1
1/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2])/(693*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}(a+b\sinh^{-1}(x)}{\sqrt{1+x^2}}\right.}{7de}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de} - \frac{8b(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c\right.}{63}$$

Mathematica [A] time = 0.119687, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{7/2}\left(8b^2(c+dx)^2\text{HypergeometricPFQ}\left(\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},-(c+dx)^2\right)-44b(c+dx)\text{Hypergeometri}\right.}{693de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcSinh[c + d*x])^2 - 44*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2
]))/(693*d*e)
```

Maple [F] time = 0.259, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{5}{2}}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2d^2e^2x^2+2\,a^2cde^2x+a^2c^2e^2+\left(b^2d^2e^2x^2+2\,b^2cde^2x+b^2c^2e^2\right)\text{arsinh}\left(dx+c\right)^2+2\left(abd^2e^2x^2+2\,abcde^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
+ 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{5}{2}} (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.238 $\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{9/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{9}{4},\frac{9}{4}\right\},\left\{\frac{11}{4},\frac{13}{4}\right\},-(c+dx)^2\right)}{315de^3}-\frac{8b(e(c+dx))^{7/2}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{35}$$

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c +
 d*x)^2])/(35*d*e^2) + (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])/(315*d*e^3)
```

Rubi [A] time = 0.216697, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3}-\frac{8b(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{35de^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c +
 d*x)^2])/(35*d*e^2) + (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])/(315*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}dx\right.}{5de}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de} - \frac{8b(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+\right.}{35de^2}$$

Mathematica [A] time = 0.101044, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{5/2}\left(8b^2(c+dx)^2\text{HypergeometricPFQ}\left(\left\{1,\frac{9}{4},\frac{9}{4}\right\},\left\{\frac{11}{4},\frac{13}{4}\right\},-(c+dx)^2\right)-36b(c+dx)\text{Hypergeometric2F1}\right.}{315de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcSinh[c + d*x])^2 - 36*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])
)/(315*d*e)
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{3}{2}}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2dex+a^2ce+\left(b^2dex+b^2ce\right)\text{arsinh}\left(dx+c\right)^2+2\left(abdex+abce\right)\text{arsinh}\left(dx+c\right)\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2 +
2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e(c+dx))^{\frac{3}{2}} (a + b \operatorname{asinh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}} (b \operatorname{arsinh}(dx+c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.239 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{7/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},-(c+dx)^2\right)}{105de^3}-\frac{8b(e(c+dx))^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\right.}{15de^2}$$

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c +
d*x)^2])/(15*d*e^2) + (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4
, 7/4}, {9/4, 11/4}, -(c + d*x)^2])/(105*d*e^3)
```

Rubi [A] time = 0.209408, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}-\frac{8b(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{15de^2}+\frac{2(e(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c +
d*x)^2])/(15*d*e^2) + (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4
, 7/4}, {9/4, 11/4}, -(c + d*x)^2])/(105*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de}-\frac{(4b)\,\text{Subst}\left(\int\frac{(ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\right.}{3de}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de}-\frac{8b(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+\right.}{15de}$$

Mathematica [A] time = 0.0848566, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{3/2}\left(8b^2(c+dx)^2\text{HypergeometricPFQ}\left(\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},-(c+dx)^2\right)-28b(c+dx)\text{Hypergeometric2}\right.}{105de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcSinh[c + d*x])^2 - 28*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c + d*x)^2] + 8*b^2
*(c + d*x)^2*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c + d*x)^2]))/
(105*d*e)
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^2\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex+ce}\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.240 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=132

$$\frac{16b^2(e(c+dx))^{5/2}\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},-(c+dx)^2\right)}{15de^3}-\frac{8b(e(c+dx))^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{3de^2}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)
^2])/(3*d*e^2) + (16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4
}, {7/4, 9/4}, -(c + d*x)^2])/(15*d*e^3)
```

Rubi [A] time = 0.20267, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3}-\frac{8b(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)
^2])/(3*d*e^2) + (16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4
}, {7/4, 9/4}, -(c + d*x)^2])/(15*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}$$
$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de} - \frac{8b(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\right.}{3de^2}$$

Mathematica [A] time = 0.061877, size = 110, normalized size = 0.83

$$\frac{2\sqrt{e(c+dx)}\left(8b^2(c+dx)^2\text{HypergeometricPFQ}\left(\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},-(c+dx)^2\right)-20b(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{15de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcSinh[c + d*x])^2 - 20*b*(c + d*x)*(a + b
*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2] + 8*b^2*(
c + d*x)^2*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, -(c + d*x)^2]))/(15
*d*e)
```

Maple [F] time = 0.337, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^2 \frac{1}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/sqrt(d*e*x
 + c*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{asinh}(c+dx))^2}{\sqrt{e(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.241 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{16b^2(e(c+dx))^{3/2}\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},-(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{e(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.}{de^2}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^
2])/(d*e^2) - (16*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1},
{5/4, 7/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rubi [A] time = 0.221698, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$-\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{2\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^
2])/(d*e^2) - (16*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1},
{5/4, 7/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$
$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+\right.}{de^2}$$

Mathematica [A] time = 0.0981019, size = 109, normalized size = 0.84

$$\frac{2\left(-4b(c+dx)\left(2b(c+dx)\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},-(c+dx)^2\right)-3\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.\right.}{3de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-3*(a + b*ArcSinh[c + d*x])^2 - 4*b*(c + d*x)*(-3*(a + b*ArcSinh[c + d*
x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2] + 2*b*(c + d*x)*Hypergeo
metricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, -(c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*
x)])
```

Maple [F] time = 0.249, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^2\,(dex+ce)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2\right)\sqrt{dex+ce}}{d^2e^2x^2+2\,cde^2x+c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^2}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

3.242 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{16b^2\sqrt{e(c+dx)}\text{HypergeometricPFQ}\left(\left\{\frac{1}{4},\frac{1}{4},1\right\},\left\{\frac{3}{4},\frac{5}{4}\right\},-(c+dx)^2\right)}{3de^3}-\frac{8b\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)\right.}{3de^2\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2])/(3*d*e^2*
Sqrt[e*(c + d*x)]) + (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4,
 1}, {3/4, 5/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rubi [A] time = 0.226133, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$\frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3}-\frac{8b\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh^{-1}(\right.}{3de(e(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2])/(3*d*e^2*
Sqrt[e*(c + d*x)]) + (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4,
 1}, {3/4, 5/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)}{3de^2\sqrt{e(c+dx)}} + \frac{16b^2}{}$$

Mathematica [A] time = 0.0752128, size = 106, normalized size = 0.79

$$-\frac{2\left(4b(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)-2b(c+dx)\text{HypergeometricPFQ}\right.\right.}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])^2 + 4*b*(c + d*x)*((a + b*ArcSinh[c + d*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2] - 2*b*(c + d*x)*Hypergeometr
icPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2
))
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^2\,(dex+ce)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2\right)\sqrt{dex+ce}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^2}{(e(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(5/2), x)
```

3.243 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{16b^2\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},1\right\},\left\{\frac{1}{4},\frac{3}{4}\right\},-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)\left(a+b\right.}{15de^2(e(c+dx))^{3/2}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2])/(15*d*e^2
*(e*(c + d*x))^(3/2)) - (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/
4}, -(c + d*x)^2])/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rubi [A] time = 0.229618, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5865, 5661, 5762}

$$-\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{15de^2(e(c+dx))^{3/2}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2])/(15*d*e^2
*(e*(c + d*x))^(3/2)) - (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/
4}, -(c + d*x)^2])/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)}{15de^2(e(c+dx))^{3/2}} - 1$$

Mathematica [A] time = 0.0863127, size = 110, normalized size = 0.82

$$-\frac{2\left(8b^2(c+dx)^2\text{HypergeometricPFQ}\left(\left\{-\frac{1}{4},-\frac{1}{4},1\right\},\left\{\frac{1}{4},\frac{3}{4}\right\},-(c+dx)^2\right)+\left(a+b\sinh^{-1}(c+dx)\right)\left(4b(c+dx)\text{Hyper}\right.\right.}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])*(3*(a + b*ArcSinh[c + d*x]) + 4*b*(c + d*x)*H
ypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2]) + 8*b^2*(c + d*x)^2*Hyperge
ometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, -(c + d*x)^2]))/(15*d*e*(e*(c + d*x
))^(5/2))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^2\,(dex+ce)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
+ c^4*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

3.244 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9de}-\frac{2b\text{Unintegrable}\left(\frac{(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{3e}$$

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^3)/(9*d*e) - (2*b*Unintegra
ble[((e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/Sqrt[1 + (c + d*x)^2],
 x])/(3*e)
```

Rubi [A] time = 0.207566, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^3)/(9*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^{7/2} \left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\right.}{3de}$$

Mathematica [A] time = 85.427, size = 0, normalized size = 0.

$$\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{7}{2}} (a+b\text{Arcsinh}\,(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(a^3d^3e^3x^3 + 3\,a^3cd^2e^3x^2 + 3\,a^3c^2de^3x + a^3c^3e^3 + \left(b^3d^3e^3x^3 + 3\,b^3cd^2e^3x^2 + 3\,b^3c^2de^3x + b^3c^3e^3\right)\text{arsinh}\left(dx + c\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d^3*e^3*x^3 + 3*a^3*c*d^2*e^3*x^2 + 3*a^3*c^2*d*e^3*x + a^3*c
^3*e^3 + (b^3*d^3*e^3*x^3 + 3*b^3*c*d^2*e^3*x^2 + 3*b^3*c^2*d*e^3*x + b^3*c
^3*e^3)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^3*e^3*x^3 + 3*a*b^2*c*d^2*e^3*x^2 +
 3*a*b^2*c^2*d*e^3*x + a*b^2*c^3*e^3)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^3*e^3
*x^3 + 3*a^2*b*c*d^2*e^3*x^2 + 3*a^2*b*c^2*d*e^3*x + a^2*b*c^3*e^3)*arcsinh
(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx + c) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.245 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{7de}-\frac{6b\text{Unintegrable}\left(\frac{(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{7e}$$

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^3)/(7*d*e) - (6*b*Unintegra
ble[((e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/Sqrt[1 + (c + d*x)^2],
 x])/(7*e)
```

Rubi [A] time = 0.203318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^3)/(7*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^{5/2} \left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{7de}-\frac{(6b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}\left(a+b\sinh^{-1}(x)\right.}{\sqrt{1+x^2}}\right.}{7de}$$

Mathematica [A] time = 99.8167, size = 0, normalized size = 0.

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A] time = 0.255, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{5}{2}} \left(a+b\text{Arcsinh}\,(dx+c)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3d^2e^2x^2 + 2\,a^3cde^2x + a^3c^2e^2 + \left(b^3d^2e^2x^2 + 2\,b^3cde^2x + b^3c^2e^2\right)\text{arsinh}\,(dx+c)^3 + 3\left(ab^2d^2e^2x^2 + 2\,ab^2cde^2x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^
2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^
2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^2*e^
2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c
*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.246 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de}-\frac{6b\text{Unintegrable}\left(\frac{(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{5e}$$

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^3)/(5*d*e) - (6*b*Unintegra
ble[((e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/Sqrt[1 + (c + d*x)^2],
 x])/(5*e)
```

Rubi [A] time = 0.209061, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^3 \, dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de}-\frac{(6b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\right.}{5de}$$

Mathematica [A] time = 67.7186, size = 0, normalized size = 0.

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A] time = 0.258, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{3}{2}}\left(a+b\text{Arcsinh}\left(dx+c\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(a^3dex + a^3ce + \left(b^3dex + b^3ce\right)\text{arsinh}\left(dx + c\right)^3 + 3\left(ab^2dex + ab^2ce\right)\text{arsinh}\left(dx + c\right)^2 + 3\left(a^2bdex + a^2bce\right)\text{ars}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3 +
3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e
)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx + c) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.247 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=79

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de}-\frac{2b\text{Unintegrable}\left(\frac{(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{e}$$

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^3)/(3*d*e) - (2*b*Unintegra
ble[((e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/Sqrt[1 + (c + d*x)^2],
 x])/e
```

Rubi [A] time = 0.202466, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(d*e)
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de}-\frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}$$

$$de$$

Mathematica [A] time = 89.1327, size = 0, normalized size = 0.

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A] time = 0.263, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^3\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2 b \operatorname{arsinh}(dx+c) + a^3\right)\sqrt{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e(c+dx)}\,(a + b \operatorname{asinh}(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex+ce}(b \operatorname{arsinh}(dx+c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.248 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de}-\frac{6b\text{Unintegrable}\left(\frac{\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{e}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^3)/(d*e) - (6*b*Unintegrable[
(Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/Sqrt[1 + (c + d*x)^2], x])/e
```

Rubi [A] time = 0.183353, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de}-\frac{(6b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}
\end{aligned}$$

Mathematica [A] time = 8.70346, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

Maple [A] time = 0.19, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(dx + c))^3 \frac{1}{\sqrt{dex + ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \,\text{arsinh}\,(dx + c)^3 + 3\,ab^2 \,\text{arsinh}\,(dx + c)^2 + 3\,a^2 b \,\text{arsinh}\,(dx + c) + a^3}{\sqrt{dex + ce}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{asinh}\,(c + dx))^3}{\sqrt{e\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.249 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{6b\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Unintegrable
[(a + b*ArcSinh[c + d*x])^2/(Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2]), x])/
e
```

Rubi [A] time = 0.192779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^2/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{ex}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 18.9413, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

Maple [A] time = 0.251, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(dx+c))^3\,(dex+ce)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^3\,\mathrm{arsinh}\,(dx+c)^3+3\,ab^2\,\mathrm{arsinh}\,(dx+c)^2+3\,a^2b\,\mathrm{arsinh}\,(dx+c)+a^3\right)\sqrt{dex+ce}}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2
), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+dx))^3}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

3.250 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2b\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}},x\right)}{e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Unintegr
able[(a + b*ArcSinh[c + d*x])^2/((e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])
, x])/e
```

Rubi [A] time = 0.208812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\
&= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}
\end{aligned}$$

Mathematica [A] time = 21.0769, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(dx+c))^3\,(dex+ce)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^3\,\mathrm{arsinh}\,(dx+c)^3+3\,ab^2\,\mathrm{arsinh}\,(dx+c)^2+3\,a^2b\,\mathrm{arsinh}\,(dx+c)+a^3\right)\sqrt{dex+ce}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c
^2*d*e^3*x + c^3*e^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+dx))^3}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(5/2), x)
```

3.251 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=81

$$\frac{6b\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Unintegr
able[(a + b*ArcSinh[c + d*x])^2/((e*(c + d*x))^(5/2)*Sqrt[1 + (c + d*x)^2])
, x])/(5*e)
```

Rubi [A] time = 0.216786, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 69.217, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(dx+c))^3 (dex+ce)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2 b \operatorname{arsinh}(dx+c) + a^3\right)\sqrt{dex+ce}}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c
^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

3.252 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{9de}-\frac{8b\text{Unintegrable}\left(\frac{(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{9e}$$

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^4)/(9*d*e) - (8*b*Unintegra
ble[((e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^3)/Sqrt[1 + (c + d*x)^2],
 x])/(9*e)
```

Rubi [A] time = 0.20604, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^4)/(9*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(9*d*e)
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{9de} - \frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\right.}{9de}$$

Mathematica [A] time = 93.7129, size = 0, normalized size = 0.

$$\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A] time = 0.257, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{7}{2}} (a+b\text{Arcsinh}\,(dx+c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4d^3e^3x^3 + 3\,a^4cd^2e^3x^2 + 3\,a^4c^2de^3x + a^4c^3e^3 + \left(b^4d^3e^3x^3 + 3\,b^4cd^2e^3x^2 + 3\,b^4c^2de^3x + b^4c^3e^3\right)\text{arsinh}\left(dx + c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d^3*e^3*x^3 + 3*a^4*c*d^2*e^3*x^2 + 3*a^4*c^2*d*e^3*x + a^4*c
^3*e^3 + (b^4*d^3*e^3*x^3 + 3*b^4*c*d^2*e^3*x^2 + 3*b^4*c^2*d*e^3*x + b^4*c
^3*e^3)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^3*e^3*x^3 + 3*a*b^3*c*d^2*e^3*x^2 +
 3*a*b^3*c^2*d*e^3*x + a*b^3*c^3*e^3)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d^3*e
^3*x^3 + 3*a^2*b^2*c*d^2*e^3*x^2 + 3*a^2*b^2*c^2*d*e^3*x + a^2*b^2*c^3*e^3)
*arcsinh(d*x + c)^2 + 4*(a^3*b*d^3*e^3*x^3 + 3*a^3*b*c*d^2*e^3*x^2 + 3*a^3*
b*c^2*d*e^3*x + a^3*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx + c) + a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.253 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{7de}-\frac{8b\text{Unintegrable}\left(\frac{(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{7e}$$

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^4)/(7*d*e) - (8*b*Unintegra
ble[((e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^3)/Sqrt[1 + (c + d*x)^2],
 x])/(7*e)
```

Rubi [A] time = 0.206969, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^4)/(7*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int (ex)^{5/2} \left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{7de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}} \, d\right.}{7de}$$

Mathematica [A] time = 122.473, size = 0, normalized size = 0.

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A] time = 0.261, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{5}{2}} (a+b\text{Arcsinh}\,(dx+c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4d^2e^2x^2 + 2\,a^4cde^2x + a^4c^2e^2 + \left(b^4d^2e^2x^2 + 2\,b^4cde^2x + b^4c^2e^2\right)\text{arsinh}\,(dx+c)^4 + 4\left(ab^3d^2e^2x^2 + 2\,ab^3cd\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d^2*e^2*x^2 + 2*a^4*c*d*e^2*x + a^4*c^2*e^2 + (b^4*d^2*e^2*x^
2 + 2*b^4*c*d*e^2*x + b^4*c^2*e^2)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^2*e^2*x^
2 + 2*a*b^3*c*d*e^2*x + a*b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d^2*
e^2*x^2 + 2*a^2*b^2*c*d*e^2*x + a^2*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 4*(a^
3*b*d^2*e^2*x^2 + 2*a^3*b*c*d*e^2*x + a^3*b*c^2*e^2)*arcsinh(d*x + c))*sqrt
(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c) + a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.254 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de}-\frac{8b\text{Unintegrable}\left(\frac{(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{5e}$$

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^4)/(5*d*e) - (8*b*Unintegra
ble[((e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^3)/Sqrt[1 + (c + d*x)^2],
 x])/(5*e)
```

Rubi [A] time = 0.205551, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$
$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}} \, d\right.}{5de}$$

Mathematica [A] time = 79.3502, size = 0, normalized size = 0.

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{3}{2}} \left(a+b\text{Arcsinh}\,(dx+c)\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4dex + a^4ce + \left(b^4dex + b^4ce\right)\operatorname{arsinh}(dx+c)^4 + 4\left(ab^3dex + ab^3ce\right)\operatorname{arsinh}(dx+c)^3 + 6\left(a^2b^2dex + a^2b^2c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4 +
4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b^2
*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c))*sq
rt(d*e*x + c*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{3}{2}}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.255 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=81

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de}-\frac{8b\text{Unintegrable}\left(\frac{(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{3e}$$

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^4)/(3*d*e) - (8*b*Unintegra
ble[((e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^3)/Sqrt[1 + (c + d*x)^2],
 x])/(3*e)
```

Rubi [A] time = 0.194064, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}}\,dx,\right.}{3de}$$

Mathematica [A] time = 113.666, size = 0, normalized size = 0.

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A] time = 0.271, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(dx+c))^4\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4 \operatorname{arsinh}(dx+c)^4 + 4ab^3 \operatorname{arsinh}(dx+c)^3 + 6a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4a^3b \operatorname{arsinh}(dx+c) + a^4\right)\sqrt{dex+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e(c+dx)}\,(a+b\operatorname{asinh}(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**4, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dex+ce}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.256 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de}-\frac{8b\text{Unintegrable}\left(\frac{\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{e}$$

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^4)/(d*e) - (8*b*Unintegrable[
(Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^3)/Sqrt[1 + (c + d*x)^2], x])/e
```

Rubi [A] time = 0.181656, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^4}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\ &= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de}-\frac{(8b)\,\text{Subst}\left(\int \frac{\sqrt{ex}(a+b\sinh^{-1}(x))^3}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 8.6694, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

Maple [A] time = 0.199, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(dx+c\right)\right)^{4}\frac{1}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{4}\,\mathrm{arsinh}\left(dx+c\right)^{4}+4\,ab^{3}\,\mathrm{arsinh}\left(dx+c\right)^{3}+6\,a^{2}b^{2}\,\mathrm{arsinh}\left(dx+c\right)^{2}+4\,a^{3}b\,\mathrm{arsinh}\left(dx+c\right)+a^{4}}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/sqrt(d*e*x + c*e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{4}}{\sqrt{e\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.257 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{8b\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Unintegrable
[(a + b*ArcSinh[c + d*x])^3/(Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2]), x])/
e
```

Rubi [A] time = 0.194081, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^3/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{ex}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 35.1356, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

Maple [A] time = 0.251, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(dx+c))^{4} (dex+ce)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{4}\,\mathrm{arsinh}\,(dx+c)^{4}+4\,ab^{3}\,\mathrm{arsinh}\,(dx+c)^{3}+6\,a^{2}b^{2}\,\mathrm{arsinh}\,(dx+c)^{2}+4\,a^{3}b\,\mathrm{arsinh}\,(dx+c)+a^{4}\right)\sqrt{dex+ce}}{d^{2}e^{2}x^{2}+2\,cde^{2}x+c^{2}e^{2}},$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^2*
e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+dx))^{4}}{(e\,(c+dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^{4}}{(dex+ce)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

3.258 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=81

$$\frac{8b\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}},x\right)}{3e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Unintegr
able[(a + b*ArcSinh[c + d*x])^3/((e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])
, x])/(3*e)
```

Rubi [A] time = 0.206197, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}\end{aligned}$$

Mathematica [A] time = 40.1042, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\left(dx+c\right))^{4} (dex+ce)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{4}\,\mathrm{arsinh}\left(dx+c\right)^{4}+4\,ab^{3}\,\mathrm{arsinh}\left(dx+c\right)^{3}+6\,a^{2}b^{2}\,\mathrm{arsinh}\left(dx+c\right)^{2}+4\,a^{3}b\,\mathrm{arsinh}\left(dx+c\right)+a^{4}\right)\sqrt{dex+}}{d^{3}e^{3}x^{3}+3\,cd^{2}e^{3}x^{2}+3\,c^{2}de^{3}x+c^{3}e^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^3*
e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{asinh}\left(c+dx\right))^{4}}{(e\left(c+dx\right))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\left(dx+c\right)+a)^{4}}{(dex+ce)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(5/2), x)
```

3.259 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=81

$$\frac{8b\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}$$

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Unintegr
able[(a + b*ArcSinh[c + d*x])^3/((e*(c + d*x))^(5/2)*Sqrt[1 + (c + d*x)^2])
, x])/(5*e)
```

Rubi [A] time = 0.207014, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 102.925, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

Maple [A] time = 0.259, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(dx+c\right)\right)^{4} \left(dex+ce\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^4 \operatorname{arsinh}\left(dx+c\right)^4+4\,ab^3 \operatorname{arsinh}\left(dx+c\right)^3+6\,a^2b^2 \operatorname{arsinh}\left(dx+c\right)^2+4\,a^3b \operatorname{arsinh}\left(dx+c\right)+a^4\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^4*
e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arsinh}\left(dx+c\right)+a\right)^{4}}{\left(dex+ce\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

3.260 $\int \sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=131

$$-\frac{3(a+bx)^2}{8b}+\frac{\sinh^{-1}(a+bx)^4}{8b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^3}{2b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}-\frac{3\sinh^{-1}(a+b}{8b}$$

```
[Out] (-3*(a + b*x)^2)/(8*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(4*b) - (3*ArcSinh[a + b*x]^2)/(8*b) - (3*(a + b*x)^2*ArcSinh[a + b*x]^2
)/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(2*b) + ArcS
inh[a + b*x]^4/(8*b)
```

Rubi [A] time = 0.188993, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5867, 5682, 5675, 5661, 5758, 30}

$$-\frac{3(a+bx)^2}{8b}+\frac{\sinh^{-1}(a+bx)^4}{8b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^3}{2b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}-\frac{3\sinh^{-1}(a+b}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```


```
[Out] (-3*(a + b*x)^2)/(8*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(4*b) - (3*ArcSinh[a + b*x]^2)/(8*b) - (3*(a + b*x)^2*ArcSinh[a + b*x]^2
)/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(2*b) + ArcS
inh[a + b*x]^4/(8*b)
```


Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\sinh^{-1}(x)^3\,dx,x,a+bx\right)}{b}$$
$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^3}{2b} + \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\sqrt{1+x^2}}\,dx,x,a\right.}{2b}$$
$$= -\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b}$$
$$= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{4b} - \frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}$$
$$= -\frac{3(a+bx)^2}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{4b} - \frac{3\sinh^{-1}(a+}{8b}$$

Mathematica [A] time = 0.108373, size = 127, normalized size = 0.97

$$\frac{4(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)^3 - 3\left(2a^2+4abx+2b^2x^2+1\right)\sinh^{-1}(a+bx)^2 + 6(a+bx)\sqrt{a^2+2ab}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-3*b*x*(2*a + b*x) + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] - 3*(1 + 2*a^2 + 4*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 4*(a +
 b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3 + ArcSinh[a + b*
x]^4)/(8*b)
```

Maple [A] time = 0.084, size = 204, normalized size = 1.6

$$\frac{1}{8b}\left(4\,(\text{Arcsinh}\,(bx+a))^3\sqrt{b^2x^2+2\,xab+a^2+1}xb - 6\,(\text{Arcsinh}\,(bx+a))^2x^2b^2 + 4\,(\text{Arcsinh}\,(bx+a))^3\sqrt{b^2x^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/8*(4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b-6*arcsinh(b*x+a)^
2*x^2*b^2+4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x
+a)^2*x*a*b+arcsinh(b*x+a)^4-6*arcsinh(b*x+a)^2*a^2+6*arcsinh(b*x+a)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)*x*b-3*b^2*x^2+6*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)*a-6*x*a*b-3*arcsinh(b*x+a)^2-3*a^2-3)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.71438, size = 495, normalized size = 3.78

$$4\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3\,b^2x^2+\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/8*(4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))^3 - 3*b^2*x^2 + log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))^4 - 6*a*b*x - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

3.261 $\int \sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=107

$$\frac{(a+bx)\sqrt{(a+bx)^2+1}}{4b}+\frac{\sinh^{-1}(a+bx)^3}{6b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{2b}-\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}-\frac{\sinh^{-1}}{}$$

```
[Out] ((a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b) - ArcSinh[a + b*x]/(4*b) - ((a + b*
x)^2*ArcSinh[a + b*x])/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
 b*x]^2)/(2*b) + ArcSinh[a + b*x]^3/(6*b)
```

Rubi [A] time = 0.118978, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5867, 5682, 5675, 5661, 321, 215}

$$\frac{(a+bx)\sqrt{(a+bx)^2+1}}{4b}+\frac{\sinh^{-1}(a+bx)^3}{6b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{2b}-\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}-\frac{\sinh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b) - ArcSinh[a + b*x]/(4*b) - ((a + b*
x)^2*ArcSinh[a + b*x])/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
 b*x]^2)/(2*b) + ArcSinh[a + b*x]^3/(6*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\sinh^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2b} + \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx,x,a\right.}{2b}$$

$$= -\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a}{}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{\sinh^{-1}(a+bx)}{4b} - \frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}$$

Mathematica [A] time = 0.0854566, size = 110, normalized size = 1.03

$$\frac{3(a+bx)\sqrt{a^2+2abx+b^2x^2+1}+6(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)^2-3\left(2a^2+4abx+2b^2x^2+1\right)\sinh}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] (3*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*(1 + 2*a^2 + 4*a*b*x + 2
*b^2*x^2)*ArcSinh[a + b*x] + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*
ArcSinh[a + b*x]^2 + 2*ArcSinh[a + b*x]^3)/(12*b)
```

Maple [A] time = 0.052, size = 167, normalized size = 1.6

$$\frac{1}{12\,b}\left(6\,(\text{Arcsinh}\,(bx+a))^2\sqrt{b^2x^2+2\,xab+a^2+1}xb-6\,\text{Arcsinh}\,(bx+a)\,x^2b^2+6\,(\text{Arcsinh}\,(bx+a))^2\sqrt{b^2x^2+2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/12*(6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b-6*arcsinh(b*x+a)
*x^2*b^2+6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x+
a)*x*a*b+2*arcsinh(b*x+a)^3-6*arcsinh(b*x+a)*a^2+3*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)*x*b+3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-3*arcsinh(b*x+a))/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.6742, size = 401, normalized size = 3.75

$$\frac{6\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/12*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))^2 + 2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))^3 - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

3.262 $\int \sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{(a+bx)^2}{4b}+\frac{\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)}{2b}+\frac{\sinh^{-1}(a+bx)^2}{4b}$$

```
[Out] -(a + b*x)^2/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*
b) + ArcSinh[a + b*x]^2/(4*b)
```

Rubi [A] time = 0.0683636, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5867, 5682, 5675, 30}

$$-\frac{(a+bx)^2}{4b}+\frac{\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)}{2b}+\frac{\sinh^{-1}(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] -(a + b*x)^2/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*
b) + ArcSinh[a + b*x]^2/(4*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b} - \frac{\text{Subst}(\int x\,dx, x, a+bx)}{2b} + \frac{\text{Subs}}{}$$
$$= -\frac{(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b} + \frac{\sinh^{-1}(a+bx)^2}{4b}$$

Mathematica [A] time = 0.0553745, size = 61, normalized size = 1.

$$\frac{2(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx) - bx(2a+bx) + \sinh^{-1}(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(2*a + b*x)) + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + ArcSinh[a + b*x]^2)/(4*b)
```

Maple [A] time = 0.047, size = 91, normalized size = 1.5

$$\frac{1}{4b}\left(2\,\text{Arcsinh}\,(bx+a)\sqrt{b^2x^2+2\,xab+a^2+1}xb - b^2x^2 + 2\,\text{Arcsinh}\,(bx+a)\sqrt{b^2x^2+2\,xab+a^2+1}a - 2\,xab + (\text{Arcsi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/4*(2*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b-b^2*x^2+2*arcsinh(b
*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-2*x*a*b+arcsinh(b*x+a)^2-a^2-1)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60704, size = 240, normalized size = 3.93

$$-\frac{b^2x^2 + 2\,abx - 2\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - \log\left(bx+a+\sqrt{b^2x^2+2\,abx+\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + 2abx + b^2x^2 + 1} \operatorname{asinh}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\, \operatorname{arsinh}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a), x)
```

3.263 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + Log[ArcSinh[a + b*x]]/(2*b)
```

Rubi [A] time = 0.119285, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5867, 5699, 3312, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + Log[ArcSinh[a + b*x]]/(2*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b} + \frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Mathematica [A] time = 0.0629579, size = 24, normalized size = 0.77

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

```
[Out] (CoshIntegral[2*ArcSinh[a + b*x]] + Log[ArcSinh[a + b*x]])/(2*b)
```

Maple [A] time = 0.052, size = 28, normalized size = 0.9

$$\frac{\text{Chi}\left(2\,\text{Arcsinh}\left(bx+a\right)\right)}{2\,b}+\frac{\ln\left(\text{Arcsinh}\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x)
```

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/2*ln(arcsinh(b*x+a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2+2abx+b^2x^2+1}}{\text{asinh}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

3.264 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^2} \, dx$

Optimal. Leaf size=36

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b} - \frac{(a+bx)^2+1}{b\sinh^{-1}(a+bx)}$$

```
[Out] -((1 + (a + b*x)^2)/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*x]
]/b
```

Rubi [A] time = 0.115471, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5867, 5696, 5669, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b} - \frac{(a+bx)^2+1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*x]
]/b
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{\textbf{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0582283, size = 47, normalized size = 1.31

$$
-\frac{a^2-\sinh^{-1}(a+bx)\textbf{Shi}\left(2\sinh^{-1}(a+bx)\right)+2abx+b^2x^2+1}{b\sinh^{-1}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + a^2 + 2*a*b*x + b^2*x^2 - ArcSinh[a + b*x]*SinhIntegral[2*ArcSinh[a
+ b*x]])/(b*ArcSinh[a + b*x]))
```

Maple [A] time = 0.049, size = 44, normalized size = 1.2

$$
\frac{2\,\text{Shi}\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\text{Arcsinh}\left(bx+a\right)-\cosh\left(2\,\text{Arcsinh}\left(bx+a\right)\right)-1}{2\,b\text{Arcsinh}\left(bx+a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/2/b*(2*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arc
sinh(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2x^2+2\,abx+a^2+1\right)^2+\left(b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+a\right)\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}+\int\frac{\left(2\,b^2x^2+\right.}{\left(b^4x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
 + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2*x + a^2
*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((2*b^2*x^2 + 4*a*b*x + 2*a^2
- 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a
^3 + (6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^4*x^4 + 8*a*
b^3*x^3 + 2*a^4 + 3*(4*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(4*a^3*b + 3*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3
*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^
2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
+ b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

3.265 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=69

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{b\sinh^{-1}(a+bx)}-\frac{(a+bx)^2+1}{2b\sinh^{-1}(a+bx)^2}$$

```
[Out] -(1 + (a + b*x)^2)/(2*b*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 + (a + b*x)
^2])/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b
```

Rubi [A] time = 0.106354, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5867, 5696, 5665, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{b\sinh^{-1}(a+bx)}-\frac{(a+bx)^2+1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -(1 + (a + b*x)^2)/(2*b*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 + (a + b*x)
^2])/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}$$

Mathematica [A] time = 0.267333, size = 85, normalized size = 1.23

$$-\frac{2(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)+a^2-2\sinh^{-1}(a+bx)^2\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+2abx+b^2x^2+1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -(1 + a^2 + 2*a*b*x + b^2*x^2 + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^
2]*ArcSinh[a + b*x] - 2*ArcSinh[a + b*x]^2*CoshIntegral[2*ArcSinh[a + b*x]]
)/(2*b*ArcSinh[a + b*x]^2)
```

Maple [A] time = 0.054, size = 63, normalized size = 0.9

$$\frac{4\,\text{Chi}\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\left(\text{Arcsinh}\left(bx+a\right)\right)^2-2\,\sinh\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\text{Arcsinh}\left(bx+a\right)-\cosh\left(2\,\text{Arcsinh}\left(bx+\right.\right.}{4\,b\left(\text{Arcsinh}\left(bx+a\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/4/b*(4*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*sinh(2*arcsinh(b*x+a))*ar
csinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arcsinh(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3
*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^5*x^5 + 15*a*b^4*x^4 +
3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*
```

a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + ((2*b^4*x
^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(4*a^3*b + a*b)
x - 1)(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (6*b^5*x^5 + 30*a*b^4*x^4 + 6*a^5
 + (60*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + 3*(20*a^3*b^2 + 7*a*b^2)*x^2 + (30*a^
4*b + 21*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*b^6*x^6
 + 36*a*b^5*x^5 + 6*a^6 + (90*a^2*b^4 + 11*b^4)*x^4 + 11*a^4 + 4*(30*a^3*b^
3 + 11*a*b^3)*x^3 + 6*(15*a^4*b^2 + 11*a^2*b^2 + b^2)*x^2 + 6*a^2 + 4*(9*a^
5*b + 11*a^3*b + 3*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^7*x^7 +
 14*a*b^6*x^6 + 2*a^7 + (42*a^2*b^5 + 5*b^5)*x^5 + 5*a^5 + 5*(14*a^3*b^4 +
5*a*b^4)*x^4 + 2*(35*a^4*b^3 + 25*a^2*b^3 + 2*b^3)*x^3 + 4*a^3 + 2*(21*a^5*
b^2 + 25*a^3*b^2 + 6*a*b^2)*x^2 + (14*a^6*b + 25*a^4*b + 12*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) + (b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^
5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3
a^3 + 3(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a
^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^7*x^6 + 6*a*b^6*x^5
 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3
+ 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*x^3 + 3*a*b^3*x^2 +
3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^4 + 4*a
*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2 + a*b^2)*x + 3*(b^6
*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b
^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*((4*b^4*x^4 + 16*a*b^3*x^3 + 4*a^4 + 2*(12*a^2*b^2 - b^2)*x
^2 - 2*a^2 + 4*(4*a^3*b - a*b)*x + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) +
 2*(8*b^5*x^5 + 40*a*b^4*x^4 + 8*a^5 + 4*(20*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4
*(20*a^3*b^2 + 3*a*b^2)*x^2 + (40*a^4*b + 12*a^2*b + b)*x + a)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 2*(12*b^6*x^6 + 72*a*b^5*x^5 + 12*a^6 + 18*(10*a^2*b^
4 + b^4)*x^4 + 18*a^4 + 24*(10*a^3*b^3 + 3*a*b^3)*x^3 + 6*(30*a^4*b^2 + 18*
a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(6*a^5*b + 6*a^3*b + a*b)*x - 1)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2) + 2*(8*b^7*x^7 + 56*a*b^6*x^6 + 8*a^7 + 4*(42*a^2
*b^5 + 5*b^5)*x^5 + 20*a^5 + 20*(14*a^3*b^4 + 5*a*b^4)*x^4 + 5*(56*a^4*b^3
+ 40*a^2*b^3 + 3*b^3)*x^3 + 15*a^3 + (168*a^5*b^2 + 200*a^3*b^2 + 45*a*b^2)
*x^2 + (56*a^6*b + 100*a^4*b + 45*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (4*b^8*x^8 + 32*a*b^7*x^7 + 4*a^8 + 14*(8*a^2*b^6 + b^6)*x^6 +
 14*a^6 + 28*(8*a^3*b^5 + 3*a*b^5)*x^5 + (280*a^4*b^4 + 210*a^2*b^4 + 17*b^
4)*x^4 + 17*a^4 + 4*(56*a^5*b^3 + 70*a^3*b^3 + 17*a*b^3)*x^3 + 2*(56*a^6*b^
2 + 105*a^4*b^2 + 51*a^2*b^2 + 4*b^2)*x^2 + 8*a^2 + 4*(8*a^7*b + 21*a^5*b +
 17*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x^8 + 8*
a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)
x^5 + 2(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*
a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x
 + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2
*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 +
 a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b
x + a^2 + 1)^(3/2) + 6(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)
*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2
)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6
+ a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35
*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*
a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

3.266 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=235

$$-\frac{3(a+bx)^4}{128b}-\frac{51(a+bx)^2}{128b}-\frac{9(a+bx)^2\sinh^{-1}(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)^3}{4b}+\frac{3\sqrt{(a+bx)^2}}{}$$

```
[Out] (-51*(a + b*x)^2)/(128*b) - (3*(a + b*x)^4)/(128*b) + (45*(a + b*x)*Sqrt[1
+ (a + b*x)^2]*ArcSinh[a + b*x])/(64*b) + (3*(a + b*x)*(1 + (a + b*x)^2)^(3
/2)*ArcSinh[a + b*x])/(32*b) - (27*ArcSinh[a + b*x]^2)/(128*b) - (9*(a + b*
x)^2*ArcSinh[a + b*x]^2)/(16*b) - (3*(1 + (a + b*x)^2)^2*ArcSinh[a + b*x]^2
)/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(8*b) + (
(a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^3)/(4*b) + (3*ArcSinh[a
+ b*x]^4)/(32*b)
```

Rubi [A] time = 0.309131, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5867, 5684, 5682, 5675, 5661, 5758, 30, 5717, 14}

$$-\frac{3(a+bx)^4}{128b}-\frac{51(a+bx)^2}{128b}-\frac{9(a+bx)^2\sinh^{-1}(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)^3}{4b}+\frac{3\sqrt{(a+bx)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```


```
[Out] (-51*(a + b*x)^2)/(128*b) - (3*(a + b*x)^4)/(128*b) + (45*(a + b*x)*Sqrt[1
+ (a + b*x)^2]*ArcSinh[a + b*x])/(64*b) + (3*(a + b*x)*(1 + (a + b*x)^2)^(3
/2)*ArcSinh[a + b*x])/(32*b) - (27*ArcSinh[a + b*x]^2)/(128*b) - (9*(a + b*
x)^2*ArcSinh[a + b*x]^2)/(16*b) - (3*(1 + (a + b*x)^2)^2*ArcSinh[a + b*x]^2
)/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(8*b) + (
(a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^3)/(4*b) + (3*ArcSinh[a
+ b*x]^4)/(32*b)
```


Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
```

```
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)^3}{4b} - \frac{3\,\text{Subst}\left(\int x\left(1+x^2\right)\right.}{}$$

$$= -\frac{3\left(1+(a+bx)^2\right)^2\sinh^{-1}(a+bx)^2}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\text{sinh}}{8b}$$

$$= \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)}{32b} - \frac{9(a+bx)^2\sinh^{-1}(a+}{16b}$$

$$= \frac{45(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{64b} + \frac{3(a+bx)\left(1+(a+bx)^2\right.}{32b}$$

$$= -\frac{51(a+bx)^2}{128b} - \frac{3(a+bx)^4}{128b} + \frac{45(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+}{64b}$$

Mathematica [A] time = 0.228042, size = 266, normalized size = 1.13

$$-\frac{3\left(6a^2+17\right)b^2x^2 - 16\sqrt{a^2+2abx+b^2x^2+1}\left(6a^2bx+2a^3+6ab^2x^2+5a+2b^3x^3+5bx\right)\sinh^{-1}(a+bx)^3 + 3\left(8a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

```
[Out] -(6*a*(17 + 2*a^2)*b*x + 3*(17 + 6*a^2)*b^2*x^2 + 12*a*b^3*x^3 + 3*b^4*x^4
- 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*b*x +
6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*(17 + 8*a^4 + 32*a^3*b*x + 40
*b^2*x^2 + 8*b^4*x^4 + 16*a*b*x*(5 + 2*b^2*x^2) + 8*a^2*(5 + 6*b^2*x^2))*Ar
cSinh[a + b*x]^2 - 16*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*
x + 6*a^2*b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^3 - 12*ArcSinh[a
+ b*x]^4)/(128*b)
```

Maple [B] time = 0.066, size = 592, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x)
```

```
[Out] 1/128*(-48-51*a^2+96*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^2*b
+80*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+102*arcsinh(b*x+a)*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*a+80*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)*x*b-240*arcsinh(b*x+a)^2*x*a*b+102*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*x*b-12*x^3*a*b^3-102*x*a*b-120*arcsinh(b*x+a)^2*x^2*b^2-3*a^4+12*a
rcsinh(b*x+a)^4-51*arcsinh(b*x+a)^2+36*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*x^2*a*b^2+36*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^2*b+
96*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^2*a*b^2+12*arcsinh(b*x+
a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3+32*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)*a^3-24*arcsinh(b*x+a)^2*x^4*b^4-18*x^2*a^2*b^2-12*x*a^3*b+32*ar
csinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3+12*arcsinh(b*x+a)*(b^2
```

```
*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3-96*arcsinh(b*x+a)^2*x^3*a*b^3-144*arcsinh
(b*x+a)^2*x^2*a^2*b^2-96*arcsinh(b*x+a)^2*x*a^3*b-51*b^2*x^2-120*arcsinh(b*
x+a)^2*a^2-24*arcsinh(b*x+a)^2*a^4-3*x^4*b^4)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.87411, size = 792, normalized size = 3.37

$$-\frac{3\,b^4x^4+12\,ab^3x^3+3\left(6\,a^2+17\right)b^2x^2-16\left(2\,b^3x^3+6\,ab^2x^2+2\,a^3+\left(6\,a^2+5\right)bx+5\,a\right)\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/128*(3*b^4*x^4 + 12*a*b^3*x^3 + 3*(6*a^2 + 17)*b^2*x^2 - 16*(2*b^3*x^3 +
 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 12*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4 + 6*(2*a^3 + 17*a)*b*x + 3*(8*b^4*x^4
+ 32*a*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*
a^2 + 17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^3*x^3
 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [A] time = 33.7999, size = 694, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-3*a**4*asinh(a + b*x)**2/(16*b) - 3*a**3*x*asinh(a + b*x)**2/4
- 3*a**3*x/32 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3
/(4*b) + 3*a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(32*b)
- 9*a**2*b*x**2*asinh(a + b*x)**2/8 - 9*a**2*b*x**2/64 + 3*a**2*x*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a**2*x*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a**2*asinh(a + b*x)**2/(16*b) -
 3*a*b**2*x**3*asinh(a + b*x)**2/4 - 3*a*b**2*x**3/32 + 3*a*b*x**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a*b*x**2*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a*x*asinh(a + b*x)**2/8 - 51
```

```
*a*x/64 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/(8*b)
+ 51*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(64*b) - 3*b**3*
x**4*asinh(a + b*x)**2/16 - 3*b**3*x**4/128 + b**2*x**3*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 3*b**2*x**3*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/32 - 15*b*x**2*asinh(a + b*x)**2/16 - 51*b*x**
2/128 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/8 + 51*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/64 + 3*asinh(a + b*x)*
*4/(32*b) - 51*asinh(a + b*x)**2/(128*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*a
sinh(a)**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

3.267 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=189

$$\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{32b}+\frac{15(a+bx)\sqrt{(a+bx)^2+1}}{64b}+\frac{\sinh^{-1}(a+bx)^3}{8b}+\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+bx)^2}{4b}+$$

```
[Out] (15*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(64*b) + ((a + b*x)*(1 + (a + b*x)^2)^
(3/2))/(32*b) - (9*ArcSinh[a + b*x])/(64*b) - (3*(a + b*x)^2*ArcSinh[a + b*
x])/(8*b) - ((1 + (a + b*x)^2)^2*ArcSinh[a + b*x])/(8*b) + (3*(a + b*x)*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)
^(3/2)*ArcSinh[a + b*x]^2)/(4*b) + ArcSinh[a + b*x]^3/(8*b)
```

Rubi [A] time = 0.190467, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5867, 5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{32b}+\frac{15(a+bx)\sqrt{(a+bx)^2+1}}{64b}+\frac{\sinh^{-1}(a+bx)^3}{8b}+\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+bx)^2}{4b}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (15*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(64*b) + ((a + b*x)*(1 + (a + b*x)^2)^
(3/2))/(32*b) - (9*ArcSinh[a + b*x])/(64*b) - (3*(a + b*x)^2*ArcSinh[a + b*
x])/(8*b) - ((1 + (a + b*x)^2)^2*ArcSinh[a + b*x])/(8*b) + (3*(a + b*x)*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)
^(3/2)*ArcSinh[a + b*x]^2)/(4*b) + ArcSinh[a + b*x]^3/(8*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2} \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2} \sinh^{-1}(a+bx)^2}{4b} - \frac{\text{Subst}\left(\int x\left(1+x^2\right) \sinh\right.}{2b}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2 \sinh^{-1}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a}{8b}$$
$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{3(a+bx)^2\sinh^{-1}(a+bx)}{8b} - \frac{\left(1+(a+bx\right.}{}$$
$$= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{3(a+bx)^2\text{si}}{8}$$
$$= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{9\sinh^{-1}(a+}{64b}$$

Mathematica [A] time = 0.155967, size = 211, normalized size = 1.12

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(6a^2bx+2a^3+6ab^2x^2+17a+2b^3x^3+17bx\right)+8\sqrt{a^2+2abx+b^2x^2+1}\left(6a^2bx+2a^3+6ab^2x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*b*x + 6*a
*b^2*x^2 + 2*b^3*x^3) - (17 + 40*a^2 + 8*a^4)*ArcSinh[a + b*x] - 8*b*x*(10*
a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*ArcSinh[a + b*x] + 8
*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b
^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^2 + 8*ArcSinh[a + b*x]^3)/(64*b)
```

Maple [B] time = 0.063, size = 479, normalized size = 2.5

$$\frac{1}{64\,b}\left(16\,(\text{Arcsinh}\,(bx+a))^2\sqrt{b^2x^2+2\,xab+a^2+1}x^3b^3 - 8\,\text{Arcsinh}\,(bx+a)\,x^4b^4 + 48\,(\text{Arcsinh}\,(bx+a))^2\sqrt{b^2x^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/64*(16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3-8*arcsinh(b
*x+a)*x^4*b^4+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^2*a*b^2-3
2*arcsinh(b*x+a)*x^3*a*b^3+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*x*a^2*b+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3-48*arcsinh(b*x+a)*x^2*a^2
*b^2+16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3+6*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)*x^2*a*b^2-32*arcsinh(b*x+a)*x*a^3*b+40*arcsinh(b*x+a)^2*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+6*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^2*b-40*a
rcsinh(b*x+a)*x^2*b^2-8*arcsinh(b*x+a)*a^4+40*arcsinh(b*x+a)^2*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*a+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-80*arcsinh(b*x+a)*x
*a*b+17*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+8*arcsinh(b*x+a)^3-40*arcsinh(b*x
+a)*a^2+17*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-17*arcsinh(b*x+a))/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.81868, size = 612, normalized size = 3.24

$$\frac{8\left(2\,b^3x^3+6\,ab^2x^2+2\,a^3+\left(6\,a^2+5\right)bx+5\,a\right)\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+8\,\log \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] 1/64*(8*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 8*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - (8*b^4*x^4 + 32*a
*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*a^2 +
17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (2*b^3*x^3 + 6*a*b^2
*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/
b
```

Sympy [A] time = 16.2136, size = 568, normalized size = 3.01

$$\begin{cases} -\frac{a^4\operatorname{asinh}(a+bx)}{8b}-\frac{a^3x\operatorname{asinh}(a+bx)}{2}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}}{32b}-\frac{3a^2bx^2\operatorname{asinh}(a+bx)}{4}+\frac{3a^2x\sqrt{a^2+2abx+b^2 \cdots}}{4} \cdots \\ x\left(a^2+1\right)^{\frac{3}{2}}\operatorname{asinh}^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(8*b) - a**3*x*asinh(a + b*x)/2 + a**3*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) + a**3*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)/(32*b) - 3*a**2*b*x**2*asinh(a + b*x)/4 + 3*a**2*
x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/4 + 3*a**2*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a**2*asinh(a + b*x)/(8*b) - a*b**2*x
**3*asinh(a + b*x)/2 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asin
h(a + b*x)**2/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a*
x*asinh(a + b*x)/4 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)**2/(8*b) + 17*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(64*b) - b**3*x**4*a
sinh(a + b*x)/8 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/4 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*b*x**2*as
inh(a + b*x)/8 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2
/8 + 17*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/64 + asinh(a + b*x)**3/(8*b)
```

```
 - 17*asinh(a + b*x)/(64*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*asinh(a)**2, T
rue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

3.268 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=106

$$-\frac{(a+bx)^4}{16b}-\frac{5(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)}{4b}+\frac{3\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)}{8b}+\frac{3\,\mathrm{si}}{}$$

```
[Out] (-5*(a + b*x)^2)/(16*b) - (a + b*x)^4/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b
*x)^2]*ArcSinh[a + b*x])/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh
[a + b*x])/(4*b) + (3*ArcSinh[a + b*x]^2)/(16*b)
```

Rubi [A] time = 0.103627, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5867, 5684, 5682, 5675, 30, 14}

$$-\frac{(a+bx)^4}{16b}-\frac{5(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)}{4b}+\frac{3\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)}{8b}+\frac{3\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-5*(a + b*x)^2)/(16*b) - (a + b*x)^4/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b
*x)^2]*ArcSinh[a + b*x])/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh
[a + b*x])/(4*b) + (3*ArcSinh[a + b*x]^2)/(16*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)}{4b} - \frac{\text{Subst}\left(\int x\left(1+x^2\right)\,dx, x, a\right.}{4b}$$
$$= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{8b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sin}{4b}$$
$$= -\frac{5(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{8b} + \frac{(a}{}$$

Mathematica [A] time = 0.090909, size = 124, normalized size = 1.17

$$\frac{-bx\left(6a^2bx+4a^3+4ab^2x^2+10a+b^3x^3+5bx\right)+2\sqrt{a^2+2abx+b^2x^2+1}\left(6a^2bx+2a^3+6ab^2x^2+5a+2b^3x^3+5bx\right)\text{s}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(10*a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)) + 2*Sqrt
[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b^2*x^
2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*ArcSinh[a + b*x]^2)/(16*b)
```

Maple [B] time = 0.055, size = 262, normalized size = 2.5

$$\frac{1}{16\,b}\left(4\,\text{Arcsinh}\,(bx+a)\,\sqrt{b^2x^2+2\,xab+a^2+1}x^3b^3 - x^4b^4 + 12\,\text{Arcsinh}\,(bx+a)\,\sqrt{b^2x^2+2\,xab+a^2+1}x^2ab^2 - 4\,x^3ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x)
```

```
[Out] 1/16*(4*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3-x^4*b^4+12*arc
sinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^2*a*b^2-4*x^3*a*b^3+12*arcsinh(
b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^2*b-6*x^2*a^2*b^2+4*arcsinh(b*x+a)
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-4*x*a^3*b+10*arcsinh(b*x+a)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*x*b-5*b^2*x^2-a^4+10*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+
```

```
1)^(1/2)*a-10*x*a*b+3*arcsinh(b*x+a)^2-5*a^2-4)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69118, size = 374, normalized size = 3.53

$$-\frac{b^4x^4 + 4\,ab^3x^3 + \left(6\,a^2 + 5\right)b^2x^2 + 2\left(2\,a^3 + 5\,a\right)bx - 2\left(2\,b^3x^3 + 6\,ab^2x^2 + 2\,a^3 + \left(6\,a^2 + 5\right)bx + 5\,a\right)\sqrt{b^2x^2 + 2\,abx +}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] -1/16*(b^4*x^4 + 4*a*b^3*x^3 + (6*a^2 + 5)*b^2*x^2 + 2*(2*a^3 + 5*a)*b*x -
2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)/b
```

Sympy [A] time = 7.3441, size = 298, normalized size = 2.81

$$\begin{cases} -\frac{a^3x}{4} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4b} - \frac{3a^2bx^2}{8} + \frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4} - \frac{ab^2x^3}{4} + \frac{3abx^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4} - \\ x\left(a^2+1\right)^{\frac{3}{2}}\operatorname{asinh}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**3*x/4 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)/(4*b) - 3*a**2*b*x**2/8 + 3*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)*asinh(a + b*x)/4 - a*b**2*x**3/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*asinh(a + b*x)/4 - 5*a*x/8 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1)*asinh(a + b*x)/(8*b) - b**3*x**4/16 + b**2*x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*asinh(a + b*x)/4 - 5*b*x**2/16 + 5*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/8 + 3*asinh(a + b*x)**2/(16*b), Ne(b, 0)), (x*
(a**2 + 1)**(3/2)*asinh(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a), x)
```

3.269 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\mathrm{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + CoshIntegral[4*ArcSinh[a + b*x]]/(
8*b) + (3*Log[ArcSinh[a + b*x]])/(8*b)
```

Rubi [A] time = 0.14352, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5867, 5699, 3312, 3301}

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\mathrm{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + CoshIntegral[4*ArcSinh[a + b*x]]/(
8*b) + (3*Log[ArcSinh[a + b*x]])/(8*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{8b} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\right.}{}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b} + \frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Mathematica [A] time = 0.286448, size = 37, normalized size = 0.79

$$\frac{4\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+\text{Chi}\left(4\sinh^{-1}(a+bx)\right)+3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] (4*CoshIntegral[2*ArcSinh[a + b*x]] + CoshIntegral[4*ArcSinh[a + b*x]] + 3*
Log[ArcSinh[a + b*x]])/(8*b)
```

Maple [A] time = 0.048, size = 42, normalized size = 0.9

$$\frac{\text{Chi}(2\,\text{Arcsinh}(bx+a))}{2\,b}+\frac{\text{Chi}(4\,\text{Arcsinh}(bx+a))}{8\,b}+\frac{3\,\ln(\text{Arcsinh}(bx+a))}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/8*Chi(4*arcsinh(b*x+a))/b+3/8*ln(arcsinh(b*x+
a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\text{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

3.270 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^2} \, dx$

Optimal. Leaf size=54

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}-\frac{\left((a+bx)^2+1\right)^2}{b\sinh^{-1}(a+bx)}$$

```
[Out] -((1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*
x]]/b + SinhIntegral[4*ArcSinh[a + b*x]]/(2*b)
```

Rubi [A] time = 0.156555, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5867, 5696, 5779, 5448, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}-\frac{\left((a+bx)^2+1\right)^2}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*
x]]/b + SinhIntegral[4*ArcSinh[a + b*x]]/(2*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sinh^{-1}(x)^2}\,dx,x,a+bx\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)}+\frac{4\,\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\sinh^{-1}(x)}\,dx,x,a+bx\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)}+\frac{4\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)}+\frac{4\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx,x,\sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)}+\frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx,x,\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,d\right.}{}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}$$

Mathematica [A] time = 0.264959, size = 70, normalized size = 1.3

$$\frac{-2\left(a^2+2abx+b^2x^2+1\right)^2+2\sinh^{-1}(a+bx)\text{Shi}\left(2\sinh^{-1}(a+bx)\right)+\sinh^{-1}(a+bx)\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*(1 + a^2 + 2*a*b*x + b^2*x^2)^2 + 2*ArcSinh[a + b*x]*SinhIntegral[2*Arc
Sinh[a + b*x]] + ArcSinh[a + b*x]*SinhIntegral[4*ArcSinh[a + b*x]])/(2*b*Ar
cSinh[a + b*x])
```

Maple [A] time = 0.05, size = 72, normalized size = 1.3

$$\frac{8\,\text{Shi}\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\text{Arcsinh}\left(bx+a\right)+4\,\text{Shi}\left(4\,\text{Arcsinh}\left(bx+a\right)\right)\text{Arcsinh}\left(bx+a\right)-4\,\cosh\left(2\,\text{Arcsinh}\left(bx+\right.\right.}{8\,b\text{Arcsinh}\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/8/b*(8*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)+4*Shi(4*arcsinh(b*x+a))*arcsi
nh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/arcsinh(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2 + 4*(a^3*b
 + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^5*x^5 + 5*a*b^4*x^4 + a^5
 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b
 + 6*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b
^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((4*b^4*x^4 + 16*a*b
^3*x^3 + 4*a^4 + 3*(8*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(8*a^3*b + 3*a*b)*x -
1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(2*b^5*x^5 + 10*a*b^4*x^4 + 2*a^
5 + (20*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + (20*a^3*b^2 + 9*a*b^2)*x^2 + (10*a^4
*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^6*x^6 + 24*a*
b^5*x^5 + 4*a^6 + 3*(20*a^2*b^4 + 3*b^4)*x^4 + 9*a^4 + 4*(20*a^3*b^3 + 9*a*
b^3)*x^3 + 6*(10*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(2*a^5*b + 3*a
^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x
^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + 2*a*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 +
a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx + a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}{\operatorname{asinh}^2\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

3.271 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{b}-\frac{\left((a+bx)^2+1\right)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{2(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b\sinh^{-1}(a+bx)}$$

```
[Out] -(1 + (a + b*x)^2)^2/(2*b*ArcSinh[a + b*x]^2) - (2*(a + b*x)*(1 + (a + b*x)
^2)^(3/2))/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b + Cosh
Integral[4*ArcSinh[a + b*x]]/b
```

Rubi [A] time = 0.280703, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5867, 5696, 5777, 5699, 3312, 3301, 5779, 5448}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{b}-\frac{\left((a+bx)^2+1\right)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{2(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```


```
[Out] -(1 + (a + b*x)^2)^2/(2*b*ArcSinh[a + b*x]^2) - (2*(a + b*x)*(1 + (a + b*x)
^2)^(3/2))/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b + Cosh
Integral[4*ArcSinh[a + b*x]]/b
```


Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} + \frac{2\,\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \sinh^{-1}\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x,\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{\textbf{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b} + \textbf{Chi}\left(\right.$$

Mathematica [A] time = 0.359694, size = 108, normalized size = 1.29

$$\frac{-\frac{\left(a^2+2abx+b^2x^2+1\right)\left(4(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)+a^2+2abx+b^2x^2+1\right)}{\sinh^{-1}(a+bx)^2} + 2\textbf{Chi}\left(2\sinh^{-1}(a+bx)\right) + 2\textbf{Chi}\left(4\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] (-(((1 + a^2 + 2*a*b*x + b^2*x^2)*(1 + a^2 + 2*a*b*x + b^2*x^2 + 4*(a + b*x
)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2)
+ 2*CoshIntegral[2*ArcSinh[a + b*x]] + 2*CoshIntegral[4*ArcSinh[a + b*x]])/
(2*b)
```

Maple [A] time = 0.052, size = 110, normalized size = 1.3

$$\frac{16\,\text{Chi}\left(2\,\text{Arcsinh}\left(bx+a\right)\right)\left(\text{Arcsinh}\left(bx+a\right)\right)^2 + 16\,\text{Chi}\left(4\,\text{Arcsinh}\left(bx+a\right)\right)\left(\text{Arcsinh}\left(bx+a\right)\right)^2 - 8\,\sinh\left(2\,\text{Arcsinh}\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/16/b*(16*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+16*Chi(4*arcsinh(b*x+a))*
arcsinh(b*x+a)^2-8*sinh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-4*sinh(4*arcsinh(b
*x+a))*arcsinh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/ar
csinh(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^7*x^7 + 2
1*a*b^6*x^6 + 3*a^7 + (63*a^2*b^5 + 8*b^5)*x^5 + 8*a^5 + 5*(21*a^3*b^4 + 8*
a*b^4)*x^4 + (105*a^4*b^3 + 80*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (63*a^5*b^2 +
 80*a^3*b^2 + 21*a*b^2)*x^2 + (21*a^6*b + 40*a^4*b + 21*a^2*b + 2*b)*x + 2*
a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^8*x^8 + 24*a*b^7*x^7 + 3*a^8
+ 2*(42*a^2*b^6 + 5*b^6)*x^6 + 10*a^6 + 12*(14*a^3*b^5 + 5*a*b^5)*x^5 + 6*(
35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 12*a^4 + 8*(21*a^5*b^3 + 25*a^3*b^3
+ 6*a*b^3)*x^3 + 6*(14*a^6*b^2 + 25*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + 6*a^2
 + 12*(2*a^7*b + 5*a^5*b + 4*a^3*b + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 +
 1) + ((4*b^6*x^6 + 24*a*b^5*x^5 + 4*a^6 + (60*a^2*b^4 + 7*b^4)*x^4 + 7*a^4
 + 4*(20*a^3*b^3 + 7*a*b^3)*x^3 + 2*(30*a^4*b^2 + 21*a^2*b^2 + b^2)*x^2 + 2
*a^2 + 4*(6*a^5*b + 7*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 +
 3*(4*b^7*x^7 + 28*a*b^6*x^6 + 4*a^7 + 3*(28*a^2*b^5 + 3*b^5)*x^5 + 9*a^5 +
 5*(28*a^3*b^4 + 9*a*b^4)*x^4 + 2*(70*a^4*b^3 + 45*a^2*b^3 + 3*b^3)*x^3 + 6
*a^3 + 6*(14*a^5*b^2 + 15*a^3*b^2 + 3*a*b^2)*x^2 + (28*a^6*b + 45*a^4*b + 1
8*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (12*b^8*x^8 + 96*
a*b^7*x^7 + 12*a^8 + 3*(112*a^2*b^6 + 11*b^6)*x^6 + 33*a^6 + 6*(112*a^3*b^5
 + 33*a*b^5)*x^5 + (840*a^4*b^4 + 495*a^2*b^4 + 31*b^4)*x^4 + 31*a^4 + 4*(1
68*a^5*b^3 + 165*a^3*b^3 + 31*a*b^3)*x^3 + (336*a^6*b^2 + 495*a^4*b^2 + 186
*a^2*b^2 + 11*b^2)*x^2 + 11*a^2 + 2*(48*a^7*b + 99*a^5*b + 62*a^3*b + 11*a*
b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^9*x^9 + 36*a*b^8*x^8 + 4*a^9
 + (144*a^2*b^7 + 13*b^7)*x^7 + 13*a^7 + 7*(48*a^3*b^6 + 13*a*b^6)*x^6 + 3*
(168*a^4*b^5 + 91*a^2*b^5 + 5*b^5)*x^5 + 15*a^5 + (504*a^5*b^4 + 455*a^3*b^
4 + 75*a*b^4)*x^4 + (336*a^6*b^3 + 455*a^4*b^3 + 150*a^2*b^3 + 7*b^3)*x^3 +
 7*a^3 + 3*(48*a^7*b^2 + 91*a^5*b^2 + 50*a^3*b^2 + 7*a*b^2)*x^2 + (36*a^8*b
 + 91*a^6*b + 75*a^4*b + 21*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (b^9*x^9 + 9*a*b^8
*x^8 + a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 4*a^7 + 28*(3*a^3*b^6 + a*b^6)*x^6 +
 6*(21*a^4*b^5 + 14*a^2*b^5 + b^5)*x^5 + 6*a^5 + 2*(63*a^5*b^4 + 70*a^3*b^4
 + 15*a*b^4)*x^4 + 4*(21*a^6*b^3 + 35*a^4*b^3 + 15*a^2*b^3 + b^3)*x^3 + 4*a
^3 + 12*(3*a^7*b^2 + 7*a^5*b^2 + 5*a^3*b^2 + a*b^2)*x^2 + (9*a^8*b + 28*a^6
*b + 30*a^4*b + 12*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b
^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3
*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*
x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2
*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2
 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b
^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)
*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2) + integrate(1/2*((16*b^6*x^6 + 96*a*b^5*x^5 + 16*a^6 +
10*(24*a^2*b^4 + b^4)*x^4 + 10*a^4 + 40*(8*a^3*b^3 + a*b^3)*x^3 + 3*(80*a^4
*b^2 + 20*a^2*b^2 - b^2)*x^2 - 3*a^2 + 2*(48*a^5*b + 20*a^3*b - 3*a*b)*x +
3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(16*b^7*x^7 + 112*a*b^6*x^6 + 16
*a^7 + (336*a^2*b^5 + 23*b^5)*x^5 + 23*a^5 + 5*(112*a^3*b^4 + 23*a*b^4)*x^4
 + (560*a^4*b^3 + 230*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (336*a^5*b^2 + 230*a^3
*b^2 + 21*a*b^2)*x^2 + (112*a^6*b + 115*a^4*b + 21*a^2*b)*x)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^2 + 12*(8*b^8*x^8 + 64*a*b^7*x^7 + 8*a^8 + 2*(112*a^2*b^6 +
```

```
 9*b^6)*x^6 + 18*a^6 + 4*(112*a^3*b^5 + 27*a*b^5)*x^5 + (560*a^4*b^4 + 270*
a^2*b^4 + 13*b^4)*x^4 + 13*a^4 + 4*(112*a^5*b^3 + 90*a^3*b^3 + 13*a*b^3)*x^
3 + (224*a^6*b^2 + 270*a^4*b^2 + 78*a^2*b^2 + 3*b^2)*x^2 + 3*a^2 + 2*(32*a^
7*b + 54*a^5*b + 26*a^3*b + 3*a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 4*(16*b^9*x^9 + 144*a*b^8*x^8 + 16*a^9 + (576*a^2*b^7 + 49*b^7)*x^7 + 49*a
^7 + 7*(192*a^3*b^6 + 49*a*b^6)*x^6 + 3*(672*a^4*b^5 + 343*a^2*b^5 + 18*b^5
)*x^5 + 54*a^5 + (2016*a^5*b^4 + 1715*a^3*b^4 + 270*a*b^4)*x^4 + (1344*a^6*
b^3 + 1715*a^4*b^3 + 540*a^2*b^3 + 25*b^3)*x^3 + 25*a^3 + 3*(192*a^7*b^2 +
343*a^5*b^2 + 180*a^3*b^2 + 25*a*b^2)*x^2 + (144*a^8*b + 343*a^6*b + 270*a^
4*b + 75*a^2*b + 4*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (16*b^10*x^1
0 + 160*a*b^9*x^9 + 16*a^10 + 2*(360*a^2*b^8 + 31*b^8)*x^8 + 62*a^8 + 16*(1
20*a^3*b^7 + 31*a*b^7)*x^7 + 7*(480*a^4*b^6 + 248*a^2*b^6 + 13*b^6)*x^6 + 9
1*a^6 + 14*(288*a^5*b^5 + 248*a^3*b^5 + 39*a*b^5)*x^5 + (3360*a^6*b^4 + 434
0*a^4*b^4 + 1365*a^2*b^4 + 61*b^4)*x^4 + 61*a^4 + 4*(480*a^7*b^3 + 868*a^5*
b^3 + 455*a^3*b^3 + 61*a*b^3)*x^3 + (720*a^8*b^2 + 1736*a^6*b^2 + 1365*a^4*
b^2 + 366*a^2*b^2 + 17*b^2)*x^2 + 17*a^2 + 2*(80*a^9*b + 248*a^7*b + 273*a^
5*b + 122*a^3*b + 17*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x
^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3
*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^
3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*
a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2
+ 9*a^2*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3
)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 +
 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^
2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b
^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^
4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b
^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx + a\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}{\operatorname{asinh}^3\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**3,x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}}{\operatorname{arsinh}(bx + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

3.272 $\int \frac{\sinh^{-1}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0690976, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.0186737, size = 15, normalized size = 1.

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Maple [A] time = 0.049, size = 14, normalized size = 0.9

$$\frac{(\text{Arcsinh}\,(bx+a))^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/4*arcsinh(b*x+a)^4/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56151, size = 78, normalized size = 5.2

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4/b
```

Sympy [A] time = 1.40799, size = 26, normalized size = 1.73

$$\begin{cases} \frac{\operatorname{asinh}^4(a+bx)}{4b} & \text{for } b \neq 0 \\ \frac{x\operatorname{asinh}^3(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*asinh(a)**3/sqrt(a**2 + 1
), True))
```

Giac [B] time = 1.71263, size = 43, normalized size = 2.87

$$\frac{\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4/b
```

3.273 $\int \frac{\sinh^{-1}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

```
[Out] ArcSinh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0680001, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ArcSinh[a + b*x]^3/(3*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.017671, size = 15, normalized size = 1.

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.04, size = 14, normalized size = 0.9

$$\frac{(\text{Arcsinh}\,(bx+a))^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/3*arcsinh(b*x+a)^3/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.58049, size = 78, normalized size = 5.2

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/b
```

Sympy [A] time = 0.988957, size = 26, normalized size = 1.73

$$\begin{cases} \frac{\operatorname{asinh}^3(a+bx)}{3b} & \text{for } b \neq 0 \\ \frac{x\operatorname{asinh}^2(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*asinh(a)**2/sqrt(a**2 + 1
), True))
```

Giac [B] time = 1.50459, size = 43, normalized size = 2.87

$$\frac{\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/b
```

3.274 $\int \frac{\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

```
[Out] ArcSinh[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.042129, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5867, 5675}

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^2/(2*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0149427, size = 15, normalized size = 1.

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^2/(2*b)
```

Maple [A] time = 0.045, size = 14, normalized size = 0.9

$$\frac{(\mathrm{Arcsinh}\,(bx+a))^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(b*x+a)^2/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70558, size = 78, normalized size = 5.2

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/b
```

Sympy [A] time = 0.820833, size = 24, normalized size = 1.6

$$\begin{cases} \frac{\operatorname{asinh}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ \frac{x \operatorname{asinh}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*asinh(a)/sqrt(a**2 + 1),
True))
```

Giac [B] time = 1.39068, size = 43, normalized size = 2.87

$$\frac{\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/b
```

3.275 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] Log[ArcSinh[a + b*x]]/b
```

Rubi [A] time = 0.0751373, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5673}

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

```
[Out] Log[ArcSinh[a + b*x]]/b
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\ &= \frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0304706, size = 11, normalized size = 1.

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

```
[Out] Log[ArcSinh[a + b*x]]/b
```

Maple [A] time = 0.043, size = 12, normalized size = 1.1

$$\frac{\ln(\operatorname{Arcsinh}(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] ln(arcsinh(b*x+a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

Fricas [B] time = 2.63541, size = 77, normalized size = 7.

$$\frac{\log\left(\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] log(log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [A] time = 1.17822, size = 22, normalized size = 2.

$$\begin{cases}\frac{\log(\operatorname{asinh}(a+bx))}{b} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((log(asinh(a + b*x))/b, Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)),
True))
```

Giac [B] time = 1.28502, size = 42, normalized size = 3.82

$$\frac{\log\left(\left|\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] log(abs(log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))))/b
```

3.276 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

Rubi [A] time = 0.0747208, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\ &= -\frac{1}{b\sinh^{-1}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0140858, size = 13, normalized size = 1.

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

Maple [A] time = 0.042, size = 14, normalized size = 1.1

$$-\frac{1}{b\mathrm{Arcsinh}\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/b/arcsinh(b*x+a)
```

Maxima [B] time = 1.32074, size = 203, normalized size = 15.62

$$-\frac{b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}+a}{\left(\left(b^2x^2+2\,abx+a^2+1\right)\left(b^2x+ab\right)+\left(b^3x^2+2\,ab^2x+a^2b+b\right)\sqrt{b^2x^2+2\,abx+a^2+1}\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/(((b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + (b^3*x^2 +
2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

Fricas [B] time = 2.67799, size = 77, normalized size = 5.92

$$-\frac{1}{b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

Sympy [A] time = 1.73364, size = 26, normalized size = 2.

$$\begin{cases} -\frac{1}{b\,\mathrm{asinh}\,(a+bx)} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\,\mathrm{asinh}^2\,(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*asinh(a + b*x)), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)**2
), True))
```

Giac [B] time = 1.37482, size = 43, normalized size = 3.31

$$-\frac{1}{b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

3.277 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

```
[Out] -1/(2*b*ArcSinh[a + b*x]^2)
```

Rubi [A] time = 0.0702373, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

```
[Out] -1/(2*b*ArcSinh[a + b*x]^2)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\ &= -\frac{1}{2b\sinh^{-1}(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0133136, size = 15, normalized size = 1.

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

```
[Out] -1/(2*b*ArcSinh[a + b*x]^2)
```

Maple [A] time = 0.043, size = 14, normalized size = 0.9

$$-\frac{1}{2\,b\,(\mathrm{Arcsinh}\,(bx+a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/2/b/arcsinh(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^5*x^
5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*
a*b^2)*x^2 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) - (b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (5*a^4*b + 6*a^
2*b + b)*x + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2
 + 4*(a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6
+ (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^3*b^3 + 7*a*b^3)*x^3 + (45*a^4
*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9*a^5*b + 14*a^3*b + 5*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/(((b^4*x^3 + 3*a*b^3*x^2 + 3*a^2
*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 3*(b^5*x^4 + 4*a*b^4*x^3
+ a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b +
2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 +
6*a^2*b^2 + b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^7*x^6 + 6*a*b^6*x^5
+ a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3 +
 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + 6*(a^5*b^2 + 2*a^3*b^2 + a
*b^2)*x + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2) + integrate(-1/2*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6
 + 3*(10*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4*(10*a^3*b^3 + 3*a*b^3)*x^3 - (2*b^2
*x^2 + 4*a*b*x + 2*a^2 + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(5*a^4*b^2
+ 3*a^2*b^2)*x^2 - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*(b
^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 12*
(a^5*b + a^3*b)*x + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^
3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(b^5*x^5 + 5*
```

```
a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2
 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(b^6*x^6 + 6*
a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3
)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b +
a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^
7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4
*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^
2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2
 + 1) + (b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*
(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4
 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9
*a^2*b^2 + b^2)*x^2 + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 1)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))), x)
```

Fricas [B] time = 2.61269, size = 82, normalized size = 5.47

$$-\frac{1}{2\,b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
```

Sympy [A] time = 2.6884, size = 29, normalized size = 1.93

$$\begin{cases} -\frac{1}{2b\operatorname{asinh}^2(a+bx)} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}^3(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(2*b*asinh(a + b*x)**2), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(
a)**3), True))
```

Giac [B] time = 1.40197, size = 43, normalized size = 2.87

$$-\frac{1}{2\,b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/2/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
```

3.278 $\int \frac{\sinh^{-1}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{3\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{3\text{PolyLog}\left(3,-e^{2\sinh^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+}{b}$$

```
[Out] ArcSinh[a + b*x]^3/b + ((a + b*x)*ArcSinh[a + b*x]^3)/(b*Sqrt[1 + (a + b*x)
^2]) - (3*ArcSinh[a + b*x]^2*Log[1 + E^(2*ArcSinh[a + b*x])])/b - (3*ArcSin
h[a + b*x]*PolyLog[2, -E^(2*ArcSinh[a + b*x])])/b + (3*PolyLog[3, -E^(2*Arc
Sinh[a + b*x])])/(2*b)
```

Rubi [A] time = 0.206168, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5867, 5687, 5714, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{3\text{PolyLog}\left(3,-e^{2\sinh^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] ArcSinh[a + b*x]^3/b + ((a + b*x)*ArcSinh[a + b*x]^3)/(b*Sqrt[1 + (a + b*x)
^2]) - (3*ArcSinh[a + b*x]^2*Log[1 + E^(2*ArcSinh[a + b*x])])/b - (3*ArcSin
h[a + b*x]*PolyLog[2, -E^(2*ArcSinh[a + b*x])])/b + (3*PolyLog[3, -E^(2*Arc
Sinh[a + b*x])])/(2*b)
```


Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(1+x^2\right)^{3/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)^2}{1+x^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\,\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{6\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2\sinh^{-1}(a+bx)}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2\sinh^{-1}(a+bx)}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2\sinh^{-1}(a+bx)}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2\sinh^{-1}(a+bx)}\right.}{b}$$

Mathematica [A] time = 0.569996, size = 128, normalized size = 1.11

$$\frac{6\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(a+bx)}\right)+3\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(a+bx)}\right)+2\sinh^{-1}(a+bx)^2\left(\frac{\left(-\sqrt{a^2+2abx+b^2x^2+1}+a+\right.}{\sqrt{a^2+2abx+b}}\right.}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*ArcSinh[a + b*x]^2*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSi
nh[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*Log[1 + E^(-2*ArcSinh[a
+ b*x])]) + 6*ArcSinh[a + b*x]*PolyLog[2, -E^(-2*ArcSinh[a + b*x])] + 3*Pol
yLog[3, -E^(-2*ArcSinh[a + b*x])])/(2*b)
```

Maple [A] time = 0.1, size = 203, normalized size = 1.8

$$-\frac{(\text{Arcsinh}(bx+a))^3}{b\left(b^2x^2+2xab+a^2+1\right)}\left(b^2x^2-\sqrt{b^2x^2+2xab+a^2+1}xb+2xab-\sqrt{b^2x^2+2xab+a^2+1}a+a^2+1\right)+2\frac{(\text{Arcsinh}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+2*x*a*b-(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^3+2*arcsinh(b*x+a)
```

```
^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-3*arcsinh(b*x
+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3,-(b*x+a+(1+(b
*x+a)^2)^(1/2))^2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\text{arsinh}\,(bx + a)^3}{b^4x^4 + 4\,ab^3x^3 + 2\left(3\,a^2 + 1\right)b^2x^2 + a^4 + 4\left(a^3 + a\right)bx + 2\,a^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3\left(a + bx\right)}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\,(bx + a)^3}{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

3.279 $\int \frac{\sinh^{-1}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{\text{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^2}{b}-\frac{2\sinh^{-1}(a+bx)\log\left(e^{2\sinh^{-1}(a+bx)}+1\right.}{b}$$

```
[Out] ArcSinh[a + b*x]^2/b + ((a + b*x)*ArcSinh[a + b*x]^2)/(b*Sqrt[1 + (a + b*x)
^2]) - (2*ArcSinh[a + b*x]*Log[1 + E^(2*ArcSinh[a + b*x])])/b - PolyLog[2,
-E^(2*ArcSinh[a + b*x])]/b
```

Rubi [A] time = 0.160093, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5867, 5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^2}{b}-\frac{2\sinh^{-1}(a+bx)\log\left(e^{2\sinh^{-1}(a+bx)}+1\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```


```
[Out] ArcSinh[a + b*x]^2/b + ((a + b*x)*ArcSinh[a + b*x]^2)/(b*Sqrt[1 + (a + b*x)
^2]) - (2*ArcSinh[a + b*x]*Log[1 + E^(2*ArcSinh[a + b*x])])/b - PolyLog[2,
-E^(2*ArcSinh[a + b*x])]/b
```


Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(1+x^2\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)}{1+x^2}\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{4\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh^{-1}(a+bx)}\right)}{b}\\
&= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh^{-1}(a+bx)}\right)}{b}\\
&= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.381727, size = 98, normalized size = 1.14

$$\frac{\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(a+bx)}\right)+\sinh^{-1}(a+bx)\left(\frac{\left(-\sqrt{a^2+2abx+b^2x^2+1}+a+bx\right)\sinh^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}-2\log\left(e^{-2\sinh^{-1}(a+bx)}+1\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (ArcSinh[a + b*x]*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSinh[a
 + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 2*Log[1 + E^(-2*ArcSinh[a + b*
```

```
x])]) + PolyLog[2, -E^(-2*ArcSinh[a + b*x])])/b
```

Maple [A] time = 0.096, size = 168, normalized size = 2.

$$-\frac{(\operatorname{Arcsinh}(bx+a))^2}{b\left(b^2x^2+2\,xab+a^2+1\right)}\left(b^2x^2-\sqrt{b^2x^2+2\,xab+a^2+1}xb+2\,xab-\sqrt{b^2x^2+2\,xab+a^2+1}a+a^2+1\right)+2\,\frac{(\operatorname{Arcsi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+2*x*a*b-(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^2+2*arcsinh(b*x+a)
^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-polylog(2,-(b*x
+a+(1+(b*x+a)^2)^(1/2))^2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^2}{b^4x^4+4\,ab^3x^3+2\left(3\,a^2+1\right)b^2x^2+a^4+4\left(a^3+a\right)bx+2\,a^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}^2(a+bx)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**2/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

3.280 $\int \frac{\sinh^{-1}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=46

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{(a+bx)^2+1}} - \frac{\log\left((a+bx)^2+1\right)}{2b}$$

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + (a + b*x)^2]) - Log[1 + (a + b*x)^
2]/(2*b)
```

Rubi [A] time = 0.0575618, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5867, 5687, 260}

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{(a+bx)^2+1}} - \frac{\log\left((a+bx)^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + (a + b*x)^2]) - Log[1 + (a + b*x)^
2]/(2*b)
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(1+x^2\right)^{3/2}}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

Mathematica [A] time = 0.0887991, size = 62, normalized size = 1.35

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{a^2+2abx+b^2x^2+1}} - \frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) - Log[1
+ a^2 + 2*a*b*x + b^2*x^2]/(2*b)
```

Maple [B] time = 0.094, size = 131, normalized size = 2.9

$$2\,\frac{\text{Arcsinh}\,(bx+a)}{b} - \frac{\text{Arcsinh}\,(bx+a)}{b\left(b^2x^2+2\,xab+a^2+1\right)}\left(b^2x^2-\sqrt{b^2x^2+2\,xab+a^2+1}xb+2\,xab-\sqrt{b^2x^2+2\,xab+a^2+1}a+a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x)
```

```
[Out] 2*arcsinh(b*x+a)/b-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+2*x*a*b-(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)-
1/b*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.80635, size = 274, normalized size = 5.96

$$\frac{2\sqrt{b^2x^2 + 2\,abx + a^2 + 1}(bx + a)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) - \left(b^2x^2 + 2\,abx + a^2 + 1\right)\log\left(b^2x^2 + 2\,abx + \right.}{2\left(b^3x^2 + 2\,ab^2x + \left(a^2 + 1\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)) - (b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1))/(b^3*x^2 + 2*a*b^2*x + (a^2 + 1)*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(a + bx)}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [A] time = 1.31941, size = 103, normalized size = 2.24

$$\frac{\left(x + \frac{a}{b}\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}} - \frac{\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] (x + a/b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

3.281 $\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable[1/((1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]), x]
```

Rubi [A] time = 0.0821379, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((1 + x^2)^(3/2)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx=\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}\sinh^{-1}(x)}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.551142, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
```

```
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]), x]
```

Maple [A] time = 0.113, size = 0, normalized size = 0.

$$\int \frac{1}{\text{Arcsinh}\,(bx+a)}\left(b^2x^2+2\,xab+a^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

```
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{\left(b^4x^4 + 4\,ab^3x^3 + 2\left(3\,a^2 + 1\right)b^2x^2 + a^4 + 4\left(a^3 + a\right)bx + 2\,a^2 + 1\right)\operatorname{arsinh}\left(bx + a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}} \operatorname{asinh}\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac
")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

3.282 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=54

$$-2\text{Unintegrable}\left(\frac{a+bx}{\left((a+bx)^2+1\right)^2\sinh^{-1}(a+bx)},x\right)-\frac{1}{b\left((a+bx)^2+1\right)\sinh^{-1}(a+bx)}$$

```
[Out] -(1/(b*(1 + (a + b*x)^2)*ArcSinh[a + b*x])) - 2*Unintegrable[(a + b*x)/((1
+ (a + b*x)^2)^2*ArcSinh[a + b*x]), x]
```

Rubi [A] time = 0.116512, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*(1 + (a + b*x)^2)*ArcSinh[a + b*x])) - (2*Defer[Subst][Defer[Int][x/
((1 + x^2)^2*ArcSinh[x]), x], x, a + b*x])/b
```

Rubi steps

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\sinh^{-1}(x)^2}\,dx,x,a+bx\right)}{b}$$

$$=-\frac{1}{b\left(1+(a+bx)^2\right)\sinh^{-1}(a+bx)}-\frac{2\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2\sinh^{-1}(x)}\,dx,x,a\right.}{b}$$

Mathematica [A] time = 2.69618, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2), x]
```

Maple [A] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\text{Arcsinh}\,(bx+a)\right)^2}\left(b^2x^2+2\,xab+a^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

```
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{\left(\left(b^2x^2 + 2\,abx + a^2 + 1\right)\left(b^2x + ab\right) + \left(b^3x^2 + 2\,ab^2x + a^2b + b\right)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="ma
xima")
```

```
[Out] -(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(((b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b^2*x + a*b) + (b^3*x^2 + 2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) - integrate
((2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + (2*b^2*x^2 + 4
*a*b*x + 2*a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + a^2 + 2*(4*a^3*b + a*b)
*x + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^
2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2
*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*
b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + (b^6*x^6 + 6*a*b^5*x^5 + a^6 + 3*(5*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4
*(5*a^3*b^3 + 3*a*b^3)*x^3 + 3*(5*a^4*b^2 + 6*a^2*b^2 + b^2)*x^2 + 3*a^2 +
6*(a^5*b + 2*a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{\left(b^4x^4 + 4\,ab^3x^3 + 2\left(3\,a^2 + 1\right)b^2x^2 + a^4 + 4\left(a^3 + a\right)bx + 2\,a^2 + 1\right)\operatorname{arsinh}\left(bx + a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)^2), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}\operatorname{asinh}^2\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2), x)
```

3.283 $\int x^3 \sinh^{-1}(ax^2)\, dx$

Optimal. Leaf size=50

$$-\frac{x^2\sqrt{a^2x^4+1}}{8a}+\frac{\sinh^{-1}(ax^2)}{8a^2}+\frac{1}{4}x^4\sinh^{-1}(ax^2)$$

```
[Out] -(x^2*Sqrt[1 + a^2*x^4])/(8*a) + ArcSinh[a*x^2]/(8*a^2) + (x^4*ArcSinh[a*x^
2])/4
```

Rubi [A] time = 0.0365349, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5902, 12, 275, 321, 215}

$$-\frac{x^2\sqrt{a^2x^4+1}}{8a}+\frac{\sinh^{-1}(ax^2)}{8a^2}+\frac{1}{4}x^4\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x^2],x]
```

```
[Out] -(x^2*Sqrt[1 + a^2*x^4])/(8*a) + ArcSinh[a*x^2]/(8*a^2) + (x^4*ArcSinh[a*x^
2])/4
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \sinh^{-1}\left(ax^2\right)\,dx &= \frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)-\frac{1}{4}\int \frac{2ax^5}{\sqrt{1+a^2x^4}}\,dx\\
&=\frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)-\frac{1}{2}a\int \frac{x^5}{\sqrt{1+a^2x^4}}\,dx\\
&=\frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)-\frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx,x,x^2\right)\\
&=-\frac{x^2\sqrt{1+a^2x^4}}{8a}+\frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}}\,dx,x,x^2\right)}{8a}\\
&=-\frac{x^2\sqrt{1+a^2x^4}}{8a}+\frac{\sinh^{-1}\left(ax^2\right)}{8a^2}+\frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0183354, size = 44, normalized size = 0.88

$$\frac{\left(2a^2x^4+1\right)\sinh^{-1}\left(ax^2\right)-ax^2\sqrt{a^2x^4+1}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x^2],x]
```

```
[Out] (-(a*x^2*Sqrt[1 + a^2*x^4]) + (1 + 2*a^2*x^4)*ArcSinh[a*x^2])/(8*a^2)
```

Maple [A] time = 0.03, size = 67, normalized size = 1.3

$$\frac{x^4\text{Arcsinh}\left(ax^2\right)}{4}-\frac{x^2}{8\,a}\sqrt{a^2x^4+1}+\frac{1}{8\,a}\ln\left(a^2x^2\frac{1}{\sqrt{a^2}}+\sqrt{a^2x^4+1}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x^2),x)
```

```
[Out] 1/4*x^4*arcsinh(a*x^2)-1/8*x^2*(a^2*x^4+1)^(1/2)/a+1/8/a*ln(x^2*a^2/(a^2)^(
1/2)+(a^2*x^4+1)^(1/2))/(a^2)^(1/2)
```

Maxima [B] time = 1.16603, size = 138, normalized size = 2.76

$$\frac{1}{4}\,x^4\,\text{arsinh}\left(ax^2\right)+\frac{1}{16}\,a\left(\frac{\log\left(a+\frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3}-\frac{\log\left(-a+\frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3}+\frac{2\,\sqrt{a^2x^4+1}}{\left(a^4-\frac{\left(a^2x^4+1\right)a^2}{x^4}\right)x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x^2) + 1/16*a*(log(a + sqrt(a^2*x^4 + 1)/x^2)/a^3 - log(-
a + sqrt(a^2*x^4 + 1)/x^2)/a^3 + 2*sqrt(a^2*x^4 + 1)/((a^4 - (a^2*x^4 + 1)*
```

```
a^2/x^4)*x^2))
```

Fricas [A] time = 2.5887, size = 115, normalized size = 2.3

$$-\frac{\sqrt{a^2x^4+1}ax^2-\left(2\,a^2x^4+1\right)\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 + 1)*log(a*x^2 + sqrt(a^2*x^4 +
1)))/a^2
```

Sympy [A] time = 1.24267, size = 42, normalized size = 0.84

$$\begin{cases} \frac{x^4\operatorname{asinh}\left(ax^2\right)}{4}-\frac{x^2\sqrt{a^2x^4+1}}{8a}+\frac{\operatorname{asinh}\left(ax^2\right)}{8a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**4*asinh(a*x**2)/4 - x**2*sqrt(a**2*x**4 + 1)/(8*a) + asinh(a*
x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.32506, size = 100, normalized size = 2.

$$\frac{1}{4}\,x^4\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{1}{8}\,a\left(\frac{\sqrt{a^2x^4+1}x^2}{a^2}+\frac{\log\left(-x^2|a|+\sqrt{a^2x^4+1}\right)}{a^2|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/8*a*(sqrt(a^2*x^4 + 1)*x^2/a^2 +
 log(-x^2*abs(a) + sqrt(a^2*x^4 + 1))/(a^2*abs(a)))
```

3.284 $\int x^2 \sinh^{-1}(ax^2)\, dx$

Optimal. Leaf size=101

$$\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{9a^{3/2}\sqrt{a^2x^4+1}}-\frac{2x\sqrt{a^2x^4+1}}{9a}+\frac{1}{3}x^3\sinh^{-1}(ax^2)$$

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(9*a) + (x^3*ArcSinh[a*x^2])/3 + ((1 + a*x^2)*Sqrt
[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(9*a^(3/
2)*Sqrt[1 + a^2*x^4])
```

Rubi [A] time = 0.043684, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5902, 12, 321, 220}

$$-\frac{2x\sqrt{a^2x^4+1}}{9a}+\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{9a^{3/2}\sqrt{a^2x^4+1}}+\frac{1}{3}x^3\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(9*a) + (x^3*ArcSinh[a*x^2])/3 + ((1 + a*x^2)*Sqrt
[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(9*a^(3/
2)*Sqrt[1 + a^2*x^4])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int x^2\sinh^{-1}\left(ax^2\right)dx &= \frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)-\frac{1}{3}\int\frac{2ax^4}{\sqrt{1+a^2x^4}}dx\\ &= \frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)-\frac{1}{3}(2a)\int\frac{x^4}{\sqrt{1+a^2x^4}}dx\\ &= -\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)+\frac{2\int\frac{1}{\sqrt{1+a^2x^4}}dx}{9a}\\ &= -\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)+\frac{\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}\end{aligned}$$

Mathematica [C] time = 0.120905, size = 75, normalized size = 0.74

$$\frac{1}{9}\left(-\frac{2\sqrt{ia}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{ia}x\right),-1\right)}{a^2}-\frac{2\left(a^2x^5+x\right)}{a\sqrt{a^2x^4+1}}+3x^3\sinh^{-1}\left(ax^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x^2],x]
```

```
[Out] ((-2*(x + a^2*x^5))/(a*Sqrt[1 + a^2*x^4]) + 3*x^3*ArcSinh[a*x^2] - (2*Sqrt[
I*a]*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/a^2)/9
```

Maple [C] time = 0.016, size = 89, normalized size = 0.9

$$\frac{x^3\text{Arcsinh}\left(ax^2\right)}{3}-\frac{2a}{3}\left(\frac{x}{3a^2}\sqrt{a^2x^4+1}-\frac{1}{3a^2}\sqrt{1-iax^2}\sqrt{1+iax^2}\text{EllipticF}\left(x\sqrt{ia},i\right)\frac{1}{\sqrt{ia}}\frac{1}{\sqrt{a^2x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x^2),x)
```

```
[Out] 1/3*x^3*arcsinh(a*x^2)-2/3*a*(1/3/a^2*x*(a^2*x^4+1)^(1/2)-1/3/a^2/(I*a)^(1/
2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*EllipticF(x*(I*a)^
(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{2}{9}x^3-2a\int\frac{x^4}{3\left(a^3x^6+ax^2+\left(a^2x^4+1\right)^{\frac{3}{2}}\right)}dx-\frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2\sqrt{a^2}x+\sqrt{2}\left(a^2\right)^{\frac{1}{4}}\right)}{2\left(a^2\right)^{\frac{1}{4}}}+1\right)-\log\right.}{12\left(a^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2/9*x^3 - 2*a*integrate(1/3*x^4/(a
^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(
2)*(2*sqrt(a^2)*x + sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1) - log(-1/2*I*sqrt
(2)*(2*sqrt(a^2)*x + sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1))/(a^2)^(3/4) - 1
/12*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*sqrt(a^2)*x - sqrt(2)*(a^2)^(1/4))/(a^2
)^(1/4) + 1) - log(-1/2*I*sqrt(2)*(2*sqrt(a^2)*x - sqrt(2)*(a^2)^(1/4))/(a^
2)^(1/4) + 1))/(a^2)^(3/4) - 1/12*sqrt(2)*log(sqrt(a^2)*x^2 + sqrt(2)*(a^2)
^(1/4)*x + 1)/(a^2)^(3/4) + 1/12*sqrt(2)*log(sqrt(a^2)*x^2 - sqrt(2)*(a^2)^
(1/4)*x + 1)/(a^2)^(3/4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \operatorname{arsinh}\left(ax^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asinh}\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x**2),x)
```

```
[Out] Integral(x**2*asinh(a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x^2), x)
```

3.285 $\int x \sinh^{-1}\left(ax^2\right) dx$

Optimal. Leaf size=34

$$\frac{1}{2}x^2 \sinh^{-1}\left(ax^2\right) - \frac{\sqrt{a^2x^4+1}}{2a}$$

```
[Out] -Sqrt[1 + a^2*x^4]/(2*a) + (x^2*ArcSinh[a*x^2])/2
```

Rubi [A] time = 0.0224494, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6715, 5653, 261}

$$\frac{1}{2}x^2 \sinh^{-1}\left(ax^2\right) - \frac{\sqrt{a^2x^4+1}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x^2],x]
```

```
[Out] -Sqrt[1 + a^2*x^4]/(2*a) + (x^2*ArcSinh[a*x^2])/2
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x \sinh^{-1}\left(ax^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int \sinh^{-1}(ax)\, dx, x, x^2\right)\\
&= \frac{1}{2}x^2 \sinh^{-1}\left(ax^2\right) - \frac{1}{2}a\,\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x^2}}\, dx, x, x^2\right)\\
&= -\frac{\sqrt{1+a^2x^4}}{2a} + \frac{1}{2}x^2 \sinh^{-1}\left(ax^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0146358, size = 34, normalized size = 1.

$$\frac{1}{2}x^2 \sinh^{-1}\left(ax^2\right) - \frac{\sqrt{a^2x^4+1}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x^2],x]
```

```
[Out] -Sqrt[1 + a^2*x^4]/(2*a) + (x^2*ArcSinh[a*x^2])/2
```

Maple [A] time = 0.001, size = 31, normalized size = 0.9

$$\frac{1}{2a}\left(x^2 a \mathrm{Arcsinh}\left(ax^2\right)-\sqrt{a^2x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x^2),x)
```

```
[Out] 1/2/a*(x^2*a*arcsinh(a*x^2)-(a^2*x^4+1)^(1/2))
```

Maxima [A] time = 1.14811, size = 41, normalized size = 1.21

$$\frac{ax^2 \operatorname{arsinh}\left(ax^2\right)-\sqrt{a^2x^4+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2*arcsinh(a*x^2) - sqrt(a^2*x^4 + 1))/a
```

Fricas [A] time = 2.65478, size = 89, normalized size = 2.62

$$\frac{ax^2 \log\left(ax^2+\sqrt{a^2x^4+1}\right)-\sqrt{a^2x^4+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - sqrt(a^2*x^4 + 1))/a
```

Sympy [A] time = 0.251971, size = 27, normalized size = 0.79

$$\begin{cases} \frac{x^2 \operatorname{asinh}\left(ax^2\right)}{2}-\frac{\sqrt{a^2x^4+1}}{2a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**2*asinh(a*x**2)/2 - sqrt(a**2*x**4 + 1)/(2*a), Ne(a, 0)), (0,
 True))
```

Giac [A] time = 1.29993, size = 54, normalized size = 1.59

$$\frac{1}{2}\,x^2\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{\sqrt{a^2x^4+1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/2*sqrt(a^2*x^4 + 1)/a
```

3.286 $\int \sinh^{-1}(ax^2)\, dx$

Optimal. Leaf size=162

$$-\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}}-\frac{2x\sqrt{a^2x^4+1}}{ax^2+1}+\frac{2(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}}+x\sinh$$

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(1 + a*x^2) + x*ArcSinh[a*x^2] + (2*(1 + a*x^2)*Sq
rt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[
a]*Sqrt[1 + a^2*x^4]) - ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*Elli
pticF[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[a]*Sqrt[1 + a^2*x^4])
```

Rubi [A] time = 0.054459, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5900, 12, 305, 220, 1196}

$$-\frac{2x\sqrt{a^2x^4+1}}{ax^2+1}-\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}}+\frac{2(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}}+x\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(1 + a*x^2) + x*ArcSinh[a*x^2] + (2*(1 + a*x^2)*Sq
rt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[
a]*Sqrt[1 + a^2*x^4]) - ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*Elli
pticF[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[a]*Sqrt[1 + a^2*x^4])
```

Rule 5900

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/Sqrt[1 + u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}\left(ax^2\right)\,dx &= x\sinh^{-1}\left(ax^2\right)-\int \frac{2ax^2}{\sqrt{1+a^2x^4}}\,dx\\ &= x\sinh^{-1}\left(ax^2\right)-(2a)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx\\ &= x\sinh^{-1}\left(ax^2\right)-2\int \frac{1}{\sqrt{1+a^2x^4}}\,dx+2\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx\\ &= -\frac{2x\sqrt{1+a^2x^4}}{1+ax^2}+x\sinh^{-1}\left(ax^2\right)+\frac{2\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}}-\frac{\left(1+ax^2\right)\sqrt{\frac{1+}{(1+}}}{\sqrt{}}\end{aligned}$$

Mathematica [C] time = 0.0047616, size = 35, normalized size = 0.22

$$x\sinh^{-1}\left(ax^2\right)-\frac{2}{3}ax^3\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-a^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2],x]
```

```
[Out] x*ArcSinh[a*x^2] - (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(a^2*x^4)])/3
```

Maple [C] time = 0.005, size = 77, normalized size = 0.5

$$x\text{Arcsinh}\left(ax^2\right)-2i\sqrt{1-iax^2}\sqrt{1+iax^2}\left(\text{EllipticF}\left(x\sqrt{ia},i\right)-\text{EllipticE}\left(x\sqrt{ia},i\right)\right)\frac{1}{\sqrt{ia}}\frac{1}{\sqrt{a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2),x)
```

```
[Out] x*arcsinh(a*x^2)-2*I/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x
^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-EllipticE(x*(I*a)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2a\int \frac{x^2}{a^3x^6+ax^2+\left(a^2x^4+1\right)^{\frac{3}{2}}}\,dx+x\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2\sqrt{a^2}x+\sqrt{2}\left(a^2\right)^{\frac{1}{4}}\right)}{2\left(a^2\right)^{\frac{1}{4}}}+1\right)-\log\left(-\frac{i\sqrt{2}\left(2\sqrt{a^2}x\right.}{2\left(a\right.}\right.\right.}{4\left(a^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] -2*a*integrate(x^2/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) + x*log(a*x^
2 + sqrt(a^2*x^4 + 1)) - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*sqrt(a^2)*x +
sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1) - log(-1/2*I*sqrt(2)*(2*sqrt(a^2)*x +
 sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1))/(a^2)^(1/4) - 1/4*I*sqrt(2)*(log(1/
2*I*sqrt(2)*(2*sqrt(a^2)*x - sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1) - log(-1
/2*I*sqrt(2)*(2*sqrt(a^2)*x - sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1))/(a^2)^
(1/4) + 1/4*sqrt(2)*log(sqrt(a^2)*x^2 + sqrt(2)*(a^2)^(1/4)*x + 1)/(a^2)^(1
/4) - 1/4*sqrt(2)*log(sqrt(a^2)*x^2 - sqrt(2)*(a^2)^(1/4)*x + 1)/(a^2)^(1/4
) - 2*x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{arsinh}\left(ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}\left(ax^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2),x)
```

```
[Out] Integral(asinh(a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{arsinh}\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2), x)
```

3.287 $\int \frac{\sinh^{-1}(ax^2)}{x}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^2)}\right)-\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2+\frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)$$

```
[Out] -ArcSinh[a*x^2]^2/4 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 + Po
lyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Rubi [A] time = 0.0622429, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\frac{1}{4}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^2)}\right)-\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2+\frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x,x]
```

```
[Out] -ArcSinh[a*x^2]^2/4 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 + Po
lyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(ax^2\right)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(ax^2\right)\right)\\ &= -\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2 - \operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(ax^2\right)\right)\\ &= -\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2 + \frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}\left(ax^2\right)}\right) - \frac{1}{2}\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\right.\right.\\ &= -\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2 + \frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}\left(ax^2\right)}\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(ax}\right.\\ &= -\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2 + \frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}\left(ax^2\right)}\right) + \frac{1}{4}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(ax^2\right)}\right)\end{aligned}$$

Mathematica [A] time = 0.0070841, size = 54, normalized size = 1.

$$\frac{1}{4}\operatorname{PolyLog}\left(2, e^{2\sinh^{-1}\left(ax^2\right)}\right) - \frac{1}{4}\sinh^{-1}\left(ax^2\right)^2 + \frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}\left(ax^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x,x]
```

```
[Out] -ArcSinh[a*x^2]^2/4 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 + Po
lyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Arcsinh}\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x,x)
```

```
[Out] int(arcsinh(a*x^2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x^2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}\left(ax^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x,x)
```

```
[Out] Integral(asinh(a*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x, x)
```

3.288 $\int \frac{\sinh^{-1}(ax^2)}{x^2} dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a^2x^4+1}}-\frac{\sinh^{-1}\left(ax^2\right)}{x}$$

```
[Out] -(ArcSinh[a*x^2]/x) + (Sqrt[a]*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/Sqrt[1 + a^2*x^4]
```

Rubi [A] time = 0.0225441, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5902, 12, 220}

$$\frac{\sqrt{a}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{a^2x^4+1}}-\frac{\sinh^{-1}\left(ax^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^2]/x) + (Sqrt[a]*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/Sqrt[1 + a^2*x^4]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^2)}{x^2}\,dx = -\frac{\sinh^{-1}(ax^2)}{x} + \int \frac{2a}{\sqrt{1+a^2x^4}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^2)}{x} + (2a)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^2)}{x} + \frac{\sqrt{a}\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}F\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}$$

Mathematica [C] time = 0.0369385, size = 42, normalized size = 0.56

$$-\frac{\sinh^{-1}(ax^2) + 2\sqrt{ia}x\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{ia}x\right), -1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^2,x]
```

```
[Out] -((ArcSinh[a*x^2] + 2*Sqrt[I*a]*x*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/x)
```

Maple [C] time = 0.006, size = 66, normalized size = 0.9

$$-\frac{\text{Arcsinh}\left(ax^2\right)}{x} + 2\,\frac{a\sqrt{1-iax^2}\sqrt{1+iax^2}\text{EllipticF}\left(x\sqrt{ia}, i\right)}{\sqrt{ia}\sqrt{a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^2,x)
```

```
[Out] -arcsinh(a*x^2)/x+2*a/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*
x^4+1)^(1/2)*EllipticF(x*(I*a)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a^2\left(\frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2\sqrt{a^2}x+\sqrt{2}(a^2)^{\frac{1}{4}}\right)}{2(a^2)^{\frac{1}{4}}}+1\right)-\log\left(-\frac{i\sqrt{2}\left(2\sqrt{a^2}x+\sqrt{2}(a^2)^{\frac{1}{4}}\right)}{2(a^2)^{\frac{1}{4}}}+1\right)\right)}{(a^2)^{\frac{3}{4}}} + \frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2\sqrt{a^2}x-\sqrt{2}(a^2)^{\frac{1}{4}}\right)}{2(a^2)^{\frac{1}{4}}}+1\right)-\log\left(\right.\right.}{(a^2)^{\frac{3}{4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*sqrt(a^2)*x + sqrt(2)*(a^2)^(1/4)
)/(a^2)^(1/4) + 1) - log(-1/2*I*sqrt(2)*(2*sqrt(a^2)*x + sqrt(2)*(a^2)^(1/4
))/(a^2)^(1/4) + 1))/(a^2)^(3/4) + I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*sqrt(a^2
```

```
)*x - sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1) - log(-1/2*I*sqrt(2)*(2*sqrt(a^
2)*x - sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1))/(a^2)^(3/4) + sqrt(2)*log(sqr
t(a^2)*x^2 + sqrt(2)*(a^2)^(1/4)*x + 1)/(a^2)^(3/4) - sqrt(2)*log(sqrt(a^2)
*x^2 - sqrt(2)*(a^2)^(1/4)*x + 1)/(a^2)^(3/4)) + 2*a*integrate(1/(a^3*x^6 +
 a*x^2 + (a^2*x^4 + 1)^(3/2)), x) - log(a*x^2 + sqrt(a^2*x^4 + 1))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\left(ax^2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}\left(ax^2\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**2,x)
```

```
[Out] Integral(asinh(a*x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\left(ax^2\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x^2, x)
```

3.289 $\int \frac{\sinh^{-1}(ax^2)}{x^3}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right)-\frac{\sinh^{-1}\left(ax^2\right)}{2x^2}$$

```
[Out] -ArcSinh[a*x^2]/(2*x^2) - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Rubi [A] time = 0.0264191, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5902, 12, 266, 63, 208}

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right)-\frac{\sinh^{-1}\left(ax^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^3,x]
```

```
[Out] -ArcSinh[a*x^2]/(2*x^2) - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^2)}{x^3}\,dx = -\frac{\sinh^{-1}(ax^2)}{2x^2} + \frac{1}{2}\int \frac{2a}{x\sqrt{1+a^2x^4}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^2)}{2x^2} + a\int \frac{1}{x\sqrt{1+a^2x^4}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^2)}{2x^2} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^4\right)$$
$$= -\frac{\sinh^{-1}(ax^2)}{2x^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^4}\right)}{2a}$$
$$= -\frac{\sinh^{-1}(ax^2)}{2x^2} - \frac{1}{2}a\tanh^{-1}\left(\sqrt{1+a^2x^4}\right)$$

Mathematica [A] time = 0.0058583, size = 33, normalized size = 1.

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right) - \frac{\sinh^{-1}(ax^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^3,x]

[Out] -ArcSinh[a*x^2]/(2*x^2) - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Maple [A] time = 0.01, size = 28, normalized size = 0.9

$$-\frac{\text{Arcsinh}(ax^2)}{2\,x^2} - \frac{a}{2}\text{Artanh}\left(\frac{1}{\sqrt{a^2x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^3,x)

[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh(1/(a^2*x^4+1)^(1/2))
```

Maxima [A] time = 1.11058, size = 62, normalized size = 1.88

$$-\frac{1}{4}a\left(\log\left(\sqrt{a^2x^4+1}+1\right) - \log\left(\sqrt{a^2x^4+1}-1\right)\right) - \frac{\text{arsinh}(ax^2)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="maxima")

[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*arcs
inh(a*x^2)/x^2
```

Fricas [B] time = 3.0286, size = 242, normalized size = 7.33

$$-\frac{ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}+1\right)-ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}-1\right)-x^2\log\left(-ax^2+\sqrt{a^2x^4+1}\right)-\left(x^2-1\right)\log\left(ax^2+\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1) + 1) - a*x^2*log(-a*x^2 + sqrt(a
^2*x^4 + 1) - 1) - x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1)) - (x^2 - 1)*log(a*x^
2 + sqrt(a^2*x^4 + 1)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}\left(ax^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**3,x)
```

```
[Out] Integral(asinh(a*x**2)/x**3, x)
```

Giac [B] time = 1.26377, size = 78, normalized size = 2.36

$$-\frac{1}{4}\,a\left(\log\left(\sqrt{a^2x^4+1}+1\right)-\log\left(\sqrt{a^2x^4+1}-1\right)\right)-\frac{\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*log(
a*x^2 + sqrt(a^2*x^4 + 1))/x^2
```

3.290 $\int \frac{\sinh^{-1}(ax^2)}{x^4}\,dx$

Optimal. Leaf size=197

$$\frac{a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}}+\frac{2a^2x\sqrt{a^2x^4+1}}{3\left(ax^2+1\right)}-\frac{2a\sqrt{a^2x^4+1}}{3x}-\frac{2a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{a}x\right)\right.}{3\sqrt{a^2x^4+1}}$$

```
[Out] (-2*a*Sqrt[1 + a^2*x^4])/(3*x) + (2*a^2*x*Sqrt[1 + a^2*x^4])/(3*(1 + a*x^2)
) - ArcSinh[a*x^2]/(3*x^3) - (2*a^(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 +
 a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4]) + (a^
(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4])
```

Rubi [A] time = 0.0896105, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5902, 12, 325, 305, 220, 1196}

$$\frac{2a^2x\sqrt{a^2x^4+1}}{3\left(ax^2+1\right)}-\frac{2a\sqrt{a^2x^4+1}}{3x}+\frac{a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}}-\frac{2a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{a}x\right)\right.}{3\sqrt{a^2x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 + a^2*x^4])/(3*x) + (2*a^2*x*Sqrt[1 + a^2*x^4])/(3*(1 + a*x^2)
) - ArcSinh[a*x^2]/(3*x^3) - (2*a^(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 +
 a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4]) + (a^
(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(ax^2\right)}{x^4}\,dx &= -\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}\int \frac{2a}{x^2\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}(2a)\int \frac{1}{x^2\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}\left(2a^3\right)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}\left(2a^2\right)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}+\frac{2a^2x\sqrt{1+a^2x^4}}{3\left(1+ax^2\right)}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}-\frac{2a^{3/2}\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.156217, size = 88, normalized size = 0.45

$$\frac{1}{3}\left(\frac{2a^2\left(E\left(i\sinh^{-1}\left(\sqrt{ia}x\right)|-1\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{ia}x\right),-1\right)\right)}{\sqrt{ia}}-\frac{2a\sqrt{a^2x^4+1}}{x}-\frac{\sinh^{-1}\left(ax^2\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] ((-2*a*Sqrt[1 + a^2*x^4])/x - ArcSinh[a*x^2]/x^3 + (2*a^2*(EllipticE[I*ArcS
inh[Sqrt[I*a]*x], -1] - EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1]))/Sqrt[I*a])/
3
```

Maple [C] time = 0.009, size = 101, normalized size = 0.5

$$-\frac{\text{Arcsinh}\left(ax^2\right)}{3\,x^3}+\frac{2\,a}{3}\left(-\frac{1}{x}\sqrt{a^2x^4+1}+ia\sqrt{1-iax^2}\sqrt{1+iax^2}\left(\text{EllipticF}\left(x\sqrt{ia},i\right)-\text{EllipticE}\left(x\sqrt{ia},i\right)\right)\frac{1}{\sqrt{ia}}\frac{1}{\sqrt{a^2x^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^4,x)
```

```
[Out] -1/3*arcsinh(a*x^2)/x^3+2/3*a*(-(a^2*x^4+1)^(1/2)/x+I*a/(I*a)^(1/2)*(1-I*a*
x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-
EllipticE(x*(I*a)^(1/2),I)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{i\sqrt{2}a^2\left(\log\left(\frac{i\sqrt{2}\left(2\sqrt{a^2}x+\sqrt{2}(a^2)^{\frac{1}{4}}\right)}{2(a^2)^{\frac{1}{4}}}+1\right)-\log\left(-\frac{i\sqrt{2}\left(2\sqrt{a^2}x+\sqrt{2}(a^2)^{\frac{1}{4}}\right)}{2(a^2)^{\frac{1}{4}}}+1\right)\right)}{12(a^2)^{\frac{1}{4}}}-\frac{i\sqrt{2}a^2\left(\log\left(\frac{i\sqrt{2}\left(2\sqrt{a^2}x-\sqrt{2}(a^2)^{\frac{1}{4}}\right)}{2(a^2)^{\frac{1}{4}}}+1\right)-\log\left(-\frac{i\sqrt{}}{}\right.\right.}{12(a^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*I*sqrt(2)*a^2*(log(1/2*I*sqrt(2)*(2*sqrt(a^2)*x + sqrt(2)*(a^2)^(1/4)
)/(a^2)^(1/4) + 1) - log(-1/2*I*sqrt(2)*(2*sqrt(a^2)*x + sqrt(2)*(a^2)^(1/4
))/(a^2)^(1/4) + 1))/(a^2)^(1/4) - 1/12*I*sqrt(2)*a^2*(log(1/2*I*sqrt(2)*(2
*sqrt(a^2)*x - sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1) - log(-1/2*I*sqrt(2)*(
2*sqrt(a^2)*x - sqrt(2)*(a^2)^(1/4))/(a^2)^(1/4) + 1))/(a^2)^(1/4) + 1/12*s
qrt(2)*a^2*log(sqrt(a^2)*x^2 + sqrt(2)*(a^2)^(1/4)*x + 1)/(a^2)^(1/4) - 1/1
2*sqrt(2)*a^2*log(sqrt(a^2)*x^2 - sqrt(2)*(a^2)^(1/4)*x + 1)/(a^2)^(1/4) +
2*a*integrate(1/3/(a^3*x^8 + a*x^4 + (a^2*x^6 + x^2)*sqrt(a^2*x^4 + 1)), x)
 - 1/3*log(a*x^2 + sqrt(a^2*x^4 + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\left(ax^2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{asinh}\left(ax^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**4,x)
```

```
[Out] Integral(asinh(a*x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax^2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x^4, x)
```

3.291 $\int \frac{\sinh^{-1}(ax^5)}{x} \, dx$

Optimal. Leaf size=54

$$\frac{1}{10}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^5)}\right)-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

```
[Out] -ArcSinh[a*x^5]^2/10 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 + P
olyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Rubi [A] time = 0.0621063, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\frac{1}{10}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^5)}\right)-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^5]/x,x]
```

```
[Out] -ArcSinh[a*x^5]^2/10 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 + P
olyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^5)}{x} dx = \frac{1}{5}\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^5)\right)$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 - \frac{2}{5}\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^5)\right)$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right) - \frac{1}{5}\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sin$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right) - \frac{1}{10}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sin}$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right) + \frac{1}{10}\text{Li}_2\left(e^{2\sinh^{-1}(ax^5)}\right)$$

Mathematica [A] time = 0.0078048, size = 54, normalized size = 1.

$$\frac{1}{10}\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax^5)}\right) - \frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^5]/x,x]
```

```
[Out] -ArcSinh[a*x^5]^2/10 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 + P
olyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\text{Arcsinh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^5)/x,x)
```

```
[Out] int(arcsinh(a*x^5)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x^5)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^5)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**5)/x,x)
```

```
[Out] Integral(asinh(a*x**5)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^5)/x, x)
```

3.292 $\int x^2 \sinh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=72

$$-\frac{1}{18}\sqrt{x+1}x^{5/2}+\frac{5}{72}\sqrt{x+1}x^{3/2}+\frac{1}{3}x^3\sinh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{x+1}\sqrt{x}+\frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (-5*Sqrt[x]*Sqrt[1 + x])/48 + (5*x^(3/2)*Sqrt[1 + x])/72 - (x^(5/2)*Sqrt[1
+ x])/18 + (5*ArcSinh[Sqrt[x]])/48 + (x^3*ArcSinh[Sqrt[x]])/3
```

Rubi [A] time = 0.0251364, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5902, 12, 50, 54, 215}

$$-\frac{1}{18}\sqrt{x+1}x^{5/2}+\frac{5}{72}\sqrt{x+1}x^{3/2}+\frac{1}{3}x^3\sinh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{x+1}\sqrt{x}+\frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[1 + x])/48 + (5*x^(3/2)*Sqrt[1 + x])/72 - (x^(5/2)*Sqrt[1
+ x])/18 + (5*ArcSinh[Sqrt[x]])/48 + (x^3*ArcSinh[Sqrt[x]])/3
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{1+x}} dx \\
&= \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{\sqrt{1+x}} dx \\
&= -\frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{36}\int \frac{x^{3/2}}{\sqrt{1+x}} dx \\
&= \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{5}{48}\int \frac{\sqrt{x}}{\sqrt{1+x}} dx \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{96}\int \frac{1}{\sqrt{x}\sqrt{1+x}} dx \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{48}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, \right. \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0206166, size = 43, normalized size = 0.6

$$\frac{1}{144}\left(\sqrt{x}\sqrt{x+1}\left(-8x^2+10x-15\right)+3\left(16x^3+5\right)\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-15 + 10*x - 8*x^2) + 3*(5 + 16*x^3)*ArcSinh[Sqrt[x]]
)/144
```

Maple [A] time = 0.01, size = 47, normalized size = 0.7

$$\frac{5}{48}\text{Arcsinh}\left(\sqrt{x}\right)+\frac{x^3}{3}\text{Arcsinh}\left(\sqrt{x}\right)+\frac{5}{72}x^{\frac{3}{2}}\sqrt{1+x}-\frac{1}{18}x^{\frac{5}{2}}\sqrt{1+x}-\frac{5}{48}\sqrt{x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(x^(1/2)),x)
```

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

Maxima [A] time = 1.69737, size = 62, normalized size = 0.86

$$\frac{1}{3}x^3\,\text{arsinh}\left(\sqrt{x}\right)-\frac{1}{18}\sqrt{x+1}x^{\frac{5}{2}}+\frac{5}{72}\sqrt{x+1}x^{\frac{3}{2}}-\frac{5}{48}\sqrt{x+1}\sqrt{x}+\frac{5}{48}\,\text{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(sqrt(x)) - 1/18*sqrt(x + 1)*x^(5/2) + 5/72*sqrt(x + 1)*x^(3
/2) - 5/48*sqrt(x + 1)*sqrt(x) + 5/48*arcsinh(sqrt(x))
```

Fricas [A] time = 2.70848, size = 128, normalized size = 1.78

$$-\frac{1}{144}\left(8x^2-10x+15\right)\sqrt{x+1}\sqrt{x}+\frac{1}{48}\left(16x^3+5\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 - 10*x + 15)*sqrt(x + 1)*sqrt(x) + 1/48*(16*x^3 + 5)*log(sqrt
(x + 1) + sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asinh}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(x**(1/2)),x)
```

```
[Out] Integral(x**2*asinh(sqrt(x)), x)
```

Giac [A] time = 1.38079, size = 68, normalized size = 0.94

$$\frac{1}{3}x^3\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{144}(2(4x-5)x+15)\sqrt{x+1}\sqrt{x}-\frac{5}{48}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(x + 1) + sqrt(x)) - 1/144*(2*(4*x - 5)*x + 15)*sqrt(x + 1)
*sqrt(x) - 5/48*log(sqrt(x + 1) - sqrt(x))
```

3.293 $\int x\sinh^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=56

$$-\frac{1}{8}\sqrt{x+1}x^{3/2}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\sqrt{x+1}\sqrt{x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (3*Sqrt[x]*Sqrt[1 + x])/16 - (x^(3/2)*Sqrt[1 + x])/8 - (3*ArcSinh[Sqrt[x]])
/16 + (x^2*ArcSinh[Sqrt[x]])/2
```

Rubi [A] time = 0.0172571, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5902, 12, 50, 54, 215}

$$-\frac{1}{8}\sqrt{x+1}x^{3/2}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\sqrt{x+1}\sqrt{x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[Sqrt[x]],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[1 + x])/16 - (x^(3/2)*Sqrt[1 + x])/8 - (3*ArcSinh[Sqrt[x]])
/16 + (x^2*ArcSinh[Sqrt[x]])/2
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{1+x}}\,dx\\
&= \frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\int\frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0166977, size = 37, normalized size = 0.66

$$\frac{1}{16}\left(\left(8x^2-3\right)\sinh^{-1}\left(\sqrt{x}\right)+\sqrt{x}\sqrt{x+1}(3-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[Sqrt[x]],x]
```

```
[Out] ((3 - 2*x)*Sqrt[x]*Sqrt[1 + x] + (-3 + 8*x^2)*ArcSinh[Sqrt[x]])/16
```

Maple [A] time = 0.004, size = 37, normalized size = 0.7

$$-\frac{3}{16}\mathrm{Arcsinh}\left(\sqrt{x}\right)+\frac{x^2}{2}\mathrm{Arcsinh}\left(\sqrt{x}\right)-\frac{1}{8}x^{\frac{3}{2}}\sqrt{1+x}+\frac{3}{16}\sqrt{x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(x^(1/2)),x)
```

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

Maxima [A] time = 1.68926, size = 49, normalized size = 0.88

$$\frac{1}{2}x^2\,\mathrm{arsinh}\left(\sqrt{x}\right)-\frac{1}{8}\sqrt{x+1}x^{\frac{3}{2}}+\frac{3}{16}\sqrt{x+1}\sqrt{x}-\frac{3}{16}\,\mathrm{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(sqrt(x)) - 1/8*sqrt(x + 1)*x^(3/2) + 3/16*sqrt(x + 1)*sqrt(
x) - 3/16*arcsinh(sqrt(x))
```

Fricas [A] time = 2.69024, size = 112, normalized size = 2.

$$-\frac{1}{16}\,(2\,x-3)\sqrt{x+1}\sqrt{x}+\frac{1}{16}\,\left(8\,x^2-3\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x - 3)*sqrt(x + 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x + 1) + sq
rt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{asinh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(x**(1/2)),x)
```

```
[Out] Integral(x*asinh(sqrt(x)), x)
```

Giac [A] time = 1.31397, size = 65, normalized size = 1.16

$$\frac{1}{2}\,x^2\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{16}\,\sqrt{x^2+x}(2\,x-3)+\frac{3}{32}\,\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(x + 1) + sqrt(x)) - 1/16*sqrt(x^2 + x)*(2*x - 3) + 3/32*lo
g(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.294 $\int \sinh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0107196, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5900, 12, 1958, 50, 54, 215}

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*ArcSinh[Sqrt[x]]
```

Rule 5900

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/Sqrt[1 + u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Funct
ionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}\left(\sqrt{x}\right)\,dx &= x\sinh^{-1}\left(\sqrt{x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\
&= x\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\
&= x\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0375523, size = 33, normalized size = 0.94

$$\frac{1}{2}\left((2x+1)\sinh^{-1}\left(\sqrt{x}\right)-\sqrt{\frac{x}{x+1}}(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[x/(1 + x)]*(1 + x)) + (1 + 2*x)*ArcSinh[Sqrt[x]])/2
```

Maple [A] time = 0.002, size = 24, normalized size = 0.7

$$\frac{1}{2}\text{Arcsinh}\left(\sqrt{x}\right)+x\text{Arcsinh}\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2)),x)
```

```
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Maxima [A] time = 1.75239, size = 31, normalized size = 0.89

$$x\,\text{arsinh}\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x+1}\sqrt{x}+\frac{1}{2}\,\text{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arcsinh(sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x) + 1/2*arcsinh(sqrt(x))
```

Fricas [A] time = 2.61349, size = 92, normalized size = 2.63

$$\frac{1}{2}(2x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x+1}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

Sympy [A] time = 0.359289, size = 29, normalized size = 0.83

$$-\frac{\sqrt{x}\sqrt{x+1}}{2}+x\operatorname{asinh}\left(\sqrt{x}\right)+\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(x + 1)/2 + x*asinh(sqrt(x)) + asinh(sqrt(x))/2
```

Giac [A] time = 1.57915, size = 54, normalized size = 1.54

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x^2+x}-\frac{1}{4}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

3.295 $\int \frac{\sinh^{-1}(\sqrt{x})}{x} \, dx$

Optimal. Leaf size=46

$$\text{PolyLog}\left(2,e^{2\sinh^{-1}(\sqrt{x})}\right)-\sinh^{-1}\left(\sqrt{x}\right)^2+2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}(\sqrt{x})}\right)$$

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Rubi [A] time = 0.0643435, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,e^{2\sinh^{-1}(\sqrt{x})}\right)-\sinh^{-1}\left(\sqrt{x}\right)^2+2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}(\sqrt{x})}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}\,dx = 2\,\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\sqrt{x}\right)\right)$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 - 4\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\sqrt{x}\right)\right)$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - 2\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\right.\right.$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) + \text{Li}_2\left(e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Mathematica [A] time = 0.0068677, size = 46, normalized size = 1.

$$\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - \sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Maple [A] time = 0.026, size = 78, normalized size = 1.7

$$-\left(\text{Arcsinh}\left(\sqrt{x}\right)\right)^2 + 2\,\text{Arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right) + 2\,\text{polylog}\left(2, -\sqrt{x}-\sqrt{1+x}\right) + 2\,\text{Arcsinh}\left(\sqrt{x}\right)\ln\left(1-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x,x)
```

```
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1+x^(1/2)+(1+x)^(1/2))+2*polylog(
2,-x^(1/2)-(1+x)^(1/2))+2*arcsinh(x^(1/2))*ln(1-x^(1/2)-(1+x)^(1/2))+2*poly
log(2,x^(1/2)+(1+x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(sqrt(x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x,x)
```

```
[Out] Integral(asinh(sqrt(x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

3.296 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^2} dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Rubi [A] time = 0.0131465, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5902, 12, 37}

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^2,x]

[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^2} dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}+\int \frac{1}{2x^{3/2}\sqrt{1+x}} dx\\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{x^{3/2}\sqrt{1+x}} dx\\
&= -\frac{\sqrt{1+x}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0092362, size = 26, normalized size = 1.

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^2,x]
```

```
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Maple [A] time = 0.005, size = 21, normalized size = 0.8

$$-\frac{1}{x}\text{Arcsinh}\left(\sqrt{x}\right)-\sqrt{1+x}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^2,x)
```

```
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.61248, size = 27, normalized size = 1.04

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x + 1)/sqrt(x) - arcsinh(sqrt(x))/x
```

Fricas [A] time = 2.66803, size = 74, normalized size = 2.85

$$-\frac{\sqrt{x+1}\sqrt{x}+\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) + sqrt(x)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**2,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**2, x)
```

Giac [A] time = 1.21315, size = 47, normalized size = 1.81

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{2}{\left(\sqrt{x+1}-\sqrt{x}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -log(sqrt(x + 1) + sqrt(x))/x + 2/((sqrt(x + 1) - sqrt(x))^2 - 1)
```

3.297 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^3}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{x+1}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x+1}}{3\sqrt{x}}$$

```
[Out] -Sqrt[1 + x]/(6*x^(3/2)) + Sqrt[1 + x]/(3*Sqrt[x]) - ArcSinh[Sqrt[x]]/(2*x^
2)
```

Rubi [A] time = 0.0170543, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5902, 12, 45, 37}

$$-\frac{\sqrt{x+1}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] -Sqrt[1 + x]/(6*x^(3/2)) + Sqrt[1 + x]/(3*Sqrt[x]) - ArcSinh[Sqrt[x]]/(2*x^
2)
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2x^{5/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{1+x}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{6}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{1+x}}{6x^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.011902, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}\sqrt{x+1}(2x-1)-3\sinh^{-1}\left(\sqrt{x}\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-1 + 2*x) - 3*ArcSinh[Sqrt[x]])/(6*x^2)
```

Maple [A] time = 0.004, size = 31, normalized size = 0.7

$$-\frac{1}{2x^2}\mathrm{Arcsinh}\left(\sqrt{x}\right)-\frac{1}{6}\sqrt{1+x}x^{-\frac{3}{2}}+\frac{1}{3}\sqrt{1+x}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.65973, size = 41, normalized size = 0.89

$$\frac{\sqrt{x+1}}{3\sqrt{x}}-\frac{\sqrt{x+1}}{6x^{\frac{3}{2}}}-\frac{\mathrm{arsinh}\left(\sqrt{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x + 1)/sqrt(x) - 1/6*sqrt(x + 1)/x^(3/2) - 1/2*arcsinh(sqrt(x))/x^
2
```

Fricas [A] time = 2.84893, size = 97, normalized size = 2.11

$$\frac{(2x-1)\sqrt{x+1}\sqrt{x}-3\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*((2*x - 1)*sqrt(x + 1)*sqrt(x) - 3*log(sqrt(x + 1) + sqrt(x)))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**3,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**3, x)
```

Giac [A] time = 1.35205, size = 70, normalized size = 1.52

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2}+\frac{2\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{3\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(x + 1) - sqrt(x))^2 - 1)
/((sqrt(x + 1) - sqrt(x))^2 - 1)^3
```

3.298 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^4} dx$

Optimal. Leaf size=62

$$\frac{4\sqrt{x+1}}{45x^{3/2}}-\frac{\sqrt{x+1}}{15x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{8\sqrt{x+1}}{45\sqrt{x}}$$

```
[Out] -Sqrt[1 + x]/(15*x^(5/2)) + (4*Sqrt[1 + x])/(45*x^(3/2)) - (8*Sqrt[1 + x])/
(45*Sqrt[x]) - ArcSinh[Sqrt[x]]/(3*x^3)
```

Rubi [A] time = 0.0215487, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5902, 12, 45, 37}

$$\frac{4\sqrt{x+1}}{45x^{3/2}}-\frac{\sqrt{x+1}}{15x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{8\sqrt{x+1}}{45\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] -Sqrt[1 + x]/(15*x^(5/2)) + (4*Sqrt[1 + x])/(45*x^(3/2)) - (8*Sqrt[1 + x])/
(45*Sqrt[x]) - ArcSinh[Sqrt[x]]/(3*x^3)
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{3}\int \frac{1}{2x^{7/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{6}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{15x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{2}{15}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{15x^{5/2}}+\frac{4\sqrt{1+x}}{45x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{4}{45}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{15x^{5/2}}+\frac{4\sqrt{1+x}}{45x^{3/2}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] time = 0.0149408, size = 39, normalized size = 0.63

$$
\frac{\sqrt{x}\sqrt{x+1}\left(-8x^2+4x-3\right)-15\sinh^{-1}\left(\sqrt{x}\right)}{45x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-3 + 4*x - 8*x^2) - 15*ArcSinh[Sqrt[x]])/(45*x^3)
```

Maple [A] time = 0.006, size = 41, normalized size = 0.7

$$
-\frac{1}{3\,x^3}\mathrm{Arcsinh}\left(\sqrt{x}\right)-\frac{1}{15}\sqrt{1+x}x^{-\frac{5}{2}}+\frac{4}{45}\sqrt{1+x}x^{-\frac{3}{2}}-\frac{8}{45}\sqrt{1+x}\frac{1}{\sqrt{x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^4,x)
```

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.66074, size = 54, normalized size = 0.87

$$
-\frac{8\sqrt{x+1}}{45\sqrt{x}}+\frac{4\sqrt{x+1}}{45\,x^{\frac{3}{2}}}-\frac{\sqrt{x+1}}{15\,x^{\frac{5}{2}}}-\frac{\mathrm{arsinh}\left(\sqrt{x}\right)}{3\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -8/45*sqrt(x + 1)/sqrt(x) + 4/45*sqrt(x + 1)/x^(3/2) - 1/15*sqrt(x + 1)/x^(
5/2) - 1/3*arcsinh(sqrt(x))/x^3
```

Fricas [A] time = 3.05551, size = 112, normalized size = 1.81

$$-\frac{\left(8x^2-4x+3\right)\sqrt{x+1}\sqrt{x}+15\log\left(\sqrt{x+1}+\sqrt{x}\right)}{45x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/45*((8*x^2 - 4*x + 3)*sqrt(x + 1)*sqrt(x) + 15*log(sqrt(x + 1) + sqrt(x)
))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**4,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**4, x)
```

Giac [A] time = 1.32342, size = 90, normalized size = 1.45

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{3x^3}+\frac{16\left(10\left(\sqrt{x+1}-\sqrt{x}\right)^4-5\left(\sqrt{x+1}-\sqrt{x}\right)^2+1\right)}{45\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*log(sqrt(x + 1) + sqrt(x))/x^3 + 16/45*(10*(sqrt(x + 1) - sqrt(x))^4 -
 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 - 1)^5
```

3.299 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^5} dx$

Optimal. Leaf size=78

$$-\frac{2\sqrt{x+1}}{35x^{3/2}}+\frac{3\sqrt{x+1}}{70x^{5/2}}-\frac{\sqrt{x+1}}{28x^{7/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{4\sqrt{x+1}}{35\sqrt{x}}$$

```
[Out] -Sqrt[1 + x]/(28*x^(7/2)) + (3*Sqrt[1 + x])/(70*x^(5/2)) - (2*Sqrt[1 + x])/
(35*x^(3/2)) + (4*Sqrt[1 + x])/(35*Sqrt[x]) - ArcSinh[Sqrt[x]]/(4*x^4)
```

Rubi [A] time = 0.0273138, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5902, 12, 45, 37}

$$-\frac{2\sqrt{x+1}}{35x^{3/2}}+\frac{3\sqrt{x+1}}{70x^{5/2}}-\frac{\sqrt{x+1}}{28x^{7/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{4\sqrt{x+1}}{35\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] -Sqrt[1 + x]/(28*x^(7/2)) + (3*Sqrt[1 + x])/(70*x^(5/2)) - (2*Sqrt[1 + x])/
(35*x^(3/2)) + (4*Sqrt[1 + x])/(35*Sqrt[x]) - ArcSinh[Sqrt[x]]/(4*x^4)
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^5}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{1}{4}\int \frac{1}{2x^{9/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{1}{8}\int \frac{1}{x^{9/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{3}{28}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{3}{35}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{2\sqrt{1+x}}{35x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{2}{35}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{2\sqrt{1+x}}{35x^{3/2}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}
\end{aligned}
$$

Mathematica [A] time = 0.0171397, size = 44, normalized size = 0.56

$$
\frac{\sqrt{x}\sqrt{x+1}\left(16x^3-8x^2+6x-5\right)-35\sinh^{-1}\left(\sqrt{x}\right)}{140x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-5 + 6*x - 8*x^2 + 16*x^3) - 35*ArcSinh[Sqrt[x]])/(14
0*x^4)
```

Maple [A] time = 0.006, size = 51, normalized size = 0.7

$$
-\frac{1}{4x^4}\mathrm{Arcsinh}\left(\sqrt{x}\right)-\frac{1}{28}\sqrt{1+x}x^{-\frac{7}{2}}+\frac{3}{70}\sqrt{1+x}x^{-\frac{5}{2}}-\frac{2}{35}\sqrt{1+x}x^{-\frac{3}{2}}+\frac{4}{35}\sqrt{1+x}\frac{1}{\sqrt{x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^5,x)
```

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

Maxima [A] time = 1.80861, size = 68, normalized size = 0.87

$$
\frac{4\sqrt{x+1}}{35\sqrt{x}}-\frac{2\sqrt{x+1}}{35x^{\frac{3}{2}}}+\frac{3\sqrt{x+1}}{70x^{\frac{5}{2}}}-\frac{\sqrt{x+1}}{28x^{\frac{7}{2}}}-\frac{\mathrm{arsinh}\left(\sqrt{x}\right)}{4x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 4/35*sqrt(x + 1)/sqrt(x) - 2/35*sqrt(x + 1)/x^(3/2) + 3/70*sqrt(x + 1)/x^(5
/2) - 1/28*sqrt(x + 1)/x^(7/2) - 1/4*arcsinh(sqrt(x))/x^4
```

Fricas [A] time = 3.1853, size = 124, normalized size = 1.59

$$\frac{\left(16\,x^3-8\,x^2+6\,x-5\right)\sqrt{x+1}\sqrt{x}-35\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/140*((16*x^3 - 8*x^2 + 6*x - 5)*sqrt(x + 1)*sqrt(x) - 35*log(sqrt(x + 1)
+ sqrt(x)))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35595, size = 111, normalized size = 1.42

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4\,x^4}+\frac{8\left(35\left(\sqrt{x+1}-\sqrt{x}\right)^6-21\left(\sqrt{x+1}-\sqrt{x}\right)^4+7\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{35\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(sqrt(x + 1) + sqrt(x))/x^4 + 8/35*(35*(sqrt(x + 1) - sqrt(x))^6 -
21*(sqrt(x + 1) - sqrt(x))^4 + 7*(sqrt(x + 1) - sqrt(x))^2 - 1)/((sqrt(x +
1) - sqrt(x))^2 - 1)^7
```

3.300 $\int x^2 \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=56

$$\frac{1}{6}ax^2\sqrt{\frac{a^2}{x^2}+1}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2)/6 + (x^3*ArcCsch[x/a])/3 - (a^3*ArcTanh[Sqrt[1 +
a^2/x^2]])/6
```

Rubi [A] time = 0.039814, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5892, 6284, 266, 51, 63, 208}

$$\frac{1}{6}ax^2\sqrt{\frac{a^2}{x^2}+1}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2)/6 + (x^3*ArcCsch[x/a])/3 - (a^3*ArcTanh[Sqrt[1 +
a^2/x^2]])/6
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh^{-1}\left(\frac{a}{x}\right) dx &= \int x^2 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) dx \\
&= \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{3}a \int \frac{x}{\sqrt{1+\frac{a^2}{x^2}}} dx \\
&= \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{12}a^3 \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) + \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}} dx, x, \sqrt{1+\frac{a^2}{x^2}}\right) \\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3 \operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0381262, size = 57, normalized size = 1.02

$$
\frac{1}{6}\left(ax^2\sqrt{\frac{a^2}{x^2}+1} + a^3\left(-\log\left(x\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)\right)\right) + 2x^3 \sinh^{-1}\left(\frac{a}{x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2 + 2*x^3*ArcSinh[a/x] - a^3*Log[(1 + Sqrt[1 + a^2/x
^2])*x])/6
```

Maple [A] time = 0.017, size = 54, normalized size = 1.

$$
-a^3\left(-\frac{x^3}{3a^3}\operatorname{Arcsinh}\left(\frac{a}{x}\right) - \frac{x^2}{6a^2}\sqrt{1+\frac{a^2}{x^2}} + \frac{1}{6}\operatorname{Artanh}\left(\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a/x),x)
```

```
[Out] -a^3*(-1/3*arcsinh(a/x)/a^3*x^3-1/6/a^2*x^2*(1+a^2/x^2)^(1/2)+1/6*arctanh(1
/(1+a^2/x^2)^(1/2)))
```

Maxima [A] time = 1.13009, size = 93, normalized size = 1.66

$$\frac{1}{3}x^3\operatorname{arsinh}\left(\frac{a}{x}\right)-\frac{1}{12}\left(a^2\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-a^2\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)-2x^2\sqrt{\frac{a^2}{x^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a/x) - 1/12*(a^2*log(sqrt(a^2/x^2 + 1) + 1) - a^2*log(sqrt(
a^2/x^2 + 1) - 1) - 2*x^2*sqrt(a^2/x^2 + 1))*a
```

Fricas [B] time = 2.95187, size = 290, normalized size = 5.18

$$\frac{1}{6}a^3\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+\frac{1}{6}ax^2\sqrt{\frac{a^2+x^2}{x^2}}+\frac{1}{3}\left(x^3-1\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\frac{1}{3}\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\frac{1}{3}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/6*a^3*log(x*sqrt((a^2 + x^2)/x^2) - x) + 1/6*a*x^2*sqrt((a^2 + x^2)/x^2)
+ 1/3*(x^3 - 1)*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/3*log(x*sqrt((a^2
+ x^2)/x^2) + a - x) - 1/3*log(x*sqrt((a^2 + x^2)/x^2) - a - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\operatorname{asinh}\left(\frac{a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a/x),x)
```

```
[Out] Integral(x**2*asinh(a/x), x)
```

Giac [A] time = 1.32705, size = 103, normalized size = 1.84

$$\frac{1}{3}x^3\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)-\frac{1}{6}\left(a^2\log\left(|a|\right)\operatorname{sgn}\left(x\right)-\frac{a^2\log\left(-x+\sqrt{a^2+x^2}\right)}{\operatorname{sgn}\left(x\right)}-\frac{\sqrt{a^2+x^2}x}{\operatorname{sgn}\left(x\right)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(a^2/x^2 + 1) + a/x) - 1/6*(a^2*log(abs(a))*sgn(x) - a^2*lo
g(-x + sqrt(a^2 + x^2))/sgn(x) - sqrt(a^2 + x^2)*x/sgn(x))*a
```

3.301 $\int x \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] (a*Sqrt[1 + a^2/x^2]*x)/2 + (x^2*ArcCsch[x/a])/2
```

Rubi [A] time = 0.0172917, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5892, 6284, 191}

$$\frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x)/2 + (x^2*ArcCsch[x/a])/2
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}\left(\frac{a}{x}\right)\,dx &= \int x\text{csch}^{-1}\left(\frac{x}{a}\right)\,dx\\
&= \frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}a\int\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\,dx\\
&= \frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}x+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.0220711, size = 29, normalized size = 0.88

$$\frac{1}{2}x\left(a\sqrt{\frac{a^2}{x^2}+1}+x\sinh^{-1}\left(\frac{a}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a/x],x]
```

```
[Out] (x*(a*Sqrt[1 + a^2/x^2] + x*ArcSinh[a/x]))/2
```

Maple [A] time = 0.005, size = 38, normalized size = 1.2

$$-a^2\left(-\frac{x^2}{2\,a^2}\mathrm{Arcsinh}\left(\frac{a}{x}\right)-\frac{x}{2\,a}\sqrt{1+\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a/x),x)
```

```
[Out] -a^2*(-1/2/a^2*x^2*arcsinh(a/x)-1/2/a*x*(1+a^2/x^2)^(1/2))
```

Maxima [A] time = 1.04402, size = 36, normalized size = 1.09

$$\frac{1}{2}\,x^2\,\mathrm{arsinh}\left(\frac{a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^2}{x^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a/x) + 1/2*a*x*sqrt(a^2/x^2 + 1)
```

Fricas [A] time = 2.67991, size = 105, normalized size = 3.18

$$\frac{1}{2}\,x^2\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^2+x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/2*a*x*sqrt((a^2 + x^2)/x^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{asinh}\left(\frac{a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a/x),x)
```

```
[Out] Integral(x*asinh(a/x), x)
```

Giac [A] time = 1.31523, size = 65, normalized size = 1.97

$$\frac{1}{2}\,x^2\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)-\frac{1}{2}\left(|a|\mathrm{sgn}\,(x)-\frac{\sqrt{a^2+x^2}}{\mathrm{sgn}\,(x)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(a^2/x^2 + 1) + a/x) - 1/2*(abs(a)*sgn(x) - sqrt(a^2 + x^2)
/sgn(x))*a
```

3.302 $\int \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=25

$$a\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+x\text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] x*ArcCsch[x/a] + a*ArcTanh[Sqrt[1 + a^2/x^2]]
```

Rubi [A] time = 0.0172984, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5892, 6278, 266, 63, 208}

$$a\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+x\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x],x]
```

```
[Out] x*ArcCsch[x/a] + a*ArcTanh[Sqrt[1 + a^2/x^2]]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6278

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(\frac{a}{x}\right)dx &= \int \operatorname{csch}^{-1}\left(\frac{x}{a}\right)dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x}\,dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,\frac{1}{x^2}\right) \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)-\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+\frac{a^2}{x^2}}\right)}{a} \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+a\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.088855, size = 77, normalized size = 3.08

$$
\frac{a\sqrt{a^2+x^2}\left(\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)-\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)\right)}{2x\sqrt{\frac{a^2}{x^2}+1}}+x\sinh^{-1}\left(\frac{a}{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x],x]
```

```
[Out] x*ArcSinh[a/x] + (a*Sqrt[a^2 + x^2]*(-Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 +
x/Sqrt[a^2 + x^2]]))/(2*Sqrt[1 + a^2/x^2]*x)
```

Maple [A] time = 0.006, size = 31, normalized size = 1.2

$$
-a\left(-\frac{x}{a}\text{Arcsinh}\left(\frac{a}{x}\right)-\text{Artanh}\left(\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x),x)
```

```
[Out] -a*(-arcsinh(a/x)/a*x-arctanh(1/(1+a^2/x^2)^(1/2)))
```

Maxima [A] time = 1.13453, size = 58, normalized size = 2.32

$$
\frac{1}{2}a\left(\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)\right)+x\,\text{arsinh}\left(\frac{a}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(sqrt(a^2/x^2 + 1) + 1) - log(sqrt(a^2/x^2 + 1) - 1)) + x*arcsinh
(a/x)
```

Fricas [B] time = 2.75216, size = 219, normalized size = 8.76

$$-a\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+(x-1)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] -a*log(x*sqrt((a^2 + x^2)/x^2) - x) + (x - 1)*log((x*sqrt((a^2 + x^2)/x^2)
+ a)/x) + log(x*sqrt((a^2 + x^2)/x^2) + a - x) - log(x*sqrt((a^2 + x^2)/x^2
) - a - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}\left(\frac{a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x),x)
```

```
[Out] Integral(asinh(a/x), x)
```

Giac [B] time = 1.32074, size = 68, normalized size = 2.72

$$\left(\log\left(|a|\right)\operatorname{sgn}\left(x\right)-\frac{\log\left(-x+\sqrt{a^2+x^2}\right)}{\operatorname{sgn}\left(x\right)}\right)a+x\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="giac")
```

```
[Out] (log(abs(a))*sgn(x) - log(-x + sqrt(a^2 + x^2))/sgn(x))*a + x*log(sqrt(a^2/
x^2 + 1) + a/x)
```

3.303 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{2}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2-\sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Rubi [A] time = 0.0619257, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5890, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2-\sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}\,dx = -\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 + 2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$
$$= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

Mathematica [A] time = 0.0073577, size = 52, normalized size = 1.

$$-\frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Maple [A] time = 0.006, size = 114, normalized size = 2.2

$$\frac{1}{2}\left(\operatorname{Arcsinh}\left(\frac{a}{x}\right)\right)^2 - \operatorname{Arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{polylog}\left(2, -\frac{a}{x}-\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{Arcsinh}\left(\frac{a}{x}\right)\ln\left(1-\frac{a}{x}-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x,x)
```

```
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1+a/x+(1+a^2/x^2)^(1/2))-polylog(2,-a/x-
(1+a^2/x^2)^(1/2))-arcsinh(a/x)*ln(1-a/x-(1+a^2/x^2)^(1/2))-polylog(2,a/x+(
1+a^2/x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\int \frac{x\log(x)}{a^3+ax^2+\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx + \log\left(a+\sqrt{a^2+x^2}\right)\log(x) - \frac{1}{2}\log(x)^2 - \frac{1}{2}\log(x)\log\left(\frac{x^2}{a^2}+1\right) - \frac{1}{4}\operatorname{Li}_2\left(-\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="maxima")
```

```
[Out] a*integrate(x*log(x)/(a^3 + a*x^2 + (a^2 + x^2)^(3/2)), x) + log(a + sqrt(a
^2 + x^2))*log(x) - 1/2*log(x)^2 - 1/2*log(x)*log(x^2/a^2 + 1) - 1/4*dilog(
```

```
-x^2/a^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\left(\frac{a}{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a/x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}\left(\frac{a}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x,x)
```

```
[Out] Integral(asinh(a/x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\left(\frac{a}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a/x)/x, x)
```

3.304 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^2} dx$

Optimal. Leaf size=29

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcCsch[x/a]/x
```

Rubi [A] time = 0.0241453, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5892, 6284, 261}

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^2,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcCsch[x/a]/x
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^2} dx &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^2} dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x} - a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x^3} dx \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0154129, size = 29, normalized size = 1.

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^2,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcSinh[a/x]/x
```

Maple [A] time = 0.003, size = 31, normalized size = 1.1

$$-\frac{1}{a}\left(\frac{a}{x}\mathrm{Arcsinh}\left(\frac{a}{x}\right)-\sqrt{1+\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^2,x)
```

```
[Out] -1/a*(a/x*arcsinh(a/x)-(1+a^2/x^2)^(1/2))
```

Maxima [A] time = 1.11267, size = 41, normalized size = 1.41

$$-\frac{\frac{a\,\mathrm{arsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*arcsinh(a/x)/x - sqrt(a^2/x^2 + 1))/a
```

Fricas [A] time = 2.68268, size = 101, normalized size = 3.48

$$-\frac{a\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-x\sqrt{\frac{a^2+x^2}{x^2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - x*sqrt((a^2 + x^2)/x^2))/(a*x)
```

Sympy [A] time = 2.66167, size = 20, normalized size = 0.69

$$\begin{cases} -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**2,x)
```

```
[Out] Piecewise((-asinh(a/x)/x + sqrt(a**2/x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.36858, size = 53, normalized size = 1.83

$$-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{x}+\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="giac")
```

```
[Out] -log(sqrt(a^2/x^2 + 1) + a/x)/x + sqrt(a^2/x^2 + 1)/a
```

3.305 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^3} dx$

Optimal. Leaf size=50

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] Sqrt[1 + a^2/x^2]/(4*a*x) - ArcCsch[x/a]/(4*a^2) - ArcCsch[x/a]/(2*x^2)
```

Rubi [A] time = 0.034073, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5892, 6284, 335, 321, 215}

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^3,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/(4*a*x) - ArcCsch[x/a]/(4*a^2) - ArcCsch[x/a]/(2*x^2)
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx\\ &= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}-\frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x^4}\,dx\\ &= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx,x,\frac{1}{x}\right)\\ &= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}}\,dx,x,\frac{1}{x}\right)}{4a}\\ &= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0224858, size = 44, normalized size = 0.88

$$\frac{ax\sqrt{\frac{a^2}{x^2}+1}-\left(2a^2+x^2\right)\sinh^{-1}\left(\frac{a}{x}\right)}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^3,x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x - (2*a^2 + x^2)*ArcSinh[a/x])/(4*a^2*x^2)
```

Maple [A] time = 0.004, size = 46, normalized size = 0.9

$$-\frac{1}{a^2}\left(\frac{a^2}{2\,x^2}\text{Arcsinh}\left(\frac{a}{x}\right)-\frac{a}{4\,x}\sqrt{1+\frac{a^2}{x^2}}+\frac{1}{4}\text{Arcsinh}\left(\frac{a}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^3,x)
```

```
[Out] -1/a^2*(1/2*a^2/x^2*arcsinh(a/x)-1/4*a/x*(1+a^2/x^2)^(1/2)+1/4*arcsinh(a/x)
)
```

Maxima [B] time = 1.10012, size = 131, normalized size = 2.62

$$\frac{1}{8}a\left(\frac{2\,x\sqrt{\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}+1\right)-a^4}-\frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}+a\right)}{a^3}+\frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}-a\right)}{a^3}\right)-\frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(2*x*sqrt(a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 + 1) - a^4) - log(x*sqrt(a^2
/x^2 + 1) + a)/a^3 + log(x*sqrt(a^2/x^2 + 1) - a)/a^3) - 1/2*arcsinh(a/x)/x
```

^2

Fricas [A] time = 2.68232, size = 130, normalized size = 2.6

$$\frac{ax\sqrt{\frac{a^2+x^2}{x^2}} - \left(2\,a^2+x^2\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)}{4\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x*sqrt((a^2 + x^2)/x^2) - (2*a^2 + x^2)*log((x*sqrt((a^2 + x^2)/x^2)
 + a)/x))/(a^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**3,x)
```

```
[Out] Integral(asinh(a/x)/x**3, x)
```

Giac [B] time = 1.41869, size = 124, normalized size = 2.48

$$-\frac{1}{8}\,a\left(\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{a^3\operatorname{sgn}(x)} - \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{a^3\operatorname{sgn}(x)} - \frac{2\sqrt{a^2+x^2}}{a^2x^2\operatorname{sgn}(x)}\right) - \frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*a*(log(a + sqrt(a^2 + x^2))/(a^3*sgn(x)) - log(-a + sqrt(a^2 + x^2))/(
a^3*sgn(x)) - 2*sqrt(a^2 + x^2)/(a^2*x^2*sgn(x))) - 1/2*log(sqrt(a^2/x^2 +
1) + a/x)/x^2
```

3.306 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=54

$$\frac{\left(\frac{a^2}{x^2}+1\right)^{3/2}}{9a^3}-\frac{\sqrt{\frac{a^2}{x^2}+1}}{3a^3}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] -Sqrt[1 + a^2/x^2]/(3*a^3) + (1 + a^2/x^2)^(3/2)/(9*a^3) - ArcCsch[x/a]/(3*
x^3)
```

Rubi [A] time = 0.0399537, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5892, 6284, 266, 43}

$$\frac{\left(\frac{a^2}{x^2}+1\right)^{3/2}}{9a^3}-\frac{\sqrt{\frac{a^2}{x^2}+1}}{3a^3}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^4,x]
```

```
[Out] -Sqrt[1 + a^2/x^2]/(3*a^3) + (1 + a^2/x^2)^(3/2)/(9*a^3) - ArcCsch[x/a]/(3*
x^3)
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{3}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}x^5}\,dx$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\,\mathrm{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3} + \frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Mathematica [A] time = 0.0281997, size = 48, normalized size = 0.89

$$\left(\frac{1}{9ax^2} - \frac{2}{9a^3}\right)\sqrt{\frac{a^2+x^2}{x^2}} - \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^4,x]
```

```
[Out] (-2/(9*a^3) + 1/(9*a*x^2))*Sqrt[(a^2 + x^2)/x^2] - ArcSinh[a/x]/(3*x^3)
```

Maple [A] time = 0.006, size = 53, normalized size = 1.

$$-\frac{1}{a^3}\left(\frac{a^3}{3\,x^3}\mathrm{Arcsinh}\left(\frac{a}{x}\right) - \frac{a^2}{9\,x^2}\sqrt{1+\frac{a^2}{x^2}} + \frac{2}{9}\sqrt{1+\frac{a^2}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^4,x)
```

```
[Out] -1/a^3*(1/3*a^3/x^3*arcsinh(a/x)-1/9*a^2/x^2*(1+a^2/x^2)^(1/2)+2/9*(1+a^2/x
^2)^(1/2))
```

Maxima [A] time = 1.20935, size = 63, normalized size = 1.17

$$\frac{1}{9}a\left(\frac{\left(\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4} - \frac{3\sqrt{\frac{a^2}{x^2}+1}}{a^4}\right) - \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*a*((a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(a^2/x^2 + 1)/a^4) - 1/3*arcsinh(a/x
)/x^3
```

Fricas [A] time = 2.75567, size = 136, normalized size = 2.52

$$-\frac{3\,a^3\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-\left(a^2x-2\,x^3\right)\sqrt{\frac{a^2+x^2}{x^2}}}{9\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*a^3*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - (a^2*x - 2*x^3)*sqrt((a^
2 + x^2)/x^2))/(a^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**4,x)
```

```
[Out] Integral(asinh(a/x)/x**4, x)
```

Giac [A] time = 1.39379, size = 101, normalized size = 1.87

$$-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{3\,x^3}-\frac{4\left(a^2-3\left(x-\sqrt{a^2+x^2}\right)^2\right)a}{9\left(a^2-\left(x-\sqrt{a^2+x^2}\right)^2\right)^3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*log(sqrt(a^2/x^2 + 1) + a/x)/x^3 - 4/9*(a^2 - 3*(x - sqrt(a^2 + x^2))^
2)*a/((a^2 - (x - sqrt(a^2 + x^2))^2)^3*sgn(x))
```

3.307 $\int x^m \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=77

$$\frac{x^{m+1}\sinh^{-1}(ax^n)}{m+1} - \frac{anx^{m+n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n+1}{2n},\frac{m+3n+1}{2n},-a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

```
[Out] (x^(1 + m)*ArcSinh[a*x^n])/(1 + m) - (a*n*x^(1 + m + n)*Hypergeometric2F1[1
/2, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))])/((1 + m)*(1 +
m + n))
```

Rubi [A] time = 0.044325, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5902, 12, 364}

$$\frac{x^{m+1}\sinh^{-1}(ax^n)}{m+1} - \frac{anx^{m+n+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{2n};\frac{m+3n+1}{2n};-a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x^n])/(1 + m) - (a*n*x^(1 + m + n)*Hypergeometric2F1[1
/2, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))])/((1 + m)*(1 +
m + n))
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \sinh^{-1}(ax^n)\,dx = \frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m} - \frac{\int \frac{anx^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m}$$
$$= \frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m} - \frac{(an)\int \frac{x^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m}$$
$$= \frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m} - \frac{anx^{1+m+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{2n};\frac{1+m+3n}{2n};-a^2x^{2n}\right)}{(1+m)(1+m+n)}$$

Mathematica [A] time = 0.0738982, size = 74, normalized size = 0.96

$$\frac{x^{m+1}\left((m+n+1)\sinh^{-1}(ax^n) - anx^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n+1}{2n},\frac{m+3n+1}{2n},-a^2x^{2n}\right)\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*((1 + m + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, (1
+ m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))]))/((1 + m)*(1 + m + n)
)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^m \text{Arcsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x^n),x)
```

```
[Out] int(x^m*arcsinh(a*x^n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x**n),x)
```

```
[Out] Integral(x**m*asinh(a*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x^n), x)
```

3.308 $\int x^2 \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=64

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2n},\frac{3(n+1)}{2n},-a^2x^{2n}\right)}{3(n+3)}$$

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), (3*(1 + n))/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Rubi [A] time = 0.0352749, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5902, 12, 364}

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2n};\frac{3(n+1)}{2n};-a^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), (3*(1 + n))/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sinh^{-1}(ax^n)\,dx &= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{1}{3}\int \frac{anx^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{1}{3}(an)\int \frac{x^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2n};\frac{3(1+n)}{2n};-a^2x^{2n}\right)}{3(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.0470922, size = 66, normalized size = 1.03

$$\frac{1}{3}x^3\sinh^{-1}(ax^n)-\frac{anx^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2n},\frac{n+3}{2n}+1,-a^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), 1 + (3 + n)/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int x^2\text{Arcsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x^n),x)
```

```
[Out] int(x^2*arcsinh(a*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{9}nx^3+\frac{1}{3}x^3\log\left(ax^n+\sqrt{a^2x^{2n}+1}\right)-an\int\frac{x^2x^n}{3\left(a^3x^{3n}+ax^n+\left(a^2x^{2n}+1\right)^{\frac{3}{2}}\right)}\,dx+n\int\frac{x^2}{3\left(a^2x^{2n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*n*x^3 + 1/3*x^3*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) - a*n*integrate(1/3
*x^2*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) + n*integrate(
1/3*x^2/(a^2*x^(2*n) + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\text{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x**n),x)
```

```
[Out] Integral(x**2*asinh(a*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsinh}(ax^n) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x^n), x)
```

3.309 $\int x \sinh^{-1}(ax^n)\, dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{anx^{n+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2n},\frac{1}{2}\left(\frac{2}{n}+3\right),-a^2x^{2n}\right)}{2(n+2)}$$

```
[Out] (x^2*ArcSinh[a*x^n])/2 - (a*n*x^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/(2*n
), (3 + 2/n)/2, -(a^2*x^(2*n))]])/(2*(2 + n))
```

Rubi [A] time = 0.0297645, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5902, 12, 364}

$$\frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{anx^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);-a^2x^{2n}\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*ArcSinh[a*x^n])/2 - (a*n*x^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/(2*n
), (3 + 2/n)/2, -(a^2*x^(2*n))]])/(2*(2 + n))
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}(ax^n)\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{1}{2}\int \frac{anx^{1+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\
&= \frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{1}{2}(an)\int \frac{x^{1+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\
&= \frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{anx^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2+n)}
\end{aligned}$$

Mathematica [A] time = 0.0458168, size = 58, normalized size = 0.89

$$\frac{x^2\left((n+2)\sinh^{-1}(ax^n)-anx^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n}+\frac{1}{2},\frac{1}{n}+\frac{3}{2},-a^2x^{2n}\right)\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*((2 + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, 1/2 + n^(-1),
 3/2 + n^(-1), -(a^2*x^(2*n))]))/(2*(2 + n))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int x\text{Arcsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x^n),x)
```

```
[Out] int(x*arcsinh(a*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}nx^2-an\int\frac{xx^n}{2\left(a^3x^{3n}+ax^n+\left(a^2x^{2n}+1\right)^{\frac{3}{2}}\right)}\,dx+\frac{1}{2}x^2\log\left(ax^n+\sqrt{a^2x^{2n}+1}\right)+n\int\frac{x}{2\left(a^2x^{2n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -1/4*n*x^2 - a*n*integrate(1/2*x*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) +
1)^(3/2)), x) + 1/2*x^2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) + n*integrate(1/
2*x/(a^2*x^(2*n) + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x**n),x)
```

```
[Out] Integral(x*asinh(a*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsinh}(ax^n) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x^n), x)
```

3.310 $\int \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=56

$$x\sinh^{-1}(ax^n)-\frac{anx^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2n},\frac{1}{2}\left(\frac{1}{n}+3\right),-a^2x^{2n}\right)}{n+1}$$

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Rubi [A] time = 0.0225214, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5900, 12, 364}

$$x\sinh^{-1}(ax^n)-\frac{anx^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Rule 5900

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/Sqrt[1 + u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(ax^n)\,dx &= x\sinh^{-1}(ax^n)-\int \frac{anx^n}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= x\sinh^{-1}(ax^n)-(an)\int \frac{x^n}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= x\sinh^{-1}(ax^n)-\frac{anx^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0249703, size = 56, normalized size = 1.

$$x\sinh^{-1}(ax^n) - \frac{anx^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2n},\frac{1}{2}\left(\frac{1}{n}+3\right),-a^2x^{2n}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \text{Arcsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n),x)
```

```
[Out] int(arcsinh(a*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-an\int \frac{x^n}{a^3x^{3n} + ax^n + \left(a^2x^{2n}+1\right)^{\frac{3}{2}}}\,dx - nx + n\int \frac{1}{a^2x^{2n}+1}\,dx + x\log\left(ax^n + \sqrt{a^2x^{2n}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) - n*
x + n*integrate(1/(a^2*x^(2*n) + 1), x) + x*log(a*x^n + sqrt(a^2*x^(2*n) +
1))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n),x)
```

```
[Out] Integral(asinh(a*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n), x)
```

3.311 $\int \frac{\sinh^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=60

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^n)}\right)}{2n}-\frac{\sinh^{-1}(ax^n)^2}{2n}+\frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

```
[Out] -ArcSinh[a*x^n]^2/(2*n) + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Rubi [A] time = 0.0662234, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^n)}\right)}{2n}-\frac{\sinh^{-1}(ax^n)^2}{2n}+\frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x,x]
```

```
[Out] -ArcSinh[a*x^n]^2/(2*n) + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^n)}{x}\,dx = \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^n)\right)}{n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^n)\right)}{n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sinh^{-1}(ax^n\right.}{n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(ax^n)}\right)}{2n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}(ax^n)}\right)}{2n}$$

Mathematica [A] time = 0.0076814, size = 60, normalized size = 1.

$$\frac{\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax^n)}\right)}{2n} - \frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x,x]
```

```
[Out] -ArcSinh[a*x^n]^2/(2*n) + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Maple [A] time = 0.003, size = 133, normalized size = 2.2

$$-\frac{(\text{Arcsinh}(ax^n))^2}{2n} + \frac{\text{Arcsinh}(ax^n)}{n}\ln\left(1+ax^n+\sqrt{1+a^2(x^n)^2}\right) + \frac{1}{n}\text{polylog}\left(2, -ax^n-\sqrt{1+a^2(x^n)^2}\right) + \frac{\text{Arcsinh}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x,x)
```

```
[Out] -1/2*arcsinh(a*x^n)^2/n+1/n*arcsinh(a*x^n)*ln(1+a*x^n+(1+a^2*(x^n)^2)^(1/2)
)+1/n*polylog(2,-a*x^n-(1+a^2*(x^n)^2)^(1/2))+1/n*arcsinh(a*x^n)*ln(1-a*x^n
-(1+a^2*(x^n)^2)^(1/2))+1/n*polylog(2,a*x^n+(1+a^2*(x^n)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-an\int \frac{x^n\log(x)}{a^3xx^{3n}+axx^n+\left(a^2xx^{2n}+x\right)\sqrt{a^2x^{2n}+1}}\,dx - \frac{1}{2}n\log(x)^2 + n\int \frac{\log(x)}{a^2xx^{2n}+x}\,dx + \log\left(ax^n+\sqrt{a^2x^{2n}+1}\right)\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) + a*x*x^n + (a^2*x*x^(2*n) + x)*sq
rt(a^2*x^(2*n) + 1)), x) - 1/2*n*log(x)^2 + n*integrate(log(x)/(a^2*x*x^(2*
n) + x), x) + log(a*x^n + sqrt(a^2*x^(2*n) + 1))*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x,x)
```

```
[Out] Integral(asinh(a*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x, x)
```

3.312 $\int \frac{\sinh^{-1}(ax^n)}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{anx^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),-a^2x^{2n}\right)}{1-n}-\frac{\sinh^{-1}(ax^n)}{x}$$

```
[Out] -(ArcSinh[a*x^n]/x) - (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, -(1 - n)/(2*n)
, (3 - n^(-1))/2, -(a^2*x^(2*n))])/(1 - n)
```

Rubi [A] time = 0.0331579, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5902, 12, 364}

$$-\frac{anx^{n-1}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}-\frac{\sinh^{-1}(ax^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) - (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, -(1 - n)/(2*n)
, (3 - n^(-1))/2, -(a^2*x^(2*n))])/(1 - n)
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^n)}{x^2}\,dx = -\frac{\sinh^{-1}(ax^n)}{x} + \int \frac{anx^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^n)}{x} + (an)\int \frac{x^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^n)}{x} - \frac{anx^{-1+n}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}$$

Mathematica [A] time = 0.0526311, size = 61, normalized size = 0.94

$$\frac{anx^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2n},\frac{n-1}{2n}+1,-a^2x^{2n}\right)}{n-1} - \frac{\sinh^{-1}(ax^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) + (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/(2*n)
, 1 + (-1 + n)/(2*n), -(a^2*x^(2*n))])/(-1 + n)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{\text{Arcsinh}(ax^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x^2,x)
```

```
[Out] int(arcsinh(a*x^n)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$an\int \frac{x^n}{a^3x^2x^{3n} + ax^2x^n + \left(a^2x^2x^{2n} + x^2\right)\sqrt{a^2x^{2n}+1}}\,dx - n\int \frac{1}{a^2x^2x^{2n} + x^2}\,dx - \frac{n + \log\left(ax^n + \sqrt{a^2x^{2n}+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n/(a^3*x^2*x^(3*n) + a*x^2*x^n + (a^2*x^2*x^(2*n) + x^2)*sq
rt(a^2*x^(2*n) + 1)), x) - n*integrate(1/(a^2*x^2*x^(2*n) + x^2), x) - (n +
 log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}\left(ax^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x**2,x)
```

```
[Out] Integral(asinh(a*x**n)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(ax^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x^2, x)
```

3.313 $\int \frac{\sinh^{-1}(ax^n)}{x^3} \, dx$

Optimal. Leaf size=68

$$-\frac{anx^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2-n)}-\frac{\sinh^{-1}\left(ax^n\right)}{2x^2}$$

```
[Out] -ArcSinh[a*x^n]/(2*x^2) - (a*n*x^(-2 + n)*Hypergeometric2F1[1/2, (1 - 2/n)/
2, (3 - 2/n)/2, -(a^2*x^(2*n))])/(2*(2 - n))
```

Rubi [A] time = 0.0390606, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5902, 12, 364}

$$-\frac{anx^{n-2}\,_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}-\frac{\sinh^{-1}\left(ax^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] -ArcSinh[a*x^n]/(2*x^2) - (a*n*x^(-2 + n)*Hypergeometric2F1[1/2, (1 - 2/n)/
2, (3 - 2/n)/2, -(a^2*x^(2*n))])/(2*(2 - n))
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^n)}{x^3}\,dx = -\frac{\sinh^{-1}(ax^n)}{2x^2} + \frac{1}{2}\int \frac{anx^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^n)}{2x^2} + \frac{1}{2}(an)\int \frac{x^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx$$
$$= -\frac{\sinh^{-1}(ax^n)}{2x^2} - \frac{anx^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}$$

Mathematica [A] time = 0.0436203, size = 62, normalized size = 0.91

$$\frac{anx^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{1}{n},\frac{3}{2}-\frac{1}{n},-a^2x^{2n}\right)-(n-2)\sinh^{-1}(ax^n)}{2(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] (-((-2 + n)*ArcSinh[a*x^n]) + a*n*x^n*Hypergeometric2F1[1/2, 1/2 - n^(-1),
3/2 - n^(-1), -(a^2*x^(2*n))])/(2*(-2 + n)*x^2)
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{\text{Arcsinh}(ax^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x^3,x)
```

```
[Out] int(arcsinh(a*x^n)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$an\int \frac{x^n}{2\left(a^3x^3x^{3n}+ax^3x^n+\left(a^2x^3x^{2n}+x^3\right)\sqrt{a^2x^{2n}+1}\right)}\,dx - n\int \frac{1}{2\left(a^2x^3x^{2n}+x^3\right)}\,dx - \frac{n+2\log\left(ax^n+\sqrt{a^2x^{2n}+1}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="maxima")
```

```
[Out] a*n*integrate(1/2*x^n/(a^3*x^3*x^(3*n) + a*x^3*x^n + (a^2*x^3*x^(2*n) + x^3
)*sqrt(a^2*x^(2*n) + 1)), x) - n*integrate(1/2/(a^2*x^3*x^(2*n) + x^3), x)
- 1/4*(n + 2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax^n)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x**3,x)
```

```
[Out] Integral(asinh(a*x**n)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax^n)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x^3, x)
```

3.314 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4 dx$

Optimal. Leaf size=153

$$-\frac{192b^3\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2-\frac{8b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-\right.\right.}{dx}$$

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^
2]))/(d*x) + 48*b^2*x*(a + I*b*ArcSin[1 - I*d*x^2])^2 - (8*b*Sqrt[(2*I)*d*x
^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*(a + I*b*ArcSin[1
- I*d*x^2])^4
```

Rubi [A] time = 0.0360296, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4814, 8}

$$-\frac{192b^3\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2-\frac{8b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-\right.\right.}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```



```
[Out] 384*b^4*x - (192*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^
2]))/(d*x) + 48*b^2*x*(a + I*b*ArcSin[1 - I*d*x^2])^2 - (8*b*Sqrt[(2*I)*d*x
^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*(a + I*b*ArcSin[1
- I*d*x^2])^4
```


Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4 dx = -\frac{8b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4+\left(48b^2\right)$$

$$= -\frac{192b^3\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2-$$

$$= 384b^4x-\frac{192b^3\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-id\right.\right.$$

Mathematica [A] time = 0.117207, size = 149, normalized size = 0.97

$$48b^2\left(-\frac{4b\sqrt{dx^2\left(dx^2+2i\right)}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x\right)+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*
(a + I*b*ArcSin[1 - I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(2*I +
d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2]))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^
2])^2)
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int \left(a + b\text{Arcsinh}\left(i + dx^2\right)\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^4 x \log\left(dx^2 + \sqrt{dx^2 + 2i}\sqrt{d}x + i\right)^4 + 4\left(x \operatorname{arsinh}\left(dx^2 + i\right) - \frac{2\left(d^{\frac{3}{2}}x^2 + 2i\sqrt{d}\right)}{\sqrt{dx^2 + 2id}}\right)a^3 b + a^4 x + \int \frac{\left(4\left(ab^3d^2 - 2b^4d^2\right)x^4 - 8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^4 + 4*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^3*b + a^4*x +
integrate(((4*(a*b^3*d^2 - 2*b^4*d^2)*x^4 - 8*a*b^3 + (12*I*a*b^3*d - 16*I*
b^4*d)*x^2 + (4*(a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 + (8*I*a*b^3*sqrt(d) -
8*I*b^4*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d
)*x + I)^3 + (6*a^2*b^2*d^2*x^4 + 18*I*a^2*b^2*d*x^2 - 12*a^2*b^2 + 6*(a^2*
b^2*d^(3/2)*x^3 + 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqr
t(d*x^2 + 2*I)*sqrt(d)*x + I)^2)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*
sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x)
```

Fricas [B] time = 2.82357, size = 648, normalized size = 4.24

$$\frac{b^4dx^2 \log\left(dx^2 + \sqrt{d^2x^4 + 2i\,dx^2} + i\right)^4 + \left(a^4 + 48\,a^2b^2 + 384\,b^4\right)dx^2 + 4\left(ab^3dx^2 - 2\sqrt{d^2x^4 + 2i\,dx^2}b^4\right)\log\left(dx^2 + \sqrt{d^2x^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^4 + (a^4 + 48*a^2*b^2
 + 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 + 2*I*d*x^2)*b^4)*log(d
```

```
*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^3 - 6*(4*sqrt(d^2*x^4 + 2*I*d*x^2)*a*
b^3 - (a^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^2
 + 4*((a^3*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 + 2*I*d*x^2)*(a^2*b^2 + 8*b
^4))*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) - 8*sqrt(d^2*x^4 + 2*I*d*x^
2)*(a^3*b + 24*a*b^3))/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^4, x)
```

3.315 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3 \, dx$

Optimal. Leaf size=129

$$24ab^2x-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4+2idx^2}}{dx}+24ib^3x\sin^{-1}\left(\right.$$

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + (24*I)*b^3*x*ArcS
in[1 - I*d*x^2] - (6*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*
x^2])^2)/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^3
```

Rubi [A] time = 0.0639983, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4814, 4840, 12, 1588}

$$24ab^2x-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4+2idx^2}}{dx}+24ib^3x\sin^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + (24*I)*b^3*x*ArcS
in[1 - I*d*x^2] - (6*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*
x^2])^2)/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^3
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3 dx = -\frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx} + x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3 + \left(24b^2\right)$$

$$= 24ab^2x - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx} + x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3 +$$

$$= 24ab^2x + 24ib^3x\sin^{-1}\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx} + x\left(\right.$$

$$= 24ab^2x + 24ib^3x\sin^{-1}\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx} + x\left(\right.$$

$$= 24ab^2x - \frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx} + 24ib^3x\sin^{-1}\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\,\mathrm{s}\right.}{dx}$$

Mathematica [A] time = 0.133959, size = 180, normalized size = 1.4

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2\left(dx^2+2i\right)}+3ib\sin^{-1}\left(1-idx^2\right)\left(a^2dx^2-4ab\sqrt{dx^2\left(dx^2+2i\right)}+8b^2dx^2\right)+3b^2}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(2*I + d*x^2)] + (3*
I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 -
 I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 - I
*d*x^2]^2 - I*b^3*d*x^2*ArcSin[1 - I*d*x^2]^3)/(d*x)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(i+dx^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^3x\log\left(dx^2+\sqrt{dx^2+2i}\sqrt{d}x+i\right)^3 + 3\left(x\,\mathrm{arsinh}\left(dx^2+i\right) - \frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2i}d}\right)a^2b + a^3x + \int \frac{\left(3\left(ab^2d^2-2b^3d^2\right)x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^3 + 3*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^2*b + a^3*x +
```

```
integrate((3*(a*b^2*d^2 - 2*b^3*d^2)*x^4 - 6*a*b^2 + (9*I*a*b^2*d - 12*I*b^
3*d)*x^2 + (3*(a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 + (6*I*a*b^2*sqrt(d) - 6*
I*b^3*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*
x + I)^2/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 +
2*I) - 2), x)
```

Fricas [A] time = 2.72941, size = 458, normalized size = 3.55

$$\frac{b^3dx^2\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)^3+\left(a^3+24\,ab^2\right)dx^2+3\left(ab^2dx^2-2\sqrt{d^2x^4+2i\,dx^2}b^3\right)\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^3 + (a^3 + 24*a*b^2)*
d*x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 + 2*I*d*x^2)*b^3)*log(d*x^2 + sqrt(
d^2*x^4 + 2*I*d*x^2) + I)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 + 2
*I*d*x^2)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) - 6*sqrt(d^2*x^
4 + 2*I*d*x^2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^3, x)
```

3.316 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x$$

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Rubi [A] time = 0.01526, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4814, 8}

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\,dx &= -\frac{4b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+\left(8b^2\right)\int \\ &= 8b^2x-\frac{4b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\end{aligned}$$

Mathematica [A] time = 0.0257147, size = 76, normalized size = 1.

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(i+dx^{2}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\left(x\,\mathrm{arsinh}\left(dx^{2}+i\right)-\frac{2\left(d^{\frac{3}{2}}x^{2}+2i\sqrt{d}\right)}{\sqrt{dx^{2}+2id}}\right)ab+\left(x\log\left(dx^{2}+\sqrt{dx^{2}+2i}\sqrt{d}x+i\right)^{2}-\int\frac{\left(4\,d^{2}x^{4}+8i\,dx^{2}+\left(4\,d^{\frac{3}{2}}x^{3}+4i\,\sqrt{d}\right.\right.}{d^{2}x^{4}+3i\,dx^{2}+\left(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 - integrate((4*
d^2*x^4 + 8*I*d*x^2 + (4*d^(3/2)*x^3 + 4*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*lo
g(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*
x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x))*b^2 + a^2*x
```

Fricas [B] time = 2.64473, size = 288, normalized size = 3.79

$$\frac{b^{2}dx^{2}\log\left(dx^{2}+\sqrt{d^{2}x^{4}+2i\,dx^{2}}+i\right)^{2}+\left(a^{2}+8\,b^{2}\right)dx^{2}-4\,\sqrt{d^{2}x^{4}+2i\,dx^{2}}ab+2\left(abdx^{2}-2\,\sqrt{d^{2}x^{4}+2i\,dx^{2}}b^{2}\right)\log\left(dx^{2}+\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^2 + (a^2 + 8*b^2)*d*x
^2 - 4*sqrt(d^2*x^4 + 2*I*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 + 2*I*
d*x^2)*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I))/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^2, x)
```

3.317 $\int \left(a + ib\sin^{-1}\left(1 - idx^2\right)\right) dx$

Optimal. Leaf size=50

$$ax - \frac{2b\sqrt{d^2x^4 + 2idx^2}}{dx} + ibx\sin^{-1}\left(1 - idx^2\right)$$

```
[Out] a*x - (2*b*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Rubi [A] time = 0.04019, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4840, 12, 1588}

$$ax - \frac{2b\sqrt{d^2x^4 + 2idx^2}}{dx} + ibx\sin^{-1}\left(1 - idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a + ib\sin^{-1}\left(1 - idx^2\right)\right) dx &= ax + (ib)\int \sin^{-1}\left(1 - idx^2\right) dx \\
&= ax + ibx\sin^{-1}\left(1 - idx^2\right) - (ib)\int -\frac{2idx^2}{\sqrt{2idx^2 + d^2x^4}} dx \\
&= ax + ibx\sin^{-1}\left(1 - idx^2\right) - (2bd)\int \frac{x^2}{\sqrt{2idx^2 + d^2x^4}} dx \\
&= ax - \frac{2b\sqrt{2idx^2 + d^2x^4}}{dx} + ibx\sin^{-1}\left(1 - idx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0239312, size = 48, normalized size = 0.96

$$ax - \frac{2b\sqrt{dx^2\left(dx^2+2i\right)}}{dx} + ibx\sin^{-1}\left(1-idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Maple [A] time = 0.014, size = 47, normalized size = 0.9

$$ax + b\left(x\mathrm{Arcsinh}\left(i+dx^2\right) - 2\,\frac{x\left(dx^2+2\,i\right)}{\sqrt{2\,idx^2+d^2x^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(I+d*x^2),x)
```

```
[Out] a*x+b*(x*arcsinh(I+d*x^2)-2/(2*I*d*x^2+d^2*x^4)^(1/2)*x*(d*x^2+2*I))
```

Maxima [A] time = 1.21383, size = 59, normalized size = 1.18

$$\left(x\,\mathrm{arsinh}\left(dx^2+i\right) - \frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2id}}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d)
)*b + a*x
```

Fricas [A] time = 2.67691, size = 138, normalized size = 2.76

$$\frac{bdx^2\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)+adx^2-2\sqrt{d^2x^4+2i\,dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a*d*x^2 - 2*sqrt(d^2*
x^4 + 2*I*d*x^2)*b)/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(I+d*x**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arsinh}\left(dx^2 + i\right) + a\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(b*arcsinh(d*x^2 + I) + a, x)
```

3.318 $\int \frac{1}{a+ib\sin^{-1}(1-idx^2)}\,dx$

Optimal. Leaf size=194

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i(a+ib\sin^{-1}(1-idx^2))}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right.\right.\right.}$$

```
[Out] (x*CosIntegral[((-I/2)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] -
 Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcS
in[1 - I*d*x^2]/2])/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Rubi [A] time = 0.0478033, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4816}

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i(a+ib\sin^{-1}(1-idx^2))}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```


```
[Out] (x*CosIntegral[((-I/2)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] -
 Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcS
in[1 - I*d*x^2]/2])/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```


Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx=\frac{x\text{Ci}\left(-\frac{i(a+ib\sin^{-1}(1-idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{S}}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\right.}$$

Mathematica [A] time = 0.711214, size = 150, normalized size = 0.77

$$\frac{x\left(\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{1}{2}\left(\sin^{-1}\left(1-idx^2\right)-\frac{ia}{b}\right)\right)+\left(-\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(\right.\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

```
[Out] (x*(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(I*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] - Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b
 - ArcSin[1 - I*d*x^2]/2]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1
 - I*d*x^2]/2]))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a + b\text{Arcsinh}\left(i + dx^2\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2)),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsinh}\left(dx^2 + i\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(d*x^2 + I) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \log\left(dx^2 + \sqrt{d^2x^4 + 2i\,dx^2} + i\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsinh}\left(dx^2+i\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(d*x^2 + I) + a), x)
```

3.319 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}\,dx$

Optimal. Leaf size=245

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(2*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*C
osIntegral[((-I/2)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(Cosh[a/(2*b)] - I*Sin
h[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin
[1 - I*d*x^2]/2])/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Rubi [A] time = 0.0426255, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4825}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(2*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*C
osIntegral[((-I/2)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(Cosh[a/(2*b)] - I*Sin
h[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin
[1 - I*d*x^2]/2])/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[(
x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x
] + Simp[(x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^
2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}\,dx=-\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}+\frac{x\text{Ci}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Mathematica [A] time = 1.3219, size = 197, normalized size = 0.8

$$\frac{\frac{x^2\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{1}{2}\left(\sin^{-1}\left(1-idx^2\right)-\frac{ia}{b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}-\frac{2b\sqrt{dx^2\left(dx^2+2i\right)}}{d\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}}{4b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])) + (x^2*
(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)]) + (Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Arc
Sin[1 - I*d*x^2]/2]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2]))/(4*b^2*x)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^2x^4+3i\,dx^2+\left(d^{\frac{3}{2}}x^3+2i\,\sqrt{d}x\right)\sqrt{dx^2+2i}-2}{2\,abd^2x^3+4i\,abdx+\left(2\,b^2d^2x^3+4i\,b^2dx+\left(2\,b^2d^{\frac{3}{2}}x^2+2i\,b^2\sqrt{d}\right)\sqrt{dx^2+2i}\right)\log\left(dx^2+\sqrt{dx^2+2i}\sqrt{d}x+i\right)+\left(2\,abd^{\frac{3}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2
)/(2*a*b*d^2*x^3 + 4*I*a*b*d*x + (2*b^2*d^2*x^3 + 4*I*b^2*d*x + (2*b^2*d^(3
/2)*x^2 + 2*I*b^2*sqrt(d))*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)
*sqrt(d)*x + I) + (2*a*b*d^(3/2)*x^2 + 2*I*a*b*sqrt(d))*sqrt(d*x^2 + 2*I))
+ integrate((2*d^3*x^6 + 6*I*d^2*x^4 + (2*d^2*x^4 + 2*I*d*x^2 - 4)*(d*x^2 +
 2*I) + 2*(2*d^(5/2)*x^5 + 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 + 2*I) +
 8*I)/(4*a*b*d^3*x^6 + 16*I*a*b*d^2*x^4 - 16*a*b*d*x^2 + (4*a*b*d^2*x^4 + 8
*I*a*b*d*x^2 - 4*a*b)*(d*x^2 + 2*I) + (4*b^2*d^3*x^6 + 16*I*b^2*d^2*x^4 - 1
6*b^2*d*x^2 + 4*(b^2*d^2*x^4 + 2*I*b^2*d*x^2 - b^2)*(d*x^2 + 2*I) + (8*b^2*
d^(5/2)*x^5 + 24*I*b^2*d^(3/2)*x^3 - 16*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*l
og(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (8*a*b*d^(5/2)*x^5 + 24*I*a*b
*d^(3/2)*x^3 - 16*a*b*sqrt(d)*x)*sqrt(d*x^2 + 2*I)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(b^2dx\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)+abdx\right)\text{integral}\left(\frac{\sqrt{d^2x^4+2i\,dx^2}}{2\,abdx^2+4i\,ab+\left(2\,b^2dx^2+4i\,b^2\right)\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)},x\right)-\sqrt{d^2x^4+2i\,dx^2}}{2\left(b^2dx\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)+abdx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*x*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a*b*d*x)*integ
ral(sqrt(d^2*x^4 + 2*I*d*x^2)/(2*a*b*d*x^2 + 4*I*a*b + (2*b^2*d*x^2 + 4*I*b
^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)), x) - sqrt(d^2*x^4 + 2*I*d*
x^2))/(b^2*d*x*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a*b*d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-2), x)
```

3.320 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}\,dx$

Optimal. Leaf size=275

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-id\right.\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-id\right.\right.\right.}$$

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(4*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^2) - x/
(8*b^2*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*CosIntegral[((-I/2)*(a + I*b*Arc
Sin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(16*b^3*(Cos[ArcSi
n[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (x*(I*Cosh[a/(2*b)] + Si
nh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2])/(16*b^3*(Cos
[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rubi [A] time = 0.0541483, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4828, 4816}

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-id\right.\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-id\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(4*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^2) - x/
(8*b^2*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*CosIntegral[((-I/2)*(a + I*b*Arc
Sin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(16*b^3*(Cos[ArcSi
n[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (x*(I*Cosh[a/(2*b)] + Si
nh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2])/(16*b^3*(Cos
[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{\int \frac{1}{a+ib\sin^{-1}(1-idx^2)}\,dx}{8b^2}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{x\text{Ci}\left(-\frac{i\left(a+ib\sin^{-1}\left(\right.\right.}{2b}\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

Mathematica [A] time = 0.552229, size = 229, normalized size = 0.83

$$\frac{-\frac{8b^2\sqrt{dx^2(dx^2+2i)}}{d\left(a+ib\sin^{-1}(1-idx^2)\right)^2} + \frac{2ix^2\left(\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{1}{2}\left(\sin^{-1}(1-idx^2)-\frac{ia}{b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)} - \frac{4b}{a+ib\sin^{-1}}}{32b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] ((-8*b^2*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])^2) - (
4*b*x^2)/(a + I*b*ArcSin[1 - I*d*x^2]) + ((2*I)*x^2*(CosIntegral[(((-I)*a)/
b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]) - (Cosh[a/(2*
b)] - I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2]))/(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(32*b^3*x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -(4*(a*d^(11/2) + 2*b*d^(11/2))*x^10 + (24*I*a*d^(9/2) + 56*I*b*d^(9/2))*x^
8 - 4*(11*a*d^(7/2) + 36*b*d^(7/2))*x^6 + (-8*I*a*d^(5/2) - 160*I*b*d^(5/2)
)*x^4 - 16*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + (4*(a*d^4 + 2*b*d^4)*x^7 + (12
*I*a*d^3 + 32*I*b*d^3)*x^5 - 8*(2*a*d^2 + 5*b*d^2)*x^3 + (-16*I*a*d - 16*I*
b*d)*x)*(d*x^2 + 2*I)^(3/2) + (12*(a*d^(9/2) + 2*b*d^(9/2))*x^8 + (48*I*a*d
^(7/2) + 120*I*b*d^(7/2))*x^6 - 8*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 + (-40*I
*a*d^(3/2) - 120*I*b*d^(3/2))*x^2 + 16*a*sqrt(d) + 16*b*sqrt(d))*(d*x^2 + 2
*I) + (4*b*d^(11/2)*x^10 + 24*I*b*d^(9/2)*x^8 - 44*b*d^(7/2)*x^6 - 8*I*b*d^
```

```
(5/2)*x^4 - 48*b*d^(3/2)*x^2 + (4*b*d^4*x^7 + 12*I*b*d^3*x^5 - 16*b*d^2*x^3
 - 16*I*b*d*x)*(d*x^2 + 2*I)^(3/2) + (12*b*d^(9/2)*x^8 + 48*I*b*d^(7/2)*x^6
 - 64*b*d^(5/2)*x^4 - 40*I*b*d^(3/2)*x^2 + 16*b*sqrt(d))*(d*x^2 + 2*I) + (1
2*b*d^5*x^9 + 60*I*b*d^4*x^7 - 92*b*d^3*x^5 - 28*I*b*d^2*x^3 - 24*b*d*x)*sq
rt(d*x^2 + 2*I) - 32*I*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x +
 I) + (12*(a*d^5 + 2*b*d^5)*x^9 + (60*I*a*d^4 + 144*I*b*d^4)*x^7 - 4*(23*a*
d^3 + 76*b*d^3)*x^5 + (-28*I*a*d^2 - 256*I*b*d^2)*x^3 - 8*(3*a*d - 8*b*d)*x
)*sqrt(d*x^2 + 2*I) - 32*I*a*sqrt(d))/(32*a^2*b^2*d^(11/2)*x^9 + 192*I*a^2*
b^2*d^(9/2)*x^7 - 384*a^2*b^2*d^(7/2)*x^5 - 256*I*a^2*b^2*d^(5/2)*x^3 + (32
*b^4*d^(11/2)*x^9 + 192*I*b^4*d^(9/2)*x^7 - 384*b^4*d^(7/2)*x^5 - 256*I*b^4
*d^(5/2)*x^3 + (32*b^4*d^4*x^6 + 96*I*b^4*d^3*x^4 - 96*b^4*d^2*x^2 - 32*I*b
^4*d)*(d*x^2 + 2*I)^(3/2) + (96*b^4*d^(9/2)*x^7 + 384*I*b^4*d^(7/2)*x^5 - 4
80*b^4*d^(5/2)*x^3 - 192*I*b^4*d^(3/2)*x)*(d*x^2 + 2*I) + (96*b^4*d^5*x^8 +
 480*I*b^4*d^4*x^6 - 768*b^4*d^3*x^4 - 384*I*b^4*d^2*x^2)*sqrt(d*x^2 + 2*I)
)*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 + (32*a^2*b^2*d^4*x^6 + 96
*I*a^2*b^2*d^3*x^4 - 96*a^2*b^2*d^2*x^2 - 32*I*a^2*b^2*d)*(d*x^2 + 2*I)^(3/
2) + (96*a^2*b^2*d^(9/2)*x^7 + 384*I*a^2*b^2*d^(7/2)*x^5 - 480*a^2*b^2*d^(5
/2)*x^3 - 192*I*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2*I) + (64*a*b^3*d^(11/2)*x^9 +
 384*I*a*b^3*d^(9/2)*x^7 - 768*a*b^3*d^(7/2)*x^5 - 512*I*a*b^3*d^(5/2)*x^3
+ (64*a*b^3*d^4*x^6 + 192*I*a*b^3*d^3*x^4 - 192*a*b^3*d^2*x^2 - 64*I*a*b^3*
d)*(d*x^2 + 2*I)^(3/2) + (192*a*b^3*d^(9/2)*x^7 + 768*I*a*b^3*d^(7/2)*x^5 -
 960*a*b^3*d^(5/2)*x^3 - 384*I*a*b^3*d^(3/2)*x)*(d*x^2 + 2*I) + (192*a*b^3*
d^5*x^8 + 960*I*a*b^3*d^4*x^6 - 1536*a*b^3*d^3*x^4 - 768*I*a*b^3*d^2*x^2)*s
qrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (96*a^2*b^
2*d^5*x^8 + 480*I*a^2*b^2*d^4*x^6 - 768*a^2*b^2*d^3*x^4 - 384*I*a^2*b^2*d^2
*x^2)*sqrt(d*x^2 + 2*I)) + integrate((d^6*x^12 + 8*I*d^5*x^10 - 27*d^4*x^8
- 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*I*d^3*x^6 - 3*d^2*x^4 + 8*I*d*x^
2 + 4)*(d*x^2 + 2*I)^2 + 96*I*d*x^2 + (4*d^(9/2)*x^9 + 20*I*d^(7/2)*x^7 - 3
0*d^(5/2)*x^5 + 2*I*d^(3/2)*x^3 - 22*sqrt(d)*x)*(d*x^2 + 2*I)^(3/2) + (6*d^
5*x^10 + 36*I*d^4*x^8 - 78*d^3*x^6 - 72*I*d^2*x^4 + 9*d*x^2 - 30*I)*(d*x^2
+ 2*I) + (4*d^(11/2)*x^11 + 28*I*d^(9/2)*x^9 - 78*d^(7/2)*x^7 - 122*I*d^(5/
2)*x^5 + 122*d^(3/2)*x^3 + 60*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 48)/(8*a*b^2
*d^6*x^12 + 64*I*a*b^2*d^5*x^10 - 192*a*b^2*d^4*x^8 - 256*I*a*b^2*d^3*x^6 +
 128*a*b^2*d^2*x^4 + (8*a*b^2*d^4*x^8 + 32*I*a*b^2*d^3*x^6 - 48*a*b^2*d^2*x
^4 - 32*I*a*b^2*d*x^2 + 8*a*b^2)*(d*x^2 + 2*I)^2 + (32*a*b^2*d^(9/2)*x^9 +
160*I*a*b^2*d^(7/2)*x^7 - 288*a*b^2*d^(5/2)*x^5 - 224*I*a*b^2*d^(3/2)*x^3 +
 64*a*b^2*sqrt(d)*x)*(d*x^2 + 2*I)^(3/2) + (48*a*b^2*d^5*x^10 + 288*I*a*b^2
*d^4*x^8 - 624*a*b^2*d^3*x^6 - 576*I*a*b^2*d^2*x^4 + 192*a*b^2*d*x^2)*(d*x^
2 + 2*I) + (8*b^3*d^6*x^12 + 64*I*b^3*d^5*x^10 - 192*b^3*d^4*x^8 - 256*I*b^
3*d^3*x^6 + 128*b^3*d^2*x^4 + (8*b^3*d^4*x^8 + 32*I*b^3*d^3*x^6 - 48*b^3*d^
2*x^4 - 32*I*b^3*d*x^2 + 8*b^3)*(d*x^2 + 2*I)^2 + (32*b^3*d^(9/2)*x^9 + 160
*I*b^3*d^(7/2)*x^7 - 288*b^3*d^(5/2)*x^5 - 224*I*b^3*d^(3/2)*x^3 + 64*b^3*s
qrt(d)*x)*(d*x^2 + 2*I)^(3/2) + (48*b^3*d^5*x^10 + 288*I*b^3*d^4*x^8 - 624*
b^3*d^3*x^6 - 576*I*b^3*d^2*x^4 + 192*b^3*d*x^2)*(d*x^2 + 2*I) + (32*b^3*d^
(11/2)*x^11 + 224*I*b^3*d^(9/2)*x^9 - 576*b^3*d^(7/2)*x^7 - 640*I*b^3*d^(5/
2)*x^5 + 256*b^3*d^(3/2)*x^3)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2
*I)*sqrt(d)*x + I) + (32*a*b^2*d^(11/2)*x^11 + 224*I*a*b^2*d^(9/2)*x^9 - 57
6*a*b^2*d^(7/2)*x^7 - 640*I*a*b^2*d^(5/2)*x^5 + 256*a*b^2*d^(3/2)*x^3)*sqrt
(d*x^2 + 2*I)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bdx^2\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)+adx^2-8\left(b^4dx\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)^2+2\,ab^3dx\log\left(dx^2+\sqrt{d^2x^4+\cdots}\right.\right.}{8\left(b^4dx\log\left(dx^2+\sqrt{d^2x^4+2i\,dx^2}+i\right)^2+2\,ab^3dx\log\left(dx^2+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a*d*x^2 - 8*(b^4
*d*x*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^2 + 2*a*b^3*d*x*log(d*x^2 +
 sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a^2*b^2*d*x)*integral(1/8/(b^3*log(d*x^2
+ sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a*b^2), x) + 2*sqrt(d^2*x^4 + 2*I*d*x^2)
*b)/(b^4*d*x*log(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I)^2 + 2*a*b^3*d*x*log
(d*x^2 + sqrt(d^2*x^4 + 2*I*d*x^2) + I) + a^2*b^2*d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arsinh}\left(dx^2+i\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-3), x)
```

3.321 $\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 dx$

Optimal. Leaf size=153

$$-\frac{192b^3\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + \right.\right.}{dx}$$

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x
^2]))/(d*x) + 48*b^2*x*(a - I*b*ArcSin[1 + I*d*x^2])^2 - (8*b*Sqrt[(-2*I)*d
*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x*(a - I*b*ArcSin[
1 + I*d*x^2])^4
```

Rubi [A] time = 0.0318743, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4814, 8}

$$-\frac{192b^3\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + \right.\right.}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x
^2]))/(d*x) + 48*b^2*x*(a - I*b*ArcSin[1 + I*d*x^2])^2 - (8*b*Sqrt[(-2*I)*d
*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x*(a - I*b*ArcSin[
1 + I*d*x^2])^4
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 dx &= -\frac{8b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 + (48b^2 \\ &= -\frac{192b^3\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 - \\ &= 384b^4x - \frac{192b^3\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + \right.\right.\end{aligned}$$

Mathematica [A] time = 0.116773, size = 149, normalized size = 0.97

$$48b^2\left(-\frac{4b\sqrt{dx^2\left(dx^2 - 2i\right)}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 + 8b^2x\right) + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x
*(a - I*b*ArcSin[1 + I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(-2*I
+ d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2]))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^2)
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^{2}\right)\right)^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^{4}x\log\left(dx^{2}+\sqrt{dx^{2}-2i}\sqrt{d}x-i\right)^{4}+4\left(x\,\text{arsinh}\left(dx^{2}-i\right)-\frac{2\left(d^{\frac{3}{2}}x^{2}-2i\sqrt{d}\right)}{\sqrt{dx^{2}-2id}}\right)a^{3}b+a^{4}x+\int\frac{\left(4\left(ab^{3}d^{2}-2b^{4}d^{2}\right)x^{4}-8a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^4 + 4*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^3*b + a^4*x +
integrate(((4*(a*b^3*d^2 - 2*b^4*d^2)*x^4 - 8*a*b^3 + (-12*I*a*b^3*d + 16*I
*b^4*d)*x^2 + (4*(a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 + (-8*I*a*b^3*sqrt(d)
+ 8*I*b^4*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt
(d)*x - I)^3 + (6*a^2*b^2*d^2*x^4 - 18*I*a^2*b^2*d*x^2 - 12*a^2*b^2 + 6*(a^
2*b^2*d^(3/2)*x^3 - 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + s
qrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*
I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x)
```

Fricas [B] time = 2.79762, size = 648, normalized size = 4.24

$$\frac{b^{4}dx^{2}\log\left(dx^{2}+\sqrt{d^{2}x^{4}-2i\,dx^{2}}-i\right)^{4}+\left(a^{4}+48\,a^{2}b^{2}+384\,b^{4}\right)dx^{2}+4\left(ab^{3}dx^{2}-2\sqrt{d^{2}x^{4}-2i\,dx^{2}}b^{4}\right)\log\left(dx^{2}+\sqrt{d^{2}x^{4}\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^4 + (a^4 + 48*a^2*b^2
 + 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 - 2*I*d*x^2)*b^4)*log(d
```

```
*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^3 - 6*(4*sqrt(d^2*x^4 - 2*I*d*x^2)*a*
b^3 - (a^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^2
 + 4*((a^3*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 - 2*I*d*x^2)*(a^2*b^2 + 8*b
^4))*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) - 8*sqrt(d^2*x^4 - 2*I*d*x^
2)*(a^3*b + 24*a*b^3))/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**4,x)

[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="giac")

[Out] integrate((b*arcsinh(d*x^2 - I) + a)^4, x)
```

3.322 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3\,dx$

Optimal. Leaf size=129

$$24ab^2x-\frac{6b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4-2idx^2}}{dx}-24ib^3x\sin^{-1}\left(1\right.$$

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - (24*I)*b^3*x*Arc
Sin[1 + I*d*x^2] - (6*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*
d*x^2])^2)/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^3
```

Rubi [A] time = 0.0588589, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4814, 4840, 12, 1588}

$$24ab^2x-\frac{6b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4-2idx^2}}{dx}-24ib^3x\sin^{-1}\left(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - (24*I)*b^3*x*Arc
Sin[1 + I*d*x^2] - (6*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*
d*x^2])^2)/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^3
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3 dx = -\frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3 + \left(24b^2\right.$$
$$= 24ab^2x - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3$$
$$= 24ab^2x - 24ib^3x\sin^{-1}\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2}{dx} + x$$
$$= 24ab^2x - 24ib^3x\sin^{-1}\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2}{dx} + x$$
$$= 24ab^2x - \frac{48b^3\sqrt{-2idx^2 + d^2x^4}}{dx} - 24ib^3x\sin^{-1}\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - \right.}{dx}$$

Mathematica [A] time = 0.133035, size = 180, normalized size = 1.4

$$\frac{adx^2\left(a^2 + 24b^2\right) - 6b\left(a^2 + 8b^2\right)\sqrt{dx^2\left(dx^2 - 2i\right)} - 3ib\sin^{-1}\left(1 + idx^2\right)\left(a^2dx^2 - 4ab\sqrt{dx^2\left(dx^2 - 2i\right)} + 8b^2dx^2\right) + 3b^2}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(-2*I + d*x^2)] - (3
*I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
 + I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
+ I*d*x^2]^2 + I*b^3*d*x^2*ArcSin[1 + I*d*x^2]^3)/(d*x)
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \left(a + b\text{Arcsinh}\left(-i + dx^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^3x\log\left(dx^2 + \sqrt{dx^2 - 2i}\sqrt{d}x - i\right)^3 + 3\left(x\,\text{arsinh}\left(dx^2 - i\right) - \frac{2\left(d^{\frac{3}{2}}x^2 - 2i\sqrt{d}\right)}{\sqrt{dx^2 - 2id}}\right)a^2b + a^3x + \int \frac{\left(3\left(ab^2d^2 - 2b^3d^2\right)x^4 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^3 + 3*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^2*b + a^3*x +
```

```
integrate((3*(a*b^2*d^2 - 2*b^3*d^2)*x^4 - 6*a*b^2 + (-9*I*a*b^2*d + 12*I*b
^3*d)*x^2 + (3*(a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 + (-6*I*a*b^2*sqrt(d) +
6*I*b^3*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d
)*x - I)^2/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2
- 2*I) - 2), x)
```

Fricas [A] time = 2.79627, size = 458, normalized size = 3.55

$$\frac{b^3dx^2\log\left(dx^2+\sqrt{d^2x^4-2i\,dx^2}-i\right)^3+\left(a^3+24\,ab^2\right)dx^2+3\left(ab^2dx^2-2\sqrt{d^2x^4-2i\,dx^2}b^3\right)\log\left(dx^2+\sqrt{d^2x^4-2i\,dx^2}-\right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^3 + (a^3 + 24*a*b^2)*
d*x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 - 2*I*d*x^2)*b^3)*log(d*x^2 + sqrt(
d^2*x^4 - 2*I*d*x^2) - I)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 - 2
*I*d*x^2)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) - 6*sqrt(d^2*x^
4 - 2*I*d*x^2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{arsinh}\left(dx^2-i\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^3, x)
```

3.323 $\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 dx$

Optimal. Leaf size=76

$$-\frac{4b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 + 8b^2x$$

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Rubi [A] time = 0.0126438, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4814, 8}

$$-\frac{4b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 + 8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 dx &= -\frac{4b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 + \left(8b^2\right) \\ &= 8b^2x - \frac{4b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2\end{aligned}$$

Mathematica [A] time = 0.0173808, size = 76, normalized size = 1.

$$-\frac{4b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 + 8b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int \left(a + b\mathrm{Arcsinh}\left(-i + dx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\left(x\,\mathrm{arsinh}\left(dx^2 - i\right) - \frac{2\left(d^{\frac{3}{2}}x^2 - 2i\sqrt{d}\right)}{\sqrt{dx^2 - 2id}}\right)ab + \left(x\log\left(dx^2 + \sqrt{dx^2 - 2i}\sqrt{d}x - i\right)^2 - \int \frac{\left(4\,d^2x^4 - 8i\,dx^2 + \left(4\,d^{\frac{3}{2}}x^3 - 4i\,\sqrt{dx}\right.\right.}{d^2x^4 - 3i\,dx^2 + \left(d^{\frac{3}{2}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 - integrate((4*
d^2*x^4 - 8*I*d*x^2 + (4*d^(3/2)*x^3 - 4*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*lo
g(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*
x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x))*b^2 + a^2*x
```

Fricas [B] time = 2.61832, size = 288, normalized size = 3.79

$$\frac{b^2dx^2\log\left(dx^2 + \sqrt{d^2x^4 - 2i\,dx^2} - i\right)^2 + \left(a^2 + 8\,b^2\right)dx^2 - 4\,\sqrt{d^2x^4 - 2i\,dx^2}ab + 2\left(abdx^2 - 2\,\sqrt{d^2x^4 - 2i\,dx^2}b^2\right)\log\left(dx^2 + \right.}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^2 + (a^2 + 8*b^2)*d*x
^2 - 4*sqrt(d^2*x^4 - 2*I*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 - 2*I*
d*x^2)*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I))/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^2, x)
```

3.324 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right) dx$

Optimal. Leaf size=50

$$ax-\frac{2b\sqrt{d^2x^4-2idx^2}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)$$

```
[Out] a*x - (2*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Rubi [A] time = 0.0387746, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4840, 12, 1588}

$$ax-\frac{2b\sqrt{d^2x^4-2idx^2}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right) dx &= ax-(ib)\int \sin^{-1}\left(1+idx^2\right) dx\\
&= ax-ibx\sin^{-1}\left(1+idx^2\right)+(ib)\int \frac{2idx^2}{\sqrt{-2idx^2+d^2x^4}}\,dx\\
&= ax-ibx\sin^{-1}\left(1+idx^2\right)-(2bd)\int \frac{x^2}{\sqrt{-2idx^2+d^2x^4}}\,dx\\
&= ax-\frac{2b\sqrt{-2idx^2+d^2x^4}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0267625, size = 48, normalized size = 0.96

$$ax-\frac{2b\sqrt{dx^2\left(dx^2-2i\right)}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Maple [A] time = 0.015, size = 48, normalized size = 1.

$$ax+b\left(x\mathrm{Arcsinh}\left(-i+dx^2\right)+2\,\frac{x\left(-dx^2+2\,i\right)}{\sqrt{-2\,idx^2+d^2x^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(-I+d*x^2),x)
```

```
[Out] a*x+b*(x*arcsinh(-I+d*x^2)+2/(-2*I*d*x^2+d^2*x^4)^(1/2)*x*(-d*x^2+2*I))
```

Maxima [A] time = 1.19206, size = 59, normalized size = 1.18

$$\left(x\operatorname{arsinh}\left(dx^2-i\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2i\sqrt{d}\right)}{\sqrt{dx^2-2id}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d)
)*b + a*x
```

Fricas [A] time = 2.64874, size = 138, normalized size = 2.76

$$\frac{bdx^2\log\left(dx^2+\sqrt{d^2x^4-2i\,dx^2}-i\right)+adx^2-2\,\sqrt{d^2x^4-2i\,dx^2}b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a*d*x^2 - 2*sqrt(d^2*
x^4 - 2*I*d*x^2)*b)/(d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(-I+d*x**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arsinh}\left(dx^2 - i\right) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(b*arcsinh(d*x^2 - I) + a, x)
```

3.325 $\int \frac{1}{a-ib\sin^{-1}\left(1+idx^2\right)}\,dx$

Optimal. Leaf size=191

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

```
[Out] -(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] +
 Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1
 + I*d*x^2])/(2*b)])/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]))
```

Rubi [A] time = 0.0228964, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4816}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] -(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] +
 Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1
 + I*d*x^2])/(2*b)])/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]))
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{a-ib\sin^{-1}\left(1+idx^2\right)}\,dx=-\frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\ldots}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\ldots\right)}$$

Mathematica [A] time = 0.639787, size = 146, normalized size = 0.76

$$\frac{x\left(\left(-\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(1+idx^2\right)\right)\right)+\left(\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(i\ldots\right.\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] (x*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*((-I)*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I*a)/b +
ArcSin[1 + I*d*x^2])/2]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 +
 I*d*x^2]/2]))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \left(a + b\text{Arcsinh}\left(-i + dx^2\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsinh}\left(dx^2 - i\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(d*x^2 - I) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \log\left(dx^2 + \sqrt{d^2x^4 - 2i\,dx^2} - i\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\,\mathrm{arsinh}\left(dx^2 - i\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(d*x^2 - I) + a), x)
```

3.326 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}\,dx$

Optimal. Leaf size=244

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-$$

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(2*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])) + (x*
CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(Cosh[a/(2*b)] + I*Sin
h[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2
])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 +
 I*d*x^2])/(2*b)])/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]))
```

Rubi [A] time = 0.0286216, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4825}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(2*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])) + (x*
CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(Cosh[a/(2*b)] + I*Sin
h[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2
])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 +
 I*d*x^2])/(2*b)])/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]))
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[(
x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x
] + Simp[(x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^
2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{2bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}+\frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Mathematica [A] time = 1.36747, size = 196, normalized size = 0.8

$$\frac{\frac{x^2\left(\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}(1+idx^2)\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}(idx^2+1)\right)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)}-\frac{2b\sqrt{dx^2(dx^2-2i)}}{d(a-ib\sin^{-1}(1+idx^2))}}{4b^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*(a - I*b*ArcSin[1 + I*d*x^2])) + (x^2
*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]) - (Cosh[a/(2*b)] - I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1
 + I*d*x^2])/2]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
)/(4*b^2*x)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^2\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^2x^4-3i\,dx^2+\left(d^{\frac{3}{2}}x^3-2i\,\sqrt{d}x\right)\sqrt{dx^2-2i}-2}{2\,abd^2x^3-4i\,abdx+\left(2\,b^2d^2x^3-4i\,b^2dx+\left(2\,b^2d^{\frac{3}{2}}x^2-2i\,b^2\sqrt{d}\right)\sqrt{dx^2-2i}\right)\log\left(dx^2+\sqrt{dx^2-2i}\sqrt{d}x-i\right)+\left(2\,abd^{\frac{3}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2
)/(2*a*b*d^2*x^3 - 4*I*a*b*d*x + (2*b^2*d^2*x^3 - 4*I*b^2*d*x + (2*b^2*d^(3
/2)*x^2 - 2*I*b^2*sqrt(d))*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)
*sqrt(d)*x - I) + (2*a*b*d^(3/2)*x^2 - 2*I*a*b*sqrt(d))*sqrt(d*x^2 - 2*I))
+ integrate((2*d^3*x^6 - 6*I*d^2*x^4 + (2*d^2*x^4 - 2*I*d*x^2 - 4)*(d*x^2 -
 2*I) + 2*(2*d^(5/2)*x^5 - 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2*I) -
 8*I)/(4*a*b*d^3*x^6 - 16*I*a*b*d^2*x^4 - 16*a*b*d*x^2 + (4*a*b*d^2*x^4 - 8
*I*a*b*d*x^2 - 4*a*b)*(d*x^2 - 2*I) + (4*b^2*d^3*x^6 - 16*I*b^2*d^2*x^4 - 1
6*b^2*d*x^2 + 4*(b^2*d^2*x^4 - 2*I*b^2*d*x^2 - b^2)*(d*x^2 - 2*I) + (8*b^2*
d^(5/2)*x^5 - 24*I*b^2*d^(3/2)*x^3 - 16*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*l
og(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (8*a*b*d^(5/2)*x^5 - 24*I*a*b
*d^(3/2)*x^3 - 16*a*b*sqrt(d)*x)*sqrt(d*x^2 - 2*I)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(b^{2}dx\log\left(dx^{2}+\sqrt{d^{2}x^{4}-2i\,dx^{2}}-i\right)+abdx\right)\text{integral}\left(\frac{\sqrt{d^{2}x^{4}-2i\,dx^{2}}}{2\,abdx^{2}-4i\,ab+\left(2\,b^{2}dx^{2}-4i\,b^{2}\right)\log\left(dx^{2}+\sqrt{d^{2}x^{4}-2i\,dx^{2}}-i\right)},x\right)-\sqrt{d^{2}x^{4}-2i\,dx^{2}}}{2\left(b^{2}dx\log\left(dx^{2}+\sqrt{d^{2}x^{4}-2i\,dx^{2}}-i\right)+abdx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*x*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a*b*d*x)*integ
ral(sqrt(d^2*x^4 - 2*I*d*x^2)/(2*a*b*d*x^2 - 4*I*a*b + (2*b^2*d*x^2 - 4*I*b
^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)), x) - sqrt(d^2*x^4 - 2*I*d*
x^2))/(b^2*d*x*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a*b*d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^{2}-i\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-2), x)
```

3.327 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3}\,dx$

Optimal. Leaf size=272

$$-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx\right.\right.\right.}$$

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(4*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^2) - x
/(8*b^2*(a - I*b*ArcSin[1 + I*d*x^2])) - (x*CosIntegral[((I/2)*(a - I*b*Arc
Sin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(16*b^3*(Cos[ArcSi
n[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) + (x*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 + I*d*x^2])/(2*b)])/(16*b^3*(Co
s[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rubi [A] time = 0.0508033, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4828, 4816}

$$-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(4*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^2) - x
/(8*b^2*(a - I*b*ArcSin[1 + I*d*x^2])) - (x*CosIntegral[((I/2)*(a - I*b*Arc
Sin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(16*b^3*(Cos[ArcSi
n[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) + (x*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 + I*d*x^2])/(2*b)])/(16*b^3*(Co
s[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2} - \frac{x}{8b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)} + \frac{\int \frac{1}{a-ib\sin^{-1}(1+idx^2)}\,dx}{8b^2}$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2} - \frac{x}{8b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)} - \frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}(1\right.}{2b}\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

Mathematica [A] time = 0.719811, size = 227, normalized size = 0.83

$$\frac{\frac{8b^2\sqrt{dx^2(dx^2-2i)}}{d\left(ia+b\sin^{-1}(1+idx^2)\right)^2} - \frac{2ix^2\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}(1+idx^2)\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}(idx^2+1)\right)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)} - \frac{4b}{a-ib\sin^{-1}}}{32b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] ((-4*b*x^2)/(a - I*b*ArcSin[1 + I*d*x^2]) + (8*b^2*Sqrt[d*x^2*(-2*I + d*x^2
)])/(d*(I*a + b*ArcSin[1 + I*d*x^2])^2) - ((2*I)*x^2*(CosIntegral[((I*a)/b
+ ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + (Cosh[a/(2*b)
] + I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]))/(Cos[
ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))/(32*b^3*x)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^2\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -(4*(a*d^(11/2) + 2*b*d^(11/2))*x^10 + (-24*I*a*d^(9/2) - 56*I*b*d^(9/2))*x
^8 - 4*(11*a*d^(7/2) + 36*b*d^(7/2))*x^6 + (8*I*a*d^(5/2) + 160*I*b*d^(5/2)
)*x^4 - 16*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + (4*(a*d^4 + 2*b*d^4)*x^7 + (-1
2*I*a*d^3 - 32*I*b*d^3)*x^5 - 8*(2*a*d^2 + 5*b*d^2)*x^3 + (16*I*a*d + 16*I*
b*d)*x)*(d*x^2 - 2*I)^(3/2) + (12*(a*d^(9/2) + 2*b*d^(9/2))*x^8 + (-48*I*a*
d^(7/2) - 120*I*b*d^(7/2))*x^6 - 8*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 + (40*I
*a*d^(3/2) + 120*I*b*d^(3/2))*x^2 + 16*a*sqrt(d) + 16*b*sqrt(d))*(d*x^2 - 2
*I) + (4*b*d^(11/2)*x^10 - 24*I*b*d^(9/2)*x^8 - 44*b*d^(7/2)*x^6 + 8*I*b*d^
```

```
(5/2)*x^4 - 48*b*d^(3/2)*x^2 + (4*b*d^4*x^7 - 12*I*b*d^3*x^5 - 16*b*d^2*x^3
 + 16*I*b*d*x)*(d*x^2 - 2*I)^(3/2) + (12*b*d^(9/2)*x^8 - 48*I*b*d^(7/2)*x^6
 - 64*b*d^(5/2)*x^4 + 40*I*b*d^(3/2)*x^2 + 16*b*sqrt(d))*(d*x^2 - 2*I) + (1
2*b*d^5*x^9 - 60*I*b*d^4*x^7 - 92*b*d^3*x^5 + 28*I*b*d^2*x^3 - 24*b*d*x)*sq
rt(d*x^2 - 2*I) + 32*I*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x -
 I) + (12*(a*d^5 + 2*b*d^5)*x^9 + (-60*I*a*d^4 - 144*I*b*d^4)*x^7 - 4*(23*a
*d^3 + 76*b*d^3)*x^5 + (28*I*a*d^2 + 256*I*b*d^2)*x^3 - 8*(3*a*d - 8*b*d)*x
)*sqrt(d*x^2 - 2*I) + 32*I*a*sqrt(d))/(32*a^2*b^2*d^(11/2)*x^9 - 192*I*a^2*
b^2*d^(9/2)*x^7 - 384*a^2*b^2*d^(7/2)*x^5 + 256*I*a^2*b^2*d^(5/2)*x^3 + (32
*b^4*d^(11/2)*x^9 - 192*I*b^4*d^(9/2)*x^7 - 384*b^4*d^(7/2)*x^5 + 256*I*b^4
*d^(5/2)*x^3 + (32*b^4*d^4*x^6 - 96*I*b^4*d^3*x^4 - 96*b^4*d^2*x^2 + 32*I*b
^4*d)*(d*x^2 - 2*I)^(3/2) + (96*b^4*d^(9/2)*x^7 - 384*I*b^4*d^(7/2)*x^5 - 4
80*b^4*d^(5/2)*x^3 + 192*I*b^4*d^(3/2)*x)*(d*x^2 - 2*I) + (96*b^4*d^5*x^8 -
 480*I*b^4*d^4*x^6 - 768*b^4*d^3*x^4 + 384*I*b^4*d^2*x^2)*sqrt(d*x^2 - 2*I)
)*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 + (32*a^2*b^2*d^4*x^6 - 96
*I*a^2*b^2*d^3*x^4 - 96*a^2*b^2*d^2*x^2 + 32*I*a^2*b^2*d)*(d*x^2 - 2*I)^(3/
2) + (96*a^2*b^2*d^(9/2)*x^7 - 384*I*a^2*b^2*d^(7/2)*x^5 - 480*a^2*b^2*d^(5
/2)*x^3 + 192*I*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2*I) + (64*a*b^3*d^(11/2)*x^9 -
 384*I*a*b^3*d^(9/2)*x^7 - 768*a*b^3*d^(7/2)*x^5 + 512*I*a*b^3*d^(5/2)*x^3
+ (64*a*b^3*d^4*x^6 - 192*I*a*b^3*d^3*x^4 - 192*a*b^3*d^2*x^2 + 64*I*a*b^3*
d)*(d*x^2 - 2*I)^(3/2) + (192*a*b^3*d^(9/2)*x^7 - 768*I*a*b^3*d^(7/2)*x^5 -
 960*a*b^3*d^(5/2)*x^3 + 384*I*a*b^3*d^(3/2)*x)*(d*x^2 - 2*I) + (192*a*b^3*
d^5*x^8 - 960*I*a*b^3*d^4*x^6 - 1536*a*b^3*d^3*x^4 + 768*I*a*b^3*d^2*x^2)*s
qrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (96*a^2*b^
2*d^5*x^8 - 480*I*a^2*b^2*d^4*x^6 - 768*a^2*b^2*d^3*x^4 + 384*I*a^2*b^2*d^2
*x^2)*sqrt(d*x^2 - 2*I)) + integrate((d^6*x^12 - 8*I*d^5*x^10 - 27*d^4*x^8
+ 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*I*d^3*x^6 - 3*d^2*x^4 - 8*I*d*x^
2 + 4)*(d*x^2 - 2*I)^2 - 96*I*d*x^2 + (4*d^(9/2)*x^9 - 20*I*d^(7/2)*x^7 - 3
0*d^(5/2)*x^5 - 2*I*d^(3/2)*x^3 - 22*sqrt(d)*x)*(d*x^2 - 2*I)^(3/2) + (6*d^
5*x^10 - 36*I*d^4*x^8 - 78*d^3*x^6 + 72*I*d^2*x^4 + 9*d*x^2 + 30*I)*(d*x^2
- 2*I) + (4*d^(11/2)*x^11 - 28*I*d^(9/2)*x^9 - 78*d^(7/2)*x^7 + 122*I*d^(5/
2)*x^5 + 122*d^(3/2)*x^3 - 60*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 48)/(8*a*b^2
*d^6*x^12 - 64*I*a*b^2*d^5*x^10 - 192*a*b^2*d^4*x^8 + 256*I*a*b^2*d^3*x^6 +
 128*a*b^2*d^2*x^4 + (8*a*b^2*d^4*x^8 - 32*I*a*b^2*d^3*x^6 - 48*a*b^2*d^2*x
^4 + 32*I*a*b^2*d*x^2 + 8*a*b^2)*(d*x^2 - 2*I)^2 + (32*a*b^2*d^(9/2)*x^9 -
160*I*a*b^2*d^(7/2)*x^7 - 288*a*b^2*d^(5/2)*x^5 + 224*I*a*b^2*d^(3/2)*x^3 +
 64*a*b^2*sqrt(d)*x)*(d*x^2 - 2*I)^(3/2) + (48*a*b^2*d^5*x^10 - 288*I*a*b^2
*d^4*x^8 - 624*a*b^2*d^3*x^6 + 576*I*a*b^2*d^2*x^4 + 192*a*b^2*d*x^2)*(d*x^
2 - 2*I) + (8*b^3*d^6*x^12 - 64*I*b^3*d^5*x^10 - 192*b^3*d^4*x^8 + 256*I*b^
3*d^3*x^6 + 128*b^3*d^2*x^4 + (8*b^3*d^4*x^8 - 32*I*b^3*d^3*x^6 - 48*b^3*d^
2*x^4 + 32*I*b^3*d*x^2 + 8*b^3)*(d*x^2 - 2*I)^2 + (32*b^3*d^(9/2)*x^9 - 160
*I*b^3*d^(7/2)*x^7 - 288*b^3*d^(5/2)*x^5 + 224*I*b^3*d^(3/2)*x^3 + 64*b^3*s
qrt(d)*x)*(d*x^2 - 2*I)^(3/2) + (48*b^3*d^5*x^10 - 288*I*b^3*d^4*x^8 - 624*
b^3*d^3*x^6 + 576*I*b^3*d^2*x^4 + 192*b^3*d*x^2)*(d*x^2 - 2*I) + (32*b^3*d^
(11/2)*x^11 - 224*I*b^3*d^(9/2)*x^9 - 576*b^3*d^(7/2)*x^7 + 640*I*b^3*d^(5/
2)*x^5 + 256*b^3*d^(3/2)*x^3)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2
*I)*sqrt(d)*x - I) + (32*a*b^2*d^(11/2)*x^11 - 224*I*a*b^2*d^(9/2)*x^9 - 57
6*a*b^2*d^(7/2)*x^7 + 640*I*a*b^2*d^(5/2)*x^5 + 256*a*b^2*d^(3/2)*x^3)*sqrt
(d*x^2 - 2*I)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bdx^2\log\left(dx^2+\sqrt{d^2x^4-2i\,dx^2}-i\right)+adx^2-8\left(b^4dx\log\left(dx^2+\sqrt{d^2x^4-2i\,dx^2}-i\right)^2+2\,ab^3dx\log\left(dx^2+\sqrt{d^2x^4-\right.\right.}{8\left(b^4dx\log\left(dx^2+\sqrt{d^2x^4-2i\,dx^2}-i\right)^2+2\,ab^3dx\log\left(dx^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a*d*x^2 - 8*(b^4
*d*x*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^2 + 2*a*b^3*d*x*log(d*x^2 +
 sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a^2*b^2*d*x)*integral(1/8/(b^3*log(d*x^2
+ sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a*b^2), x) + 2*sqrt(d^2*x^4 - 2*I*d*x^2)
*b)/(b^4*d*x*log(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I)^2 + 2*a*b^3*d*x*log
(d*x^2 + sqrt(d^2*x^4 - 2*I*d*x^2) - I) + a^2*b^2*d*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-3), x)
```

3.328 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=348

$$-\frac{15\sqrt{\pi}\sqrt{-\frac{i}{b}}b^3x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{15\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\cdots\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\cdots\right.\right.}$$

```
[Out] 15*b^2*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] - (5*b*Sqrt[(2*I)*d*x^2 + d^2*x^
4]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x
^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[
1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)/b]*(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (15*Sqrt[(-I)/b
]*b^3*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/
Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] -
Sin[ArcSin[1 - I*d*x^2]/2])
```

Rubi [A] time = 0.111911, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4814, 4811}

$$-\frac{15\sqrt{\pi}\sqrt{-\frac{i}{b}}b^3x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{15\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\cdots\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] - (5*b*Sqrt[(2*I)*d*x^2 + d^2*x^
4]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x
^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[
1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)/b]*(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (15*Sqrt[(-I)/b
]*b^3*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/
Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] -
Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
```

```
d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}\,dx=-\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}+\left(15b^2\right.$$

$$=15b^2x\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}-\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}{dx}+x\left(a+\right.$$

Mathematica [A] time = 0.270713, size = 337, normalized size = 0.97

$$\frac{15b^2x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1}}{\sqrt{\pi}}\right.\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[(-I)/b]*Sqrt[a +
I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]
)/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + Sqrt[Pi]*FresnelS[(Sqrt[(-I
)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a
/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(5/2), x)
```

3.329 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=312

$$-\frac{3\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4+2idx^2}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+\frac{3\sqrt{\pi}\sqrt{ib}bx\left(-\mathrm{si}\right.}{\cos\left(\frac{1}{2}\right.}$$

```
[Out] (-3*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*Sqrt[I*b]*b*Sqrt[Pi]*x*Fresnel
C[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)]
- Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
 - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*
Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rubi [A] time = 0.105041, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4814, 4819}

$$-\frac{3\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4+2idx^2}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+\frac{3\sqrt{\pi}\sqrt{ib}bx\left(-\mathrm{si}\right.}{\cos\left(\frac{1}{2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```


```
[Out] (-3*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*Sqrt[I*b]*b*Sqrt[Pi]*x*Fresnel
C[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)]
- Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
 - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*
Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```


Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}dx=-\frac{3b\sqrt{2idx^2+d^2x^4}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}+\left(3b^2\right)$$

$$=-\frac{3b\sqrt{2idx^2+d^2x^4}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}+\frac{3\sqrt{ib}}{\mathrm{c}}$$

Mathematica [A] time = 0.215801, size = 258, normalized size = 0.83

$$\frac{3\sqrt{\pi}b^2x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(2*I + d*x^2)]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x) +
x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*b^2*Sqrt[Pi]*x*(-(FresnelS[Sqrt[
a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi]
)]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2+i\right)+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(3/2), x)
```

3.330 $\int \sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx$

Optimal. Leaf size=263

$$-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+i}}{}\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin\right.\right.}$$

```
[Out] x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sq
rt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]
))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
 - (Sqrt[(-I)/b]*b*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1
- I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
```

Rubi [A] time = 0.0270555, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4811}

$$-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+i}}{}\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sq
rt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]
))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
 - (Sqrt[(-I)/b]*b*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1
- I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
 d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx=x\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}+\frac{\sqrt{\pi}xS\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Mathematica [A] time = 0.0523471, size = 259, normalized size = 0.98

$$\frac{x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2}}{\sqrt{\pi}}\right.\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a
+ I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + S
qrt[Pi]*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]
*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int\sqrt{a+b\text{Arcsinh}\left(i+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\,\text{arsinh}\left(dx^2+i\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}\left(dx^2 + i\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

3.331 $\int \frac{1}{\sqrt{a+ib\sin^{-1}(1-idx^2)}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

```
[Out] -((Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcS
in[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(
Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rubi [A] time = 0.0299369, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4819}

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcS
in[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(
Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}\,dx=-\frac{\sqrt{\pi}xS\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}xC\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right.\right.\right.}$$

Mathematica [A] time = 0.0037899, size = 180, normalized size = 0.78

$$\frac{\sqrt{\pi}x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] (Sqrt[Pi]*x*(-(FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[P
i])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 -
I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I
*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+b\text{Arcsinh}\left(i+dx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\,\text{arsinh}\left(dx^2+i\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}\left(dx^2 + i\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

3.332 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/}}{\cos}$$

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Rubi [A] time = 0.0524787, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4822}

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/}}{\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} - \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}xC\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx\right.\right.}$$

Mathematica [A] time = 0.366427, size = 291, normalized size = 1.

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} - \frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)} + \frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x}{\cos\left(\frac{1}{2}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-3/2), x)
```

3.333 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(3*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))
- x/(3*b^2*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[a
 + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a
/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d
*x^2]/2])) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I
*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[Arc
Sin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rubi [A] time = 0.0722763, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4819}

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(3*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))
- x/(3*b^2*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[a
 + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a
/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d
*x^2]/2])) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I
*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[Arc
Sin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} + \frac{\int \frac{1}{\sqrt{a+ib\sin^{-1}(1-}}}{3b^2}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} - \frac{\sqrt{\pi}xS\left(\frac{\sqrt{a}}{}\right.}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\right.\right.}$$

Mathematica [A] time = 0.77374, size = 308, normalized size = 0.94

$$-\frac{\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\sqrt{ib}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)} + \frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)} + \frac{b\sqrt{dx^2(dx^2+2i)}}{dx\left(a+ib\sin^{-1}(1-idx^2)\right)^{3/2}} + \frac{}{\sqrt{a-}}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -((b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) +
 x/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*Ar
cSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))
/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) + (S
qrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*
(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] -
 Sin[ArcSin[1 - I*d*x^2]/2])))/(3*b^2)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.334 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{d^2x^4+2idx^2}}{15b^3dx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\left(-\right.}{15b}$$

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(5*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2))
- x/(15*b^2*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) - Sqrt[(2*I)*d*x^2 + d^2*x
^4]/(15*b^3*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (((-I)/b)^(3/2)*Sqrt[P
i]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(C
osh[a/(2*b)] - I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[
ArcSin[1 - I*d*x^2]/2])) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b
]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2
*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rubi [A] time = 0.0881781, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4822}

$$-\frac{\sqrt{d^2x^4+2idx^2}}{15b^3dx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\left(-\right.}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(5*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2))
- x/(15*b^2*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) - Sqrt[(2*I)*d*x^2 + d^2*x
^4]/(15*b^3*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (((-I)/b)^(3/2)*Sqrt[P
i]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(C
osh[a/(2*b)] - I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[
ArcSin[1 - I*d*x^2]/2])) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b
]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2
*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
```

```
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} + \frac{\int \frac{1}{\left(a+ib\sin^{-1}\left(1-\right.\right.}}{15b^2}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{\sqrt{2idx}}{15b^3dx\sqrt{a+i}}$$

Mathematica [A] time = 0.920518, size = 365, normalized size = 0.94

$$\frac{-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{x^2\left(-\left(a+ib\sin\right.\right.}{}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(2*I + d*x^2)])/d - x^2*(a + I*b*ArcSin[1 - I*d*x^2]) +
(Sqrt[d*x^2*(2*I + d*x^2)]*((-I)*a + b*ArcSin[1 - I*d*x^2])^2)/(b*d))/(x*(a
 + I*b*ArcSin[1 - I*d*x^2])^(5/2)) - (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(S
qrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I
*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
+ (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 -
 I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(15*b^2)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(i+dx^2\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^2+i\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.335 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=348

$$-\frac{15\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{15\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\cdots}}{\sqrt{\pi}}\cdots\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+\cdots\right.\right.\right.}$$

```
[Out] 15*b^2*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] - (5*b*Sqrt[(-2*I)*d*x^2 + d^2*x
^4]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1
+ I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[A
rcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (15*b^2*Sqrt[Pi]*x*F
resnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*
b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]))
```

Rubi [A] time = 0.0971646, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4814, 4811}

$$-\frac{15\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{15\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\cdots}}{\sqrt{\pi}}\cdots\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+\cdots\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] - (5*b*Sqrt[(-2*I)*d*x^2 + d^2*x
^4]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1
+ I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[A
rcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (15*b^2*Sqrt[Pi]*x*F
resnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*
b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]))
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
```

```
d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}dx=-\frac{5b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}+($$

$$=15b^2x\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}-\frac{5b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}{dx}+x$$

Mathematica [A] time = 0.277757, size = 337, normalized size = 0.97

$$\frac{15b^2x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(}}{\sqrt{\pi}}\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[I/b]*Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqr
t[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi]*FresnelC[(Sqrt[I/b]*Sqr
t[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])
))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^2\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{arsinh}\left(dx^2-i\right)+a\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{arsinh}\left(dx^2 - i\right) + a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(5/2), x)
```

3.336 $\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^{3/2} dx$

Optimal. Leaf size=310

$$-\frac{3\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)} - \frac{3b\sqrt{d^2x^4 - 2idx^2}\sqrt{a - ib\sin^{-1}\left(1 + idx^2\right)}}{dx} - \frac{3\sqrt{\pi}\sqrt{-ib}b}{\text{co}}$$

```
[Out] (-3*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2])) - (3*Sqrt[(-I)*b]*b*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 +
 I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rubi [A] time = 0.0962973, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4814, 4819}

$$-\frac{3\sqrt{\pi}b^2x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)} - \frac{3b\sqrt{d^2x^4 - 2idx^2}\sqrt{a - ib\sin^{-1}\left(1 + idx^2\right)}}{dx} - \frac{3\sqrt{\pi}\sqrt{-ib}b}{\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2])) - (3*Sqrt[(-I)*b]*b*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 +
 I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}\,dx=-\frac{3b\sqrt{-2idx^2+d^2x^4}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}+\left(3b^2\right)\int$$

$$=-\frac{3b\sqrt{-2idx^2+d^2x^4}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}-\frac{3b^2\sqrt{}}{\sqrt{-ib}\,(}$$

Mathematica [A] time = 0.252598, size = 255, normalized size = 0.82

$$-\frac{3\sqrt{\pi}(-ib)^{3/2}x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}\sqrt{-ib}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right.\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(-2*I + d*x^2)]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x) +
 x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*((-I)*b)^(3/2)*Sqrt[Pi]*x*(-(Fr
esnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(
2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqr
t[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Cos[ArcSin[1 + I*
d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{arsinh}\left(dx^2-i\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{arsinh}\left(dx^2 - i\right) + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(3/2), x)
```

3.337 $\int \sqrt{a-ib\sin^{-1}(1+idx^2)}\,dx$

Optimal. Leaf size=262

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2\right)\right.}$$

```
[Out] x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/
(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sq
rt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(
Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] -
Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rubi [A] time = 0.0275163, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4811}

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/
(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sq
rt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(
Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] -
Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
 d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int\sqrt{a-ib\sin^{-1}(1+idx^2)}\,dx=x\sqrt{a-ib\sin^{-1}(1+idx^2)}+\frac{\sqrt{\pi}xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)\right)}-$$

Mathematica [A] time = 0.0518128, size = 259, normalized size = 0.99

$$\frac{x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+}}{\sqrt{\pi}}\right.\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2]
 - Sin[ArcSin[1 + I*d*x^2]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*
ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi
]*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/
(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[Ar
cSin[1 + I*d*x^2]/2]))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int\sqrt{a+b\text{Arcsinh}\left(-i+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\,\text{arsinh}\left(dx^2-i\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(1/2),x)

[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \operatorname{arsinh}\left(dx^2 - i\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

3.338 $\int \frac{1}{\sqrt{a-ib\sin^{-1}(1+idx^2)}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx\right.\right.\right.}$$

```
[Out] -((Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt
[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d
*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(
2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2]))
```

Rubi [A] time = 0.0276035, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4819}

$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt
[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d
*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(
2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2]))
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}\,dx=-\frac{\sqrt{\pi}xC\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}xS\left(\frac{\sqrt{a-ib\sin^{-1}(}}{\sqrt{-ib}\sqrt{}}\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}(1\right.\right.}$$

Mathematica [A] time = 0.003929, size = 180, normalized size = 0.78

$$\frac{\sqrt{\pi}x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (Sqrt[Pi]*x*(-(FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqr
t[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(
Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\text{Arcsinh}\left(-i+dx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\text{arsinh}\left(dx^2-i\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{arsinh}\left(dx^2 - i\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

3.339 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cos\right.}{\cos\left(\frac{1}{2}\sin\right.}$$

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rubi [A] time = 0.0486732, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4822}

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cos\right.}{\cos\left(\frac{1}{2}\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}+\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sin\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right.\right.}$$

Mathematica [A] time = 0.356835, size = 291, normalized size = 1.

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\right.}{\cos\left(\frac{1}{2}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^2\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^2-i\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(3/2),x)

[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\operatorname{arsinh}\left(dx^2 - i\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-3/2), x)
```

3.340 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{\pi}\sqrt{-ib}x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{3b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}}}{\sqrt{-ib}}\right.}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(3*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))
 - x/(3*b^2*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(3*Sqrt[(-I)*b]*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[
1 + I*d*x^2]/2])) - (Sqrt[(-I)*b]*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3
*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rubi [A] time = 0.0725293, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4819}

$$-\frac{\sqrt{\pi}\sqrt{-ib}x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{3b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}}}{\sqrt{-ib}}\right.}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```


```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(3*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))
 - x/(3*b^2*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(3*Sqrt[(-I)*b]*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[
1 + I*d*x^2]/2])) - (Sqrt[(-I)*b]*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3
*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```


Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}} + \frac{\int \frac{1}{\sqrt{a-ib\sin^{-1}\left(1+idx^2\right.}}}{3b^2}$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}} - \frac{\sqrt{\pi}xS\left(\frac{\sqrt{a-ib}}{}\right.}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\mathrm{s}\right.\right.}$$

Mathematica [A] time = 0.786813, size = 308, normalized size = 0.94

$$-\frac{\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)} + \frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)} + \frac{b\sqrt{dx^2\left(dx^2-2i\right)}}{dx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} + \frac{}{\sqrt{a-ib}}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

```
[Out] -((b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))
+ x/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*A
rcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b
)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]
)) + (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*S
qrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 +
I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(3*b^2)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(-i+dx^2\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^2-i\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.341 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{d^2x^4-2idx^2}}{15b^3dx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}x\left(\sin\right.}{15b^3\left(\cos\left(\frac{1}{2}\right.\right.}$$

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(5*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2))
 - x/(15*b^2*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) - Sqrt[(-2*I)*d*x^2 + d^2
*x^4]/(15*b^3*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - ((I/b)^(3/2)*Sqrt[Pi
]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[
a/(2*b)] + I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcS
in[1 + I*d*x^2]/2])) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b
^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rubi [A] time = 0.0813284, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4822}

$$-\frac{\sqrt{d^2x^4-2idx^2}}{15b^3dx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}x\left(\sin\right.}{15b^3\left(\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(5*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2))
 - x/(15*b^2*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) - Sqrt[(-2*I)*d*x^2 + d^2
*x^4]/(15*b^3*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - ((I/b)^(3/2)*Sqrt[Pi
]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[
a/(2*b)] + I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcS
in[1 + I*d*x^2]/2])) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b
^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
```

```
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{7/2}}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}+\frac{\int\frac{}{\left(a-ib\sin^{-1}\right.}}{1}$$

$$=-\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}-\frac{\sqrt{-}}{15b^3dx\sqrt{a}}$$

Mathematica [A] time = 0.914323, size = 370, normalized size = 0.95

$$\frac{\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}x\left(\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x^2\left(-\left(a-ib\sin^{-1}\left(1+idx\right.\right.\right.}{}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(-2*I + d*x^2)])/d - x^2*(a - I*b*ArcSin[1 + I*d*x^2]) +
 (Sqrt[d*x^2*(-2*I + d*x^2)]*(I*a + b*ArcSin[1 + I*d*x^2])^2)/(b*d))/(x*(a
- I*b*ArcSin[1 + I*d*x^2])^(5/2)) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelC[(Sqrt[I/
b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*((-I)*Cosh[a/(2*b)] + Sinh[
a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) +
(Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]
)/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(15*b^2)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(a+b\text{Arcsinh}\left(-i+dx^2\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\,\text{arsinh}\left(dx^2-i\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.342 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),
x]
```

Rubi [A] time = 0.0442882, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.0889824, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A] time = 0.46, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\text{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(b \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.343 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=261

$$\frac{3b^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{PolyLog}\left(2,e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}+\frac{3b}{}$$

```
[Out] -(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c) - ((a + b*ArcSinh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (3*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2
, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*ArcSi
nh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/(2*c) + (3*b^3*PolyLog[4, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/(4*c)
```

Rubi [A] time = 0.225726, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6681, 5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3b^3\text{P}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c) - ((a + b*ArcSinh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (3*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2,
E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*ArcSinh[
Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])])/(2*c) - (3*b^3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])])/(4*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{(3b)\,\text{Subs}}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+b\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+b\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+b\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{3b\left(a+b\right.}{}$$

Mathematica [A] time = 0.0572943, size = 244, normalized size = 0.93

$$\frac{6b^2\text{PolyLog}\left(3, e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right) - 6b\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2 - 3b^3\text{Po}}{4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/b - 4*(a + b*ArcSinh[Sqrt[1
 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]
)] - 6*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] - 3*b^
3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(4*c)
```

Maple [B] time = 0.72, size = 1175, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)+1/4*b^3/c*arcsinh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^4-b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(-c*x+1)
^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-3*b^3/c*arcsinh((-c*x+1)^(
1/2)/(c*x+1)^(1/2))^2*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(
c*x+1))^(1/2))+6*b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(-c
*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-6*b^3/c*polylog(4,-(-
c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-b^3/c*arcsinh((-c*x+
1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*
x+1))^(1/2))-3*b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(-c*
x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+6*b^3/c*arcsinh((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))-6*b^3/c*polylog(4,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))+a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*a
rcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1
+(-c*x+1)/(c*x+1))^(1/2))-6*a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*p
olylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+6*a*b^2/
c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-3*a*b
^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1
/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-6*a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1
/2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+6*a
*b^2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+3
/2*a^2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*arcsinh((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+
1))^(1/2))-3*a^2*b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c
*x+1))^(1/2))-3*a^2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)
^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-3*a^2*b/c*polylog(2,(-c*x+
1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^3\log(cx+1)-b^3\log(-cx+1)\right)\log\left(\sqrt{2}+\sqrt{-cx+1}\right)^3}{2c}+\int\frac{\left(\sqrt{2}b^3+\sqrt{-cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^3/c + integrate(1/8*((sqrt(2)*b^3
 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^3 - 6*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a
*b^2)*log(c*x + 1)^2 - 6*(4*sqrt(2)*a*b^2 - 2*(sqrt(2)*b^3 + sqrt(-c*x + 1)
*b^3)*log(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b
^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2 + 12*(sq
rt(2)*a^2*b + sqrt(-c*x + 1)*a^2*b)*log(c*x + 1) - 6*(4*sqrt(2)*a^2*b + 4*s
qrt(-c*x + 1)*a^2*b + (sqrt(2)*b^3 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^2 - 4
*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a*b^2)*log(c*x + 1))*log(sqrt(2) + sqrt(-c
*x + 1)))/(sqrt(2)*c^2*x^2 + (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab^2\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^3}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arcsinh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

3.344 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=194

$$\frac{b\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-$$

```
[Out] -(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c) - ((a + b*ArcSinh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^
(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, E^(-2*ArcSi
nh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rubi [A] time = 0.215653, antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 5659, 3716, 2190, 2531, 2282, 6589}

$$-\frac{b\,\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c) - ((a + b*ArcSinh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2
*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, E^(2*ArcSinh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$=-\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx,x,\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$=\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}+\frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx,x,\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$=\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{(2b)\,\text{Subs}}{}$$

$$=\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}-\frac{b\left(a+b\,\text{si}\right.}{}$$

$$=\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}-\frac{b\left(a+b\,\text{si}\right.}{}$$

$$=\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}-\frac{b\left(a+b\,\text{si}\right.}{}$$

Mathematica [A] time = 0.0595205, size = 187, normalized size = 0.96

$$\frac{-6b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+3b^3\text{PolyLog}\left(3,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)+2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{6bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*(a + b*ArcSinh[Sqrt[1 - c
*x]/Sqrt[1 + c*x]] - 3*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
]) - 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 3*b^3*PolyLog[3, E^(2*ArcSinh[Sqrt[1
- c*x]/Sqrt[1 + c*x]])])/(6*b*c)
```

Maple [B] time = 0.007, size = 649, normalized size = 3.4

$$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}+\frac{b^2}{3c}\left(\text{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^3-\frac{b^2}{c}\left(\text{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2\ln\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+1/3*b^2/c*arcsinh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3-b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(-c*x+1)
^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-2*b^2/c*arcsinh((-c*x+1)^(
1/2)/(c*x+1)^(1/2))*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*
x+1))^(1/2))+2*b^2/c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c
*x+1))^(1/2))-b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(
1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-2*b^2/c*arcsinh((-c*x+1)^(1/
2)/(c*x+1)^(1/2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1
))^(1/2))+2*b^2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1
))^(1/2))+a*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-2*a*b/c*arcsinh((-c
*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c
*x+1))^(1/2))-2*a*b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(
c*x+1))^(1/2))-2*a*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^
(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-2*a*b/c*polylog(2,(-c*x+1)^
(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^2\log(cx+1)-b^2\log(-cx+1)\right)\log\left(\sqrt{2}+\sqrt{-cx+1}\right)^2}{2c}+\int-\frac{\left(\sqrt{2}b^2+\sqrt{-cx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2/c + integrate(-1/4*((sqrt(2)*b^
```

```
2 + sqrt(-c*x + 1)*b^2)*log(c*x + 1)^2 - 4*(sqrt(2)*a*b + sqrt(-c*x + 1)*a*
b)*log(c*x + 1) + 2*(4*sqrt(2)*a*b - 2*(sqrt(2)*b^2 + sqrt(-c*x + 1)*b^2)*l
og(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-
c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1)))/(sqrt(2)*c^2*x^2 +
 (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2 \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,ab \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arcsinh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

3.345 $\int \frac{a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=133

$$\frac{b\text{PolyLog}\left(2,e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

```
[Out] -(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c) - ((a + b*ArcSinh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c + (b*PolyLog[2, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*
c)
```

Rubi [A] time = 0.121832, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {206, 6681, 5659, 3716, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c) - ((a + b*ArcSinh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x
]])])/c - (b*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{Subst}\left(\int \text{lo}\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{Subst}\left(\int \frac{\text{log}}{}\right.}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\text{Li}_2\left(e^{2\sinh^{-1}\left(\right.}\right.}{2c}$$

Mathematica [A] time = 0.0279931, size = 127, normalized size = 0.95

$$\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-2b\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\right)-b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*(a + b*ArcSinh[Sqrt[1 - c*x]/
Sqrt[1 + c*x]] - 2*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])]) -
```

```
b^2*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*b*c)
```

Maple [A] time = 0.006, size = 263, normalized size = 2.

$$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{b}{2c}\left(\mathrm{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2-\frac{b}{c}\mathrm{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\ln\left(1+\sqrt{-cx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+1/2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2-b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*
x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1
)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2
))*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-b/c*polylo
g(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}b\left(\frac{2(\log(cx+1)-\log(-cx+1))\log(cx+1)-\log(cx+1)^2+2\log(cx+1)\log(-cx+1)-\log(-cx+1)^2-4(\mathrm{lo}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(2) + sqrt(-c*x + 1)))/c + 8*integrate(-1/4*(sqrt(2)*log(c*x
+ 1) - sqrt(2)*log(-c*x + 1))/(sqrt(2)*c*x + (c*x - 1)*sqrt(-c*x + 1) - sqr
t(2)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b\,\mathrm{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b \operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.346 $\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
, x]
```

Rubi [A] time = 0.0492467, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.0934967, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\text{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(c^2x^2-1\right)\left(b\,\mathrm{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{ac^2x^2+\left(bc^2x^2-b\right)\mathrm{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\,\mathrm{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.347 $\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^
2), x]
```

Rubi [A] time = 0.0451338, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.774385, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\text{Arcsinh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(\sqrt{2}+\sqrt{-cx+1}\right)}{2\sqrt{2}abc^2x-2\sqrt{2}abc-4\sqrt{-cx+1}abc-\left(\sqrt{2}b^2c^2x-\sqrt{2}b^2c-2\sqrt{-cx+1}b^2c\right)\log(cx+1)+2\left(\sqrt{2}b^2c^2x-\sqrt{2}b^2c-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] -4*(sqrt(2) + sqrt(-c*x + 1))/(2*sqrt(2)*a*b*c^2*x - 2*sqrt(2)*a*b*c - 4*sq
rt(-c*x + 1)*a*b*c - (sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x + 1)*
b^2*c)*log(c*x + 1) + 2*(sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x +
1)*b^2*c)*log(sqrt(2) + sqrt(-c*x + 1))) - integrate((4*c*x + (sqrt(2)*c*x
- 3*sqrt(2))*sqrt(-c*x + 1) - 4)/(2*a*b*c^3*x^3 - 6*a*b*c^2*x^2 + 6*a*b*c*x
 - 4*(a*b*c*x - a*b)*(c*x - 1) - 2*a*b - (b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*b
^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)*b^2*c^2*x^2 - 2*sqr
t(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(c*x + 1) + 2*(b^2*c^3*x^3 -
 3*b^2*c^2*x^2 + 3*b^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)
*b^2*c^2*x^2 - 2*sqrt(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(sqrt(2)
 + sqrt(-c*x + 1)) - 4*(sqrt(2)*a*b*c^2*x^2 - 2*sqrt(2)*a*b*c*x + sqrt(2)*a
*b)*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{1}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\left(abc^2x^2-ab\right)\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsinh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\,\mathrm{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

3.348 $\int \sinh^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=76

$$\frac{\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b} - \frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1 - e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[Out] -ArcSinh[c*E^(a + b*x)]^2/(2*b) + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Rubi [A] time = 0.0756375, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2282, 5659, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b} - \frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1 - e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -ArcSinh[c*E^(a + b*x)]^2/(2*b) + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(ce^{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(cx)}{x}\, dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int x\coth(x)\, dx, x, \sinh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\, dx, x, \sinh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right) dx, x, \sinh\right.}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\, dx, x, e^{2\sinh^{-1}(ce}\right.}{2b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.491021, size = 76, normalized size = 1.

$$
\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b} - \frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -ArcSinh[c*E^(a + b*x)]^2/(2*b) + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Maple [A] time = 0.023, size = 166, normalized size = 2.2

$$
-\frac{\left(\text{Arcsinh}\left(ce^{bx+a}\right)\right)^2}{2b} + \frac{\text{Arcsinh}\left(ce^{bx+a}\right)}{b}\ln\left(1+ce^{bx+a}+\sqrt{1+c^2\left(e^{bx+a}\right)^2}\right) + \frac{1}{b}\text{polylog}\left(2,-ce^{bx+a}-\sqrt{1+c^2\left(e^{bx+a}\right)^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*exp(b*x+a)),x)
```

```
[Out] -1/2*arcsinh(c*exp(b*x+a))^2/b+1/b*arcsinh(c*exp(b*x+a))*ln(1+c*exp(b*x+a)+
(1+c^2*exp(b*x+a)^2)^(1/2))+1/b*polylog(2,-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2
)^(1/2))+1/b*arcsinh(c*exp(b*x+a))*ln(1-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2)^(
```

```
1/2))+1/b*polylog(2,c*exp(b*x+a)+(1+c^2*exp(b*x+a)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-bc\int \frac{xe^{(bx+a)}}{c^3e^{(3\,bx+3\,a)}+ce^{(bx+a)}+\left(c^2e^{(2\,bx+2\,a)}+1\right)^{\frac{3}{2}}}\,dx+x\log\left(ce^{(bx+a)}+\sqrt{c^2e^{(2\,bx+2\,a)}+1}\right)-\frac{2\,bx\log\left(c^2e^{(2\,bx+2\,a)}+1\right)+}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) + c*e^(b*x + a) + (c^2*e^
(2*b*x + 2*a) + 1)^(3/2)), x) + x*log(c*e^(b*x + a) + sqrt(c^2*e^(2*b*x + 2
*a) + 1)) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c^2*e^(2*b*x +
 2*a)))/b
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*exp(b*x+a)),x)
```

```
[Out] Integral(asinh(c*exp(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}\left(ce^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*e^(b*x + a)), x)
```

3.349 $\int e^{\sinh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=165

$$\frac{(3-4a^2)\,ae^{2\sinh^{-1}(a+bx)}}{16b^4}+\frac{(3-4a^2)\,a\sinh^{-1}(a+bx)}{8b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}-\frac{(1-6a^2)\,e^{3\sinh^{-1}(a+bx)}}{24b^4}+\frac{3ae^{-2\sinh^{-1}(a+}}{16b^4}$$

```
[Out] 1/(48*b^4*E^(3*ArcSinh[a + b*x])) + (3*a)/(16*b^4*E^(2*ArcSinh[a + b*x])) -
 (1 - 6*a^2)/(8*b^4*E^ArcSinh[a + b*x]) + (a*(3 - 4*a^2)*E^(2*ArcSinh[a + b
*x]))/(16*b^4) - ((1 - 6*a^2)*E^(3*ArcSinh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcSinh[a + b*x]))/(32*b^4) + E^(5*ArcSinh[a + b*x])/(80*b^4) + (a*(3 - 4*a
^2)*ArcSinh[a + b*x])/(8*b^4)
```

Rubi [A] time = 0.172355, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5898, 2282, 12, 1628}

$$\frac{(3-4a^2)\,ae^{2\sinh^{-1}(a+bx)}}{16b^4}+\frac{(3-4a^2)\,a\sinh^{-1}(a+bx)}{8b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}-\frac{(1-6a^2)\,e^{3\sinh^{-1}(a+bx)}}{24b^4}+\frac{3ae^{-2\sinh^{-1}(a+}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] 1/(48*b^4*E^(3*ArcSinh[a + b*x])) + (3*a)/(16*b^4*E^(2*ArcSinh[a + b*x])) -
 (1 - 6*a^2)/(8*b^4*E^ArcSinh[a + b*x]) + (a*(3 - 4*a^2)*E^(2*ArcSinh[a + b
*x]))/(16*b^4) - ((1 - 6*a^2)*E^(3*ArcSinh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcSinh[a + b*x]))/(32*b^4) + E^(5*ArcSinh[a + b*x])/(80*b^4) + (a*(3 - 4*a
^2)*ArcSinh[a + b*x])/(8*b^4)
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x^3\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3 dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{16b^3x^4} dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{x^4} dx, x, e^{\sinh^{-1}(a+bx)}\right)}{16b^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}-\frac{6a}{x^3}-\frac{2(-1+6a^2)}{x^2}+\frac{2a(3-4a^2)}{x}+2a(3-4a^2)x+2(-1+6a^2)x^2-6ax^3+x^4\right) dx,\right.}{16b^4}$$
$$= \frac{e^{-3\sinh^{-1}(a+bx)}}{48b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}-\frac{(1-6a^2)e^{-\sinh^{-1}(a+bx)}}{8b^4}+\frac{a(3-4a^2)e^{2\sinh^{-1}(a+bx)}}{16b^4}-\frac{(1-6}{}$$

Mathematica [A] time = 0.0810424, size = 119, normalized size = 0.72

$$\frac{-\sqrt{a^2+2abx+b^2x^2+1}\left(2\left(3a^2-4\right)b^2x^2+\left(29-6a^2\right)abx+6a^4-83a^2-6ab^3x^3-24b^4x^4+16\right)+15a\left(3-4a^2\right)\sin}{120b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(16 - 83*a^2
 + 6*a^4 + a*(29 - 6*a^2)*b*x + 2*(-4 + 3*a^2)*b^2*x^2 - 6*a*b^3*x^3 - 24*b
^4*x^4) + 15*a*(3 - 4*a^2)*ArcSinh[a + b*x])/(120*b^4)
```

Maple [A] time = 0.006, size = 322, normalized size = 2.

$$\frac{x^2}{5b^2}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}-\frac{7ax}{20b^3}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}+\frac{9a^2}{20b^4}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}-\frac{a^3x}{2b^3}\sqrt{b^2x^2+2xab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x)
```

```
[Out] 1/5*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-7/20*a/b^3*x*(b^2*x^2+2*a*b*x+a^2
+1)^(3/2)+9/20*a^2/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a^3/b^3*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*x-1/2*a^4/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a^3/b^3
*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3/8*
a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3/8*a^2/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+3/8*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-2/15/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/5*b*x^5+1/4*x^4*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.84998, size = 321, normalized size = 1.95

$$\frac{24\,b^5x^5 + 30\,ab^4x^4 + 15\left(4\,a^3 - 3\,a\right)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \left(24\,b^4x^4 + 6\,ab^3x^3 - 2\left(3\,a^2 - 4\right)b^2x^2 - 6\,a}{120\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + 15*(4*a^3 - 3*a)*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)) + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 - 4)*b^2*
x^2 - 6*a^4 + (6*a^3 - 29*a)*b*x + 83*a^2 - 16)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [A] time = 1.32659, size = 234, normalized size = 1.42

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4 + \frac{1}{120}\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(\left(2\left(3\left(4\,x + \frac{a}{b}\right)x - \frac{3\,a^2b^5 - 4\,b^5}{b^7}\right)x + \frac{6\,a^3b^4 - 29\,ab^4}{b^7}\right)x - \frac{6\,a^4b^3 - 83\,a^2b^3 +}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/120*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*(4*x
 + a/b)*x - (3*a^2*b^5 - 4*b^5)/b^7)*x + (6*a^3*b^4 - 29*a*b^4)/b^7)*x - (6
*a^4*b^3 - 83*a^2*b^3 + 16*b^3)/b^7) + 1/8*(4*a^3 - 3*a)*log(-a*b - (x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*abs(b))
```

3.350 $\int e^{\sinh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=115

$$-\frac{\left(1-4a^2\right)e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{\left(1-4a^2\right)\sinh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}-\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}$$

```
[Out] -1/(16*b^3*E^(2*ArcSinh[a + b*x])) - a/(2*b^3*E^ArcSinh[a + b*x]) - ((1 - 4
*a^2)*E^(2*ArcSinh[a + b*x]))/(16*b^3) - (a*E^(3*ArcSinh[a + b*x]))/(6*b^3)
 + E^(4*ArcSinh[a + b*x])/(32*b^3) - ((1 - 4*a^2)*ArcSinh[a + b*x])/(8*b^3)
```

Rubi [A] time = 0.12525, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5898, 2282, 12, 1628}

$$-\frac{\left(1-4a^2\right)e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{\left(1-4a^2\right)\sinh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}-\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] -1/(16*b^3*E^(2*ArcSinh[a + b*x])) - a/(2*b^3*E^ArcSinh[a + b*x]) - ((1 - 4
*a^2)*E^(2*ArcSinh[a + b*x]))/(16*b^3) - (a*E^(3*ArcSinh[a + b*x]))/(6*b^3)
 + E^(4*ArcSinh[a + b*x])/(32*b^3) - ((1 - 4*a^2)*ArcSinh[a + b*x])/(8*b^3)
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x^2\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2 dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+2ax-x^2)^2(1+x^2)}{8b^2x^3} dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+2ax-x^2)^2(1+x^2)}{x^3} dx, x, e^{\sinh^{-1}(a+bx)}\right)}{8b^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1+4a^2}{x}+(-1+4a^2)\,x-4ax^2+x^3\right) dx, x, e^{\sinh^{-1}(a+bx)}\right)}{8b^3}$$
$$= -\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{(1-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}-\frac{(1-$$

Mathematica [A] time = 0.105329, size = 102, normalized size = 0.89

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(-2a^2bx+2a^3+a\left(2b^2x^2-13\right)+6b^3x^3+3bx\right)+8ab^3x^3+3(2a-1)(2a+1)\sinh^{-1}(a+bx)+6b}{24b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2*a^3 + 3*b*x
 - 2*a^2*b*x + 6*b^3*x^3 + a*(-13 + 2*b^2*x^2)) + 3*(-1 + 2*a)*(1 + 2*a)*Ar
cSinh[a + b*x])/(24*b^3)
```

Maple [A] time = 0.005, size = 264, normalized size = 2.3

$$\frac{x}{4b^2}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}-\frac{5a}{12b^3}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}+\frac{a^2x}{2b^2}\sqrt{b^2x^2+2xab+a^2+1}+\frac{a^3}{2b^3}\sqrt{b^2x^2+2xab+a^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x)
```

```
[Out] 1/4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-5/12*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+1/2*a^2/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*a^3/b^3*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+1/2*a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))/(b^2)^(1/2)-1/8/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/8/b^3*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/8/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/4*b*x^4+1/3*x^3*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.89097, size = 266, normalized size = 2.31

$$\frac{6\,b^4x^4+8\,ab^3x^3-3\left(4\,a^2-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(6\,b^3x^3+2\,ab^2x^2+2\,a^3-\left(2\,a^2-3\right)bx-13\,a\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 - 3*(4*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 - 3)*b*x -
 13*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+bx+\sqrt{a^2+2abx+b^2x^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [A] time = 1.36013, size = 189, normalized size = 1.64

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3+\frac{1}{24}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\,x+\frac{a}{b}\right)x-\frac{2\,a^2b^3-3\,b^3}{b^5}\right)x+\frac{2\,a^3b^2-13\,ab^2}{b^5}\right)-\frac{\left(4\,a^2-1\right)\log\left(-ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*x + a
/b)*x - (2*a^2*b^3 - 3*b^3)/b^5)*x + (2*a^3*b^2 - 13*a*b^2)/b^5) - 1/8*(4*a
^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(
b^2*abs(b))
```

3.351 $\int e^{\sinh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=67

$$-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}+\frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}$$

```
[Out] 1/(4*b^2*E^ArcSinh[a + b*x]) - (a*E^(2*ArcSinh[a + b*x]))/(4*b^2) + E^(3*Ar
cSinh[a + b*x])/(12*b^2) - (a*ArcSinh[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.070877, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5898, 2282, 12, 1628}

$$-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}+\frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcSinh[a + b*x]) - (a*E^(2*ArcSinh[a + b*x]))/(4*b^2) + E^(3*Ar
cSinh[a + b*x])/(12*b^2) - (a*ArcSinh[a + b*x])/(2*b^2)
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{4bx^2}dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{x^2}dx, x, e^{\sinh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2a}{x}-2ax+x^2\right)dx, x, e^{\sinh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}$$

Mathematica [A] time = 0.0987441, size = 73, normalized size = 1.09

$$\frac{1}{6}\left(\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(-a^2+abx+2b^2x^2+2\right)}{b^2}-\frac{3a\sinh^{-1}(a+bx)}{b^2}+3ax^2+2bx^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 - a^2 + a*b*x +
2*b^2*x^2))/b^2 - (3*a*ArcSinh[a + b*x])/b^2)/6
```

Maple [A] time = 0.003, size = 138, normalized size = 2.1

$$\frac{1}{3b^2}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}-\frac{ax}{2b}\sqrt{b^2x^2+2xab+a^2+1}-\frac{a^2}{2b^2}\sqrt{b^2x^2+2xab+a^2+1}-\frac{a}{2b}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x)
```

```
[Out] 1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-1/2*a/b*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-1/2*a^2/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a/b*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/3*b*x^3+1/2*a*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60614, size = 212, normalized size = 3.16

$$\frac{2\,b^3x^3 + 3\,ab^2x^2 + 3\,a\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + \left(2\,b^2x^2 + abx - a^2 + 2\right)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + 3*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)) + (2*b^2*x^2 + a*b*x - a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x,x)
```

```
[Out] Integral(x*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [A] time = 1.34604, size = 143, normalized size = 2.13

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2 + \frac{1}{6}\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(\left(2\,x + \frac{a}{b}\right)x - \frac{a^2b - 2\,b}{b^3}\right) + \frac{a\log\left(-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)|b|\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*x + a/b)*
x - (a^2*b - 2*b)/b^3) + 1/2*a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*abs(b))/(b*abs(b))
```

3.352 $\int e^{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sinh^{-1}(a+bx)}{2b}+\frac{e^{2\sinh^{-1}(a+bx)}}{4b}$$

```
[Out] E^(2*ArcSinh[a + b*x])/(4*b) + ArcSinh[a + b*x]/(2*b)
```

Rubi [A] time = 0.0174833, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5896, 2282, 12, 14}

$$\frac{\sinh^{-1}(a+bx)}{2b}+\frac{e^{2\sinh^{-1}(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x],x]
```

```
[Out] E^(2*ArcSinh[a + b*x])/(4*b) + ArcSinh[a + b*x]/(2*b)
```

Rule 5896

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{e^{2\sinh^{-1}(a+bx)}}{4b} + \frac{\sinh^{-1}(a+bx)}{2b}$$

Mathematica [A] time = 0.0323047, size = 46, normalized size = 1.48

$$\frac{(a+bx)\left(\sqrt{a^2+2abx+b^2x^2+1}+a+bx\right)+\sinh^{-1}(a+bx)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x],x]
```

```
[Out] ((a + b*x)*(a + b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ArcSinh[a + b*x]
)/(2*b)
```

Maple [B] time = 0.003, size = 89, normalized size = 2.9

$$ax+\frac{2\,b^2x+2\,ab}{4\,b^2}\sqrt{b^2x^2+2\,xab+a^2+1}+\frac{1}{2}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a+(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] a*x+1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*b*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.5868, size = 169, normalized size = 5.45

$$\frac{b^2x^2+2\,abx+\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)-\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a) - log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+bx+\sqrt{(a+bx)^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(a + b*x + sqrt((a + b*x)**2 + 1), x)
```

Giac [B] time = 1.48247, size = 108, normalized size = 3.48

$$\frac{1}{2}\,bx^2+ax+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(x+\frac{a}{b}\right)-\frac{\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x + a/b) - 1/2*log
(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/abs(b)
```

3.353 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=89

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)+a\sinh^{-1}(a+bx)+a\log(x)+bx$$

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] - Sqrt[1 + a^2
]*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2
])] + a*Log[x]
```

Rubi [A] time = 0.120651, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5907, 14, 734, 843, 619, 215, 724, 206}

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)+a\sinh^{-1}(a+bx)+a\log(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x,x]
```


```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] - Sqrt[1 + a^2
]*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2
])] + a*Log[x]
```


Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\sinh^{-1}(a+bx)}}{x}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x}\,dx\\ &= \int \left(b+\frac{a}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\right)dx\\ &= bx+a\log(x)+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\,dx\\ &= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\log(x)-\frac{1}{2}\int \frac{-2\left(1+a^2\right)-2abx}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx\\ &= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\log(x)-\left(-1-a^2\right)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx+(ab)\int \frac{}{\sqrt{1+}}\\ &= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\log(x)-\left(2\left(1+a^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{4\left(1+a^2\right)-x^2}\,dx,x,\frac{2\left(1+a^2\right.}{\sqrt{1+a^2+}}\right.\\ &= bx+\sqrt{1+a^2+2abx+b^2x^2}+a\sinh^{-1}(a+bx)-\sqrt{1+a^2}\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x}}\right.\end{aligned}$$

Mathematica [A] time = 0.0740409, size = 99, normalized size = 1.11

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)+\left(\sqrt{a^2+1}+a\right)\log(x)+a\sinh$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] + (a + Sqrt[1
+ a^2])*Log[x] - Sqrt[1 + a^2]*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
```

```
a^2 + 2*a*b*x + b^2*x^2]]
```

Maple [A] time = 0.004, size = 126, normalized size = 1.4

$$\sqrt{b^2x^2+2\,xab+a^2+1}+ab\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,xab+a^2+1}\right)\frac{1}{\sqrt{b^2}}-\sqrt{a^2+1}\ln\left(\frac{1}{x}\left(2\,a^2+2+2\,xab+2\,\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x)
```

```
[Out] (b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b*x+a*ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.62354, size = 335, normalized size = 3.76

$$bx-a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+a\log\left(x\right)+\sqrt{a^2+1}\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2}\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] b*x - a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*log(x) + sqrt
(a^2 + 1)*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 - sq
rt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.354 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}-\frac{ab\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}}+b\sinh^{-1}(a+bx)-\frac{a}{x}+b\log(x)$$

```
[Out] -(a/x) - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/x + b*ArcSinh[a + b*x] - (a*b*Ar
cTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])
/Sqrt[1 + a^2] + b*Log[x]
```

Rubi [A] time = 0.105395, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5907, 14, 732, 843, 619, 215, 724, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}-\frac{ab\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}}+b\sinh^{-1}(a+bx)-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/x + b*ArcSinh[a + b*x] - (a*b*Ar
cTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])
/Sqrt[1 + a^2] + b*Log[x]
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^2}\,dx\\
&= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\right)dx\\
&= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\,dx\\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\int \frac{2ab+2b^2x}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx\\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+(ab)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx+b^2\int \frac{}{\sqrt{1+a^2+}}\\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)-(2ab)\operatorname{Subs}\\
&= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\sinh^{-1}(a+bx)-\frac{ab\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\sqrt{1+a^2}}+b\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.158718, size = 110, normalized size = 1.11

$$
b\sinh^{-1}(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2+1}+\frac{abx\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\sqrt{a^2+1}}+\left(-\frac{a}{\sqrt{a^2+1}}-1\right)bx\log(x)+a}{x}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] b*ArcSinh[a + b*x] - (a + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 - a/Sqrt[
1 + a^2])*b*x*Log[x] + (a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
```

```
a^2 + 2*a*b*x + b^2*x^2]])/Sqrt[1 + a^2])/x
```

Maple [B] time = 0.009, size = 267, normalized size = 2.7

$$-\frac{1}{\left(a^2+1\right)x}\left(b^2x^2+2\,xab+a^2+1\right)^{\frac{3}{2}}+2\,\frac{ab\sqrt{b^2x^2+2\,xab+a^2+1}}{a^2+1}+\frac{a^2b^2}{a^2+1}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,xab+a^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x)
```

```
[Out] -1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+2*a*b/(a^2+1)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+a^2*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-a*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*x+b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)-a/x+b*ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72342, size = 437, normalized size = 4.41

$$\frac{\sqrt{a^2+1}abx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-\left(a^2+1\right)bx\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2}\right.}{\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*a*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) - (
a^2 + 1)*b*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (a^2 + 1)*
b*x*log(x) - a^3 - (a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
 1) - a)/((a^2 + 1)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**2,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.355 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=116

$$-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)x^2}-\frac{b^2\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{a}{2x^2}-\frac{b}{x}$$

```
[Out] -a/(2*x^2) - b/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2
*(1 + a^2)*x^2) - (b^2*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(3/2))
```

Rubi [A] time = 0.0858755, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5907, 14, 720, 724, 206}

$$-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)x^2}-\frac{b^2\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -a/(2*x^2) - b/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2
*(1 + a^2)*x^2) - (b^2*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(3/2))
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^3}\,dx \\
&= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\right)dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)x^2}+\frac{b^2\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx}{2\left(1+a^2\right)} \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)x^2}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{4\left(1+a^2\right)-x^2}\,dx,x,\frac{2\left(1+a^2\right)+2abx}{\sqrt{1+a^2+2abx+b^2x^2}}\right)}{1+a^2} \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)x^2}-\frac{b^2\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2\left(1+a^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.19399, size = 129, normalized size = 1.11

$$\frac{1}{2}\left(-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{\left(a^2+1\right)x^2}-\frac{b^2\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\left(a^2+1\right)^{3/2}}+\frac{b^2\log(x)}{\left(a^2+1\right)^{3/2}}-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
/((1 + a^2)*x^2) + (b^2*Log[x])/(1 + a^2)^(3/2) - (b^2*Log[1 + a^2 + a*b*x
+ Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(3/2))/2
```

Maple [B] time = 0.009, size = 457, normalized size = 3.9

$$-\frac{1}{\left(2a^2+2\right)x^2}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}+\frac{ab}{2\left(a^2+1\right)^2x}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}-\frac{a^2b^2}{\left(a^2+1\right)^2}\sqrt{b^2x^2+2xab+a^2+1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x)
```

```
[Out] -1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/2*a*b/(a^2+1)^2/x*(b^2*x^2
+2*a*b*x+a^2+1)^(3/2)-a^2*b^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a
^3*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)+1/2*a^2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(
```

```
b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/2*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)*x-1/2*a*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)+1/2*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*
b^3/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^
2)^(1/2)-1/2*b^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2))/x)-b/x-1/2*a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63687, size = 423, normalized size = 3.65

$$\frac{\sqrt{a^2+1}b^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-a^5-\left(a^3+a\right)b^2x^2-2\,a^3-2\left(a^4+2\,a^2+1\right)bx}{2\left(a^4+2\,a^2+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/
x) - a^5 - (a^3 + a)*b^2*x^2 - 2*a^3 - 2*(a^4 + 2*a^2 + 1)*b*x - (a^4 + (a^
3 + a)*b*x + 2*a^2 + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - a)/((a^4 + 2*a^
2 + 1)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**3,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.356 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^4} \, dx$

Optimal. Leaf size=156

$$\frac{ab\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)^2x^2}-\frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{3\left(a^2+1\right)x^3}+\frac{ab^3\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{5/2}}-\frac{a}{3x^3}-\frac{b}{2x^2}$$

```
[Out] -a/(3*x^3) - b/(2*x^2) + (a*b*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2])/(2*(1 + a^2)^2*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(3*(1 + a
^2)*x^3) + (a*b^3*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2
*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(5/2))
```

Rubi [A] time = 0.108295, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5907, 14, 730, 720, 724, 206}

$$\frac{ab\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)^2x^2}-\frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{3\left(a^2+1\right)x^3}+\frac{ab^3\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{5/2}}-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(2*x^2) + (a*b*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2])/(2*(1 + a^2)^2*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(3*(1 + a
^2)*x^3) + (a*b^3*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2
*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(5/2))
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
```

```
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^4}\,dx\\
&= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\right)dx\\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx\\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{(ab)\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx}{1+a^2}\\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{\left(ab^3\right)\int \frac{}{x}}{2}\\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+\frac{\left(ab^3\right)\text{Sub}}{}\\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+\frac{ab^3\tanh}{}
\end{aligned}$$

Mathematica [A] time = 0.106884, size = 162, normalized size = 1.04

$$\frac{1}{6}\left(-\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(a^2\left(4-b^2x^2\right)+a^3bx+2a^4+abx+2b^2x^2+2\right)}{\left(a^2+1\right)^2x^3}+\frac{3ab^3\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+}\right.}{\left(a^2+1\right)^{5/2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 + a
*b*x + a^3*b*x + 2*b^2*x^2 + a^2*(4 - b^2*x^2)))/((1 + a^2)^2*x^3) - (3*a*b
^3*Log[x])/(1 + a^2)^(5/2) + (3*a*b^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*S
qrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(5/2))/6
```

Maple [B] time = 0.009, size = 501, normalized size = 3.2

$$-\frac{1}{\left(3a^2+3\right)x^3}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}+\frac{ab}{2\left(a^2+1\right)^2x^2}\left(b^2x^2+2xab+a^2+1\right)^{\frac{3}{2}}-\frac{a^2b^2}{2\left(a^2+1\right)^3x}\left(b^2x^2+2xab+a^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x)
```

```
[Out] -1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3+1/2*a*b/(a^2+1)^2/x^2*(b^2*x
^2+2*a*b*x+a^2+1)^(3/2)-1/2*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/
2)+a^3*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*a^4*b^4/(a^2+1)^3*ln
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*a^3
*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)+1/2*a^2*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*a^2
*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-1/2*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a^2*b^4/(a^
2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2
)+1/2*a*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/x)-1/2*b/x^2-1/3*a/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.64109, size = 533, normalized size = 3.42

$$\frac{3\sqrt{a^2+1}ab^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-2\,a^7+\left(a^4-a^2-2\right)b^3x^3-6\,a^5-6\,a^3-3\,(}{6\left(a^6+3\,a^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + 1)*a*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) +
 a)/x) - 2*a^7 + (a^4 - a^2 - 2)*b^3*x^3 - 6*a^5 - 6*a^3 - 3*(a^6 + 3*a^4 +
 3*a^2 + 1)*b*x - (2*a^6 - (a^4 - a^2 - 2)*b^2*x^2 + 6*a^4 + (a^5 + 2*a^3 +
 a)*b*x + 6*a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a)/((a^6 + 3*a^4
 + 3*a^2 + 1)*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**4,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.357 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^5} dx$

Optimal. Leaf size=207

$$\frac{(1-4a^2)\,b^2\,(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{8\,(a^2+1)^3\,x^2}+\frac{5ab\,(a^2+2abx+b^2x^2+1)^{3/2}}{12\,(a^2+1)^2\,x^3}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{4\,(a^2+1)\,x^4}+\frac{(1-4a}{}$$

```
[Out] -a/(4*x^4) - b/(3*x^3) + ((1 - 4*a^2)*b^2*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 +
2*a*b*x + b^2*x^2])/(8*(1 + a^2)^3*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/
2)/(4*(1 + a^2)*x^4) + (5*a*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(12*(1 +
 a^2)^2*x^3) + ((1 - 4*a^2)*b^4*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sq
rt[1 + a^2 + 2*a*b*x + b^2*x^2])])/(8*(1 + a^2)^(7/2))
```

Rubi [A] time = 0.170881, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5907, 14, 744, 806, 720, 724, 206}

$$\frac{(1-4a^2)\,b^2\,(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{8\,(a^2+1)^3\,x^2}+\frac{5ab\,(a^2+2abx+b^2x^2+1)^{3/2}}{12\,(a^2+1)^2\,x^3}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{4\,(a^2+1)\,x^4}+\frac{(1-4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(3*x^3) + ((1 - 4*a^2)*b^2*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 +
2*a*b*x + b^2*x^2])/(8*(1 + a^2)^3*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/
2)/(4*(1 + a^2)*x^4) + (5*a*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(12*(1 +
 a^2)^2*x^3) + ((1 - 4*a^2)*b^4*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sq
rt[1 + a^2 + 2*a*b*x + b^2*x^2])])/(8*(1 + a^2)^(7/2))
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^5}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^5}\,dx \\
&= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\right)dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\,dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}-\frac{\int \frac{\left(5ab+b^2x\right)\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx}{4\left(1+a^2\right)} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}+\frac{5ab\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{12\left(1+a^2\right)^2x^3}-\frac{\left(\left(1-4a^2\right)b^2\right)\int \frac{\sqrt{1+a}}{}}{4\left(1+a^2\right.} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}+ \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}+ \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}+
\end{aligned}
$$

Mathematica [A] time = 0.73903, size = 192, normalized size = 0.93

$$\frac{1}{24}\left(-\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(\frac{a(2a^2-13)b^3x^3}{(a^2+1)^3}-\frac{(2a^2-3)b^2x^2}{(a^2+1)^2}+\frac{2abx}{a^2+1}+6\right)}{x^4}-\frac{3(2a-1)(2a+1)b^4\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2}\right.}{(a^2+1)^{7/2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(6 + (2*a*b*x)
/(1 + a^2) - ((-3 + 2*a^2)*b^2*x^2)/(1 + a^2)^2 + (a*(-13 + 2*a^2)*b^3*x^3)
/(1 + a^2)^3))/x^4 + (3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[x])/(1 + a^2)^(7/2) -
(3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(7/2))/24
```

Maple [B] time = 0.01, size = 841, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x)
```

```
[Out] -1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4+5/12*a*b*(b^2*x^2+2*a*b*x+a^
2+1)^(3/2)/(a^2+1)^2/x^3-5/8*a^2*b^2/(a^2+1)^3/x^2*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+5/8*a^3*b^3/(a^2+1)^4/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-5/4*a^4*b^4/(a^
2+1)^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/8*a^5*b^5/(a^2+1)^4*ln((b^2*x+a*b)/(
b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+5/8*a^4*b^4/(a^2+1)^(
7/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-
5/8*a^3*b^5/(a^2+1)^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-5/8*a^3*b^5/(a^2+1)^4
*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+7/8*
a^2*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/4*a^3*b^5/(a^2+1)^3*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/4*a^2*b^
4/(a^2+1)^(5/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/x)+1/8*b^2/(a^2+1)^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/8*b^3/(a^2
+1)^3*a/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/8*b^5/(a^2+1)^3*a*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)*x+1/8*b^5/(a^2+1)^3*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/8*b^4/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)-1/8*b^5/(a^2+1)^2*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)+1/8*b^4/(a^2+1)^(3/2)*ln((2*a^2+2+2*x*a*b+2*(a^2+1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/4*a/x^4-1/3*b/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.87863, size = 683, normalized size = 3.3

$$3\left(4a^2-1\right)\sqrt{a^2+1}b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-6a^9-\left(2a^5-11a^3-13a\right)b^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*a^2 - 1)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt
(a^2 + 1) + a)/x) - 6*a^9 - (2*a^5 - 11*a^3 - 13*a)*b^4*x^4 - 24*a^7 - 36*a
^5 - 24*a^3 - 8*(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 - 1
1*a^3 - 13*a)*b^3*x^3 + 24*a^6 - (2*a^6 + a^4 - 4*a^2 - 3)*b^2*x^2 + 36*a^4
 + 2*(a^7 + 3*a^5 + 3*a^3 + a)*b*x + 24*a^2 + 6)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 6*a)/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**5,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.358 $\int e^{\sinh^{-1}(a+bx)^2} x^3 \, dx$

Optimal. Leaf size=359

$$-\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^4}-\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^4}+\frac{3\sqrt{\pi}a^2\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^4}+\frac{3\sqrt{\pi}}{}$$

```
[Out] -(Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1
- ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcSinh[a + b*x]])/(32*
b^4*E^4) - (Sqrt[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[P
i]*Erfi[1 + ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcSinh[a + b
*x]])/(32*b^4*E^4) - (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16*b
^4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1
/4)) - (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) + (
3*a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt
[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi
[(3 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rubi [A] time = 0.742937, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5898, 6741, 12, 6742, 5513, 2234, 2204, 5514}

$$-\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^4}-\frac{\sqrt{\pi}a^3\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^4}+\frac{3\sqrt{\pi}a^2\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^4}+\frac{3\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] -(Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1
- ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcSinh[a + b*x]])/(32*
b^4*E^4) - (Sqrt[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[P
i]*Erfi[1 + ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcSinh[a + b
*x]])/(32*b^4*E^4) - (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16*b
^4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1
/4)) - (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) + (
3*a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt
[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi
[(3 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\sinh^{-1}(a+bx)^2} x^3\,dx &= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3 dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))^3}{b^3}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))^3\,dx, x, \sinh^{-1}(a+bx)\right)}{b^4}\\
&= \frac{\text{Subst}\left(\int \left(-a^3e^{x^2}\cosh(x)+3a^2e^{x^2}\cosh(x)\sinh(x)-3ae^{x^2}\cosh(x)\sinh^2(x)+e^{x^2}\cosh(x)\sin\cdots\right.\right.}{b^4}\\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^3(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^4}-\frac{(3a)\,\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2(x)\,dx,\cdots\right.}{b^4}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{16}e^{-4x+x^2}+\frac{1}{8}e^{-2x+x^2}-\frac{1}{8}e^{2x+x^2}+\frac{1}{16}e^{4x+x^2}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^4}-\frac{(3a)\,\text{Subst}\left(\int\cdots\right.}{\cdots}\\
&= -\frac{\text{Subst}\left(\int e^{-4x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{16b^4}+\frac{\text{Subst}\left(\int e^{4x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{16b^4}+\frac{\text{Subst}\left(\int\cdots\right.}{\cdots}\\
&= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{16b^4e^4}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{16b^4e^4}-\frac{(3a)\cdots}{\cdots}\\
&= -\frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{16b^4e}+\frac{3a^2\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^4e}+\frac{\sqrt{\pi}\,\text{erfi}\left(2-\sinh^{-1}(a+bx\cdots\right.}{32b^4e^4}
\end{aligned}
$$

Mathematica [A] time = 0.303135, size = 198, normalized size = 0.55

$$\sqrt{\pi}\left(-8e^{15/4}a^3\text{Erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+12e^3a^2\text{Erfi}\left(\sinh^{-1}(a+bx)+1\right)+2e^{15/4}\left(4a^2-3\right)a\text{Erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(2*a*(-3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcSinh[a + b*x]] + 2*(-1 +
 6*a^2)*E^3*Erfi[1 - ArcSinh[a + b*x]] + 6*a*E^(7/4)*Erfi[3/2 - ArcSinh[a +
 b*x]] + Erfi[2 - ArcSinh[a + b*x]] + 6*a*E^(15/4)*Erfi[1/2 + ArcSinh[a + b
*x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcSinh[a + b*x]] - 2*E^3*Erfi[1 + ArcSinh
[a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcSinh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
 ArcSinh[a + b*x]] + Erfi[2 + ArcSinh[a + b*x]]))/(32*b^4*E^4)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int e^{(\text{Arcsinh}(bx+a))^2}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3e^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3e^{\left(\text{arsinh}(bx+a)^2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arcsinh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(asinh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

3.359 $\int e^{\sinh^{-1}(a+bx)^2} x^2 \, dx$

Optimal. Leaf size=251

$$\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^3}-\frac{\sqrt{\pi}a\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4eb^3}-\frac{\sqrt{\pi}a\text{Erf}}{}$$

```
[Out] -(a*Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(4*b^3*E) - (a*Sqrt[Pi]*Erfi[1 + A
rcSinh[a + b*x]])/(4*b^3*E) + (Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/
(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(
1/4)) + (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) -
(Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[Pi
]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2
*ArcSinh[a + b*x])/2])/(16*b^3*E^(9/4))
```

Rubi [A] time = 0.516042, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5898, 6741, 12, 6742, 5513, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}a^2\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^3}-\frac{\sqrt{\pi}a\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4eb^3}-\frac{\sqrt{\pi}a\text{Erf}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x^2,x]
```


```
[Out] -(a*Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(4*b^3*E) - (a*Sqrt[Pi]*Erfi[1 + A
rcSinh[a + b*x]])/(4*b^3*E) + (Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/
(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(
1/4)) + (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) -
(Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[Pi
]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2
*ArcSinh[a + b*x])/2])/(16*b^3*E^(9/4))
```


Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2} x^2\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2 dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(a-\sinh(x))^2}{b^2} dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(a-\sinh(x))^2 dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(a^2e^{x^2}\cosh(x)-2ae^{x^2}\cosh(x)\sinh(x)+e^{x^2}\cosh(x)\sinh^2(x)\right) dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x \ldots}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{8}e^{-3x+x^2}-\frac{1}{8}e^{-x+x^2}-\frac{e^{x+x^2}}{8}+\frac{1}{8}e^{3x+x^2}\right) dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \left(-\frac{1}{4}e^{\ldots}\right.\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int e^{-3x+x^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e^{-x+x^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e \ldots\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{a\,\text{Sub}\ldots}{\ldots}$$

$$= -\frac{a\sqrt{\pi}\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4b^3e} - \frac{a\sqrt{\pi}\text{erfi}\left(1+\sinh^{-1}(a+bx)\right)}{4b^3e} + \frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}\left(-3+2\sinh^{-1}(a\ldots\right.\right.}{16b^3e^{9/4}}$$

Mathematica [A] time = 0.183048, size = 138, normalized size = 0.55

$$-\frac{\sqrt{\pi}\left(-4e^2a^2\text{Erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+e^2\left(4a^2-1\right)\text{Erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+4e^{5/4}a\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)+4e\ldots\right.}{16e\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -(Sqrt[Pi]*((-1 + 4*a^2)*E^2*Erfi[1/2 - ArcSinh[a + b*x]] + 4*a*E^(5/4)*Erf
i[1 - ArcSinh[a + b*x]] + Erfi[3/2 - ArcSinh[a + b*x]] + E^2*Erfi[1/2 + Arc
Sinh[a + b*x]] - 4*a^2*E^2*Erfi[1/2 + ArcSinh[a + b*x]] + 4*a*E^(5/4)*Erfi[
1 + ArcSinh[a + b*x]] - Erfi[3/2 + ArcSinh[a + b*x]]))/(16*b^3*E^(9/4))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \mathrm{e}^{(\mathrm{Arcsinh}(bx+a))^2} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{\left(\mathrm{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2 e^{\left(\mathrm{arsinh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(arcsinh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{\mathrm{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x**2,x)
```

```
[Out] Integral(x**2*exp(asinh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

3.360 $\int e^{\sinh^{-1}(a+bx)^2} x\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\pi}\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\text{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{8eb^2}-\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^2}-\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\sin\right.\right.}{4\sqrt[4]{e}}$$

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcSin
h[a + b*x]])/(8*b^2*E) - (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rubi [A] time = 0.276839, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5898, 6741, 12, 6742, 5513, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\text{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{8eb^2}-\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^2}-\frac{\sqrt{\pi}a\text{Erfi}\left(\frac{1}{2}\left(2\sin\right.\right.}{4\sqrt[4]{e}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcSin
h[a + b*x]])/(8*b^2*E) - (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2} x\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))}{b}dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{\text{Subst}\left(\int \left(-ae^{x^2}\cosh(x)+e^{x^2}\cosh(x)\sinh(x)\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \sinh^{-1}(a+\right.}{b^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx, x, \text{si}\right.}{b^2}$$

$$= -\frac{\text{Subst}\left(\int e^{-2x+x^2}dx, x, \sinh^{-1}(a+bx)\right)}{4b^2} + \frac{\text{Subst}\left(\int e^{2x+x^2}dx, x, \sinh^{-1}(a+bx)\right)}{4b^2} - \frac{a\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}dx, x, \sinh^{-1}(a+bx)\right)}{4b^2e} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}dx, x, \sinh^{-1}(a+bx)\right)}{4b^2e} - \frac{a\,\text{Su}}{}$$

$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^2e} + \frac{\sqrt{\pi}\,\text{erfi}\left(1+\sinh^{-1}(a+bx)\right)}{8b^2e} - \frac{a\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+b\right.\right.}{4b^2\sqrt[4]{e}}$$

Mathematica [A] time = 0.0932882, size = 76, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(2e^{3/4}a\text{Erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)-2e^{3/4}a\text{Erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+\text{Erfi}\left(\sinh^{-1}(a+\right.\right.}{8eb^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*(2*a*E^(3/4)*Erfi[1/2 - ArcSinh[a + b*x]] + Erfi[1 - ArcSinh[a +
b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[1 + ArcSinh[a + b*x
```

```
]]))/(8*b^2*E)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \mathrm{e}^{(\mathrm{Arcsinh}(bx+a))^2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{\left(\mathrm{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(xe^{\left(\mathrm{arsinh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(arcsinh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{\mathrm{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x,x)
```

```
[Out] Integral(x*exp(asinh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int xe^{\left(\mathrm{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

3.361 $\int e^{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b}+\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b}$$

```
[Out] (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erfi
[(1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4))
```

Rubi [A] time = 0.055106, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5896, 5513, 2234, 2204}

$$\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b}+\frac{\sqrt{\pi}\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erfi
[(1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 5896

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int\left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int e^{x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}$$
$$= \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}$$

Mathematica [A] time = 0.0324645, size = 44, normalized size = 0.68

$$\frac{\sqrt{\pi}\left(\text{Erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)\right)}{4\sqrt[4]{e}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*(Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[(-1 + 2*ArcSinh[a + b*x])/2]
))/(4*b*E^(1/4))
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int e^{(\text{Arcsinh}(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2),x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{\left(\operatorname{arsinh}(bx+a)^{2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] integral(e^(arcsinh(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\operatorname{asinh}^{2}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2),x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(\operatorname{arsinh}(bx+a)^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="giac")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

3.362 $\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\frac{e^{\sinh^{-1}(a+bx)^2}}{x},x\right)$$

```
[Out] CannotIntegrate[E^ArcSinh[a + b*x]^2/x, x]
```

Rubi [A] time = 0.0382415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcSinh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx=\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Mathematica [A] time = 0.128316, size = 0, normalized size = 0.

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x,x]
```

```
[Out] Integrate[E^ArcSinh[a + b*x]^2/x, x]
```

Maple [A] time = 0.009, size = 0, normalized size = 0.

$$\int \frac{e^{(\mathrm{Arcsinh}(bx+a))^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)/x,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arcsinh(b*x + a)^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{asinh}^2(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)/x,x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

3.363 $\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$

Optimal. Leaf size=16

$$\text{CannotIntegrate}\left(\frac{e^{\sinh^{-1}(a+bx)^2}}{x^2},x\right)$$

```
[Out] CannotIntegrate[E^ArcSinh[a + b*x]^2/x^2, x]
```

Rubi [A] time = 0.0373086, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx = \int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$$

Mathematica [A] time = 0.428082, size = 0, normalized size = 0.

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[E^ArcSinh[a + b*x]^2/x^2, x]
```

Maple [A] time = 0.008, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{(\mathrm{Arcsinh}(bx+a))^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(arcsinh(b*x + a)^2)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\operatorname{asinh}^2(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)/x**2,x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

3.364 $\int \frac{\sinh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=60

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(a+bx)}\right)}{2d}-\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}$$

```
[Out] -ArcSinh[a + b*x]^2/(2*d) + (ArcSinh[a + b*x]*Log[1 - E^(2*ArcSinh[a + b*x]
)])/d + PolyLog[2, E^(2*ArcSinh[a + b*x])]/(2*d)
```

Rubi [A] time = 0.095525, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5865, 12, 5659, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(a+bx)}\right)}{2d}-\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -ArcSinh[a + b*x]^2/(2*d) + (ArcSinh[a + b*x]*Log[1 - E^(2*ArcSinh[a + b*x]
)])/d + PolyLog[2, E^(2*ArcSinh[a + b*x])]/(2*d)
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\sinh^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{d}$$
$$= -\frac{\sinh^{-1}(a+bx)^2}{2d} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(a+bx)\right)}{d}$$
$$= -\frac{\sinh^{-1}(a+bx)^2}{2d} + \frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sinh^{-1}\right.}{d}$$
$$= -\frac{\sinh^{-1}(a+bx)^2}{2d} + \frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(a+bx}\right.}{2d}$$
$$= -\frac{\sinh^{-1}(a+bx)^2}{2d} + \frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}(a+bx)}\right)}{2d}$$

Mathematica [A] time = 0.0165718, size = 52, normalized size = 0.87

$$\frac{\text{PolyLog}\left(2, e^{2\sinh^{-1}(a+bx)}\right) - \sinh^{-1}(a+bx)\left(\sinh^{-1}(a+bx) - 2\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (-(ArcSinh[a + b*x]*(ArcSinh[a + b*x] - 2*Log[1 - E^(2*ArcSinh[a + b*x])]))
 + PolyLog[2, E^(2*ArcSinh[a + b*x])])/(2*d)
```

Maple [A] time = 0.037, size = 125, normalized size = 2.1

$$-\frac{(\text{Arcsinh}\,(bx+a))^2}{2\,d} + \frac{\text{Arcsinh}\,(bx+a)}{d}\ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right) + \frac{1}{d}\text{polylog}\left(2, -bx-a-\sqrt{1+(bx+a)^2}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] -1/2*arcsinh(b*x+a)^2/d+1/d*arcsinh(b*x+a)*ln(1+b*x+a+(1+(b*x+a)^2)^(1/2))+
1/d*polylog(2,-b*x-a-(1+(b*x+a)^2)^(1/2))+1/d*arcsinh(b*x+a)*ln(1-b*x-a-(1+
(b*x+a)^2)^(1/2))+1/d*polylog(2,b*x+a+(1+(b*x+a)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsinh}(bx+a)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsinh(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{\operatorname{asinh}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(asinh(a + b*x)/(a + b*x), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arsinh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

3.365 $\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\text{Shi}\left(\sinh^{-1}(x)\right)$$

```
[Out] SinhIntegral[ArcSinh[x]]
```

Rubi [A] time = 0.0595927, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5779, 3298}

$$\text{Shi}\left(\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

```
[Out] SinhIntegral[ArcSinh[x]]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx &= \text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(x)\right)\\ &= \text{Shi}\left(\sinh^{-1}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0583648, size = 3, normalized size = 1.

$$\text{Shi}\left(\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

```
[Out] SinhIntegral[ArcSinh[x]]
```

Maple [A] time = 0.047, size = 4, normalized size = 1.3

$$\mathrm{Shi}\left(\mathrm{Arcsinh}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(x)/(x^2+1)^(1/2),x)
```

```
[Out] Shi(arcsinh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+1}\,\mathrm{arsinh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{x^2+1}\,\mathrm{arsinh}\left(x\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+1}\,\mathrm{asinh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(x)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x**2 + 1)*asinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+1}\,\mathrm{arsinh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

3.366 $\int x^3 \sinh^{-1}\left(a+bx^4\right) dx$

Optimal. Leaf size=45

$$\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{\left(a+bx^4\right)^2+1}}{4b}$$

```
[Out] -Sqrt[1 + (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.0485548, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 5863, 5653, 261}

$$\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{\left(a+bx^4\right)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x^4],x]
```

```
[Out] -Sqrt[1 + (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \sinh^{-1}\left(a+bx^4\right) dx = \frac{1}{4}\text{Subst}\left(\int \sinh^{-1}(a+bx)\, dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\, dx, x, a+bx^4\right)}{4b}$$
$$= \frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\, dx, x, a+bx^4\right)}{4b}$$
$$= -\frac{\sqrt{1+\left(a+bx^4\right)^2}}{4b} + \frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}$$

Mathematica [A] time = 0.0245283, size = 41, normalized size = 0.91

$$\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)-\sqrt{\left(a+bx^4\right)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x^4],x]
```

```
[Out] (-Sqrt[1 + (a + b*x^4)^2] + (a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$\frac{1}{4\,b}\left(\left(bx^4+a\right)\text{Arcsinh}\left(bx^4+a\right)-\sqrt{1+\left(bx^4+a\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x^4+a),x)
```

```
[Out] 1/4/b*((b*x^4+a)*arcsinh(b*x^4+a)-(1+(b*x^4+a)^2)^(1/2))
```

Maxima [A] time = 1.11223, size = 50, normalized size = 1.11

$$\frac{\left(bx^4+a\right)\text{arsinh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arcsinh(b*x^4 + a) - sqrt((b*x^4 + a)^2 + 1))/b
```

Fricas [A] time = 2.42898, size = 151, normalized size = 3.36

$$\frac{\left(bx^4+a\right)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/b
```

Sympy [A] time = 1.12979, size = 61, normalized size = 1.36

$$\begin{cases} \frac{a \operatorname{asinh}\left(a+bx^4\right)}{4b} + \frac{x^4 \operatorname{asinh}\left(a+bx^4\right)}{4} - \frac{\sqrt{a^2+2abx^4+b^2x^8+1}}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{asinh}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x**4+a),x)
```

```
[Out] Piecewise((a*asinh(a + b*x**4)/(4*b) + x**4*asinh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

Giac [B] time = 1.39041, size = 142, normalized size = 3.16

$$\frac{1}{4}x^4 \log\left(bx^4 + a + \sqrt{\left(bx^4+a\right)^2 + 1}\right) - \frac{1}{4}b\left(\frac{a\log\left(-ab - \left(x^4|b| - \sqrt{b^2x^8 + 2\,abx^4 + a^2 + 1}\right)|b|\right)}{b|b|} + \frac{\sqrt{b^2x^8 + 2\,abx^4 + a^2}}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 + 1)) - 1/4*b*(a*log(-a*b - (x^4
*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))*abs(b))/(b*abs(b)) + sqrt(b^
2*x^8 + 2*a*b*x^4 + a^2 + 1)/b^2)
```

3.367 $\int x^{-1+n}\sinh^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=46

$$\frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}-\frac{\sqrt{(a+bx^n)^2+1}}{bn}$$

```
[Out] -(Sqrt[1 + (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Rubi [A] time = 0.0504628, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 5863, 5653, 261}

$$\frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}-\frac{\sqrt{(a+bx^n)^2+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

```
[Out] -(Sqrt[1 + (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\sinh^{-1}(a+bx^n)\,dx = \frac{\text{Subst}\left(\int \sinh^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,a+bx^n\right)}{bn}$$
$$= -\frac{\sqrt{1+(a+bx^n)^2}}{bn} + \frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}$$

Mathematica [A] time = 0.0370183, size = 41, normalized size = 0.89

$$\frac{(a+bx^n)\sinh^{-1}(a+bx^n)-\sqrt{(a+bx^n)^2+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

```
[Out] (-Sqrt[1 + (a + b*x^n)^2] + (a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int x^{n-1}\text{Arcsinh}(a+bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arcsinh(a+b*x^n),x)
```

```
[Out] int(x^(n-1)*arcsinh(a+b*x^n),x)
```

Maxima [A] time = 1.13509, size = 53, normalized size = 1.15

$$\frac{(bx^n+a)\,\text{arsinh}(bx^n+a)-\sqrt{(bx^n+a)^2+1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arcsinh(b*x^n + a) - sqrt((b*x^n + a)^2 + 1))/(b*n)
```

Fricas [B] time = 2.78078, size = 443, normalized size = 9.63

$$\frac{(b\cosh(n\log(x))+b\sinh(n\log(x))+a)\log\left(b\cosh(n\log(x))+b\sinh(n\log(x))+a+\sqrt{\frac{2ab+(a^2+b^2+1)\cosh(n\log(x))}{\cosh(n\log(x))-\text{si}}}\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a^2
+ b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asinh(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35281, size = 153, normalized size = 3.33

$$-\frac{b\left(\frac{a\log\left(-ab-\left(x^{n}|b|-\sqrt{b^{2}x^{2n}+2abx^{n}+a^{2}+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^{2}x^{2n}+2abx^{n}+a^{2}+1}}{b^{2}}\right)-x^{n}\log\left(bx^{n}+a+\sqrt{(bx^{n}+a)^{2}+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -(b*(a*log(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1))*ab
s(b))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1)/b^2) - x^n*log(b
*x^n + a + sqrt((b*x^n + a)^2 + 1)))/n
```

3.368 $\int \sinh^{-1}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=49

$$\frac{c\tanh^{-1}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

```
[Out] ((a + b*x)*ArcCsch[a/c + (b*x)/c])/b + (c*ArcTanh[Sqrt[1 + (a/c + (b*x)/c)^
(-2)]])/b
```

Rubi [A] time = 0.0322443, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5892, 6314, 372, 266, 63, 207}

$$\frac{c\tanh^{-1}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcCsch[a/c + (b*x)/c])/b + (c*ArcTanh[Sqrt[1 + (a/c + (b*x)/c)^
(-2)]])/b
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6314

```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcCsch[c + d*
x])/d, x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(\frac{c}{a+bx}\right)dx &= \int \operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)dx\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}\,dx\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x}\,dx,x,\frac{a}{c}+\frac{bx}{c}\right)}{b}\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}\right)}{2b}\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\tanh^{-1}\left(\sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}
\end{aligned}
$$

Mathematica [B] time = 0.117619, size = 131, normalized size = 2.67

$$
\frac{(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2+c^2}{(a+bx)^2}}\left(c\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2+c^2}}\right)+a\tanh^{-1}\left(\frac{\sqrt{(a+bx)^2+c^2}}{c}\right)\right)}{b\sqrt{a^2+2abx+b^2x^2+c^2}}+x\sinh^{-1}\left(\frac{c}{a+bx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c/(a + b*x)],x]
```

```
[Out] x*ArcSinh[c/(a + b*x)] + ((a + b*x)*Sqrt[(a^2 + c^2 + 2*a*b*x + b^2*x^2)/(a
 + b*x)^2]*(c*ArcTanh[(a + b*x)/Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]] + a*Ar
cTanh[Sqrt[c^2 + (a + b*x)^2]/c]))/(b*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2])
```

Maple [A] time = 0.02, size = 46, normalized size = 0.9

$$
-\frac{c}{b}\left(-\frac{bx+a}{c}\text{Arcsinh}\left(\frac{c}{bx+a}\right)-\text{Artanh}\left(\frac{1}{\sqrt{1+\frac{c^2}{(bx+a)^2}}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c/(b*x+a)),x)
```

```
[Out] -1/b*c*(-arcsinh(c/(b*x+a))/c*(b*x+a)-arctanh(1/(1+1/(b*x+a)^2*c^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ic\left(\log\left(\frac{i(b^2x+ab)}{bc}+1\right)-\log\left(-\frac{i(b^2x+ab)}{bc}+1\right)\right)}{2b}+\frac{2bx\log\left(c+\sqrt{b^2x^2+2abx+a^2+c^2}\right)+a\log\left(b^2x^2+2abx+a^2+\cdots\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*I*c*(log(I*(b^2*x + a*b)/(b*c) + 1) - log(-I*(b^2*x + a*b)/(b*c) + 1))
/b + 1/2*(2*b*x*log(c + sqrt(b^2*x^2 + 2*a*b*x + a^2 + c^2)) + a*log(b^2*x^
2 + 2*a*b*x + a^2 + c^2) - 2*(b*x + a)*log(b*x + a))/b + integrate((b^2*c*x
^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c + c^3 + (b^2*x^2 + 2*a*b*x + a
^2 + c^2)^(3/2)), x)
```

Fricas [B] time = 2.62695, size = 532, normalized size = 10.86

$$\frac{bx\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2+c^2}{b^2x^2+2abx+a^2}}+c}{bx+a}\right)+a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2+c^2}{b^2x^2+2abx+a^2}}-a+c\right)-a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2\cdots}{b^2x^2+2abx+\cdots}}\right.\cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x
 + a^2)) + c)/(b*x + a)) + a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
 a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a + c) - a*log(-b*x + (b*x + a)*sq
rt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a - c) - c*
log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) - a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asinh}\left(\frac{c}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c/(b*x+a)),x)
```

```
[Out] Integral(asinh(c/(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsinh}\left(\frac{c}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c/(b*x + a)), x)
```

3.369 $\int \frac{x}{\sinh^{-1}(\sinh(x))} dx$

Optimal. Leaf size=27

$$\sinh^{-1}(\sinh(x)) + \log\left(\sinh^{-1}(\sinh(x))\right)\left(x\sqrt{\cosh^2(x)}\,\mathbf{sech}(x) - \sinh^{-1}(\sinh(x))\right)$$

```
[Out] ArcSinh[Sinh[x]] + Log[ArcSinh[Sinh[x]]]*(-ArcSinh[Sinh[x]] + x*Sqrt[Cosh[x
]^2]*Sech[x])
```

Rubi [F] time = 0.0393858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\sinh^{-1}(\sinh(x))} dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcSinh[Sinh[x]],x]
```

```
[Out] Defer[Int][x/ArcSinh[Sinh[x]], x]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(\sinh(x))} dx = \int \frac{x}{\sinh^{-1}(\sinh(x))} dx$$

Mathematica [A] time = 0.519165, size = 28, normalized size = 1.04

$$x\sqrt{\cosh^2(x)}\,\mathbf{sech}(x)\log\left(\sinh^{-1}(\sinh(x))\right) - \sinh^{-1}(\sinh(x))\left(\log\left(\sinh^{-1}(\sinh(x))\right) - 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[Sinh[x]],x]
```

```
[Out] -(ArcSinh[Sinh[x]]*(-1 + Log[ArcSinh[Sinh[x]]])) + x*Sqrt[Cosh[x]^2]*Log[Ar
cSinh[Sinh[x]]]*Sech[x]
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{x}{\mathrm{Arcsinh}\left(\sinh(x)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(sinh(x)),x)
```

```
[Out] int(x/arcsinh(sinh(x)),x)
```

Maxima [A] time = 1.81796, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 2.20826, size = 4, normalized size = 0.15

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{asinh}(\sinh(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(sinh(x)),x)
```

```
[Out] Integral(x/asinh(sinh(x)), x)
```

Giac [A] time = 1.35227, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="giac")
```

```
[Out] x
```

3.370 $\int \frac{\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Rubi [A] time = 0.0665338, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5894, 5675}

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5894

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)^n}{\sqrt{1+x^2}}\,dx, x, \sqrt{-1+bx^2}\right)}{bx}\\ &= \frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.0396998, size = 37, normalized size = 1.

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \left(\operatorname{Arcsinh}\left(\sqrt{bx^2-1}\right)\right)^n \frac{1}{\sqrt{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

```
[Out] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(sqrt(b*x^2 - 1))^n/sqrt(b*x^2 - 1), x)
```

Fricas [B] time = 2.78902, size = 282, normalized size = 7.62

$$\frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\sinh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 - 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 - 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh((b*x**2-1)**(1/2))**n/(b*x**2-1)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(sqrt(b*x^2 - 1))^n/sqrt(b*x^2 - 1), x)
```

3.371 $\int \frac{1}{\sqrt{-1+bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

```
[Out] (Sqrt[b*x^2]*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/(b*x)
```

Rubi [A] time = 0.0615313, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5894, 5673}

$$\frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

```
[Out] (Sqrt[b*x^2]*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/(b*x)
```

Rule 5894

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)}\,dx &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx, x, \sqrt{-1+bx^2}\right)}{bx}\\ &= \frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{-1+bx^2}\right)\right)}{bx}\end{aligned}$$

Mathematica [A] time = 0.0208927, size = 24, normalized size = 0.83

$$\frac{x\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

```
[Out] (x*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/Sqrt[b*x^2]
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \left(\text{Arcsinh}\left(\sqrt{bx^2-1}\right)\right)^{-1} \frac{1}{\sqrt{bx^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

```
[Out] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2-1}\,\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 - 1)*arcsinh(sqrt(b*x^2 - 1))), x)
```

Fricas [A] time = 2.4263, size = 80, normalized size = 2.76

$$\frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))/(b*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2-1}\,\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh((b*x**2-1)**(1/2))/(b*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2-1}\,\mathrm{arsinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 - 1)*arcsinh(sqrt(b*x^2 - 1))), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```